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Sticky Inflation: Monetary Policy when Debt Drags Inflation Expectations

Saki Bigio

Nicolas Caramp

Dejanir Silva

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Saki Bigio* Nicolas Caramp† Dejanir Silva‡

UCLA and NBER UC Davis Purdue

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Abstract

We append the expectation of a monetary-fiscal reform into a standard New Keynesian model. If a reform occurs, monetary policy will temporarily aid debt sustainability through a temporary burst in inflation. The anticipation of a possible reform links debt levels with inflation expectations. As a result, interest rates have two effects: they influence demand and affect expected inflation in opposite directions. The expectations effect is linked to the impact of interest rates on public debt. While lowering inflation in the short term is possible through demand control, inflation tends to rise again due to their impact on inflation expectations (sticky inflation). Optimal monetary policy may allow negative real interest rates after fiscal shocks, temporarily breaking away from the Taylor principle. We assess whether the Federal Reserve's "staying behind the curve" was the right strategy during the recent post-Pandemic inflation surge.

*Bigio is at the Economics Department at UCLA. Email: sbigio@econ.ucla.edu.

†Caramp is at the Department of Economics at UC Davis. Email: ncaramp@ucdavis.edu

‡Silva is at the Mitchell E. Daniels, Jr. School of Business, Purdue University. Email: dejanir@purdue.edu

1. Introduction

Persistent inflation became a global phenomenon in the wake of the COVID-19 pandemic. As inflation gathered momentum, central banks maintained low interest rates, defying early warning calls and drawing harsh criticism. This criticism is rooted in a traditional view: that a prompt rate hike would have prevented inflation expectations from de-anchoring, and that by contracting aggregate demand, central banks signal resolve and avoid more painful future corrections. This traditional view, however, overlooks that during significant events such as wars, natural disasters, or political turmoil, inflation often surges alongside rising national debt levels.[1] Under these circumstances, central banks face an atypical challenge: rate hikes can increase the burden of public debt, potentially leading people to expect further inflation as a means of stabilizing that debt.

This paper examines inflation dynamics when agents anticipate scenarios where public debt may need to be stabilized through inflationary finance. Our analysis is prompted by the sharp rise in medium-term inflationary disaster expectations following the COVID-19 pandemic, as reported by Hilscher, Raviv and Reis (2022). This rise was likely the result of expectations of inflationary finance. In the U.S., for example, Hazell and Hobler (2024) link single electoral events to jumps in inflation expectations. Gomez Cram, Kung and Lustig (2023) find that fiscal news affect Treasury prices through changes in inflation expectations. We aim to provide a straightforward analysis of how the possibility of future inflationary finance impairs current monetary policy.

To that end, we study a paper-and-pencil New Keynesian model in which agents anticipate a possible monetary-fiscal reform. In the event of such a reform, the monetary authority temporarily allows higher inflation, resulting in negative real interest rates. After the reform, debt and inflation are stabilized. The key tension is that, prior to the reform, increases in nominal interest rates carry two opposing effects: the first is a conventional decrease in aggregate demand, and the second, the novelty, is the anticipation of a burst in inflation due to a greater debt burden. The increase in expected inflation modifies the transmission mechanism of monetary policy.

The paper makes three contributions. First, it portrays the phenomenon of *sticky inflation*. Sticky inflation occurs when efforts to reduce inflation through interest rate hikes, while initially effective, fail because the increased debt burden causes inflation to resurge in the medium term. That is, under sticky inflation, attempts to control inflation with temporary

[1] See e.g. the historical evidence in Hall and Sargent (2021,2022).

rate hikes backfire: The phenomenon occurs because with greater debt, agents anticipate more inflation if a monetary-fiscal reform scenario materializes. This anticipation appears in the Euler equation and Phillips curve as *endogenous* demand and cost-push shocks, respectively. Since expected inflation depends on the debt stock, the effects of interest-rate increases are persistent.

Second, the paper derives optimal policy prescriptions under sticky inflation. While the Taylor principle suggests raising real interest rates aggressively whenever inflation spikes, many central bankers ignored that principle. This paper shows that, under sticky inflation, such underreaction is actually optimal for a central bank with the standard objectives of stabilizing inflation and output. Paradoxically, the more hawkish a central bank is, the less it responds by raising rates after increases in fiscal deficits.

Third, we produce a policy counterfactual for the post-pandemic U.S. inflation. The model contends that if the Federal Reserve had followed the Taylor principle's prescription, inflation would have been higher with much higher national debt levels.

The paper is organized into four core sections. Section 2 presents the framework and shows we can represent the equilibrium in a 4-equation system akin to the 3-equation system in the New Keynesian model. A key feature of this representation is that debt enters the Phillips curve and Euler equations. This section clarifies that sticky inflation occurs because the Phillips curve features a backward-looking component associated with the path of debt. The section also clarifies that sticky inflation does not rely on the equilibrium determination implied by the fiscal theory of the price level (FTPL); rather, it is a feature of anticipated inflationary finance.

Section 3 is devoted to characterizing inflationary dynamics and fleshing out the sticky inflation phenomenon. For that, we present three policy exercises. First, we characterize sticky inflation in the context of policy-rate paths that aim to close the output gap. This first exercise shows that stabilizing output generates a prolonged inflationary episode fueled by inflation expectations associated with higher debt levels. A second exercise shows that temporary increases in nominal rates aimed at controlling inflation may only be successful on impact. As long as surpluses do not revert the path of debt, inflation returns with greater force. Likewise, a policy that aims to stabilize debt permanently leads to an explosion in inflation and an undesirable overheating of the economy. The three exercises demonstrate that attempting to stabilize one outcome variable (inflation, output gap, or debt) destabilizes the other variables.

The policy exercises showcase that when debt levels add inflationary pressure through expectations, the effects of interest rates on debt financing affect inflation dynamics. The feedback from debt to inflation expectations breaks, in an endogenous way, the possibility of jointly stabilizing output and inflation, the so-called divine coincidence.

Lack of divine coincidence furthermore leads to a non-trivial optimal monetary policy analysis, which we investigate in Section 4. Under sticky inflation, minimizing the expected square of inflation and the output gap has an indirect representation in terms of the squared deviation of debt relative to a "natural debt level"—i.e., an inflation-neutral debt level. Under commitment, we obtain an optimal real-interest path after a fiscal shock. We find that it is optimal for the central bank to underreact to the fiscal shock, ultimately moving nominal rates less than one-to-one with inflation. We contrast the optimal responses of dovish and hawkish central banks—who only care about output and inflation stabilization, respectively. A novel finding is that a hawkish central bank should be less responsive in the short run and allow greater inflation to burst after a fiscal shock: Knowing that debt permanently impacts inflation, a more hawkish central bank prefers front-loading inflation to flatten the debt trajectory. Lowering the debt trajectory mitigates the sticky inflation component of inflation.

Section 5 is where we confront the theory with the data and evaluate counterfactuals where the Fed would have followed the Taylor principle. We discipline the calibration using the pass-through of fiscal shocks to inflation expectations. We decompose the recent inflation surge into shocks associated with primary deficits, supply shocks, bond valuation shocks, and deviations from the Taylor rule. Contrary to conventional wisdom, we show that expansionary monetary policy shocks contributed to inflation only for a few quarters, but by eroding debt, it depressed inflation in the medium term.

Section 6 concludes with a discussion on the importance of the timing of reforms and the challenge of balancing the sticky inflation mechanism with signals about the central bank's anti-inflationary stance.

Literature review. Monetary and fiscal policy interactions have been studied in formal models since Sargent and Wallace (1981). The topic is now part of textbook material that studies seigniorage financing and abstracts from nominal rigidities (e.g., Ljungqvist and Sargent, 2018). Fiscal and monetary interactions were swept aside in the standard versions of the New Keynesian model, where a combination of Ricardian equivalence and Taylor rules decouple inflation from the government budget constraint. A separate tradition evolved from the FTPL, the idea that the price level may adjust to erode nominal debt, making it sustainable given the present value of real primary surpluses (e.g. Leeper, 1991; Woodford, 1998; Cochrane, 1998). The government budget constraint naturally found its way back into New Keynesian models through the FTPL (e.g., Sims, 2011; Leeper and Leith, 2016; Cochrane, 2018a; Caramp and Silva, 2023, among many others). With nominal rigidities, there is an additional channel of monetary/fiscal interactions that works through changes in the real interest rate.

The government budget constraint also finds its way into the New Keynesian model in our paper through expected jumps in inflation upon a reform event, not through the determination of time-zero inflation. Unlike FTPL, which posits monetary policy as subordinate to fiscal policy, here, monetary policy remains autonomous, both before and after any potential fiscal-monetary reform. Thus, the prevalent equilibrium determination of FTPL models is not present here. However, our analysis does share the FTPL's recognition of the importance of fiscal-monetary interactions. Our framework is notably agnostic about the specifics of a monetary-fiscal reform: it could stem from the monetary authority's voluntary cooperation with the Treasury, a negotiated compromise, or temporary surrender of autonomy (e.g, Chung, Davig and Leeper, 2007; Bianchi, Faccini and Melosi, 2023).[2] The reform's exact nature—or eventual occurrence—is not central. What is central is that the mere anticipation of a reform will lead to sticky inflation.

The anticipation of a monetary-fiscal reform does bring our work closer to models that allow switches between regimes where deficits are financed with taxes or inflation, (e.g. Chung et al., 2007; Bianchi and Ilut, 2017). Relative to this literature, we have in common that agent's beliefs about the possibility of inflationary finance have effects in the present. Our contribution to this literature is that we identify, formalize, and characterize the sticky inflation phenomenon. We contend that sticky inflation is also present in many of those models and its presence does not rely on the equilibrium selection germane to the FTPL. The presence of monetary-fiscal interactions in a setting where the FTPL does not select equilibrium is also a feature of Angeletos, Lian and Wolf (2024), who emphasize the role of having non-Ricardian agents.

Related work also includes Bianchi and Melosi (2019), who introduces a monetary authority that commits to accepting sufficient inflation to stabilize the debt-to-output ratio after exceptionally large shocks. In this model, fiscal expansions can be backed, and paid for with future revenues, or unbacked, and financed with inflation. Jacobson, Leeper and

[2] As pointed out by Cochrane (2023), there is an observational equivalence between the fiscal-monetary reforms here and fiscal shocks in models of the FTPL-tradition.

Preston (2019) provide evidence that an unbacked fiscal expansion was implemented in the aftermath of the Great Depression, while Bianchi et al. (2023) present evidence that unbacked fiscal shocks account for a significant fraction of U.S. inflation dynamics. Crucially, this work emphasizes the dynamics after the shocks, whereas ours emphasizes their *anticipation*.

On the normative front, our work also relates to Leeper, Leith and Liu (2021), who study optimal policies in settings with long-term debt, distortionary taxes, and inflationary finance. Here, the monetary policy must confront the fact that its current effect on real rates will permanently affect the path of debt and, consequently, future inflation. Key to this feature is the agents' expectations that the central bank can only influence through the level of debt. In this sense, our model is closely related to Caballero and Simsek (2022), who show that a monetary authority might choose to accommodate the private sector's beliefs, even if it disagrees with them.

Notably, the post-COVID-19 inflation surge renewed the interest in understanding the drivers of inflation from analytic and quantitative standpoints.[3] On the quantitative front, Blanchard and Bernanke (2023), Gagliardone and Gertler (2023), Shapiro (2024), and Giannone and Primiceri (2024) take versions of the New Keynesian model models that decompose the drivers of inflation into labor-market shocks and energy shocks. Benigno and Eggertsson (2023) emphasize the role of the non-linearity of the Phillips curve. This first wave of decompositions follows the New Keynesian tradition that typically abstracts away from how debt financing impairs monetary policy, indirectly attributing demand shocks to fiscal variables. Our study is explicit about fiscal-monetary interactions and shows that these may also appear as cost-push shocks. Thus, our decomposition is part of a second wave of studies, including Barro and Bianchi (2024), Angeletos et al. (2024), and Smets and Wouters (2024), that is explicit about fiscal-monetary interactions.

2. Model

[3] On the analytical front, recent work has emphasized channels unrelated to fiscal shocks: employer-worker tensions Lorenzoni and Werning (2023b,a); Guerreiro, Hazell, Lian and Patterson (2024), hiring frictions Michaillat and Saez (2024), and supply side constraints Comin, Johnson and Jones (2023)

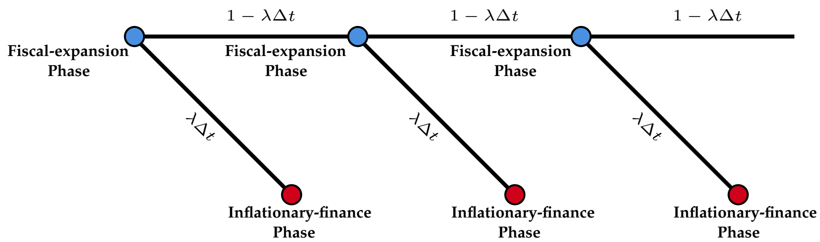


Figure 1: Timeline of events

Note: Over a small time interval Δt, the economy switches to the inflationary-finance phase with probability $\lambda\Delta t$, and stays in the fiscal-expansion phase with the remaining probability.

2.1 Environment

We cast the model in continuous time, $t \in [0, \infty)$. The economy starts at a *fiscal-expansion phase* where the government runs primary deficits. With Poisson intensity λ, the economy switches to an *inflationary-finance phase* that lasts for a predetermined amount of time, T^*. In the inflationary-finance phase, government debt is reduced through a mix of fiscal and monetary tools—which we also call fiscal-monetary reform. After the inflationary-finance phase is over, deficits, debt, output, and inflation are stabilized forever. Figure 1 summarizes the timeline of events.

The economy is populated by households, firms, and a government. Each group has views about the fiscal-monetary arrival rates, possibly differing from the objective ones. Next, we describe the agents' behavior, relegating derivations to Appendix A. We discuss the modeling assumptions at the end of the section.

Notation. We index variables in the inflationary-finance phase using an asterisk ($*$) superscript whereas variables during the fiscal-expansion phase do not carry that superscript. For example, π_t represents inflation at time t of the fiscal-expansion phase whereas π_t^* is inflation at time t since the start of the inflationary-finance phase. Variables in the steady state are denoted by an upper bar. For example, consumption in a steady state is denoted by $\overline{C}$.

Government. The government is comprised of fiscal and monetary authorities. The fiscal authority sends lump-sum transfers T_t—taxes if $T_t < 0$—to households and issues short-term real debt B_t. The monetary authority sets the nominal interest rate i_t.

The government's flow budget constraint is given by

$$\dot{B}_t = (i_t - \pi_t)B_t + T_t, \tag{1}$$

given $B_0 > 0$, where π_t denotes the inflation rate and i_t the nominal interest rate. Fiscal transfers, which equal primary deficits—or surpluses when negative—satisfy the following rule:

$$T_t = -\rho B_t - \gamma(B_t - \overline{B}) + \Psi_t, \tag{2}$$

where ρ denotes the interest rate that prevails in a zero-inflation steady state, $\overline{B}$ is the steady-state level of debt, and Ψ_t corresponds to a fiscal shock. Importantly, $\gamma \geq 0$ controls the strength of fiscal responses—primary surpluses—to the level of government debt. If $\gamma > 0$, debt is mean reverting to $\overline{B}$; if $\gamma = 0$, transitory fiscal shocks lead debt to stabilize at different levels.

During the *fiscal-expansion phase*, there are ongoing fiscal pressures, $\Psi_t > 0$. Meanwhile, the monetary authority's instrument, the nominal rate i_t, satisfies a Taylor rule:

$$i_t = \rho + \phi\pi_t + u_t. \tag{3}$$

We focus on the case where the Taylor coefficient ϕ and the fiscal rule coefficient γ are such that the economy is always in an active monetary regime, following the Leeper (1991) terminology. The disturbance u_t allows the monetary authority to respond freely to the fiscal expansion.[4] These choices allow us to analyze an independent monetary authority that freely chooses interest rates—deviating by u_t from the Taylor rule.

When the economy switches to the inflationary-finance phase, the government sets $\Psi_t = 0$, and the monetary authority commits to set a constant real interest rate for the duration of the reform, that is, for a time interval of length T^*. The rate is set to whatever level is necessary to bring debt to a target level B^n. Once debt reaches B^n, monetary policy implements a zero inflation target, and the economy permanently reaches its steady-state level. T^* is fixed regardless of the debt levels. This assumption translates debt levels into a period of future low policy rates, which, in turn, lead to inflationary bursts.

[4]The disturbance u_t captures the *response* of the monetary authority to the fiscal expansion, so we refer to it as a disturbance to the policy rule instead of a shock.

Households. In the fiscal-expansion phase, households form expectations of the arrival time of the inflationary-finance phase. The household's problem is given by

$$V_t(B_t) = \max_{[C_s, N_s]_{s \geq t}} \mathbb{E}_t^h \left[\int_t^{\tilde{t}} e^{-\rho(s-t)} \left(\log C_s - \frac{N_s^{1+\varphi}}{1+\varphi} \right) dt + e^{-\rho \tilde{t}} \tilde{V}_{\tilde{t}}(B_{\tilde{t}}) \right],$$

subject to

$$\dot{B}_t = r_t B_t + \frac{W_t}{P_t} N_t + D_t + T_t - C_t,$$

and the No-Ponzi condition $\lim_{T \to \infty} \mathbb{E}_t^h [\eta_T B_T] \geq 0$, given the household's stochastic discount factor (SDF) η_t. B_t denotes the real value of bonds held by households, $r_t = i_t - \pi_t$ is the real interest rate, W_t is the nominal wage, P_t is the price level, and D_t are the firm's dividends. The random time $\tilde{t}$ denotes the arrival time of the reform, and $\tilde{V}_t$ denotes the value function after the reform.

Households believe that the monetary-fiscal reform occurs with Poisson intensity λ_h. The key object from the demand side is the households' Euler equation:

$$\frac{\dot{C}_t}{C_t} = \underbrace{(i_t - \pi_t - \rho)}_{\text{standard term}} + \lambda_h \underbrace{\left[\frac{C_t}{C_t^J} - 1 \right]}_{\text{reform risk}}. \tag{4}$$

where C_t^J denotes consumption at the instant of a fiscal-monetary reform. This Euler equation includes a standard term associated with the gap between real interest rates and the discount rate dictating consumption growth. The second term captures a risk adjustment for the monetary-fiscal reform. The adjustment is given by the jump in marginal utilities the instant the economy enters an inflationary-finance phase.[5] The expectation of a reform provokes a change in the household's discount factor, a central variable in the New Keynesian model. The usual intra-temporal labor-supply condition holds: $W_t/P_t = C_t N_t^{\varphi}$.

Firms. Production follows the structure of the standard New Keynesian model. The economy has two types of firms: final-goods and intermediate-goods producers index by $i \in [0, 1]$. Final goods are produced by competitive firms using a constant-elasticity of substitution production function over intermediate inputs. As usual, the demand for intermediate good i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} Y_t$, where $P_{i,t}$ is the price of intermediate i,

[5] Similar terms appear with other forms of uncertainty, such as the uninsurable idiosyncratic income risk of McKay, Nakamura and Steinsson (2016) or the aggregate disaster risk in Caramp and Silva (2021).

$P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price level, and Y_t is the aggregate output.

Intermediate-goods producers operate the technology $Y_{i,t} = AN_{i,t}$, where $N_{i,t}$ denotes labor input and A is a productivity factor. They compete monopolistically, and they are subject to quadratic price-adjustment costs. The problem of intermediate-goods firm i is

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t^f \left[\int_t^{\tilde{t}} \frac{\eta_s}{\eta_t} \left(\frac{P_{i,s}}{P_{i,t}} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A} - \frac{\varphi}{2} \pi_{i,s}^2 \right) ds + \frac{\eta_{\tilde{t}}}{\eta_t} \tilde{Q}_{i,\tilde{t}}(P_{i,\tilde{t}}) \right], \tag{5}$$

subject to their demand schedule, $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$, where φ is a price adjustment cost parameter. As in the household's problem, $\tilde{t}$ is the random arrival time of a reform.

Firms have beliefs about the arrival rate of the monetary-fiscal reform, which are potentially different from those of households. Firms believe that the monetary-fiscal reform occurs with Poisson intensity λ_f. The key object of this supply-side block is a modified New Keynesian Phillips curve (NKPC):

$$\dot{\pi}_t = \underbrace{(i_t - \pi_t)\,\pi_t + \epsilon\varphi^{-1}\left((1-\epsilon^{-1}) - \frac{W_t}{P_t}\right) Y_t}_{\text{standard term}} + \underbrace{\lambda_f \frac{\eta_t^J}{\eta_t}\left(\pi_t - \pi_t^J\right)}_{\text{reform risk}} \tag{6}$$

Like the Euler equation, the firm's Phillips curve features a standard term associated with marginal costs. However, it is also modified by a second term associated with the beliefs about the reform. Firms anticipate that if a monetary-fiscal reform occurs, inflation will jump to π_t^J—which we dub the *jump inflation* term. The intuition is that because adjusting prices immediately is costly, firms reduce price-setting costs by raising prices today. The jump in inflation is adjusted by η_t^J, which translates the probability of a reform to a risk-adjusted probability.

2.2 A 4-equation log-linear representation

Part of the appeal of the standard New Keynesian model is its log-linear representation into a tractable 3-equation system. Here, we present a tractable 4-equation log-linear representation that includes the feedback of fiscal variables on inflation expectations—the log-linear approximation is around the zero-inflation constant-debt steady state.

Steady state and log-linear deviations. The steady-state corresponds to the case $\Psi_t = 0$ and $u_t = 0$, so $B_t = \overline{B}$, $C_t = \overline{C}$, $i_t = \rho$, and $\pi_t = 0$, where $\overline{B}$ corresponds to the initial condition for government debt and $\overline{C}$ is the steady-state level of consumption.

In this system, lower-case variables denote log-linear deviations. We also define $b_t \equiv \frac{B_t - \overline{B}}{\overline{B}}$, and the output gap $x_t \equiv \frac{Y_t - \overline{Y}}{\overline{Y}}$. In turn, $\psi_t \equiv \Psi_t / \overline{B}$ denotes the fiscal shock scaled by debt.

Dynamics: Inflationary-finance phase. Once the inflationary-finance phase begins, fiscal shocks ψ_t^* and the parameter controlling the fiscal response γ are set to zero. In turn, the monetary authority implements a constant real interest rate r^* for T^* periods, as needed to bring debt to a level that no longer requires a fiscal response to stabilize it. Hence, during the inflationary-finance phase, debt evolves according to $b_t^* = b_0^* + (r^* - \rho)t$ for $t \leq T^*$. To ensure that debt reaches the sustainable level after T^* periods, monetary policy must set the real interest rate to:

$$r^* = \rho - \frac{b_0^* - b^n}{T^*}, \tag{7}$$

where $b^n \equiv \frac{B^n - \overline{B}}{\overline{B}}$ denotes the *natural or neutral debt level*, that is, the debt level for which no fiscal response is needed to keep debt constant. This is also the debt level at which inflation and output would jump to zero at the start of an inflationary-finance phase. Once the target debt level is reached by the end of the reform, the monetary authority implements a zero inflation target, that is, $\{x_{T^*}^*, \pi_{T^*}^*\} = \{0, 0\}$.

Given the terminal condition at the end of the reform, we can roll back the Euler equation and NKPC to obtain:

$$x_t^* = (r^* - \rho)(t - T^*) = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right), t \in [0, T^*], \tag{8}$$

and

$$\pi_t^* = \kappa(r^* - \rho)\int_t^{T^*} \exp(-\rho(s - t))(s - T^*)ds. \tag{9}$$

Since at any moment t debt does not jump when the economy switches phases, debt at the start of an inflationary-finance phase equals debt at the end of the fiscal-expansion phase, $b_0^* = b_t$. Thus, using the expression for the real rate, given in (7), and using (8) and (9), we can write inflation and the output gaps at the instant of the fiscal-monetary reform in terms of the *debt gap* $b_t - b^n$ at the instant of the switch:

$$\pi^*(b_t) \equiv \kappa\Phi(b_t - b^n) \quad \text{and} \quad x^*(b_t) \equiv b_t - b^n, \tag{10}$$

where

$$\Phi \equiv \int_0^{T^*} \exp(-\rho s)\left(1 - \frac{s}{T^*}\right) ds > 0.$$

Inflation and the output gap at the instant of the reform, given by (10), in general will differ from their values the instant prior to the reform. Thus, these variables jump at the start of the inflationary-finance phase. The jump size depends on the debt gap. The larger the gap, the lower the real interest rate and the higher the inflation. The coefficient Φ controls the pass-through from debt to inflation. It captures the increase in the inflation rate required to bring debt to its neutral level during an inflationary-finance phase as current debt increases.

Dynamics: fiscal-expansion phase. The system of linearized Euler equation, NKPC, and government budget constraint is:

$$\dot{x}_t = i_t - \pi_t - \rho + \lambda_h x_t - \lambda_h(b_t - b^n) \tag{11}$$

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f\kappa\Phi(b_t - b^n) \tag{12}$$

$$\dot{b}_t = i_t - \pi_t - \rho - \gamma b_t + \psi_t. \tag{13}$$

Here, $\kappa > 0$ is the slope of the Phillips curve and a function of other parameters. The Taylor rule (Eq. 3) completes the 4-equation system.

Without beliefs about a monetary-fiscal reform, the model collapses to the standard formulation of the New Keynesian model with monetary dominance: the debt dynamics are entirely decoupled from the inflationary and the output gap dynamics. Moreover, divine coincidence holds: $\{x_t, \pi_t, i_t - \rho\} = \{0, 0, 0\}$ is a solution to the New Keynesian block.

Determinacy, implementation, and monetary dominance. Next, we provide the conditions for local determinacy.

Proposition 1 (Determinacy and implementability)**.** *Consider a given path of monetary disturbances u_t and fiscal shock ψ_t. Assume that $\gamma \in (0, \rho + \lambda_f + \lambda_h)$. Then,*

I. *Determinacy.* *There exists a unique bounded equilibrium if and only if*

$$[\gamma - \lambda_h (1 + \lambda_f \Phi)] (\phi - 1) > -\gamma \frac{\rho + \lambda_f}{\kappa} \lambda_h. \tag{14}$$

II. *Implementability.* *Let $\hat{i}_t$ denote a given path of nominal interest rates and $(\hat{x}_t, \hat{\pi}_t, \hat{b}_t)$ that satisfies the Euler equation* (11)*, the NKPC* (12)*, and the government's flow budget constraint* (13)*. Suppose that $u_t = \hat{i}_t - \rho - \phi\hat{\pi}_t$, with ϕ satisfying condition* (14)*, such that we can write the policy rule as*

$$i_t = \hat{i}_t + \phi(\pi_t - \hat{\pi}_t). \tag{15}$$

Then, the unique solution to the system (11)*-*(13) *and* (15) *is given by $x_t = \hat{x}_t$, $\pi_t = \hat{\pi}_t$, and $b_t = \hat{b}_t$.*

Condition (14) generalizes the Taylor principle to our setting.[6] For the rest of the paper, condition (14) is satisfied during the fiscal-expansion phase. An implication is that monetary policy is active in the sense of Leeper (1991). Appendix B further shows that fiscal policy is passive when $\gamma \geq 0$. Because these are the opposite assumptions of fiscally dominant regimes, the mechanism is not the mechanism that prevails under the FTPL.

The second part of Proposition 1 shows how a time-varying inflation target implements any allocation satisfying the equilibrium conditions. A similar approach can be used to implement the equilibrium outcomes in the inflationary-finance phase by assuming the monetary authority follows the policy rule: $i_t^* = \rho + \phi\pi_t^* + u_t^*$, given the same coefficient ϕ. Moreover, disturbances to the Taylor rule are regime-dependent, but the coefficients are fixed.[7]

Integral representation Given an arbitrary path for the real rate $r_t = i_t - \pi_t$, we can characterize the system in closed form. The path of debt satisfies:

$$b_t = e^{-\gamma t} b_0 + \int_0^t e^{-\gamma(t-s)} (\psi_s + r_s - \rho) ds. \tag{16}$$

Debt accumulates through two forces: fiscal pressures, ψ_s, and real interest rates that exceed the natural rate ρ. The parameter γ controls the mean reversion in government debt.

[6] When $\lambda_h = 0$, we recover the standard Taylor principle: equilibrium determinacy requires $\phi > 1$. With $\lambda_h > 0$, determinacy can be achieved with a relaxed condition: $\phi \leq 1$.

[7] This feature is in contrast to the literature on regime-dependent rules—see e.g. Davig and Leeper (2007) and Farmer, Waggoner and Zha (2009).

Uncertainty about the reform leads to a *discounted Euler equation*:

$$x_t = -\int_t^{\infty} e^{-\lambda_h(s-t)}(r_s - (\rho - \lambda_h(b_t - b^n)))ds. \tag{17}$$

This equation states that changes in future interest rates are discounted by λ_h. The second term is an *expectation effect*. The term captures how the path of debt enters isomorphically to changes in the natural interest rate.[8]

Integrating the NKPC forward, we obtain the inflation rate

$$\pi_t = \kappa \int_t^{\infty} e^{-(\rho+\lambda_f)(s-t)} x_s ds + \kappa\Phi\lambda_f \int_t^{\infty} e^{-(\rho+\lambda_f)(s-t)}(b_s - b^n)ds. \tag{18}$$

As in the standard New Keynesian model, inflation is given by forward-looking components. One component equals the expected present value of output gaps in the fiscal-expansion phase. However, there is a second component associated with the expectation of a jump in inflation after a reform.

The integral representations in (17) and (18) reveal the key insight of this paper. Recall that debt appears in the integral formulations because it summarizes the expectations of jumps in inflation and output conditional on reform. The presence of debt changes the transmission mechanism of monetary policy relative to the standard New Keynesian model because, unlike inflation or the output gap, debt is a backward-looking variable.

The appearance of a backward-looking variable in the New Keynesian model has important implications for monetary policy analysis. Consider a shock to real rates induced by monetary policy in the standard New Keynesian model: to compute inflation and output at any moment, we need only information on rates from that moment onward. Thus, if the shock vanishes with time, so will its effects. This is not true when the backward-looking behavior of debt is present. To compute inflation and output at any moment, we need information on debt every moment onward. However, debt depends on the entire history of rates, not only rates going forward. As a result, monetary policy in the past can affect today's outcomes. The sticky inflation phenomenon we highlight in this paper critically depends on this feature.

A salient feature of the NKPC, displayed in (18), is its expectation component inherits some history dependence. Several empirical studies document such effects: Hazell, Herreno, Nakamura and Steinsson (2022) estimate a similar NKPC that contains a term captur-

[8] See e.g. Leeper and Zha (2003) for a definition and discussion of expectation-formation effects.

ing long-term inflation expectations and find that most of the variation in inflation comes from that term.[9] Likewise, Coibion, Gorodnichenko and Weber (2022) argue that news about future debt leads households to anticipate higher inflation, both in the short-run and the long run.[10]

Discussion: modeling assumptions and outcomes. Two key assumptions merit discussion: the nature of fiscal-monetary reforms and the expectations of economic agents regarding these reforms. The random arrival of inflationary-finance phase reforms reflects the inherent uncertainty in the political process that determines how the fiscal burden of large debt levels is resolved. These reforms depend on negotiations, capabilities, and decisions made by political actors who may seek compromises with monetary policymakers. Reforms could also be caused by self-fulfilling episodes where the rollover of national debt fails.

For simplicity, we assume a single outcome for any reform. However, in reality, reforms are complex, and their outcomes are uncertain. This complexity could be modeled by adding uncertainty regarding the magnitude of inflationary effects, perhaps through a compound Poisson process. In our framework, this extension would only alter the parameter Φ.

As in Caballero and Simsek (2022), we allow for the possibility that households' and firms' beliefs differ from the actual probabilities of a reform. Given that fiscal expansions and subsequent reforms are rare, it can be challenging for any entity—households, firms, monetary authorities—or even modelers—to accurately assess the likelihood of policy changes. This justifies allowing differences in beliefs. For much of the analysis, we examine versions of the model where household beliefs about reforms are "turned off," creating simplified cases that clarify the role of household and firm beliefs.

3. Three policy experiments

This section considers three policy experiments. The experiments are designed to show that once the expectation of a monetary-fiscal reform lurks in the background, monetary policy can no longer jointly stabilize output and inflation. The meaning of this result is profound:

[9] Hazell et al. (2022) attribute fluctuations in the expectations component to *permanent* changes in the conduct of monetary policy—permanent changes in output gap targets. Equation (18) shows that *temporary* fiscal shocks can rationalize that evidence since they will provoke movements in the expectation component.

[10] Similarly, Li, Fu and Xie (2022) shows that inflation expectations respond to fiscal shocks and predict future debt levels, consistent with our mechanism.

it breaks divine coincidence. Lack of divine coincidence crucially depends on the firms' expectation: if firms do not expect a reform, it is possible to jointly stabilize the output gap and inflation even if households do. To focus squarely on the role of firm expectations, most of the section assumes that $\lambda_h = 0$. We also temporarily focus on the case with a fiscal stabilizer, $\gamma = 0$, for tractability. We dispense these assumptions at the end of the section.

For the rest of the analytical formulations, we assume the fiscal shock is exponentially decaying, the continuous-time analog of AR(1) processes in discrete time: hence, $\psi_t = e^{-\theta_\psi t}\psi_0$.

Policy I: Output gap stabilization. In the first experiment, monetary policy aims to stabilize output during the fiscal-expansion phase. That is, monetary policy implements a zero output gap, $x_t = 0$.

To stabilize the output gap, the real rate must satisfy $r_t = \rho$. Given the fiscal shock, government debt is increasing over time: $b_t = b^{lr} - \psi_t/\theta_\psi$, where $b^{lr} \equiv b_0 + \psi_0/\theta_\psi$ denotes the long-run debt level in the fiscal-expansion phase.

The proposition below shows that the expectation effects induced by the fiscal shock lead to an increasing path of inflation over time.

Proposition 2 (Inflation under output gap stabilization)**.** *Suppose $x_t = 0$ in the fiscal-expansion phase. Then, inflation is*

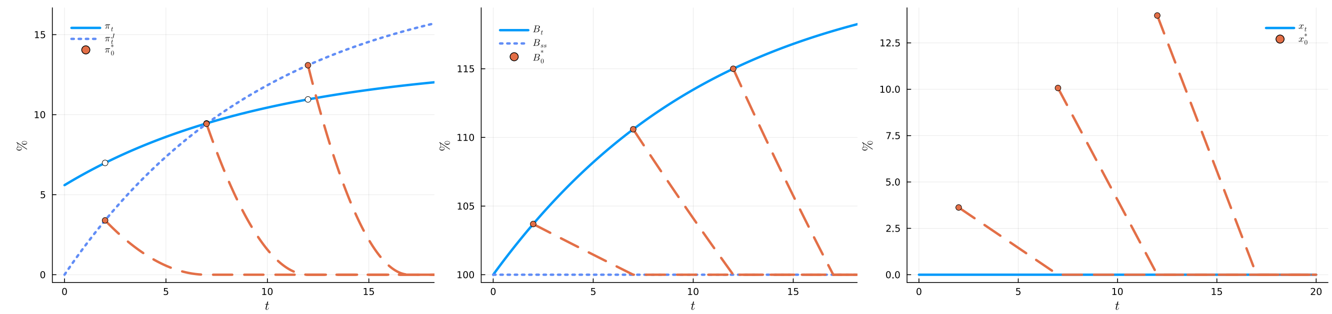
$$\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}\left[b_t - b^n + \frac{\psi_t}{\rho+\lambda+\theta_\psi}\right]. \tag{19}$$

Moreover, inflation increases over time, $\dot{\pi}_t = \frac{\kappa\lambda\Phi}{\rho+\lambda+\theta_\psi}\psi_t > 0$, and converges to a positive level, $\lim_{t\to\infty}\pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}(b^{lr} - b^n) > 0$.

Proposition 2 shows that to stabilize output, monetary policy must live with an ever-growing inflation. Before a reform, inflation increases initially in proportion to the primary deficits. However, inflation persists even after deficits dissipate. Moreover, inflation lasts until a reform happens. That is, *inflation is sticky*. Sticky inflation occurs because the jump inflation component in the Phillips curve, $\pi_t^J = \kappa\Phi(b_t - b^n)$, reflects the expected burst in inflation that trails the path of debt in a inflationary-finance phase.

Proposition 2 tells us something meaningful: an independent monetary policy focused on stabilizing output must live with inflation that trails debt, a fiscal variable. That is, the sole belief, rational or not, of a future compromise to aid debt stabilization is enough to destabilize inflation in a monetary independent regime.

Figure 2 shows the typical paths of inflation, debt, and the output gap, during the fiscal-expansion phase and the inflationary-finance phase. In Panel (a), we find an example of



(a) Inflation (b) Debt (c) Output Gap

Figure 2: Pre and Post Reform Equilibrium Objects

Note: Red dashed lines correspond to reform paths that occur at different points in time.

a path of inflation (solid blue curve) plotted together with its corresponding jump inflation term (blue dotted curve) and the paths that start with different arrival dates of the inflationary-finance phase (red dashed curves). When the inflationary-finance phase is initiated, inflation jumps to the jump inflation term. Prior to the reform, inflation increased steadily because of the present value of all future jump inflation terms. If the reform happens early, the early resolution of reform uncertainty may lead inflation to drop.[11] However, inflation will jump to a higher level if the reform is postponed past a date. In either case, inflation is increasingly converging to a higher level until a reform. Panel (b) shows the corresponding paths of debt. Prior to a reform, the national debt is also growing. Upon a reform, it trends downwards to its target level in exactly T^* time. The speed of the debt reduction is faster the later the date of a reform. This reflects that since inflation must reduce debt to the same level during the same time interval, later reforms require greater bursts in inflation to stabilize debt. Panel (c) shows the output gap. Again, the later the reform, the greater the overheating needed to stabilize debt. The increasing path of inflation shows that postponing reforms is increasingly costly.

Policy II: Inflation stabilization. In the previous exercise, monetary policy focuses exclusively on stabilizing output. In the next exercise, it attempts to combat inflation by temporarily raising rates, such that $r_t - \rho = e^{-\theta_r t}(r_0 - \rho)$, for a given initial rate $r_0 > \rho$ and persistence parameter $\theta_r > 0$.

It is useful to express results relative to the previous experiment. For that, we use a

[11] If the reform happens early, inflation drops because the initial jump inflation is lower than the net present expected discounted value of future jump inflation.

superscript og to denote the variable paths in the previous exercise. With mean reverting shocks to r_t, the output gap follows:

$$\dot{x}_t = r_t - \rho \Rightarrow x_t = -\frac{1}{\theta_r}(r_t - \rho). \tag{20}$$

As $r_t > \rho$, the output gap is negative during the fiscal-expansion phase.[12] In turn, the path of debt is

$$b_t = b_t^{og} + \frac{1 - e^{-\theta_r t}}{\theta_r}(r_0 - \rho), \tag{21}$$

where $b_t^{og} = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$.

We solve for inflation using the NKPC—using equation (18). A policy that fights inflation deviates from the output-gap stabilization solution through the sum of two effects: a fight-inflation effect and a jump-inflation effect. Formally:

Lemma 1. *Suppose $r_0 > \rho$. With mean-reverting real interest rates $\dot{r}_t = -\theta_r(r_t - \rho)$, inflation is given by:*

$$\pi_t - \pi_t^{og} = F_t^\pi + J_t^\pi.$$

where F_t^π and J_t^π are, correspondingly, fight and jump inflation components given by:

$$F_t^\pi = -\frac{\kappa}{\theta_r}\frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}(r_0 - \rho) < 0 \quad and \quad J_t^\pi = \frac{\lambda\kappa\Phi}{\theta_r}\left[\frac{1}{\rho + \lambda} - \frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}\right](r_0 - \rho) > 0.$$

The first term, the fight-inflation term F_t^π, captures the standard effect of contractionary policy through aggregate demand. The term is negative since $r_t > \rho$ and converges to zero as the contractionary stimulus vanishes. Thus, the increase in r_t above the natural rate ρ has a mitigating effect on inflation, as in standard versions of the New Keynesian model.

The second term, the jump inflation J_t^π, is the expected present value of inflation surges after possible reforms, which depends on the path of debt, $\pi_t^J = \kappa\Phi(b_t - b^n)$. The jump inflation term is always positive and, furthermore, builds up with time. Jump inflation is related to the increase in the fiscal burden from an increase in the real interest rate. A greater fiscal burden builds up with time: debt grows with the past accumulation of interests—above the natural debt level. Hence, the additional fiscal cost is never repaid and, thus, added to the debt stock. Through debt accumulation, current rate hikes feedback into present inflation by expecting a greater burst in future inflation. In a nutshell, the debt accumulation resulting

[12] We used the terminal condition $\lim_{t\to\infty} x_t = 0$, a form of long-run neutrality.

from higher real rates pressures current prices upward.

Whereas the fight-inflation term pressures prices downward, the jump inflation does the opposite. The fight-inflation term vanishes over time, but the jump-inflation term continues to build up. Hence, which effect dominates depends on the persistence of shocks and the horizon ahead of the stimulus. These observations lead to the following :

Proposition 3 (Stepping on a Rake)**.** *Suppose $r_0 > \rho$. The rate increase reduces inflation on impact, i.e., $\pi_0 < \pi_0^{og}$ iff:*

$$\theta_r < \frac{\rho + \lambda}{\lambda \Phi}.$$

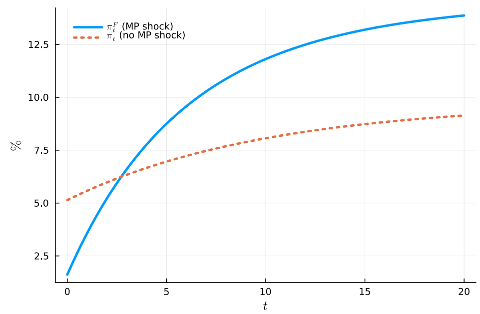
However, in that case, there always exists a $\hat{T} > 0$ such that $\pi_t > \pi_t^{og}$ for $t > \hat{T}$.

The proposition shows two things: First, to be successful in the present, monetary policy must commit to a sufficiently persistent contractionary policy stance. Indeed, monetary policy can fight inflation in the short run, provided the policy is sufficiently persistent.

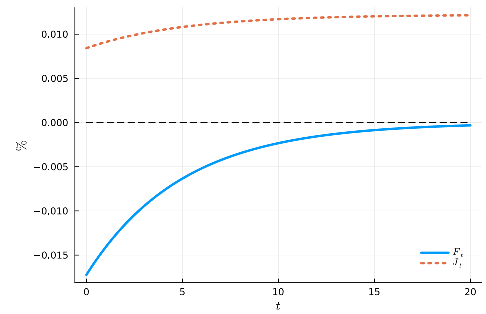
Second, although monetary policy may succeed in fighting inflation in the short run, it faces an unpleasant "stepping-on-a-rake" result: eventually, inflation will come back and stronger. Once again, *inflation is sticky!* The reason is that the contractionary effect on the output gap eventually fades away, whereas the effect on the government debt builds up over time.

Figure 3 shows paths of inflation, debt, and the output gap, considering an attempt to fight inflation in a fiscal-expansion phase. In Panel (a), we find an example of two paths of inflation: a baseline (dashed)—without a monetary policy disturbance—and a counterfactual (solid) corresponding to a temporary increase in policy rates. The figure shows Proposition 3 at play. While the anti-inflationary strategy is successful early on, inflation returns a year into the policy. Panel (b) shows why: it plots the fight inflation (solid) and jump inflation (dashed) components. The fight inflation component is initially strong but eventually fades away with the vanishing impulse on aggregate demand. The jump inflation component is initially weaker but builds up. Panel (c) shows the path of debt, which picks up with the higher real interest rates. Finally, Panel (d) shows the contractionary effect on the output gap.

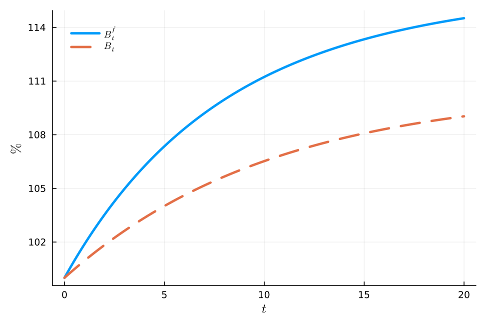
All in all, with a lurking expected fiscal-monetary reform, attempts to curtail inflation have standard short-run effects. Unlike the canonical New Keynesian model, they lead to higher inflation in the long run. This pattern is similar to the "stepping-on-a-rake" result in Sims (2011). However, that paper's pattern follows from changes in the valuation of



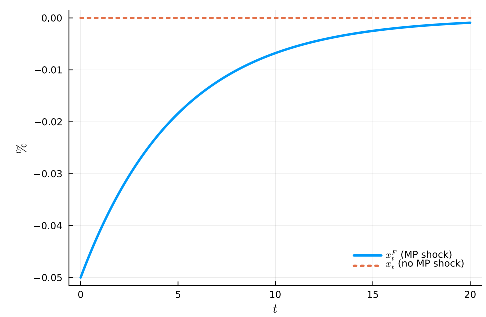
(a) Inflation



(b) Decomposition



(c) Debt



(d) Output gap

Figure 3: Equilibrium Paths with and without Contractionary Monetary Shock

nominal long-term debt, a feature that plays no role here.[13]

While monetary policy cannot fully stabilize inflation with temporary movements in the output gap, it could do so if it were to induce a permanent decline in output. Indeed, permanent inflation stabilization can be achieved if debt follows an increasing path while the output gap becomes more and more negative with time.[14]

Policy III: Debt stabilization. In the third policy experiment, monetary policy attempts to stabilize debt. Stabilizing debt requires the real rate to neutralize the effects of deficits: $r_t - \rho = -\psi_t$, so $b_t = b_0$. Thus, the output gap is: $x_t = \psi_t/\theta_\psi$ given the terminal condition $\lim_{t\to\infty} x_t = 0$. In this case, inflation follows:

$$\pi_t = \frac{\kappa}{\theta_\psi} \frac{\psi_t}{\rho + \lambda + \theta_\psi}.$$

[13] As shown by Cochrane (2018b), long-term bonds are strictly necessary to obtain the result.

[14] In this case, the output gap must offset movements in the government debt, $x_t = -\lambda\Phi(b_t - b^n)$. This condition requires that the real rate be $r_t - \rho = -\frac{\lambda\Phi}{1+\lambda\Phi}\psi_t$, in which case debt is given by $b_t = b_0 + \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\frac{\psi_0}{1+\lambda\Phi}$.

All in all, to stabilize the debt, the monetary authority must overheat the economy in proportion to the trajectory of primary deficits.

Debt stabilizers and expected reforms by households. The previous exercises abstract away from households' expectation effects, $\lambda_h = 0$, and automatic debt stabilizers, $\gamma = 0$. The point of this subsection is to explain the role of these features. For clarity, we return to the first exercise and study a monetary policy that stabilizes output, $x_t = 0$, and take $b_0 = b^n = 0$ to simplify expressions.

With households expecting a reform, stabilizing output in the fiscal-expansion phase requires setting the real rate to $r_t - \rho = \lambda_h(b_t - b^n)$. A high interest rate is necessary to offset the expansionary effects of a positive debt gap. Given this real rate, debt will follow:

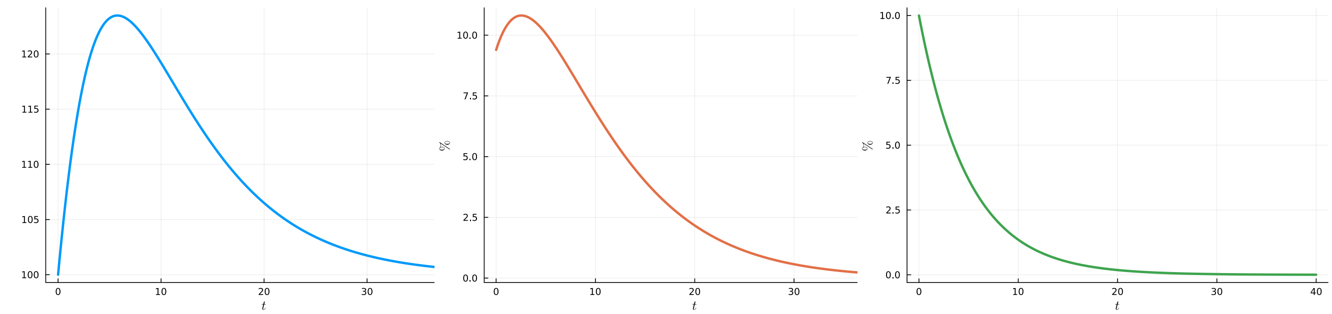
$$\dot{b}_t = -(\gamma - \lambda_h)b_t + \psi_t \Rightarrow b_t = \frac{e^{-(\gamma-\lambda_h)t} - e^{-\theta_\psi t}}{\theta_\psi + \lambda_h - \gamma}\psi_0,$$

Thus, provided $\lambda_h < \gamma$, debt eventually reverts to its initial level. In turn, inflation is given by the net present value of its jump inflation term, which is given by

$$\pi_t = \frac{\psi_0}{\theta_\psi + \lambda_h - \gamma}\left[\frac{e^{-(\gamma-\lambda_h)t}}{\rho + \lambda + \gamma - \lambda_h} - \frac{e^{-\theta_\psi t}}{\rho + \lambda + \theta_\psi}\right] > 0.$$

These expressions explain the role of λ_h and γ. A positive λ_h leads to real rates that increase with debt, further increasing debt. A positive γ leads to an offsetting response of primary surpluses, which reduces debt. When $\lambda_h < \gamma$, inflation also reverts to its steady state. In this case, the automatic debt stabilizer is enough to let primary surpluses bring down debt over time despite the extra fiscal pressures caused by monetary policy setting the real rate above the discount factor ρ. Figure 4 illustrates this point: a fiscal shock causes a sharp increase in government debt and an inflation bout. As the fiscal shock dissipates, the automatic debt stabilizer successfully brings debt down, and inflation recedes. We can also see the sticky-inflation phenomenon, as the inflationary effects of the fiscal shock persist even after ψ_t nearly reaches zero.

When $\gamma = \lambda_h$, we recover the case of Section 3, where output-gap stabilization leads to an increasing path of government debt and inflation. This is because despite the debt stabilizer, households expect the reform, and their beliefs enter as positive demand shocks. The real rate increase needed to stabilize output offsets the debt stabilizer's effect. Thus, fiscal shocks have a permanent effect on debt and inflation. When $\gamma < \lambda_h$, the feedback between debt and



(a) Government debt (b) Inflation (c) Fiscal shock

Figure 4: Equilibrium with households' expectation effects and debt stabilizer

the real rates needed to stabilize output causes debt and inflation to spiral out of control. Fiscal sustainability requires primary surpluses to react strongly to debt when real rates move with debt.

This last exercise shows that adding household expectations and an automatic stabilizer does not change the main message of the output stabilization exercise. This is true for all experiments in the section. Appendix A.2 actually develops all the experiments in this section and shows that the same lessons carry through in general: attempts to stabilize one variable, the output gap, inflation, or debt, destabilize the other variables in the system.

Taking stock: sticky inflation as endogenous fiscal cost-push shocks. We have seen that it is impossible to stabilize output and inflation jointly due to a lurking monetary-fiscal reform. Divine coincidence fails even in the absence of cost-push shocks, in contrast to standard versions of the New Keynesian model. Indeed, sticky inflation is a form of endogenous *fiscal cost-push shock*.[15] As with standard cost-push shocks, rate hikes that induce an output contraction are required to offset the inflationary effects of an anticipated fiscal-monetary reform. Despite this similarity, sticky inflation brings a greater challenge: The endogenous nature of the fiscal cost-push shocks means that those rate hikes will return to haunt the central banker. This complicates the optimal policy analysis of the next section.

[15] This result is reminiscent of the endogenous cost-push shock in Guerrieri, Lorenzoni, Straub and Werning (2021). While they rely on asymmetric sectorial shocks, we focus instead on the role of expectation effects.

4. Optimal Policy

In this section, we study optimal monetary policy during the fiscal-expansion phase. As discussed above, the expectation of a reform breaks divine coincidence. Moreover, changes in current real rates carry permanent effects on real debt. Thus, a benevolent monetary authority faces a non-trivial trade-off between stabilizing output, inflation, and debt.

4.1 The optimal policy problem

We consider a standard approximation to the household's welfare function in which the planner minimizes the expected present value of squared deviations of output and inflation from their steady-state values. The only policy instrument is the path of nominal interest rates during the fiscal-expansion phase. The planner commits to a path of interest rates.

We present the optimal policy without automatic debt stabilizers, $\gamma = 0$, and no households' expectation effects, $\lambda_h = 0$. This is the most tractable version, so we relegate more general solutions to the appendix. The planner and the firms' beliefs coincide. Hence, we write $\lambda_f = \lambda$.

The planner's objective. Once an inflationary-finance phase initiates, the planner has no control over inflation or output, but we can still compute the value of its welfare objective. Starting with a debt level b_0^*, the value of the planner's objective is proportional to the square deviation of debt from its neutral level:

$$\mathcal{P}^*(b_0^*) = \int_0^{T^*} e^{-\rho t}(\alpha x_t^{*2} + \beta \pi_t^{*2})dt = \Upsilon \cdot (b_0^* - b^n)^2$$

where[16]

$$\Upsilon \equiv \left(\alpha + \beta(\kappa\Phi)^2\right) \int_0^{T^*} e^{-\rho t} \left(1 - \frac{t}{T^*}\right)^2 dt.$$

At the beginning of the fiscal-expansion phase, the planner's objective function can be written as

$$\mathcal{P} = -\frac{1}{2}\mathbb{E}\left[\int_0^{\tau} e^{-\rho t}\left(\alpha x_t^2 + \beta \pi_t^2\right) dt + e^{-\rho\tau}\mathcal{P}_\tau^*(b_\tau)\right],$$

where τ denotes the random time the economy switches to an inflationary-finance phase.

[16] We use that $x_t^* = (b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ and $\pi_t^* = \kappa\Phi(b_0^* - b^n)\left(1 - \frac{t}{T^*}\right)$ to obtain $\mathcal{P}^*(b_0^*) = \Upsilon(b_0^* - b^n)^2$.

Given the arrival time is exponentially distributed, we obtain:

$$\mathcal{P} = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\cdot\Upsilon\cdot(b_t - b^n)^2\right]dt.$$

This objective tells us that even though only output and inflation directly affect the planner's objective, the influence of government debt on the inflationary-finance phase creates an endogenous *debt-stabilization motive*. In other words, in addition to inflation and output, the planner wants to minimize deviations of government debt from its natural level; the weight on debt does not come from the planner's concern about budgetary affairs but because debt will affect inflation once a monetary-fiscal reform starts. Debt will also affect inflation prior to the reform through expectation effects.

The presence of a debt-stabilization motive distinguishes this problem from the classic analysis of Barro (1979), or its modern formulations (Aiyagari, Marcet, Sargent and Seppälä, 2002), where the planner uses fluctuations in government debt to smooth variations in distortionary taxes. In our setting, deviations of the government debt from its natural level are costly.

Competitive equilibria. The planner's problem involves choosing a competitive equilibrium. A competitive equilibrium corresponds to a bounded solution to the system (11)-(13) given b_0, a path of a path of fiscal shock ψ_t, and a path of real interest rates.

For any given initial condition for the output gap, inflation satisfies:

$$\pi_0 = \kappa\frac{x_0 + \lambda\Phi(b_0 - b^n)}{\rho+\lambda} + \frac{\kappa}{\rho+\lambda}\int_0^\infty e^{-(\rho+\lambda)t}\left[(1+\lambda\Phi)(r_t-\rho) + \lambda\Phi\psi_t\right]dt. \quad (22)$$

Thus, the set of competitive equilibria can be indexed by a path of real interest rates $\{r_t\}_0^\infty$ and an initial output gap x_0—the monetary authority can implement a particular equilibrium using the conditions provided in Proposition 1. While the planner can freely choose the initial output gap, it cannot independently choose *both* the output gap and inflation.

Debt expropriation and lack of a classical solution. As often occurs in optimal Ramsey problems, the planner may have incentives to expropriate private agents at time zero. Debt is real, and prices are sticky, so expropriation cannot occur through a price level jump. Instead, the planner can effectively choose an arbitrarily negative real return r_t on debt for an infinitesimal period, which leads to a downward jump in government debt in period zero.

This would amount to an instantaneous debt deflation.

The above observation implies that a classical solution to the planner's problem, one where states follow a continuous path, does not exist. The possibility of an "expropriation-like" debt path occurs because the model does not penalize extremely low rates. To avoid the possibility of expropriation, we introduce penalties on past promises, similar to the approach in Marcet and Marimon (2019) and Dávila and Schaab (2023).[17]

Solution without instantaneous debt deflation. Following Dávila and Schaab (2023), we consider a penalized version of the problem in which the planner faces a penalty associated with the choice of the initial value for each forward-looking variable, namely x_0 and π_0. By appropriately choosing the penalties, we ensure there is no expropriation. The penalty itself does not affect directly the path of inflation and output. Its effect on the optimal solution is entirely mediated by the impact on the initial debt level. The planner's problem can be written as follows:

Problem 1 (Commitment Problem)**.** *The planner's problem is*

$$\max_{[x_t,\pi_t,b_t,r_t]_0^\infty} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]dt+\xi_x x_0+\xi_\pi\pi_0, \tag{23}$$

subject to the equilibrium system conditions (11-13) and the initial condition for inflation, (22)*, given* b_0 *and fiscal shock's path,* ψ_t*.*

The integral above is the original objective, and the last two terms capture the penalties on the initial output gap, ξ_x, and initial inflation, ξ_π. Absent the penalties, the initial value of the co-states for inflation, output gap, and debt are all zero. In that case, there would be a discontinuous jump in the value of debt at $t=0$. We choose the values of ξ_x and ξ_π such that $\lim_{t\to 0} b_t = b_0$, while the initial value of the co-states on the output gap and inflation is still equal to zero.

The following proposition characterizes interest rates under the optimal policy.

Proposition 4 (Interest rates.)**.** *The paths of real and nominal interest rates under the optimal*

[17] We show in the Appendix C.1 that a classical solution with smooth state variables does not exist without a penalty. Formally, it is optimal to have a Dirac mass on interest rates in period zero, and $\lim_{t\to 0} b_t \neq b_0$. See Arutyunov, Karamzin and Pereira (2019) for a discussion of control problems lacking classical solutions.

policy are:

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t \quad \textit{and } i_t - \rho = \left[1-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t.$$

The proposition provides a solution to the optimal path of real rates as a linear rule. An important implication is that it is optimal to *reduce* the real interest rate in response to the fiscal shock. That is, to the extent the shock is inflationary, i.e., if $\pi_t \geq 0$, real rates should fall below the natural rate ρ. Moreover, nominal rates move less than one-to-one with inflation. This result sharply contrasts with the standard prescription based on the Taylor rule, which dictates the importance of nominal rates moving more than one-to-one with inflation.[18] In contrast, Proposition 4 shows that it is optimal to *underreact* to the shock.

To understand the intuition behind this result, consider a change in real rates that raises both x_t and b_t by one unit, while keeping their values at other dates constant.[19] This reduces the planners objective by the amount

$$\mathcal{C}_t = \alpha x_t + \lambda\Upsilon(b_t - b^n) + \beta\kappa(1+\lambda\Phi)\int_0^t \pi_s ds.$$

The first two terms reflect the direct impact of changing x_t and b_t, while the last term captures the indirect impact through inflation in all past dates. Under the optimal policy, the marginal cost of changing x_t and b_t is equalized across all periods, so $\dot{\mathcal{C}}_t = 0$. This implies that it is optimal to reduce the real rate when inflation is high, so the first two terms offset the impact on welfare of having high inflation. Therefore, nominal rates must react less than one-to-one to inflation under the optimal policy.

Dynamics under optimal policy. To characterize the dynamics under the optimal policy, we use the optimal real interest rule, combined with the fact that $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, and collapse the solution into a bivariate system in π_t and b_t:

$$\begin{bmatrix}\dot{\pi}_t \\ \dot{b}_t\end{bmatrix} = \begin{bmatrix}\rho+\lambda & -\kappa(1+\lambda\Phi) \\ -\hat{\beta} & 0\end{bmatrix}\begin{bmatrix}\pi_t \\ b_t - b^n\end{bmatrix} + \begin{bmatrix}\kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t\end{bmatrix}, \tag{24}$$

[18] Of course, in our solution, the planner still uses the *threat* of reacting to movements in inflation more than one-to-one off the equilibrium to ensure the equilibrium is locally unique.

[19] Given x_0, and the log utility assumption, an increase in real rates raise the output gap and government debt by the same amount.

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. The eigenvalues of this system are:

$$\overline{\omega} = \frac{\rho + \lambda + \sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} > 0, \qquad \underline{\omega} = \frac{\rho + \lambda - \sqrt{(\rho+\lambda)^2 + 4\kappa(1+\lambda\Phi)\hat{\beta}}}{2} < 0.$$

There is one positive and one negative eigenvalue. Hence, there is a unique bounded solution for any given x_0.

The optimality condition for the initial output gap is:

$$\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t + \frac{\beta\kappa}{\rho+\lambda}\pi_t\right] = 0. \tag{25}$$

This condition says that the planner sets the discounted value of a combination of output and inflation to zero, depending on the relative weight of output and inflation on welfare. Therefore, if inflation is on average positive, it is optimal to choose x_0 such that the present value of the output gap is negative, counteracting the inflationary pressures.

4.2 Hawks vs. doves

It is instructive to consider extreme cases where the planner only cares about inflation or only about output, which we associate with *hawkish* and *dovish* central banks. In both cases, the planner assigns a positive endogenous weight to debt stabilization. To simplify the message, we set $b_0 = b^n = 0$. We characterize optimal policy for generic values of α and β in Appendix C.2.

Doves. Consider first the case where the central bank does not place a weight on inflation, so $\beta = 0$.

Proposition 5 (Optimal policy: Doves)**.** *If $\beta = 0$, then,*

(i) Inflation:

$$\pi_t = \frac{\kappa}{\overline{\omega}}\frac{\alpha\lambda\Phi}{\alpha+\lambda\Upsilon}\frac{\psi_0}{\overline{\omega}+\theta_\psi} + \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{1-e^{-\theta_\psi t}}{\theta_\psi(\overline{\omega}+\theta_\psi)}\psi_0. \tag{26}$$

where $\pi_t > 0$ and $\dot{\pi}_t > 0$.

(ii) Output gap:

$$x_t = \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\lambda+\theta_\psi} - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}. \tag{27}$$

(iii) Government debt:

$$b_t = \frac{\alpha}{\lambda\Upsilon + \alpha}\frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{28}$$

Proposition 5 characterizes the optimal reaction of a dovish central bank to a fiscal shock. The dovish central bank faces a trade-off between stabilizing the output gap in the fiscal-expansion phase and stabilizing the output gap in the inflationary-finance phase, which ultimately requires influencing the government debt. The optimal response of the monetary authority is to partially offset the effects of the fiscal shock on debt:

$$r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t.$$

Intuitively, starting from an equilibrium where $r_t = \rho$ and the output gap is constant, a reduction of real rates has a first-order benefit of reducing debt and only a second-order cost of distorting the output gap. Hence, the planner has an incentive at the margin to stabilize debt. The magnitude of the adjustment depends on the relative weight of debt stabilization on welfare. When λ is close to zero, it is unlikely the economy will switch to the inflationary-finance phase, and the planner minimally reacts to the shock. In this case, the output gap is close to zero, and government debt absorbs most of the fiscal shock. When λ is large, the planner offsets most of the fiscal shock, dampening the debt response. Given the planner only cares about the output gap, there is no attempt to stabilize inflation, which ends up being positive and increasing over time.

Hawks. Consider next the case where the planner only cares about inflation, so $\alpha = 0$.

Proposition 6 (Optimal policy: Hawks)**.** *Suppose $\alpha = 0$. Then,*

(i) Inflation:

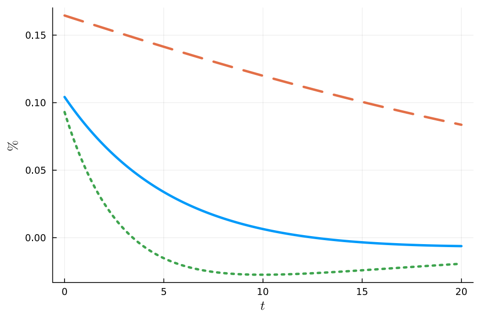
$$\pi_t = \kappa\frac{\psi_t - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}e^{\underline{\omega}t}\psi_0}{(\overline{\omega} + \theta_\psi)(\underline{\omega} + \theta_\psi)}, \tag{29}$$

where $\pi_0 > 0$, $\dot{\pi}_0 < 0$, and $\pi_t < 0$ for t sufficiently large.

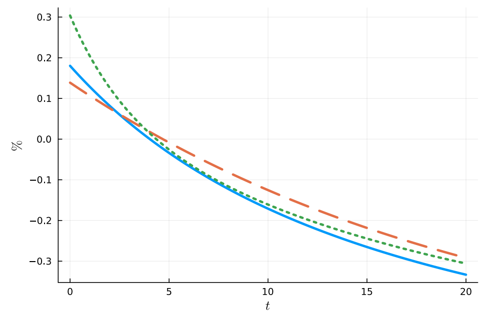
(ii) Output gap:

$$x_t = \frac{\psi_0}{\rho + \lambda + \theta_\psi}\left[\frac{\overline{\omega}}{\overline{\omega} + \theta_\psi} + \frac{\rho + \lambda + \theta_\psi}{\overline{\omega} + \theta_\psi}\right] - \beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon}p_t - \frac{\psi_0 - \psi_t}{\theta_\psi}, \tag{30}$$

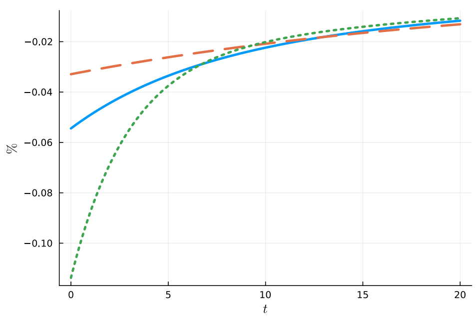
where $p_t = \int_0^t \pi_s ds$ is the price level at date t.



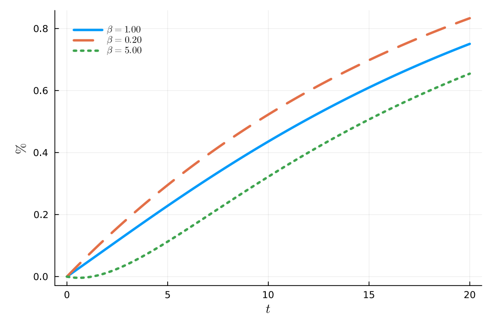
(a) Inflation



(b) Output gap



(c) Real rates



(d) Government debt

Figure 5: Equilibrium dynamics under optimal policy

(iii) Government debt

$$b_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} p_t, \tag{31}$$

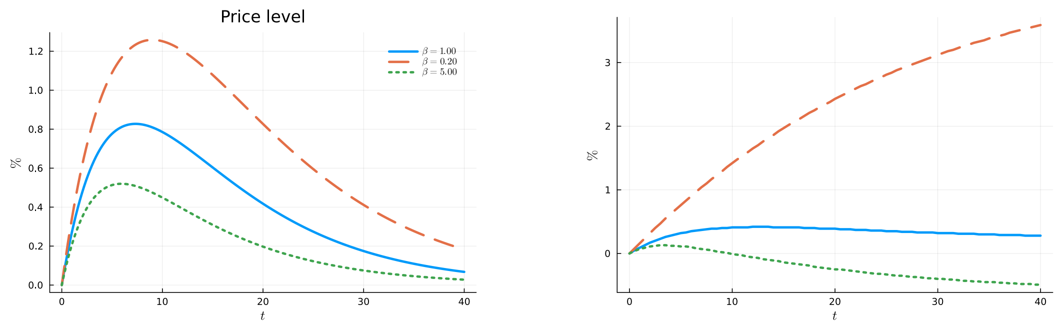
where $\dot{b}_0 < 0$ *and* $\lim_{t\to\infty} b_t > 0$.

The hawkish central bank faces a trade-off between stabilizing inflation in the fiscal-expansion phase and stabilizing it in the inflationary-finance phasethrough its effect on government debt. Give $\pi_0 > 0$, it is again optimal to reduce real rates on impact:

$$r_t - \rho = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon} \pi_t - \psi_t. \tag{32}$$

Interestingly, a hawkish central bank initially reduces real rates more aggressively than its dovish counterpart.[20] With lower interest rates, the planner slows down the accumulation of future debt, which reduces inflation expectations in all previous periods. Therefore, the sticky-inflation mechanism creates an incentive for the planner to *front-load* inflation. Low

[20]Recall that for a dovish central bank $r_0 - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_0$, which is greater than $-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}\pi_0 - \psi_0$.



(a) Exogenous cost-push shock

(b) Fiscal cost-push shock

Figure 6: Price level: exogenous cost-push shock vs fiscal shock

real rates initially raise the output gap, creating some short-run inflationary pressures, but it slows down debt accumulation and reduce future inflation.

Discussion: Hawks vs. doves. Figure 5 shows the optimal policy for different values of β, the welfare weight on inflation, for a fixed weight on the output gap, which we normalize to $\alpha = 1$. The case $\beta = 1$ corresponds to a planner who gives equal weight to inflation and the output gap, while the case $\beta > 1$ ($\beta < 1$) corresponds to a planner who gives more weight to inflation (output gap). A striking feature is that the optimal real interest is below its natural level, regardless of β. Therefore, the planner always finds optimal to move nominal rates less than one-to-one with inflation.

Paradoxically, a hawkish central bank achieves lower inflation, despite having lower real rates and a more overheated economy. By reducing the pace of debt accumulation, the planner counteracts the inflationary pressures coming from de-anchored expectations caused by the fiscal shock.

A comparison with textbook cost-push shocks. As we explained above, sticky inflation takes the form of an endogenous fiscal cost-push shock. However, the dynamics under the optimal policy differ from the optimal policy with exogenous cost-push.

Figure 6 contrasts both responses. A first difference refers to the behavior of the output gap. The optimal response to exogenous cost-push shocks is to allow a recession. In contrast, the planner engineers a boom in response to the endogenous fiscal cost-push shock here, as shown in Panel (b) of Figure 5.

The second difference is the behavior of the price level. In the textbook exercise, it is optimal to promise a sufficiently long period of deflation that brings the price level back to its pre-shock level, see Panel (a) of Figure 6. *Price-level targeting* is optimal in the textbook exercise, a distinguishing feature of optimal policy under commitment—see, e.g., Woodford (2010) for a discussion. Price-level targeting is sub-optimal under sticky inflation, as shown in Panel (b).

4.3 Robustness to commitment assumptions

We have seen that optimal monetary-policy with commitment reduces real rates in responses to a fiscal shock if sticky-inflation is prevalent. Is this a robust feature or does it depend on commitment assumptions? To answer this question, we consider two polar opposites of the time-zero commitment case studied: First, we present the solution under *discretion*. Second, we consider optimal policy under the *timeless perspective*. We show that under both scenarios, it is still optimal to reduce real rates in response to the fiscal shock. This shows that under-reaction is a robust feature of optimal policies under sticky inflation.

Discretion. To capture the idea of discretion in continuous time, we assume that the planner has commitment over a random time interval and takes as given the actions of future planners.[21] Formally, assume that with Poisson intensity $\overline{\lambda}$, the monetary control is surrendered to a new planner. This implies that, in expectation, the planner has control over $\frac{1}{\overline{\lambda}}$ periods. We are interested in the limit as $\overline{\lambda} \to \infty$.[22] The next proposition characterizes the optimal policy.

Proposition 7 (Discretion). *As $\overline{\lambda} \to \infty$, the real interest rate under the optimal policy is given by $r_t - \rho = -\psi_t$. Moreover, the output gap is $x_t = 0$.*

Proposition 7 shows that, under discretion, the real rate is also below its natural level upon a fiscal shock. With an arbitrarily short planning horizon, the planner cannot directly control inflation, which depends on future decisions and has no incentive to distort the output gap. Hence, the planner fully stabilizes debt to influence future decisions. Behind the scenes, the planner sets the output gap to zero and promises a decline over time, given

[21] For a similar formulation of a problem with discretion in continuous time, see e.g., Harris and Laibson (2013) and Dávila and Schaab (2023).

[22] This corresponds to the continuous-time analog of the case of discretion in discrete time, where the planner controls policy over a single period.

the low interest rate. Once a new planner arrives, the planner does not keep this promise, and sets the output gap again to zero.[23]

Timeless perspective. Finally, we consider next the case of optimal policy under the timeless perspective, in the sense of Woodford (1999). When the planner commits to a time-zero plan, it sets the value of the co-states for the forward-looking variables equal to zero at $t = 0$. Under the timeless perspective, the initial value of co-states equal the corresponding value for a planner who started its planning in the distant past.[24] The next proposition shows that the timeless perspective and commitment solutions actually coincide when $b_0 = b^n$.

Proposition 8 (Timeless perspective)**.** *Suppose that $b_0 = b^n$, such that government debt is at its natural level when the fiscal shock is announced. Then, the optimal policy when the planner commits to a time-zero plan coincides with the optimal policy under the timeless perspective.*

An implication of Proposition 8 is that the solution to the Ramsey problem satisfies a *self-consistency* property: output and inflation can be described by time-invariant functions of the exogenous shock, ψ_t, a predetermined variable, b_t, and variables describing history-dependence, the co-states on the forward-looking variables. From the point of view of a planner who started planning in the distant past, there is no incentive to have output and inflation deviate from these time-invariant functions.[25] Once again, we find that a reduction in real rates after a fiscal shock is a robust feature of optimal policies under sticky inflation.

5. Staying behind the curve?

In this final study, we compare the observed dynamics of the U.S. economy in the post-COVID-19 period with the counterfactual scenario where the Fed follows a Taylor rule. The exercise is motivated by the policy debates ongoing in the aftermath of the COVID-19 pandemic.

[23] In Appendix D.4, we consider the case of partial commitment, where the planner takes the initial value of the output gap as given. Optimal policy with partial commitment coincides with the case of full commitment and a dovish central bank. In this case, it is also optimal for the real rate to be below the natural level.

[24] For a formal discussion of this procedure, see the discussion in Giannoni and Woodford (2017).

[25] The assumption that $b_0 = b^n$ is important, as we would observe dynamics under the solution to the Ramsey problem even in the absence of shocks, so the optimal policy would be time-dependent and deviate from the solution under the timeless perspective. This observation motivates our focus on the case $b_0 = b^n$

5.1 The debate

To set the stage, we present some data patterns from the period. In response to the COVID-19 crisis, the United States implemented an unprecedented fiscal expansion, resulting in the highest level of government debt (normalized by GDP) in the post-war era. Panel (a) of Figure 7 shows the large increase in primary deficits in the aftermath of the COVID-19 crisis, reaching 25% of GDP at its peak. Panel (b) shows how large deficits—coupled with disruptions to production—substantially increased the debt-to-GDP ratio. Panel (c) shows a burst in inflation that persisted for two years. The increase in inflation was unlike any other of the last 40 years. An important aspect of this episode was that real interest rates remained remarkably low; Panel (c) also shows that the 1-year (ex-ante) interest rate was negative for over two years after the beginning of the fiscal expansion.

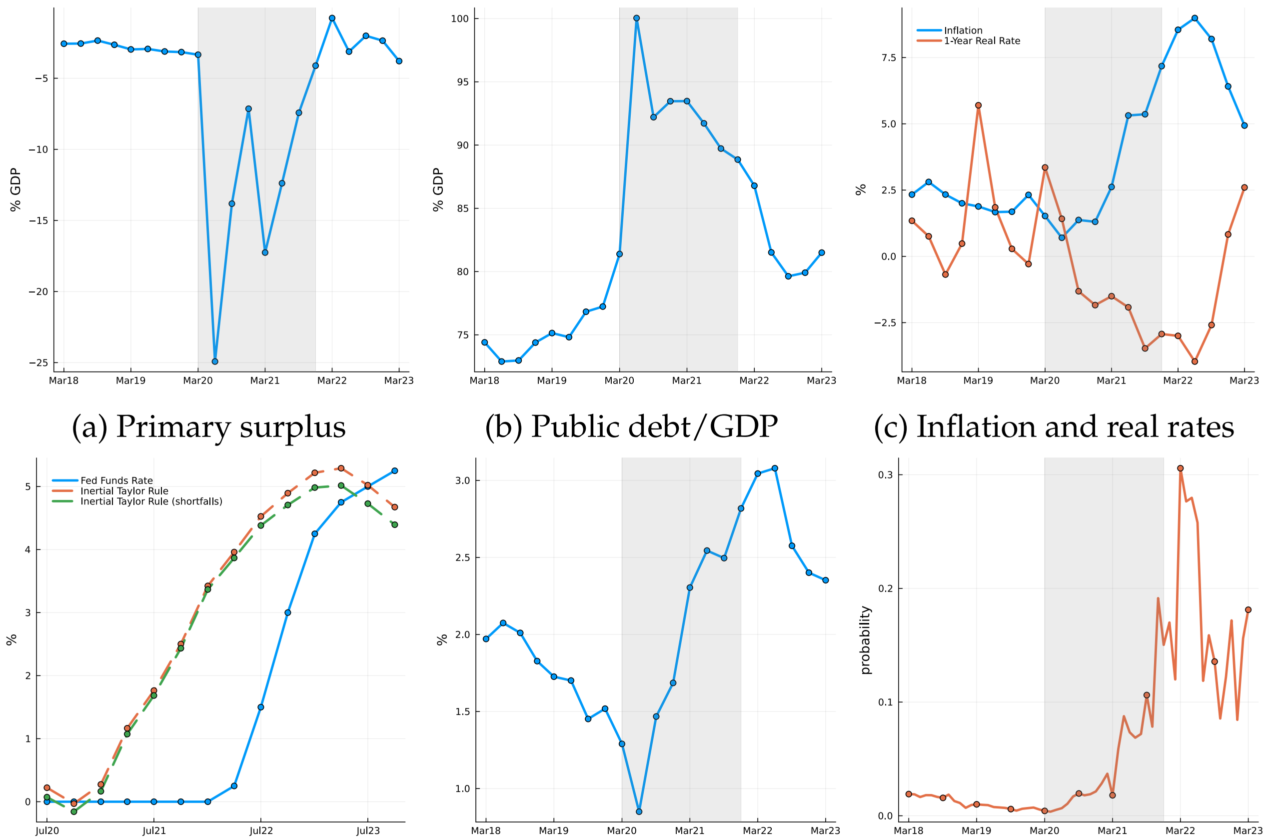
Low real rates reflected the Fed's response to the inflationary pressures: Panel (d) shows that the Federal Reserve kept its nominal policy rate target low even after the surge in inflation. Panel (d) also shows the nominal rates dictated by two versions of a Taylor rule.[26] The figure illustrates the extent of the Fed's *underreaction* relative to the Taylor rule.[27] The Fed's underreaction period was also marked by a persistent increase in inflation expectations, as shown in Panels (e) and (f). In particular, Panel (e) shows the increase in the 5-year breakeven inflation, the difference between the yield on a nominal bond and the yield on an inflation-protected bond (TIPS).[28] Panel (f) shows that the market-implied inflation-disaster probability, as measured by Hilscher et al. (2022), also increased substantially during this period.

The Fed's underreaction when inflation expectations were rising led many commentators to state the Fed was staying "behind the curve." This was a call to a more aggressive stance for fears that the Fed had lost control over inflation expectations and a subsequent painful recession would be necessary to get inflation back to target—see e.g. Bordo, Taylor and Cochrane (2023) for an account of the debate. Evidently, the Fed ignored the advice. Did it make a mistake by deviating from the Taylor rule? Did it risk triggering an inflationary spiral? The next exercise investigates whether following the Taylor rule would have been the correct policy response in the context of our model.

[26] These two versions of the Taylor exemplify the rules discussed by the Fed's Monetary Policy Report during this period. For an assessment of these rules, see Papell and Prodan-Boul (2024).

[27] Bocola, Dovis, Jørgensen and Kirpalani (2024) provide complementary evidence of the Fed's underreaction based on movements in bond prices.

[28] The breakeven inflation is not an unbiased measure of inflation expectation, as it incorporates a risk premium. Survey-based measures showed patterns similar to the market-based ones.



(a) Primary surplus (b) Public debt/GDP (c) Inflation and real rates

(d) Taylor rules (e) 5-year breakeven inflation (f) 5-year inflation disaster

Figure 7: Pre- and Post-COVID-19 Data

Note: Panel (a) shows the primary surplus to GDP ratio. Panel (b) shows the market debt held by the public plus central bank reserves over GDP. Panel (c) shows year-over-year CPI inflation and the Federal Reserve of Cleveland estimate of the 1-year (ex-ante) real rate. Panel (d shows the lower limit of the Federal Funds target range and the predicted nominal rate for two specifications of inertial Taylor rules. Panel (e) shows the 5-year breakeven inflation. Panel (f) shows the probability of inflation exceeding 4% on average for the next five years, a so-called inflation disaster, as estimated by Hilscher et al. (2022) based on inflation option prices.

5.2 Taylor rules vs. realized policies

We use the model to assess the quantitative relevance of our sticky-inflation channel in shaping the dynamics of debt and inflation following the COVID-19 pandemic. To that end, we provide a historical shock decomposition and counterfactual analysis. We employ a discretized version of the model which we use to construct a Kalman filter to obtain shocks. We present the details in Appendix E.

We focus on four shocks: a fiscal shock, which captures the exogenous fiscal expansion;

Table 1: Calibration of the Model

Parameter	**Symbol**	**Value**	**Description**
Discount rate	ρ	0.0022	Real-rate average (1990-2019)
Elasticity of Intertemporal Substitution	σ	0.5	Attanasio and Weber (1995)
Slope of the NKPC	κ	0.0138	Hazell et al. (2022)
Taylor coefficient	ϕ_π	1.2	Moderate response calibration
Fiscal rule	γ	0.038	Bianchi et al. (2023)
Initial debt to quarterly GDP ratio	b^n	0.7683*4	Debt to GDP in 2019Q4
Quarters of high inflation in Phase II	T^*	16	Hazell and Hobler (2024)
Probability of Phase II	λ_f	0.015	Hilscher et al. (2022)

a standard cost-push or markup shock, which reflects the sectorial reallocations and bottlenecks experienced in this period; a monetary shock, representing the deviations of monetary policy from our specified rule; and a shock in the government's budget constraint, capturing unmodeled revaluation effects as in Bianchi and Melosi (2017), or a residual capturing asset purchases by the fed, other sources of funding, and the approximation error of the linearization, as in Hall and Sargent (2024). We then use the identified shock to compute the dynamics of an economy subject to the same fiscal, return, and markup shocks, but where the monetary authority no longer deviates from the Taylor rule—the "Taylor" scenario.

Calibration. We calibrate the discretized version of the model. We treat variable values during 2009Q4 as a steady-state target. Table 1 summarizes the calibration. We adopt standard calibrations for parameters commonly used in the New Keynesian literature. We set the discount rate, ρ, to reflect the average real interest rate in the U.S. from 1990 to 2019 of 0.88% per year. We set the elasticity of intertemporal substitution, σ, to 0.5, roughly in line with the evidence by Attanasio and Weber (1995). We set the slope of the NKPC, κ, to 0.0138, which is the value in the empirical work of Hazell et al. (2022). We find a range of values for the Taylor rule inflation coefficient, ϕ_π, in the literature from 1.2 to 1.5. We set the coefficient to the lower bound of 1.2 to capture a moderate monetary policy response to inflation deviations from an inflation target. This choice is to bring the actual and realized interest-rate path coefficients as close as possible.

The rest of the parameters merit further discussion: We set the fiscal rule coefficient, γ, to 0.038, following Bianchi et al. (2023). This parameter represents the repayment rate of deficits, which, in turn, governs the mean reversion in public debt. We set the inflation-neutral debt level, b_n, to 0.7683×4 so that the debt-to-quarterly-GDP ratio in 2019Q4 was at its neutral level. Thus, under the calibration the sticky inflation channel was muted before the pandemic.

We set the probability of a monetary-fiscal reform, λ_f to 0.015 which translates into an annual probability of 6% or, equivalently, of observing a monetary-fiscal reform once every 15 years. This choice is consistent with the inflation disaster risk in Hilscher et al. (2022).[29]

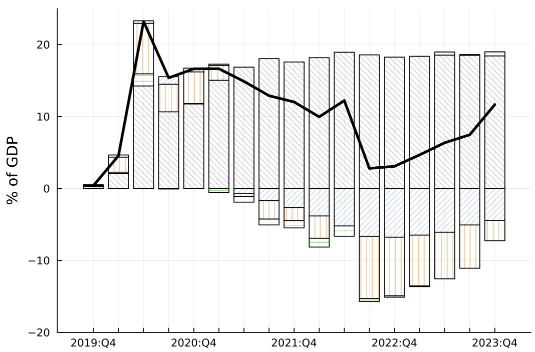
The duration of the fiscal consolidationphase, T^*, governs the pass-through from the debt-gap to inflation. We set T^* to 16 quarters so that the implied pass-through is consistent with the empirical impulse responses to fiscal events in Hazell and Hobler (2024).[30]

Finally, we assume shocks are i.i.d. and set the standard deviation of all shocks to the same value of 1% per year. This last assumption allows us to remain agnostic about the likelihood of shocks when we back them out using the Kalman filter. Lowering the variance of one or the other shocks significantly narrows the likelihood function, biasing the filtering exercise. Our motivation is that the COVID-19 episode was unique, and thus, we cannot use past information to infer the dispersion of shocks. Setting a large variance for each shock and making them i.i.d. allows the episode's data to "speak for itself."

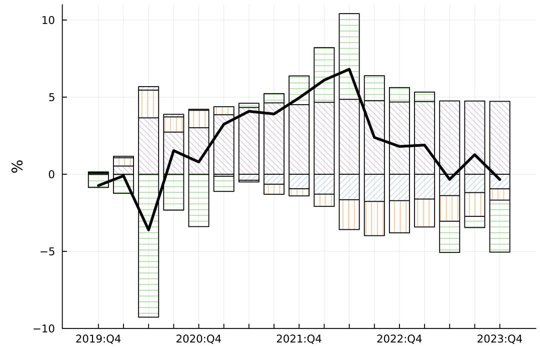
Shock decomposition. Next, we conduct a shock decomposition analysis using historical time series data for the market value of debt to GDP, primary deficits to GDP ratio, inflation, and nominal policy rates—see Figure 7. The results of the shock decomposition are illustrated in Figure 8. Panels (a) through (c) display the contribution of shocks to the paths of government debt path, inflation, and nominal policy rates, respectively. We report the debt-to-GDP ratio as deviations from its neutral level (2019Q4), the inflation rate as deviations from a 2% inflation target, and the nominal rate as deviations from the discount rate plus the inflation target. Figure 9 presents the identified structural shocks.

[29] Recall that this parameter represents the likelihood that the economy will experience an inflation burst. Using inflation swap contracts, Hilscher et al. (2022) report probabilities that inflation will exceed $4\% - 6\%$ thresholds.

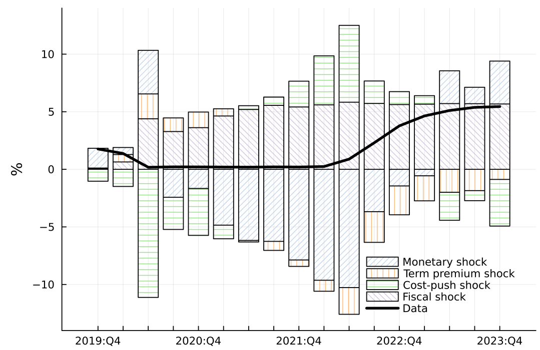
[30] That paper uses electoral outcomes in the senate race in Georgia to proxy for the expectation of the Biden stimulus plan. Their study shows a pass-through of 0.18% inflation over the next 2 years to a 1% increase in the deficit-to-GDP ratio.



(a) Public Debt to GDP



(b) Inflation



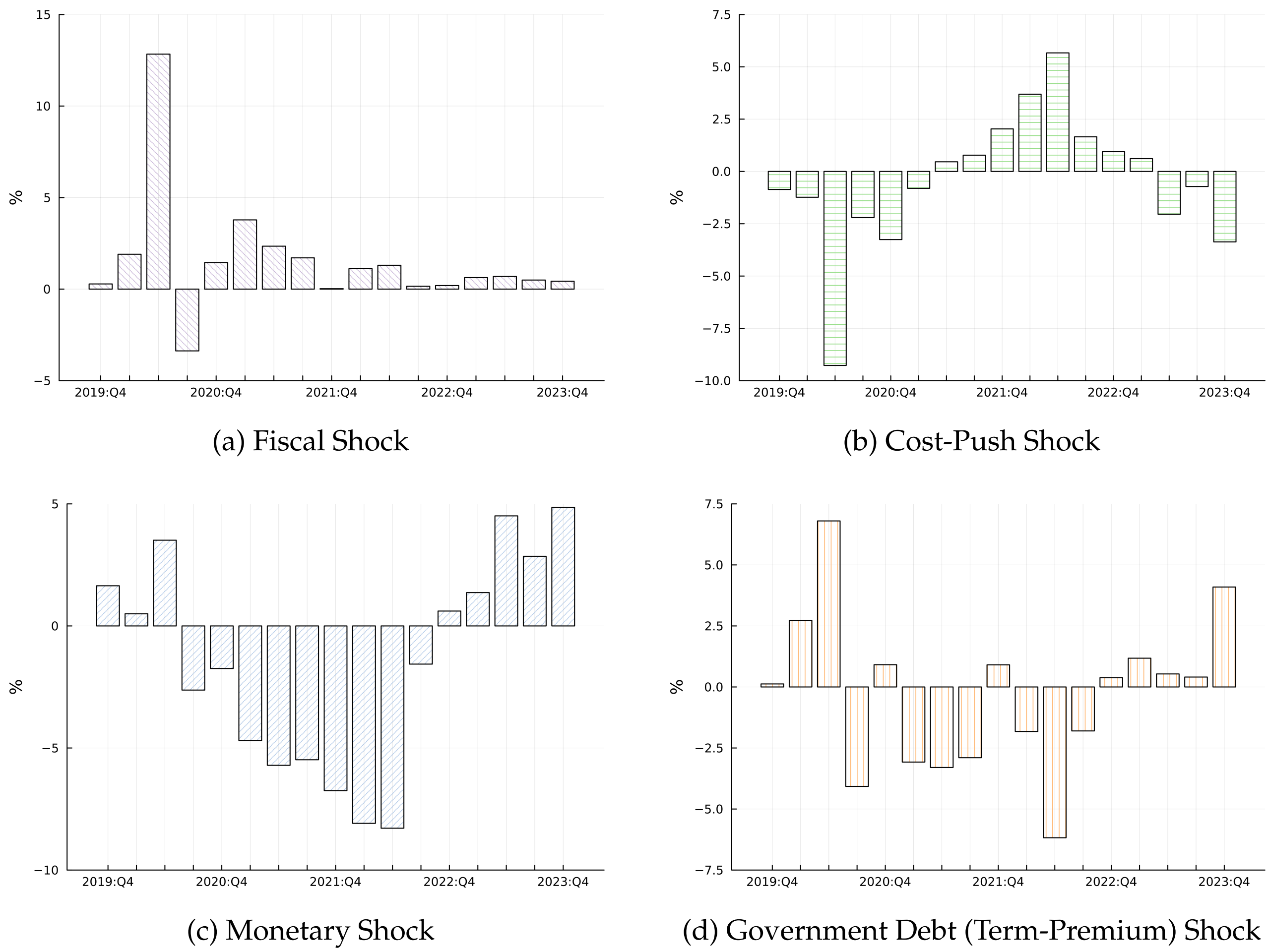
(c) Federal Funds Rate

Figure 8: COVID-19 Shock Decomposition

Next, we describe what determines the dynamics of each of the main macro variables.

Government Debt: A key feature of the debt-to-GDP path is the sharp increase during the second quarter of 2020, following the onset of the pandemic. This spike is primarily driven by abnormally large fiscal shocks, as government spending surged in response to the crisis (Figure 8, Panel a). Additionally, revaluation shocks, which capture the flattening of the yield curve, contributed to the increase in market debt. Cost-push shocks had a smaller impact, predominantly through a reduction in inflation during the early phase of the pandemic.

Monetary policy played a limited role in the initial stages of the crisis. In 2020Q2, monetary policy was constrained by the zero lower bound in its ability to offset deflationary pressures. As the pandemic progressed, fiscal shocks continued to expand, especially during the quarters following 2021Q1, which coincided with the fiscal stimulus measures implemented by the Biden administration. The contribution of fiscal shocks to debt levels remained signif-



(a) Fiscal Shock

(b) Cost-Push Shock

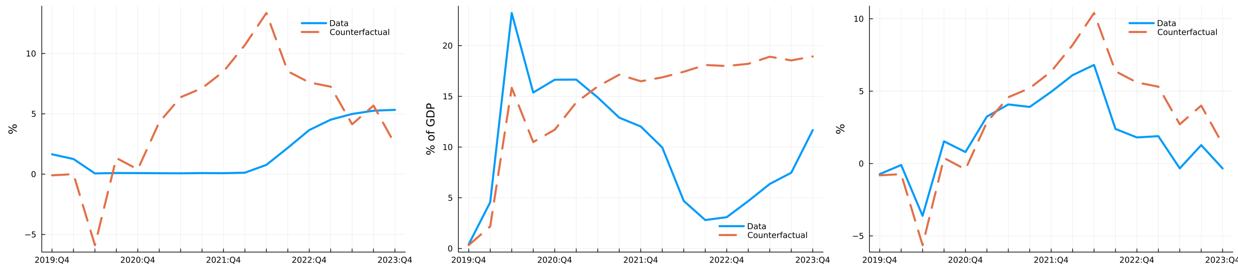
(c) Monetary Shock

(d) Government Debt (Term-Premium) Shock

Figure 9: COVID-19 Shock Decomposition: The Shocks

icant throughout this period. By 2021Q3, monetary policy shocks substantially contributed to reducing government debt. How so becomes clear from the decomposition of inflation and policy rates.

Inflation: Panel (b) presents the decomposition of inflation. The cost-push shock had a significant deflationary impact in 2020Q2, resulting in an annualized inflation rate of -5%. This deflationary trend persisted until the end of 2020. By 2021, the effect of these shocks reversed, likely arising from supply bottlenecks and the Ukranian war, as suggested by other studies. By 2023Q4, the cost-push shocks began to dissipate. During this period, the sticky-inflation component (represented by the backslash bars in the figure) becomes more prominent. Sticky inflation arose due to the persistent fiscal deficits, contributing to



(a) Federal Funds Rate (b) Public Debt to GDP (c) Inflation

Figure 10: COVID-19 Counterfactual Monetary Policy

debt accumulation. As debt deviated further from its target, inflation expectations gained momentum, amplifying inflationary pressures. According to our model, the fiscal shock, though short-lived, contributed almost 5% to annual inflation throughout the period. That inflation started to stabilize in 2022 can be attributed to the deviations of monetary policy from the prescriptions of the Taylor rule.

Monetary Policy Rates: Starting from 2020Q4, the Federal Reserve deviated from the Taylor rule and stayed "behind the curve." Indeed, Panel (c) indicates that nominal rates should have been much higher given the cost-push and fiscal shocks. However, the most notable aspect of the decomposition is that despite the expansionary stance of monetary policy—see the slash bars in panel c—, the effect on inflation was deflationary (see the slash bars in Panel b). This apparent paradox is nothing but the sticky inflation channel at work. Because the lower policy rates eased the debt burden, as shown in Panel (a), the Federal Reserve indirectly mitigated the sticky-inflation component in the Phillips curve. Lower debt levels helped temper inflation expectations, thus counteracting the inflationary pressures from fiscal and cost-push shocks.

Counterfactuals. Using the filtered shocks, we simulate a counterfactual scenario in which the Fed would have followed the Taylor rule. This counterfactual analysis allows us to explore what would have happened to debt and inflation if the Fed had responded more aggressively during the inflation surge, as advised by its critics. The results are shown in Figure 10.

Panel (a) illustrates the actual (solid) and counterfactual (dashed) nominal interest rate paths. Had the Federal Reserve adhered to the Taylor principle, nominal rates would have been reduced more rapidly in the early phases of the pandemic and increased much more aggressively to rising inflation starting in 2020Q4. This more forceful response, while countering inflation, would have significantly increased the burden of government debt, as shown in Panel (b). The more aggressive stance would have led to a more persistent increase in the debt-to-GDP ratio due to the higher debt servicing cost.

Interestingly, the counterfactual inflation path shows that, despite the more aggressive anti-inflationary policy, inflation would have actually been significantly *higher*. Again, the apparent paradox arises from the interaction of two opposing forces: the reduction in inflation through lower demand stimulus and the countervailing effect of the sticky inflation component, amplified by the larger debt burden. Thus, while the Taylor rule would have curbed demand-driven inflation, the resulting increase in debt would have fueled inflationary pressures through the fiscal channel, ultimately offsetting the benefits of the rule's tighter policy.

Given the optimal policy prescriptions derived in the previous section, we conclude that the Federal Reserve's decision to stay "behind the curve" was appropriate. Our counterfactual shows that adhering to the Taylor rule would have resulted in suboptimal outcomes, with higher debt and moderate inflation. All in all, the exercise shows that admitting the possibility that inflation expectations can be dragged by debt alters the conventional wisdom regarding the optimal response to monetary policy.

It is important to note that the Taylor rule does not necessarily indicate that the Federal Reserve intentionally acted to ease the debt burden. Rather, the Fed's gradualism doctrine, characterized by measured responses to unfolding events, aligned well with the optimal policy in this context. Our model indicates that the Federal Reserve's decision to resist calls for a faster tightening of monetary policy was ultimately the right course of action.

6. Conclusion

This paper offers new insights regarding fiscal-monetary interactions in New Keynesian models. First, we demonstrated that in an environment where monetary-fiscal reform is anticipated, attempts to curb inflation can backfire, as expectations of greater inflation upon reform create "sticky inflation." Second, we showed that due to this stickiness, optimal policy should balance inflation and debt objectives, often keeping real interest rates low after fiscal shocks.

This analysis suggests additional policy implications. If a fiscal-monetary reform is inevitable, it is preferable to enact it sooner rather than later. We did not explore the possibility that early anti-inflation efforts signal that monetary policy will resist future inflationary financing. However, without signaling effects, such efforts are futile. Understanding how medium-term inflation expectations respond to signaling, possibly by incorporating policy stance attention as in Bassetto and Miller (2022), is thus essential.

Signaling effects, a missing feature in this paper, may be particularly important to understand the eventual success of the Volker disinflation. Nevertheless, understanding sticky inflation is particularly relevant in today's high-debt environment. Sticky inflation also rationalizes the repeated failures to curb inflation in countries like Argentina, Brazil, and Turkey, where orthodox central bankers often raised real interest rates with limited long-term success. Our theory suggests that temporary measures are unlikely to overcome sticky inflation unless expectations of monetary-fiscal reform dissipate. We hope developed economies heed this lesson.

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A. Derivations

A.1 Derivations for Section 2

Households. The household problem is given by

$$V_t(B_t) = \max_{[C_s, N_s]_{s \geq t}} \mathbb{E}_t \left[\int_t^{t^*} e^{-\rho(s-t)} \left[u(C_s) - h(N_s) \right] ds + e^{-\rho(t^* - t)} V_{t^*}^*(B_t^*) \right], \tag{33}$$

subject to

$$\dot{B}_t = (i_t - \pi_t) B_t + \frac{W_t}{P_t} N_t + D_t + T_t - C_t, \tag{34}$$

and a No-Ponzi condition, where t^* denotes the arrival time for a Poisson process with intensity $\lambda \geq 0$, B_t denotes the real valued of bonds held by households, W_t is the nominal wage, P_t is the price level, D_t are dividends payed by firms, T_t denotes fiscal transfers.

The HJB equation for this problem is given by

$$\rho V = u(C) - h(N) + \dot{V} + V_B \left[(i - \pi) B + \frac{W}{P} N + T - C \right] + \lambda [V^* - V], \tag{35}$$

where $\dot{V}$ denotes the time derivative of the value function conditional on no-switching.

The first-order conditions are given by

$$u'(C) = V_B, \qquad h'(N) = V_B \frac{W}{P}. \tag{36}$$

The envelope condition is given by

$$\rho V_B = V_B(i - \pi) + \dot{V}_B + V_{BB} \left[(i - \pi) B + \frac{W}{P} N + T - C \right] + \lambda [V_B^* - V_B]. \tag{37}$$

Combining the envelope condition with the optimality condition for consumption, we obtain

$$0 = (i - \pi - \rho) + \frac{u''(C) C}{u'(C)} \frac{\dot{C}_t}{C_t} + \lambda \left[\frac{u'(C^*)}{u'(C)} - 1 \right] \Rightarrow \frac{\dot{C}}{C} = \sigma^{-1}(i - \pi - \rho) + \frac{\lambda}{\sigma} \left[\frac{u'(C^*)}{u'(C)} - 1 \right], \tag{38}$$

where $\sigma = -\frac{u''(C)C}{u'(C)}$.

The optimality condition for labor can be written as

$$\frac{h'(N)}{u'(C)} = \frac{W}{P}. \tag{39}$$

Firms. There are two types of firms in the economy: final-goods producers and intermediate-goods producers. Final goods are produced by competitive firms according to the production function $Y_t = \left(\int_0^1 Y_{i,t}^{\frac{\epsilon}{\epsilon-1}} di\right)^{\frac{\epsilon-1}{\epsilon}}$, where $Y_{i,t}$ denotes the output of intermediate $i \in [0,1]$. The demand for intermediate i is given by $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$, where $P_{i,t}$ is the price of intermediate i, $P_t = \left(\int_0^1 P_{i,t}^{1-\epsilon} di\right)^{\frac{1}{1-\epsilon}}$ is the price level, and Y_t is the aggregate output.

Intermediate-goods producers have monopoly over their variety and operate the technology $Y_{i,t} = A_t N_{i,t}$, where $N_{i,t}$ denotes labor input. Firms are subject to quadratic adjustment costs on price changes, so the problem of intermediate i is given by

$$Q_{i,t}(P_i) = \max_{[\pi_{i,s}]_{s\geq t}} \mathbb{E}_t \left[\int_t^{t^*} \frac{\eta_s}{\eta_t} \left(\frac{P_{i,s}}{P_{i,t}} Y_{i,s} - \frac{W_s}{P_s} \frac{Y_{i,s}}{A_s} - \frac{\varphi}{2} \pi_{i,s}^2 \right) ds + \frac{\eta_{t^*}}{\eta_t} Q^*_{i,t}(P^*_{i,t}) \right], \tag{40}$$

subject to $Y_{i,t} = \left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t$ and $\dot{P}_{i,t} = \pi_{i,t} P_{i,t}$, given $P_{i,t} = P_i$ and $\eta_t = e^{-\rho t} u'(C_t)$, where φ is the adjustment cost parameter.

The HJB equation for this problem is

$$0 = \max_{\pi_{i,t}} \eta_t \left(\frac{P_{i,t}}{P_t} Y_{i,t} - \frac{W_t}{P_t} \frac{Y_{i,t}}{A} - \frac{\varphi}{2} \pi_{i,t}^2 \right) dt + \mathbb{E}_t[d(\eta_t Q_{i,t})], \tag{41}$$

where $\frac{\mathbb{E}_t[d(\eta_t Q_{i,t})]}{\eta_t dt} = -(i_t - \pi_t) Q_{i,t} + \frac{\partial Q_{i,t}}{\partial P_{i,t}} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial t} + \lambda \frac{\eta_t^*}{\eta_t} \left[Q^*_{i,t} - Q_{i,t} \right]$.

The first-order condition is given by

$$\frac{\partial Q_{i,t}}{\partial P_i} P_{i,t} = \varphi \pi_{i,t}.$$

The change in π_t conditional on no switching in state is then given by

$$\left(\frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} \right) P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} P_{i,t} = \varphi \dot{\pi}_{i,t}. \tag{42}$$

The envelope condition with respect to $P_{i,t}$ is given by

$$0 = \left((1-\epsilon) \frac{P_{i,t}}{P_t} + \epsilon \frac{W_t}{P_t A} \right) \left(\frac{P_{i,t}}{P_t} \right)^{-\epsilon} \frac{Y_t}{P_{i,t}} + \frac{\partial^2 Q_{i,t}}{\partial t \partial P_i} + \frac{\partial^2 Q_{i,t}}{\partial P_i^2} \pi_{i,t} P_{i,t} + \frac{\partial Q_{i,t}}{\partial P_i} \pi_{i,t} - (i_t - \pi_t) \frac{\partial Q_{i,t}}{\partial P_i} + \lambda \frac{\eta_t^*}{\eta_t} \left(\frac{\partial Q^*_{i,t}}{\partial P_i} - \frac{\partial Q_{i,t}}{\partial P_i} \right). \tag{43}$$

Multiplying the expression above by $P_{i,t}$ and using Equation (42), we obtain

$$0 = \left((1-\epsilon)\frac{P_{i,t}}{P_t} + \epsilon\frac{W_t}{P_t A}\right)\left(\frac{P_{i,t}}{P_t}\right)^{-\epsilon} Y_t + \varphi\dot{\pi}_t - (i_t - \pi_t)\varphi\pi_{i,t} + \lambda\varphi\frac{\eta_t^*}{\eta_t}\left(\pi_{i,t}^* - \pi_{i,t}\right).$$

Rearranging the expression above, we obtain the non-linear New Keynesian Phillips curve

$$\dot{\pi}_t = (i_t - \pi_t)\,\pi_t + \lambda\frac{\eta^*}{\eta_t}\left(\pi_t - \pi_t^*\right) - \frac{\epsilon\varphi^{-1}}{A}\left(\frac{W_t}{P_t} - (1-\epsilon^{-1})A\right)Y_t.$$

Government and market clearing. The government flow budget constraint is given by

$$\dot{B}_t^g = (i_t - \pi_t)B_t^g + T_t, \tag{44}$$

where B_t^g denotes the real value of government debt. The government must also satisfy the No-Ponzi condition $\lim_{T\to\infty}\mathbb{E}_t[\eta_T B_T^g] = 0$.

The market clearing condition is given by

$$C_t = Y_t, \qquad N_t = \int_0^1 N_{i,t}di, \qquad B_t = B_t^g. \tag{45}$$

A.2 Derivations for Section 3

In this section, we revisit our three policy experiments in the context of the more general version of the model, which includes households' expectation effects and a debt-stabilization term. In this case, the dynamic system describing the evolution of output, inflation, and debt is given by

$$\dot{x}_t = r_t - \rho + \lambda_h x_t - \lambda_h(b_t - b^n) \tag{46}$$

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f\kappa\Phi(b_t - b^n) \tag{47}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t. \tag{48}$$

Output gap stabilization. Consider first the case of output-gap stabilization, so $x_t = 0$ for all t. This requires that the interest rate is given by

$$r_t - \rho = \lambda_h(b_t - b^n). \tag{49}$$

The law of motion of debt is then given by

$$\dot{b}_t = -\left(\gamma - \lambda_h\right)\left(b_t - b^n\right) + \psi_t. \tag{50}$$

Solving the differential equation above, we obtain

$$b_t - b^n = e^{-(\gamma-\lambda_h)t}(b_0 - b^n) + \int_0^t e^{-(\gamma-\lambda_h)(t-s)}\psi_s ds. \tag{51}$$

Assuming $\psi_t = e^{-\theta_\psi t}\psi_0$, we obtain

$$b_t - b^n = e^{-(\gamma-\lambda_h)t}(b_0 - b^n) + \frac{e^{-(\gamma-\lambda_h)t} - e^{-\theta_\psi t}}{\theta_\psi - (\gamma - \lambda_h)}\psi_0. \tag{52}$$

Notice that b_t converges back to the steady state if $\gamma > \lambda_h$.

Inflation is given by

$$\pi_t = \lambda_f \kappa \Phi \int_t^\infty e^{-(\rho+\lambda_f)(s-t)}(b_s - b^n)ds. \tag{53}$$

Plugging the value of b_t into the expression above, we obtain

$$\pi_t = \lambda_f \kappa \Phi \left[\frac{e^{-(\gamma-\lambda_h)t}}{\rho + \lambda_f + \gamma - \lambda_h} \left(b_0 - b^n + \frac{\psi_0}{\theta_\psi - (\gamma - \lambda_h)} \right) - \frac{e^{-\theta_\psi t}}{\rho + \lambda_f + \theta_\psi} \frac{\psi_0}{\theta_\psi - (\gamma - \lambda_h)} \right]. \tag{54}$$

Notice that debt and inflation depend on γ and λ_h only through their difference. Hence, if we assume that $\gamma = \lambda_h > 0$, we obtain the same values of b_t and π if we assume $\gamma = \lambda_h = 0$, which corresponds to the case in Section 3. If $\gamma > \lambda_h$, then eventually debt and inflation eventually return to their steady-state levels.

Inflation stabilization. Next, we will consider the case of inflation stabilization. Suppose the real rate is given by $r_t - \rho = e^{-\theta_r t}(r_0 - \rho)$. In this case, debt is given by:

$$b_t - b^n = \underbrace{e^{-\gamma t}(b_0 - b^n) + \frac{e^{-\gamma t} - e^{-\theta_\psi t}}{\theta_\psi - \gamma}\psi_0}_{b_t^p} + \underbrace{\frac{e^{-\gamma t} - e^{-\theta_r t}}{\theta_r - \gamma}(r_0 - \rho)}_{b_t^r}. \tag{55}$$

The first term corresponds to the level of debt if the monetary authority implements a passive policy of setting the real rate equal to its natural level at all periods, $r_t = \rho$, and the second term captures the impact on debt of changing the real rate.

The output gap is given by

$$x_t = -\int_t^\infty e^{-\lambda_h(s-t)}(r_s - \rho)ds + \lambda_h \int_t^\infty e^{-\lambda_h(s-t)}(b_s - b^n)ds. \tag{56}$$

The output gap can be expressed as follows

$$x_t = x_t^p - \frac{r_t - \rho}{\lambda_h + \theta_r} + \frac{\lambda_h}{\theta_r - \gamma}\left[\frac{e^{-\gamma t}}{\lambda_h + \gamma} - \frac{e^{-\theta_r t}}{\lambda_h + \theta_r}\right](r_0 - \rho), \tag{57}$$

where

$$x_t^p = \lambda_h\left[\frac{e^{-\gamma t}}{\lambda_h + \gamma}\left(b_0 - b^n + \frac{\psi_0}{\theta_\psi - \gamma}\right) - \frac{\psi_t}{(\lambda_h + \theta_\psi)(\theta_\psi - \gamma)}\right]. \tag{58}$$

There are now two opposing effects on the output gap. Higher rates reduce the output gap through the usual intertemporal substitution channel. However, higher real rates push debt up, which creates a positive output gap in the inflationary-finance phase. This expectation tends to increase the output gap today. Therefore, the presence of this expectation attenuates the response of output to higher real rates.

Inflation is given by

$$\pi_t = \kappa\int_t^\infty e^{-(\rho+\lambda_f)(s-t)}x_s ds + \lambda_f\kappa\Phi\int_t^\infty e^{-(\rho+\lambda_f)(s-t)}(b_s - b^n)ds. \tag{59}$$

Inflation is given by

$$\pi_t = \pi_t^p + F_t + J_t^x + J_t^b, \tag{60}$$

where $\pi_t^p \equiv \kappa\int_t^\infty e^{-(\rho+\lambda_f)(s-t)}x_s^p ds + \lambda_f\kappa\Phi\int_t^\infty e^{-(\rho+\lambda_f)(s-t)}b_s^p ds$, and

$$F_t \equiv -\frac{\kappa}{\lambda_h + \theta_r}\frac{r_t - \rho}{\rho + \lambda_f + \theta_r} < 0 \tag{61}$$

$$J_t^x \equiv \frac{\kappa\lambda_h}{\theta_r - \gamma}\left[\frac{e^{-\gamma t}}{(\lambda_h + \gamma)(\rho + \lambda_f + \gamma)} - \frac{e^{-\theta_r t}}{(\lambda_h + \theta_r)(\rho + \lambda_f + \theta_r)}\right](r_0 - \rho) > 0 \tag{62}$$

$$J_t^b \equiv \frac{\kappa\lambda_f\Phi}{\theta_r - \gamma}\left[\frac{e^{-\gamma t}}{\rho + \lambda_f + \gamma} - \frac{e^{-\theta_r t}}{\rho + \lambda_f + \theta_r}\right](r_0 - \rho) > 0. \tag{63}$$

The fight-inflation term dominates at period zero if the following condition is satisfied

$$\frac{1}{\rho + \lambda_f + \theta_r} > \frac{\lambda_f\Phi}{\rho + \lambda_f + \gamma}\frac{\lambda_h + \theta_r}{\rho + \lambda_f + \theta_r} + \frac{\lambda_h}{\theta_r - \gamma}\left[\frac{\lambda_h + \theta_r}{(\lambda_h + \gamma)(\rho + \lambda_f + \gamma)} - \frac{1}{\rho + \lambda_f + \theta_r}\right]. \tag{64}$$

We can write the expression above as follows

$$\theta_r < \frac{\rho + \lambda_f + \gamma - \lambda_h\left[\lambda_f\Phi + \frac{\lambda_h - \gamma + \rho + \lambda_f}{\lambda_h + \gamma}\right]}{\lambda_f\Phi + \frac{\lambda_h}{\lambda_h + \gamma}}. \tag{65}$$

Notice that we recover the condition for the fight inflation to dominate at $t = 0$ when $\lambda_h = \gamma = 0$.

If $\theta_r > \gamma$, such that the response of taxes to government debt is not too strong, then the jump inflation term eventually dominates, consistent with the stepping-on a-rake result.

Debt stabilization. We consider next the case where the monetary authority stabilizes government debt, $b_t = 0$. For simplicity, we focus on the case $b^n = 0$. The real interest rate is then given by $r_t - \rho = -\psi_t$. This corresponds to the previous case with $r_0 - \rho = -\psi_0$ and $\theta_r = \theta_\psi$. Given the low real rate, for λ_h sufficiently small, we have a positive output gap and inflation on impact.

B. Proofs

Proof of Proposition 1

Proof. We first show that fiscal policy is passive, that is, for any Lebesgue integrable path for (x_t, π_t, i_t), government debt is bounded if and only if $\gamma \geq 0$. Note that in the fiscal consolidation phase and the inflationary-finance phase, government debt is bounded by construction. In the fiscal-expansion phase, from equation (16) we get

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)}(i_s - \pi_s - \rho + \psi_s) ds.$$

Notice that since $\gamma \geq 0$, $e^{-\gamma t} \leq 1$ for all $t \geq 0$. Then

$$\lim_{t\to\infty} b_t = \lim_{t\to\infty} e^{-\gamma t} b_0 + \lim_{t\to\infty} \int_0^t e^{-\gamma(t-s)}(i_s - \pi_s - \rho + \psi_s) ds \leq \lim_{t\to\infty} b_0 + \lim_{t\to\infty} \int_0^t (i_s - \pi_s - \rho + \psi_s) ds < \infty,$$

where the last inequality follows from (i_t, π_t) being Lebesgue integrable.

For **I.**, notice that the dynamic system is given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} (\rho + \lambda_f) & -\kappa & -\lambda_f \kappa \Phi \\ (\phi - 1) & \lambda_h & -\lambda_h \\ (\phi - 1) & 0 & -(\gamma - \rho) \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t \end{bmatrix} + \begin{bmatrix} \lambda_f \kappa \Phi b^n \\ u_t + \lambda_h b^n \\ u_t + \psi_t \end{bmatrix}.$$

The equilibrium is uniquely determined if the matrix above has two eigenvalues with positive real components and an eigenvalue with a non-positive real component. The eigenvalues of the system above satisfies the characteristic equation:

$$f(\lambda) \equiv \lambda^3 + \underbrace{[\gamma - (\rho + \lambda_f + \lambda_h)]}_{\equiv a} \lambda^2 + \underbrace{[(\phi - 1)\kappa(1 + \lambda_f \Phi) + \lambda_h(\rho + \lambda_f) - \gamma(\rho + \lambda_f + \lambda_h)]}_{\equiv b} \lambda +$$
$$\underbrace{[\gamma(\rho + \lambda_f)\lambda_h + (\phi - 1)\kappa[\gamma - (\lambda_h + \lambda_h \lambda_f \Phi)]]}_{\equiv c} = 0.$$

Using Descartes' rule of signs, we get that $c > 0$ is a necessary condition for determinacy. To see this, suppose $c < 0$. Then, two options exist for the number of sign changes of $f(\lambda)$: one and three. This implies that there can be either 1 or 3 roots with a positive real part. Since we need two roots with positive real part for determinacy, we can rule out those cases.

Next, we show that $c > 0$ is a sufficient condition for determinacy. Because $\gamma < \rho + \lambda_f + \lambda_h$, $a < 0$. Then, we are guaranteed two sign changes. Using the Routh-Hurwitz criterion, not all roots of f are negative, completing the proof.

Part **II.** is immediately true by construction.

Proof of Proposition 2

Proof. From equation (18), given $x_t = 0$ and $\pi_t^J = \kappa\Phi(b_t - b^n)$, inflation is given by

$$\pi_t = \kappa\lambda\Phi \int_t^\infty e^{-(\rho+\lambda)(s-t)}(b_s - b^n)ds. \tag{66}$$

Debt is given by $b_s = b_0 + \frac{1-e^{-\theta_\psi s}}{\theta_\psi}\psi_0 = b_t + \frac{1-e^{-\theta_\psi (s-t)}}{\theta_\psi}\psi_t$. We can then write inflation as follows:

$$\pi_t = \frac{\kappa\lambda\Phi}{\rho + \lambda}\left[b_t - b^n + \frac{\psi_t}{\rho + \lambda + \theta_\psi}\right]. \tag{67}$$

The limit of the expression above as $t \to \infty$ is $\lim_{t\to\infty} \pi_t = \frac{\kappa\lambda\Phi}{\rho+\lambda}(b^{lr} - b^n)$. Differentiating the expression above with respect to time, we obtain

$$\dot{\pi}_t = \frac{\kappa\lambda\Phi}{\rho + \lambda}\left[\psi_t - \frac{\theta_\psi \psi_t}{\rho + \lambda + \theta_\psi}\right] = \frac{\kappa\lambda\Psi}{\rho + \lambda + \theta_\psi}\psi_t > 0. \tag{68}$$

□

Proof of Lemma 1.

Proof. From equation (18), inflation is given by

$$\pi_t = \kappa \int_t^{\infty} e^{-(\rho+\lambda)(s-t)} x_s ds + \lambda\kappa\Phi \int_t^{\infty} e^{-(\rho+\lambda)(s-t)} (b_s - b^n) ds, \tag{69}$$

where $x_t = -\frac{1}{\theta_r}(r_t - \rho)$ and $b_t = b_t^{og} + \frac{1-e^{-\theta_r t}}{\theta_r}(r_0 - \rho)$.

We can then write inflation as follows:

$$\pi_t = \pi_t^{og} \underbrace{-\frac{\kappa(r_t - \rho)}{\theta_r(\rho + \lambda + \theta_r)}}_{F_t} + \underbrace{\frac{\kappa\lambda\Phi}{\theta_r}\left[\frac{1}{\rho+\lambda} - \frac{e^{-\theta_r t}}{\rho + \lambda + \theta_r}\right](r_0 - \rho)}_{J_t}, \tag{70}$$

where $\pi_t^{og} = \kappa\lambda\Phi \int_t^{\infty} e^{-(\rho+\lambda)(s-t)} (b_s^{og} - b^n) ds$.

□

Proof of Proposition 3.

Proof. The fight-inflation strategy is successful at bringing inflation down at $t = 0$ if:

$$-F_0^{\pi} > J_0^{\pi} \iff \frac{\kappa(r_0 - \rho)}{\theta_r(\rho + \lambda + \theta_r)} > \frac{\kappa\lambda\Phi}{\theta_r}\left[\frac{1}{\rho+\lambda} - \frac{1}{\rho+\lambda+\theta_r}\right](r_0 - \rho).$$

We can write the inequality above as follows:

$$1 > \frac{\lambda\Phi}{\rho+\lambda}\theta_r \iff \theta_r < \frac{\rho+\lambda}{\lambda\Phi}. \tag{71}$$

Notice that $\lim_{t\to\infty} F_t^{\pi} = 0$ and $\lim_{t\to\infty} J_t^{\pi} = \frac{\kappa\lambda\Phi}{\theta_r(\rho+\lambda)}(r_0 - \rho) > 0$. Hence, there exits $\hat{T} > 0$ such that for $t > \hat{T}$ the following inequality holds:

$$-F_t^{\pi} < J_t^{\pi}. \tag{72}$$

Hence, $\pi_t > \pi_t^{og}$ for $t > \hat{T}$. □

Proof of Proposition 4

Proof. The Hamiltonian for problem 1 is given by

$$\mathcal{H}_t = -\frac{1}{2}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] + \mu_{\pi,t}\left[(\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n)\right] + \mu_{b,t}\left[r_t - \rho + \psi_t\right]$$
$$+ \mu_{x,t}[r_t - \rho] + (\mu_{x,0} + \xi_x)(\rho+\lambda)x_0 + (\mu_{\pi,0} + \xi_\pi)\left[\kappa x_0 + \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(r_t - \rho)\right], \tag{73}$$

The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda)\mu_{\pi,t} = \beta\pi_t - \mu_{\pi,t}(\rho+\lambda) \tag{74}$$
$$\dot{\mu}_{b,t} - (\rho+\lambda)\mu_{b,t} = \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{75}$$
$$\dot{\mu}_{x,t} - (\rho+\lambda)\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}. \tag{76}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = -\xi, \tag{77}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(\mu_{\pi,0} + \xi_\pi)$.

The optimality condition for the initial output gap:

$$(\rho+\lambda)(\mu_{x,0} + \xi_x) + \kappa(\mu_{\pi,0} + \xi_\pi) = 0. \tag{78}$$

We will choose $\xi_x = -\kappa\frac{\mu_{\pi,0}+\xi_\pi}{\rho+\lambda}$, such that $\mu_{x,0} = 0$. We show below that we can set $\mu_{\pi,0} = 0$ without loss of generality.

The optimality condition for interest rates imply that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha x_t + \lambda\Upsilon(b_t - b^n) = \kappa(1+\lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) + (\rho+\lambda)\xi. \tag{79}$$

Differentiating the expression above with respect to time, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) = -\kappa(1+\lambda\Phi)\beta\pi_t. \tag{80}$$

Rearranging the expression above, we obtain the real interest rate

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t, \tag{81}$$

and the nominal interest rate is given by

$$i_t = \rho + \left[1 - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t. \tag{82}$$

□

Proof of Propositions 5 and 6

Proof. The proof of Proposition 9 derives the solution to the optimal policy problem for arbitrary values of α and β. Here, we specialize the general formulas to the case of doves, $\beta = 0$, and hawks $\alpha = 0$.

Doves. Suppose $\beta = 0$. In this case, initial inflation is given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\frac{\alpha\Phi}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi} + \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{|\underline{\omega}|\psi_0}{(\rho+\lambda+\theta_\psi)(\overline{\omega}+\theta_\psi)} + \lambda\Phi(b_0 - b^n)\right], \tag{83}$$

Using the fact that $b_0 = b^n$ and that $\underline{\omega} = 0$, the expression above simplifies to

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\frac{\alpha\lambda\Phi}{\alpha+\lambda\Upsilon}\frac{\psi_0}{\overline{\omega}+\theta_\psi}. \tag{84}$$

Inflation is then given by

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi - \Upsilon)}{\lambda\Upsilon+\alpha}\frac{1-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})\theta_\psi}\psi_0 + \pi_0. \tag{85}$$

The initial value of the output gap is given by

$$x_0 = \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\lambda+\theta_\psi}. \tag{86}$$

The real rate is given by $r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t$, then output gap is given by

$$x_t = \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\lambda+\theta_\psi} - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}. \tag{87}$$

The government debt is given by

$$b_t = \frac{\alpha}{\lambda\Upsilon + \alpha}\frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{88}$$

Hawks. Suppose $\alpha = 0$. In this case, initial inflation is given by

$$\pi_0 = \frac{\kappa}{\theta_\psi + \overline{\omega}}\frac{\psi_0}{\rho + \lambda + \theta_\psi} > 0. \tag{89}$$

Inflation at t is given by

$$\pi_t = -\kappa\frac{e^{\underline{\omega}t} - e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + e^{\underline{\omega}t}\pi_0. \tag{90}$$

Combining the previous two expressions, we obtain

$$\pi_t = \kappa\frac{\psi_t - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}e^{\underline{\omega}t}\psi_0}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}. \tag{91}$$

Suppose $\theta_\psi > |\underline{\omega}|$, then the numerator is negative for t sufficiently large, and the denominator is positive, so $\lim_{t\to\infty} < 0$. If $\theta_\psi < |\underline{\omega}|$, then the numerator is positive for t sufficiently large, and the denominator is negative, so again $\lim_{t\to\infty} < 0$.

The derivative of inflation with respect to time is given by

$$\dot{\pi}_t = -\kappa\frac{\underline{\omega}e^{\underline{\omega}t} + \theta_\psi e^{-\theta_\psi t}}{(\theta_\psi + \overline{\omega})(\theta_\psi + \underline{\omega})}\psi_0 + e^{\underline{\omega}t}\frac{\kappa}{\theta_\psi + \overline{\omega}}\frac{\underline{\omega}\psi_0}{\rho + \lambda + \theta_\psi} \tag{92}$$

$$= -\frac{\kappa}{\theta_\psi + \overline{\omega}}\left[\frac{\theta_\psi}{\theta_\psi + \underline{\omega}}\psi_t - \frac{|\underline{\omega}|}{\theta_\psi + \underline{\omega}}\frac{\overline{\omega}e^{\underline{\omega}t}\psi_0}{\rho + \lambda + \theta_\psi}\right]. \tag{93}$$

The term in brackets is always positive, so $\dot{\pi}_0 < 0$. Notice that inflation is decreasing at $t = 0$. If $\theta_\psi < |\underline{\omega}|$, so the fiscal shock is very persistent, then inflation is eventually increasing. If $\theta_\psi > |\underline{\omega}$, then inflation is decreasing even for large t.

The initial output gap

$$x_0 = \frac{\overline{\omega}}{\theta_\psi + \overline{\omega}}\left[\frac{1}{\rho + \lambda + \theta_\psi} + \frac{1}{\overline{\omega}}\right]\psi_0. \tag{94}$$

The real interest rate is given by $r_t - \rho = -\frac{\beta}{\lambda\Upsilon}\kappa(1+\lambda\Phi)\pi_t - \psi_t$. Output gap is given by

$$x_t = x_0 - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}p_t - \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{95}$$

The government debt is given by

$$b_t = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}p_t. \tag{96}$$

Debt is initially decreasing, as $\dot{b}_t = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon}\pi_0 < 0$. The price level is given by

$$p_t = \kappa\frac{\frac{\psi_0-\psi_t}{\theta_\psi} - \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}\frac{1-e^{\underline{\omega}t}}{|\underline{\omega}|}\psi_0}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}. \tag{97}$$

Taking the limit as $t \to \infty$, we obtain

$$\lim_{t\to\infty} p_t = \kappa\frac{\frac{1}{\theta_\psi} + \frac{\overline{\omega}}{\rho+\lambda+\theta_\psi}\frac{1}{\underline{\omega}}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})} = \kappa\frac{\rho+\lambda}{(\theta_\psi+\overline{\omega})(\theta_\psi+\rho+\lambda)\theta_\psi}\frac{1}{\underline{\omega}} < 0. \tag{98}$$

Therefore, $\lim_{t\to\infty} b_t > 0$. □

Proof of Proposition 7.

Proof. The planner's objective is given by

$$\mathcal{P}_0(b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\overline{\lambda}\mathcal{P}_t(b_t)dt. \tag{99}$$

The planner's problem consists of maximizing the objective above subject to the constraints

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho.$$

We also include a penalty on π_0 and x_0, as in the case with full commitment.

Optimality conditions The optimality conditions are given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{100}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{101}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}, \tag{102}$$

where $\mathcal{P}_{b,t}(b_t)$ denotes the partial derivative of $\mathcal{P}_t(b_t)$ with respect to debt.

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\xi, \tag{103}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\theta}\xi_\pi$.

The optimality condition for x_0 is given by

$$\mu_{x,0} = 0. \tag{104}$$

Standard envelope arguments imply that

$$\mu_{b,t} = \mathcal{P}_{b,t}(b_t). \tag{105}$$

The discretion limit. Consider the limit as $\overline{\lambda} \to \infty$, so each planner has commitment only over an infinitesimal amount of time. In the limit, the co-states on π_t and x_t are given by

$$\mu_{\pi,t} = 0, \qquad \mu_{x,t} = 0. \tag{106}$$

Integrating the expression for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,t} = -\int_t^\infty e^{-(\rho+\lambda+\overline{\lambda})(s-t)}\left[\alpha x_s + \kappa\mu_{\pi,s}\right] ds \Rightarrow \lim_{\overline{\lambda}\to\infty} \overline{\lambda}\mu_{x,t} = -\alpha x_t, \tag{107}$$

using the fact that $\lim_{\overline{\lambda}\to\infty} \mu_{\pi,t} = 0$. Hence, from the optimality condition for x_0, we obtain $x_0 = 0$. Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mu_{b,t} + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{108}$$

where we used the envelope condition for b_t

Given $\mu_{b,t} = -\xi - \mu_{x,t}$, and combining the previous two expressions, we obtain

$$(\rho + \lambda)\xi = \lambda \Upsilon (b_t - b^n). \tag{109}$$

Therefore, the interest rate is given by

$$r_t - \rho = -\psi_t. \tag{110}$$

□

Proof of Proposition 8.

Proof. The dynamics under the optimal policy are characterized by the conditions:

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda \kappa \Phi (b_t - b^n) \tag{111}$$

$$\dot{b}_t = r_t - \rho - \gamma (b_t - b^n) + \psi_t \tag{112}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^* (b_t - b^n) \tag{113}$$

$$\dot{\mu}_{\pi,t} = \beta \pi_t \tag{114}$$

$$\dot{\mu}_{b,t} = (\rho + \lambda)\mu_{b,t} + \lambda \Upsilon (b_t - b^n) + \kappa \lambda \Phi \mu_{\pi,t} \tag{115}$$

$$\dot{\mu}_{x,t} = (\rho + \lambda)\mu_{x,t} + \alpha x_t + \kappa \mu_{\pi,t}, \tag{116}$$

where the real rate is given by

$$r_t - \rho = -\beta \frac{\kappa(1 + \lambda \Phi)}{\lambda \Upsilon + \alpha} \pi_t - \frac{\lambda \Upsilon}{\lambda \Upsilon + \alpha} \psi_t, \tag{117}$$

given the initial value of debt, b_0, and the boundary conditions $\mu_{x,0} = \mu_{\pi,0} = 0$.

Consider the case without a fiscal shock, $\psi_t = 0$, and denote the co-states in this case with no shocks by $\mu_{x,t}^{ns}$ and $\mu_{\pi,t}^{ns}$. The optimal policy under the timeless perspective corresponds to the solution to the system above when we replace the initial conditions by the long-run values of these multipliers: $\mu_{x,0} = \lim_{t \to \infty} \mu_{x,t}^{ns}$ and $\mu_{\pi,0} = \lim_{t \to \infty} \mu_{\pi,t}^{ns}$ (see Giannoni and Woodford (2017) for a discussion in the context a general model). This is equivalent to the problem of a planner who started its planning in a distant past, so the multipliers had time to converge to their long-run values.

Even without shocks, the limits $\lim_{t \to \infty} \mu_{x,t}^{ns}$ and $\lim_{t \to \infty} \mu_{\pi,t}^{ns}$ will not be equal to zero, provided that $b_0 \neq b^n$. However, in the case $b_0 = b^n$, the solution to the system above in the absence of shocks is simply $\pi_t = x_t = b_t = \mu_{\pi,t} = \mu_{x,t} = \mu_{b,t} = 0$. Hence, we have that $\lim_{t\to\infty} \mu^{ns}_{x,t} = 0$ and $\lim_{t\to\infty} \mu^{ns}_{\pi,t} = 0$, so the boundary conditions for the problem under the timeless perspective coincide with the time-zero commitment solution. □

Proof of Proposition 9.

Proof. The matrix of eigenvectors and its inverse are given by

$$V = \begin{bmatrix} \frac{\kappa(1+\lambda\Phi)}{\underline{\omega}} & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & 1 \end{bmatrix}, \qquad V^{-1} = \frac{\overline{\omega}|\underline{\omega}|}{(\overline{\omega}-\underline{\omega})\kappa(1+\lambda\Phi)} \begin{bmatrix} -1 & \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \\ 1 & \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \end{bmatrix}. \tag{118}$$

Let $Z_t = [\pi_t, b_t]'$ denote the vector of endogenous variables, A the matrix of coefficients, and U_t the vector of coefficients. We can then write the dynamic system as $\dot{Z}_t = AZ_t + U_t$. We can write the matrix of coefficients as $A = V\Lambda V^{-1}$, where Λ is a diagonal matrix with the eigenvalues. Using the matrix eigendecomposition, we can decouple the system using the transformation: $z_t \equiv V^{-1}Z_t$ and $u_t \equiv V^{-1}U_t$. This gives us the system of decoupled differential equations:

$$\dot{z}_{1,t} = \overline{\omega} z_{1,t} + u_{1,t}, \qquad \dot{z}_{2,t} = \underline{\omega} z_{2,t} + u_{2,t}. \tag{119}$$

Integrating the first equation forward and the second backwards, we obtain

$$z_{1,t} = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds, \qquad z_{2,t} = e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{120}$$

Rotating the system back to its original coordinates, we obtain

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{|\underline{\omega}|} \int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}} \left[e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds \right], \tag{121}$$

and

$$b_t - b^n = -\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds + e^{\underline{\omega}t} z_{2,0} + \int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds. \tag{122}$$

The disturbances $u_{1,t}$ and $u_{2,t}$ are given by

$$u_{1,t} = \frac{|\underline{\omega}|}{\overline{\omega} - \underline{\omega}} \left[\frac{\alpha}{\lambda \Upsilon + \alpha} \psi_t - \frac{\overline{\omega}}{(1 + \lambda \Phi)} (\hat{\psi}_t + b_0 - b^n - x_0) \right] \tag{123}$$

$$u_{2,t} = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}} \left[\frac{\alpha}{\lambda \Upsilon + \alpha} \psi_t + \frac{|\underline{\omega}|}{(1 + \lambda \Phi)} (\hat{\psi}_t + b_0 - b^n - x_0) \right], \tag{124}$$

where $\hat{\psi}_t = \frac{1 - e^{-\theta_\psi t}}{\theta_\psi} \psi_0$ if $\theta_\psi > 0$ and $\hat{\psi}_t = \psi_0 t$ if $\theta_\psi = 0$.

The forward integral of $u_{1,t}$ is given by

$$\int_t^\infty e^{-\overline{\omega}(s-t)} u_{1,s} ds = \frac{|\underline{\omega}|}{\overline{\omega} - \underline{\omega}} \left[\left(\frac{\alpha}{\lambda \Upsilon + \alpha} - \frac{\overline{\omega}}{(1 + \lambda \Phi)} \frac{1}{\theta_\psi} \right) \frac{\psi_t}{\overline{\omega} + \theta_\psi} - \frac{\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0}{1 + \lambda \Phi} \right]. \tag{125}$$

The backward integral of $u_{2,t}$ is given by

$$\int_0^t e^{\underline{\omega}(t-s)} u_{2,s} ds = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}} \left[\left(\frac{\alpha}{\lambda \Upsilon + \alpha} - \frac{|\underline{\omega}|}{(1 + \lambda \Phi)} \frac{1}{\theta_\psi} \right) \frac{e^{\underline{\omega} t} - e^{-\theta_\psi t}}{\theta_\psi + \underline{\omega}} \psi_0 + \frac{|\underline{\omega}|}{(1 + \lambda \Phi)} (\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0) \frac{1 - e^{\underline{\omega} t}}{|\underline{\omega}|} \right] \tag{126}$$

From the expression for $z_{1,0}$, we obtain

$$\begin{aligned} \pi_0 &= \frac{\kappa(1 + \lambda \Phi)}{\overline{\omega}} \left[(b_0 - b^n) + \frac{\overline{\omega} - \underline{\omega}}{|\underline{\omega}|} \int_0^\infty e^{-\overline{\omega} t} u_{1,t} dt \right] \\ &= \frac{\kappa(1 + \lambda \Phi)}{\overline{\omega}} \left[(b_0 - b^n) + \left(\frac{\alpha}{\lambda \Upsilon + \alpha} + \frac{\overline{\omega}}{(1 + \lambda \Phi)} \frac{1}{\theta_\psi} \right) \frac{\psi_0}{\overline{\omega} + \theta_\psi} - \frac{\frac{\psi_0}{\theta_\psi} + b_0 - b^n - x_0}{1 + \lambda \Phi} \right] \end{aligned} \tag{127}$$

We can then write initial inflation as follows:

$$\pi_0 = \frac{\kappa}{\overline{\omega}} \left[\lambda \Phi (b_0 - b^n) + x_0 + \frac{\alpha \Phi - \Upsilon}{\alpha + \lambda \Upsilon} \frac{\lambda \psi_0}{\overline{\omega} + \theta_\psi} \right].$$

The initial value for $z_{2,t}$ is given by

$$z_{2,0} = \frac{\overline{\omega}}{\overline{\omega} - \underline{\omega}} \left[\frac{|\underline{\omega}|}{\kappa(1 + \lambda \Phi)} \pi_0 + b_0 - b^n \right].$$

Inflation is then given by

$$\pi_t = \frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\left(\frac{\alpha}{\lambda\Upsilon+\alpha}+\frac{\overline{\omega}}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{\psi_t}{\overline{\omega}+\theta_\psi}-\frac{\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0}{1+\lambda\Phi}\right] \tag{128}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[e^{\underline{\omega}t}\left[\frac{|\underline{\omega}|}{\kappa(1+\lambda\Phi)}\pi_0+b_0-b^n\right]+\left(\frac{\alpha}{\lambda\Upsilon+\alpha}-\frac{|\underline{\omega}|}{(1+\lambda\Phi)}\frac{1}{\theta_\psi}\right)\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{\theta_\psi+\underline{\omega}}\psi_0\right] \tag{129}$$

$$+\frac{\kappa(1+\lambda\Phi)}{\overline{\omega}-\underline{\omega}}\left[\frac{1-e^{\underline{\omega}t}}{(1+\lambda\Phi)}\left(\frac{\psi_0}{\theta_\psi}+b_0-b^n-x_0\right)\right]. \tag{130}$$

After some rearrangement, we obtain

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}\psi_0+e^{\underline{\omega}t}\pi_0. \tag{131}$$

Boundary conditions. The optimality condition for x_0 involves the co-states for x and π. Solving the equation for $\mu_{\pi,t}$ backward, we obtain

$$\mu_{\pi,t} = \mu_{\pi,0}+\beta\int_0^t \pi_s ds. \tag{132}$$

Solving the equation for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,0} = -\int_0^\infty e^{-(\rho+\theta)t}\left[\kappa\mu_{\pi,t}+\alpha x_t\right]dt \tag{133}$$

$$= -\frac{\kappa}{\rho+\theta}\mu_{\pi,0}-\frac{\kappa\beta}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt-\int_0^\infty e^{-(\rho+\theta)t}\alpha x_t dt. \tag{134}$$

The optimality condition for x_0 is given by

$$0 = \mu_{x,0}+\frac{\kappa}{\rho+\theta}\mu_{\pi,0} = -\int_0^\infty e^{-(\rho+\theta)t}\left[\frac{\beta}{\rho+\theta}\pi_t+\alpha x_t\right]. \tag{135}$$

Using the fact that $x_t = x_0+\hat{r}_t$, we obtain

$$\int_0^\infty e^{-(\rho+\theta)t}x_t dt = \frac{x_0}{\rho+\theta}+\frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}(r_t-\rho)dt. \tag{136}$$

The optimality condition for x_0 can then be written as

$$0 = \frac{\alpha}{\rho+\theta}x_0 + \frac{1}{\rho+\theta}\int_0^\infty e^{-(\rho+\theta)t}\left[\kappa\beta\pi_t + \alpha\left(-\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t\right)\right]dt. \tag{137}$$

Rearranging the expression above, we obtain

$$\alpha x_0 = \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\int_0^\infty e^{-(\rho+\theta)t}\psi_t dt. \tag{138}$$

The present discounted value of inflation is given by

$$\int_0^\infty e^{-(\rho+\theta)t}\pi_t dt = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{(\lambda\Upsilon+\alpha)(\theta_\psi+\overline{\omega})}\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \frac{\pi_0}{\rho+\theta+|\underline{\omega}|}. \tag{139}$$

Combining the previous two equations, we obtain

$$\alpha x_0 = \frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\pi_0}{\rho+\theta+|\underline{\omega}|} \tag{140}$$

$$+ \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi}. \tag{141}$$

Using the fact that $\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\lambda\Phi(b_0-b^n) + x_0 + \frac{\alpha\Phi-\Upsilon}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi}\right]$, we obtain

$$x_0 = \frac{\frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\underline{\omega}|)}\left[\frac{1}{\rho+\theta+\theta_\psi}+\frac{1}{\overline{\omega}}\right] + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi} + \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\kappa\lambda\Phi(b_0-b^n)}{\overline{\omega}(\rho+\theta+|\underline{\omega}|)}}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho+\theta+|\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}}. \tag{142}$$

Initial inflation is then given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\frac{\frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\underline{\omega}|)}\frac{1}{\rho+\theta+\theta_\psi} + \frac{\alpha^2\Phi}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi} + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{|\underline{\omega}|\psi_0}{(\rho+\theta+\theta_\psi)(\overline{\omega}+\theta_\psi)} + \alpha\lambda\Phi(b_0-b^n)}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho+\theta+|\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}}\right]. \tag{143}$$

Notice that the numerator is positive. The denominator is positive for α large or β large. In these cases, a fiscal shock leads to more inflation and higher output gap.

Output gap. The output gap is given by

$$x_t = x_0 - \beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha} p_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}, \tag{144}$$

where $p_t = \int_0^t \pi_s ds$.

Government debt. Government debt is given by

$$b_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon + \alpha} p_t + \frac{\alpha}{\lambda\Upsilon + \alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{145}$$

□

□

C. Optimal policy

C.1 The planner's problem

Planner's problem. We can write the planner's problem as follows:

$$\max_{\{[x_t,\pi_t,b_t,r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2+\beta\pi_t^2+\lambda\Upsilon(b_t-b^n)^2\right]dt, \tag{146}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t-b^n) \tag{147}$$

$$\dot{b}_t = r_t - \rho + \psi_t \tag{148}$$

$$\dot{x}_t = r_t - \rho, \tag{149}$$

given b_0 and the initial value for inflation.

The lack of a classical solution. It turns out that a classical solution, where the states are continuous functions of time, does not exist. The issue of non-existence of a solution can be seen more clearly in the case $\beta = 0$, where inflation drops out of the problem. For simplicity, assume that $b_0 = b_n = 0$. The optimality condition for r_t is given by

$$\alpha x_t + \lambda\Upsilon b_t = 0, \tag{150}$$

for all $t \geq 0$. The optimality condition for x_0 is given by

$$\mu_{x,0} = 0 \iff -\alpha\int_0^\infty e^{-(\rho+\lambda)t}x_t dt = 0. \tag{151}$$

Let (x_t^*, b_t^*) denote a candidate solution, where b_t^* is a differentiable function of time satisfying $b_0^* = 0$. Differentiating the optimality condition for r_t with respect to time, we obtain

$$r_t - \rho = -\frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\psi_t \Rightarrow \hat{r}_t = -\frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\hat{\psi}_t. \tag{152}$$

As $x_t = x_0 + \hat{r}_t$, the optimality condition for x_0 implies that the following condition must hold:

$$\frac{x_0}{\rho+\lambda} + \int_0^\infty e^{-(\rho+\lambda)t}\hat{r}_t dt = 0 \Rightarrow x_0 = \frac{\lambda\Upsilon}{\alpha+\lambda\Upsilon}\int_0^\infty e^{-(\rho+\lambda)t}\psi_t dt > 0. \tag{153}$$

However, from the optimality condition for the interest rate at $t = 0$, we obtain:[31]

$$\alpha x_0 + \lambda \Upsilon b_0 = 0 \Rightarrow x_0 = 0, \tag{154}$$

which contradicts the fact that $x_0 > 0$.

Incentive for expropriation. While a classical solution to this problem does not exist, a generalized solution with discontinuous states exists. In a classical solution, b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds \tag{155}$$

The integral above is equal to zero at $t = 0$, so $b_0 = 0$. Following the approach in optimal impulsive control, consider the following generalization:[32]

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \int_{[0,t]} \overline{r}_s d\mu, \tag{156}$$

where μ denotes a Borel measure on $\mathbb{R}_+$. For example, if μ is a Dirac measure with weight on zero, then b_t is given by

$$b_t = \int_0^t (r_s - \rho + \psi_s) ds + \overline{r}_0. \tag{157}$$

In this case, government debt can immediately jump at zero, provided $\overline{r}_0 \neq 0$.

Define $\hat{r}_t \equiv \int_0^\infty (r_s - \rho) ds + \int_{[0,t]} \overline{r}_s d\mu$, so $x_t = x_0 + \hat{r}_t$ and $b_t = \hat{r}_t + \hat{\psi}_t$. In a classical solution, $\hat{r}_t$ must be an absolutely continuous function satisfying $\hat{r}_0 = 0$, while it is a bounded variation function in the context of optimal impulsive control, where $\hat{r}_0$ can take any value. Without the constraint that $\hat{r}_0 = 0$, the planner's problem becomes particularly simple:

$$\max_{\{x_0, [\hat{r}_t]_0^\infty\}} -\frac{1}{2} \int_0^\infty e^{-(\rho+\lambda)t} \left[\alpha \left(x_0 + \hat{r}_t \right)^2 + \lambda \Upsilon \left(\hat{r}_t + \hat{\psi}_t \right)^2 \right] dt, \tag{158}$$

with optimality conditions

$$\alpha x_t + \lambda \Upsilon b_t = 0, \qquad -\alpha \int_0^\infty e^{-(\rho+\lambda)t} x_t dt = 0. \tag{159}$$

[31] Notice that the optimality condition for the interest rate must hold at $t = 0$. From continuity of x_t and b_t, if $\alpha x_t + \lambda \Upsilon b_t > 0$ for $t = 0$, there exists $t_1 > 0$ such that this inequality holds for $t \in [0, t_1)$. By reducing interest rates in this interval, we can improve the planner's objective.

[32] See Arutyunov et al. (2019) for a discussion of optimal impulsive control theory.

The solution in this case takes the form:

$$r_t - \rho = -\frac{\lambda\Upsilon}{\alpha + \lambda\Upsilon}\psi_t, \qquad x_0 = \frac{\lambda\Upsilon}{\alpha + \lambda\Upsilon}\int_0^\infty e^{-(\rho+\lambda)t}\psi_t dt, \qquad b_0 = -\frac{\alpha}{\lambda\Upsilon}x_0. \tag{160}$$

Hence, government debt jumps immediately down on impact, which requires $\overline{r}_0 = -\frac{\alpha}{\lambda\Upsilon}x_0$ and μ to be a Dirac measure with weight in zero. Intuitively, the planner has an incentive to expropriate part of the debt by having the real interest rate be very negative over a small period (the impulse from the Dirac measure).

C.2 Characterization of the optimal policy

The penalized planner's problem. To deal with the incentive to expropriate, we introduce a penalty associated with the initial value of each forward-looking variable:

$$\max_{\{[\pi_t, b_t, x_t, r_t]_0^\infty\}} -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda)t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \xi_x x_0 + \xi_\pi \pi_0, \tag{161}$$

subject to

$$\dot{\pi}_t = (\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho, \tag{162}$$

given b_0 and the initial value for inflation. We will choose the penalty ξ_x and ξ_π such that there is no discontinuity in b_t at $t = 0$, and the co-state for the output gap is equal to zero at $t = 0$.

Optimality conditions. The Hamiltonian to this problem is given by

$$\begin{aligned}\mathcal{H}_t = &-\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right] + \mu_{\pi,t}\left[(\rho+\lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n)\right] + \mu_{b,t}\left[r_t - \rho + \psi_t\right] \\ &+ \mu_{x,t}[r_t - \rho] + (\mu_{x,0} + \xi_x)(\rho+\lambda)x_0 + (\mu_{\pi,0} + \xi_\pi)\left[\kappa x_0 + \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(r_t - \rho)\right],\end{aligned} \tag{163}$$

The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\lambda)\mu_{\pi,t} = \beta\pi_t - \mu_{\pi,t}(\rho+\lambda) \tag{164}$$

$$\dot{\mu}_{b,t} - (\rho+\lambda)\mu_{b,t} = \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{165}$$

$$\dot{\mu}_{x,t} - (\rho+\lambda)\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}. \tag{166}$$

The optimality condition for the real interest rate is

$$\mu_{b,t} + \mu_{x,t} = -\xi, \tag{167}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\lambda}(\mu_{\pi,0} + \xi_\pi)$.

The optimality condition for the initial output gap:

$$(\rho+\lambda)(\mu_{x,0} + \xi_x) + \kappa(\mu_{\pi,0} + \xi_\pi) = 0. \tag{168}$$

We will choose $\xi_x = -\kappa\frac{\mu_{\pi,0}+\xi_\pi}{\rho+\lambda}$, such that $\mu_{x,0} = 0$. We show below that we can set $\mu_{\pi,0} = 0$ without loss of generality.

Real and nominal rates. The optimality condition for interest rates imply that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha x_t + \lambda\Upsilon(b_t - b^n) = \kappa(1+\lambda\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) + (\rho+\lambda)\xi. \tag{169}$$

Differentiating the expression above with respect to time, we obtain

$$\alpha(r_t - \rho) + \lambda\Upsilon(r_t - \rho + \psi_t) = -\kappa(1+\lambda\Phi)\beta\pi_t. \tag{170}$$

Rearranging the expression above, we obtain the real interest rate

$$r_t - \rho = -\beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t, \tag{171}$$

and the nominal interest rate is given by

$$i_t = \rho + \left[1 - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}\right]\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\psi_t. \tag{172}$$

Dynamics under the optimal policy. Using the expression for $x_t = x_0 + b_t - b_0 - \hat{\psi}_t$, we can write a dynamic system for π_t and b_t

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho+\lambda & -\kappa(1+\lambda\Phi) \\ -\hat{\beta} & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ b_t - b^n \end{bmatrix} + \begin{bmatrix} \kappa(\hat{\psi}_t + b_0 - b^n - x_0) \\ \frac{\alpha}{\lambda\Upsilon+\alpha}\psi_t \end{bmatrix}, \tag{173}$$

where $\hat{\beta} \equiv \frac{\beta\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}$ and $\hat{\psi}_t = \frac{1-e^{-\theta_\psi t}}{\theta_\psi}\psi_0$. As b_0 is given and π_0 can jump, there is a unique bounded solution to the system above if the system has a positive eigenvalue and a negative eigenvalue. The eigenvalues of the system satisfy the condition

$$(\rho+\lambda-\omega)(-\omega) - \hat{\beta}\kappa(1+\lambda\Phi) = 0 \Rightarrow \omega^2 - [\rho+\lambda]\,\omega - \kappa(1+\lambda\Phi)\hat{\beta} = 0.$$

Denote the eigenvalues of the system by $\overline{\omega} > 0$ and $\underline{\omega} < 0$, where

$$\overline{\omega} = \frac{\rho+\lambda+\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}, \qquad \underline{\omega} = \frac{\rho+\lambda-\sqrt{(\rho+\lambda)^2+4\kappa(1+\lambda\Phi)\hat{\beta}}}{2}. \tag{174}$$

We provide next a characterization of inflation and output gap under the optimal policy.

Proposition 9 (Optimal policy: general case)**.** *Suppose the planner implements the optimal policy given welfare weights $\alpha \geq 0$ and $\beta \geq 0$. Then,*

1. *Inflation is given by*

$$\pi_t = \frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{e^{\underline{\omega}t}-e^{-\theta_\psi t}}{(\theta_\psi+\overline{\omega})(\theta_\psi+\underline{\omega})}\psi_0 + e^{\underline{\omega}t}\pi_0, \tag{175}$$

where initial inflation is given by

$$\pi_0 = \frac{\kappa}{\overline{\omega}}\left[\frac{\frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\underline{\omega}|)}\frac{1}{\rho+\theta+\theta_\psi} + \frac{\alpha^2\Phi}{\alpha+\lambda\Upsilon}\frac{\lambda\psi_0}{\overline{\omega}+\theta_\psi} + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{|\underline{\omega}|\psi_0}{(\rho+\theta+\theta_\psi)(\overline{\omega}+\theta_\psi)} + \alpha\lambda\Phi(b_0-b^n)}{\alpha - \frac{\kappa\beta}{\overline{\omega}(\rho+\theta+|\underline{\omega}|)}\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}}\right]. \tag{176}$$

2. *Output gap is given by*

$$x_t = x_0 - \beta\frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha}p_t - \frac{\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0-\psi_t}{\theta_\psi}, \tag{177}$$

where $p_t = \int_0^t \pi_s ds$, and the initial output gap is given by

$$\alpha x_0 = \frac{\beta}{\theta_\psi+\overline{\omega}}\left(\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\right)^2\frac{\psi_0}{(\rho+\theta+|\lambda|)(\rho+\theta+\theta_\psi)} + \beta\frac{\kappa\lambda(\alpha\Phi-\Upsilon)}{\lambda\Upsilon+\alpha}\frac{\pi_0}{\rho+\theta+|\underline{\omega}|} + \frac{\alpha\lambda\Upsilon}{\lambda\Upsilon+\alpha}\frac{\psi_0}{\rho+\theta+\theta_\psi}. \tag{178}$$

3. *The government debt is given by*

$$b_t = -\beta \frac{\kappa(1+\lambda\Phi)}{\lambda\Upsilon+\alpha} p_t + \frac{\alpha}{\lambda\Upsilon+\alpha} \frac{\psi_0 - \psi_t}{\theta_\psi}. \tag{179}$$

D. Optimal policy with expectation effects and cost-push shocks

D.1 Implementability

Suppose that $\theta_h = 0$, so the equilibrium dynamics is described by the dynamic system:

$$\dot{\pi}_t = (\rho+\theta)\pi_t - \kappa x_t - \kappa\Phi\theta^* b_t - v_t, \qquad \dot{x}_t = r_t - \rho, \qquad \dot{b}_t = r_t - \rho - \gamma b_t + \psi_t, \tag{180}$$

for a given path of real rates, the initial condition for the output gap x_0, the evolution of the fiscal shock ψ_t, and a cost-push shock v_t.

Proposition 10 (Implementability). *Given a path of real rates $[r_t]_0^\infty$ and an initial condition for the output gap, x_0, and for government debt, b_0, then initial inflation is given by*

$$\pi_0 = \kappa \left[\frac{x_0}{\rho+\theta} + \frac{\theta^*\Phi b_0}{\rho+\theta+\gamma} + \int_0^\infty e^{-(\rho+\theta)t} \left(\frac{r_t-\rho}{\rho+\theta} + \frac{\theta^*\Phi}{\rho+\theta+\gamma}(r_t - \rho + \psi_t) + v_t \right) dt \right]. \tag{181}$$

Proof. Integrating the law of motion of debt, we obtain

$$b_t = e^{-\gamma t} b_0 + \hat{r}_{\gamma,t} + \hat{\psi}_{\gamma,t}, \tag{182}$$

where $\hat{r}_{\gamma,t} \equiv \int_0^t e^{-\gamma(t-s)}(r_s - \rho)ds$ and $\hat{\psi}_{\gamma,t} \equiv \int_0^t e^{-\gamma(t-s)}\psi_s ds$.

Output gap is given by

$$x_t = x_0 + \hat{r}_t, \tag{183}$$

where $\hat{r}_t \equiv \hat{r}_{0,t}$. Initial inflation is given by

$$\pi_0 = \kappa \left[\frac{x_0}{\rho+\theta} + \frac{\theta^*\Phi b_0}{\rho+\theta+\gamma} + \int_0^\infty e^{-(\rho+\theta)t} \left(\hat{r}_t + \theta^*\Phi(\hat{r}_{\gamma,t} + \hat{\psi}_{\gamma,t}) + v_t \right) dt \right]. \tag{184}$$

Applying integration by parts, we obtain the expression for initial inflation. □

D.2 Optimal policy

The optimal policy problem is given by

$$\max_{[\pi_t, b_t, x_t, r_t]_0^\infty} -\frac{1}{2}\int_0^\infty e^{-(\rho+\theta)t}\left[\alpha(x_t - x^*)^2 + \beta\pi_t^2 + \theta^*\Upsilon(b_t - b^n)^2\right]dt + \xi_x x_0 + \xi_\pi \pi_0. \tag{185}$$

subject to

$$\dot{\pi}_t = (\rho+\theta)\pi_t - \kappa x_t - \kappa\Phi\theta^*(b_t - b^n) - v_t, \qquad \dot{x}_t = r_t - \rho, \qquad \dot{b}_t = r_t - \rho - \gamma b_t + \psi_t, \tag{186}$$

given the initial condition for inflation, where ξ_x and ξ_π denote the penalty on the initial value of output and inflation.

The Hamiltonian for this problem is given by

$$\mathcal{H}_t = -\frac{1}{2}\left[\alpha(x_t - x^*)^2 + \beta\pi_t^2 + \theta^*\Upsilon(b_t - b^n)^2\right] + (\rho+\theta)\xi_x x_0 + \mu_{x,t}\left[r_t - \rho\right] + \mu_{x,0}(\rho+\theta)x_0 + \mu_{b,t}\left[r_t - \rho - \gamma b_t + \psi\right.$$
$$+ \mu_{\pi,t}\left[(\rho+\theta)\pi_t - \kappa x_t - \theta^*\kappa\Phi(b_t - b^n) - v_t\right] + (\mu_{\pi,0} + \xi_\pi)\,\kappa\left[x_0 + \left(1 + \frac{(\rho+\theta)\theta^*\Phi}{\rho+\theta+\gamma}\right)\frac{r_t - \rho}{\rho+\theta}\right]. \tag{187}$$

Optimality conditions. The dynamics of the co-states are given by

$$\dot{\mu}_{\pi,t} - (\rho+\theta)\mu_{\pi,t} = \beta\pi_t - (\rho+\theta)\mu_{\pi,t} \tag{188}$$
$$\dot{\mu}_{b,t} - (\rho+\theta)\mu_{b,t} = \theta^*\Upsilon(b_t - b^n) + \theta^*\kappa\Phi\mu_{\pi,t} + \gamma\mu_{b,t} \tag{189}$$
$$\dot{\mu}_{x,t} - (\rho+\theta)\mu_{x,t} = \alpha(x_t - x^*) + \kappa\mu_{\pi,t}. \tag{190}$$

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\frac{\kappa\left[\mu_{\pi,0} + \xi_\pi\right]}{\rho+\theta}\left(1 + \frac{(\rho+\theta)\theta^*\Phi}{\rho+\theta+\gamma}\right). \tag{191}$$

The optimality condition for the initial value of the output gap is given by

$$(\rho+\theta)(\mu_{x,0} + \xi_x) + \kappa(\mu_{\pi,0} + \xi_\pi) = 0. \tag{192}$$

Real interest rates. The next proposition gives the real interest rate.

Proposition 11 (Real interest rate)**.** *The real interest rate is given by*

$$r_t - \rho = -\beta\frac{\kappa(1+\theta^*\Phi)}{\theta^*\Upsilon+\alpha}\pi_t - \frac{\theta^*\Upsilon}{\theta^*\Upsilon+\alpha}\psi_t - \frac{\gamma}{\theta^*\Upsilon+\alpha}\left[(\rho+\theta+\gamma)\mu_{b,t} + \theta^*\kappa\Phi\mu_{\pi,t} - \theta^*\Upsilon b^n\right]. \quad (193)$$

Proof. The optimality condition for the interest rate implies that $\dot{\mu}_{b,t} + \dot{\mu}_{x,t} = 0$. From the law of motion of the co-states, we obtain

$$\alpha(x_t - x^*) + \theta^*\Upsilon(b_t - b^n) = -\kappa(1+\theta^*\Phi)\mu_{\pi,t} - \gamma\mu_{b,t} + \kappa\left(1 + \frac{(\rho+\theta)\theta^*\Phi}{\rho+\theta+\gamma}\right)(\mu_{\pi,0} + \xi_\pi). \quad (194)$$

Rearranging the expression above, we obtain

$$\alpha(x_t - x^*) + \theta^*\Upsilon(b_t - b^n) = \kappa(1+\theta^*\Phi)\left(\mu_{\pi,0} - \mu_{\pi,t}\right) - \gamma\left[\mu_{b,t} + \frac{\kappa\theta^*\Phi}{\rho+\theta+\gamma}\mu_{\pi,0}\right] + \kappa\left(1 + \frac{(\rho+\theta)\theta^*\Phi}{\rho+\theta+\gamma}\right)\xi_\pi. \quad (195)$$

Differentiating the expression above, we obtain

$$\alpha(r_t - \rho) + \theta^*\Upsilon(r_t - \rho + \psi_t - \gamma b_t) = -\beta\kappa(1+\theta^*\Phi)\pi_t - \gamma\dot{\mu}_{b,t}. \quad (196)$$

Rearranging the expression above, and using the dynamics for $\mu_{b,t}$, we obtain the real interest rate. □

Dynamic system. Equilibrium dynamics under the optimal policy satisfies the dynamic system:

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \\ \dot{\mu}_{b,t} \\ \dot{\mu}_{\pi,t} \end{bmatrix} = \begin{bmatrix} \rho+\theta & -\kappa & -\kappa\Phi\theta^* & 0 & 0 \\ -\hat{\beta} & 0 & 0 & -\frac{\gamma(\rho+\theta+\gamma)}{\theta^*\Upsilon+\alpha} & -\frac{\gamma\kappa\theta^*\Phi}{\theta^*\Upsilon+\alpha} \\ -\hat{\beta} & 0 & -\gamma & -\frac{\gamma(\rho+\theta+\gamma)}{\theta^*\Upsilon+\alpha} & -\frac{\gamma\kappa\theta^*\Phi}{\theta^*\Upsilon+\alpha} \\ 0 & 0 & \theta^*\Upsilon & \rho+\theta+\gamma & \kappa\theta^*\Phi \\ \beta & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t \\ \mu_{b,t} \\ \mu_{\pi,t} \end{bmatrix} + \begin{bmatrix} 0.0 \\ -\frac{\theta^*\Upsilon}{\theta^*\Upsilon+\alpha} \\ \frac{\alpha}{\theta^*\Upsilon+\alpha} \\ 0.0 \\ 0.0 \end{bmatrix}\psi_t + \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} v_t + \begin{bmatrix} \kappa\theta^*\Phi \\ \frac{\gamma\theta^*\Upsilon}{\theta^*\Upsilon+\alpha} \\ \frac{\gamma\theta^*\Upsilon}{\theta^*\Upsilon+\alpha} \\ -\theta^*\Upsilon \\ 0 \end{bmatrix} b^n, \quad (197)$$

where $\hat{\beta} \equiv \beta\frac{\kappa(1+\theta^*\Phi)}{\theta^*\Upsilon+\alpha}$, given the boundary conditions:

$$\mu_{x,0} + \xi_x = -\frac{\kappa}{\rho+\theta}(\mu_{\pi,0} + \xi_\pi), \qquad \mu_{b,0} - \xi_x = -\frac{\kappa\theta^*\Phi}{\rho+\theta+\gamma}(\mu_{\pi,0} + \xi_\pi). \quad (198)$$

Proposition 12 (Dynamic system). *Let V and Λ denote the matrix of eigenvectors and a diagonal matrix with the eigenvalues of the dynamic system (197), respectively, and denote the vector of endogenous variables by $Z_t = [\pi_t, x_t, b_t, \mu_{b,t}, \mu_{\pi,t}]'$. Assume that V is diagonalizable with real eigenvalues. Then, Z_t is given by*

$$Z_t = V_1 z_{1,t} + V_2 z_{2,t}, \tag{199}$$

where $V = [V_1\ V_2]$, $\Lambda = diag(\Lambda_1, \Lambda_2)$, Λ_1 is a diagonal matrix with positive eigenvalues, Λ_2 is a diagonal matrix with non-positive eigenvalues, and

$$z_{1,t} = -\int_t^\infty \exp\left(-\Lambda_1(s-t)\right)\left[u_1^\psi \psi_s + u_1^v v_s + u_1^n b^n\right] ds, \tag{200}$$

and

$$z_{2,t} = \exp\left(\Lambda_2 t\right) z_{2,0} + \int_0^t \exp\left(\Lambda_2(t-s)\right)\left[u_2^\psi \psi_s + u_2^v v_s + u_2^n b^n\right] ds. \tag{201}$$

Proof. Let $Z_t = [\pi_t, x_t, b_t, \mu_{b,t}, \mu_{\pi,t}]'$, so we can write the system above in matrix form:

$$\dot{Z}_t = AZ_t + U^\psi \psi_t + U^v v_t + U^n b^n. \tag{202}$$

Assuming the matrix A is diagonalizable, we can write the eigendecomposition $A = V\Lambda V^{-1}$ and obtain a decoupled system under new coordinates:

$$\dot{z}_t = \Lambda z_t + u^\psi \psi_t + u^v v_t + u^n b^n, \tag{203}$$

where $z_t = V^{-1} Z_t$ and $u^j = V^{-1} U^j$, for $j \in \{\psi, v, n\}$. Let $z_t = [z_{1,t}', z_{2,t}']'$, where $z_{1,t}$ is associated with the positive eigenvalues, and $z_{2,t}$ is associated with the non-positive eigenvalues (assuming the eigenvalues are real-valued). Solving forward the differential equation for $z_{1,t}$, we obtain

$$z_{1,t} = -\int_t^\infty \exp\left(-\Lambda_1(s-t)\right)\left[u_1^\psi \psi_s + u_1^v v_s + u_1^n b^n\right] ds. \tag{204}$$

When ψ_t is exponentially decaying, we obtain

$$z_{1,t} = -\left[\Lambda_1 + \theta_\psi I\right]^{-1} u_1^\psi \psi_t - \left[\Lambda_1 + \theta_v I\right]^{-1} u_1^v v_t - \Lambda_1^{-1} u_1^n b^n. \tag{205}$$

Solving backward the differential equation for $z_{2,t}$, we obtain

$$z_{2,t} = \exp\left(\Lambda_2 t\right) z_{2,0} + \int_0^t \exp\left(\Lambda_2(t-s)\right)\left[u_2^\psi \psi_s + u_2^v v_s + u_2^n b^n\right] ds. \tag{206}$$

When ψ_t is exponentially decaying, we obtain

$$z_{2,t} = \exp(\Lambda_2 t)\, z_{2,0} + [\Lambda_2 + \theta_\psi I]^{-1} [\exp(\Lambda_2 t) - \exp(-\theta_\psi I t)]\, u_2^\psi \psi_0$$
$$+ [\Lambda_2 + \theta_v I]^{-1} [\exp(\Lambda_2 t) - \exp(-\theta_v I t)]\, u_2^v v_0 + \Lambda_2^{-1} [\exp(\Lambda_2 t) - I]\, u_2^n b^n. \quad (207)$$

Rotating the system back to the original coordinates, we obtain

$$Z_t = V_1 z_{1,t} + V_2 z_{2,t}. \quad (208)$$

The vector $z_{1,t}$ captures the dependence on the exogenous shocks, while $z_{2,0}$ captures the effect of past promises.

□

Boundary conditions. The next proposition characterizes the boundary conditions

Proposition 13 (Boundary conditions)**.** *The optimality condition for x_0 and for the interest rate evaluated at zero are given:*

$$\frac{\kappa \xi_\pi}{\rho + \theta} + \xi_x = \int_0^\infty e^{-(\rho+\theta)t} \left[\alpha(x_t - x^*) + \frac{\kappa\beta}{\rho+\theta}\pi_t \right] dt, \quad (209)$$

$$\frac{\kappa\theta^* \Phi \xi_\pi}{\rho + \theta + \gamma} - \xi_x = \int_0^\infty e^{-(\rho+\theta+\gamma)t} \left[\theta^* \Upsilon (b_t - b^n) + \frac{\kappa\theta^*\Phi\beta\pi_t}{\rho+\theta+\gamma} \right] dt. \quad (210)$$

Proof. The boundary conditions can be written as

$$\frac{\kappa}{\rho+\theta}(\mu_{\pi,0} + \xi_\pi) + \xi_x = \int_0^\infty e^{-(\rho+\theta)t} [\alpha(x_t - x^*) + \kappa\mu_{\pi,t}]\, dt = -\mu_{x,0} \quad (211)$$

$$\frac{\kappa\theta^*\Phi}{\rho+\theta+\gamma}(\mu_{\pi,0} + \xi_\pi) - \xi_x = \int_0^\infty e^{-(\rho+\theta+\gamma)t} [\theta^*\Upsilon(b_t - b^n) + \kappa\theta^*\Phi\mu_{\pi,t}]\, dt = -\mu_{b,0}, \quad (212)$$

Using the fact that $\mu_{\pi,t} = \mu_{\pi,0} + \int_0^t \beta\pi_s ds$, we obtain the two boundary conditions. □

To obtain $\mu_{x,0} = 0$, the value of the co-state in the timeless perspective, the following condition must be satisfied:

$$\xi_x = -\frac{\kappa\xi_\pi}{\rho+\theta}. \quad (213)$$

This implies that $-\mu_{b,0}$ is given by

$$\left[\frac{\kappa\theta^*\Phi}{\rho+\theta+\gamma}+\frac{\kappa}{\rho+\theta}\right]\xi_\pi=\int_0^\infty e^{-(\rho+\theta+\gamma)t}\left[\theta^*\Upsilon(b_t-b^n)+\frac{\kappa\theta^*\Phi\beta\pi_t}{\rho+\theta+\gamma}\right]dt. \tag{214}$$

Irrelevance of $\mu_{\pi,0}$. We show next that the system is independent of $\mu_{\pi,0}$, which will allow us to normalize it to zero. Define the adjusted co-states:

$$\tilde{\mu}_{\pi,t}\equiv\mu_{\pi,t}-\mu_{\pi,0}, \qquad \tilde{\mu}_{x,t}\equiv\mu_{x,t}+\frac{\kappa}{\rho+\theta}\mu_{\pi,0}, \qquad \tilde{\mu}_{b,t}\equiv\mu_{b,t}+\frac{\kappa\theta^*\Phi}{\rho+\theta+\gamma}\mu_{\pi,0}. \tag{215}$$

The law of motion of the adjusted co-states is given by

$$\dot{\tilde{\mu}}_{\pi,t}=\beta\pi_t \tag{216}$$
$$\dot{\tilde{\mu}}_{b,t}-(\rho+\theta+\gamma)\tilde{\mu}_{b,t}=\theta^*\Upsilon(b_t-b^n)+\theta^*\kappa\Phi\tilde{\mu}_{\pi,t} \tag{217}$$
$$\dot{\tilde{\mu}}_{x,t}-(\rho+\theta)\tilde{\mu}_{x,t}=\alpha(x_t-x^*)+\kappa\tilde{\mu}_{\pi,t}. \tag{218}$$

The optimality condition for the interest rate is then given by

$$\tilde{\mu}_{x,t}+\tilde{\mu}_{b,t}=-\frac{\kappa\xi_\pi}{\rho+\theta}\left(1+\frac{(\rho+\theta)\theta^*\Phi}{\rho+\theta+\gamma}\right). \tag{219}$$

The optimality condition for the initial value of the output gap is given by

$$(\rho+\theta)(\tilde{\mu}_{x,0}+\xi_x)+\kappa(\tilde{\mu}_{\pi,0}+\xi_\pi)=0. \tag{220}$$

The dynamic system for the equilibrium variables can be equivalently written in terms of the adjusted co-states $(\tilde{\mu}_{\pi,t},\tilde{\mu}_{x,t},\tilde{\mu}_{b,t})$. As $\tilde{\mu}_{\pi,0}=0$, we can assume that $\mu_{\pi,0}=0$ without loss of generality.

Determination of initial conditions. We have three initial conditions for the system above: $\mu_{x,0}=0$, $\mu_{\pi,0}=0$, and the initial value of debt b_0. it remains to write $\mu_{x,0}$ in terms of the remaining variables. The output gap can be written as

$$x_t=V_{x,1}z_{1,t}+V_{x,2}z_{2,t} \tag{221}$$

The average value of $z_{1,t}$ is given by

$$\overline{z}_1 = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t} z_{1,t} dt = -\left[\Lambda_1 + \theta_\psi I\right]^{-1} u_1^\psi \frac{(\rho+\theta)\psi_0}{\rho+\theta+\theta_\psi} - \left[\Lambda_1 + \theta_v I\right]^{-1} u_1^v \frac{(\rho+\theta)v_t}{\rho+\theta+\theta_v} - \Lambda_1^{-1} u_1^n b^n. \tag{222}$$

The average value of $z_{2,t}$ is given by

$$\overline{z}_2 = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t} z_{2,t} dt = \left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} z_{2,0} + \tilde{z}_2, \tag{223}$$

where

$$\begin{aligned}\tilde{z}_{2,0} &= \left[\Lambda_2 + \theta_\psi I\right]^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} - \frac{\rho+\theta}{\rho+\theta+\theta_\psi} I\right] u_2^\psi \psi_0 \\ &+ \left[\Lambda_2 + \theta_v I\right]^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} - \frac{\rho+\theta}{\rho+\theta+\theta_v} I\right] u_2^v v_0 + \Lambda_2^{-1} \left[\left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} - I\right] u_2^n b^n\end{aligned} \tag{224}$$

The optimality condition for x_0 can be written as follows:

$$0 = \alpha(\overline{x} - x^*) + \frac{\kappa\beta}{\rho+\theta}\overline{\pi}, \tag{225}$$

where $\overline{x} = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t} x_t dt$ and $\overline{\pi} = (\rho+\theta)\int_0^\infty e^{-(\rho+\theta)t} \pi_t dt$. The other boundary conditions can be written as

$$0 = V_{\mu_\pi 1} z_{1,0} + V_{\mu_\pi 2} z_{2,0}, \qquad b_0 = V_{b1} z_{1,0} + V_{b2} z_{2,0}. \tag{226}$$

Let's assume that the system has two positive eigenvalues and three non-positive eigenvalues. Then, the $z_{2,0}$ is a three-dimensional vector that can be determined using the initial conditions for $\mu_{x,t}$, $\mu_{\pi,t}$, and b_t:

$$\underbrace{\begin{bmatrix} \alpha x^* \\ 0 \\ b_0 \end{bmatrix}}_{d_0} = \underbrace{\begin{bmatrix} \left(\alpha V_{x2} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 2}\right)\left[I - \frac{1}{\rho+\theta}\Lambda_2\right]^{-1} \\ V_{\mu_\pi 2} \\ V_{b2} \end{bmatrix}}_{D} z_{2,0} + d_1, \tag{227}$$

where

$$d_1 = \begin{bmatrix} \alpha V_{x1} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 1} \\ V_{\mu_\pi 1} \\ V_{b1} \end{bmatrix} z_{1,0} + \begin{bmatrix} \left(\alpha V_{x2} + \frac{\kappa\beta}{\rho+\theta} V_{\pi 2}\right) \tilde{z}_{2,0} \\ 0 \\ 0 \end{bmatrix}. \tag{228}$$

The initial condition for $z_{2,t}$ is then given by

$$z_{2,0} = D^{-1} \left[d_0 - d_1 \right]. \tag{229}$$

D.3 Perturbation solution

Let $r = \{r_t \in \mathbb{R} : t \geq 0\}$ denote the path of real interest rates. Define $w_t = (\pi_t, x_t, b_t)$ as the vector of non-policy variables and $w = \{w_t \in \mathbb{R}^3 : t \geq 0\}$ as the path of w_t. We say that a path of non-policy variables w is feasible if there exists a path of real interest rates r such that w is a *bounded solution* to the system of differential equations:

$$\dot{\pi}_t = (\rho + \lambda_f)\pi_t - \kappa x_t - \lambda_f \kappa \Phi(b_t - b^n) \equiv g_\pi(w_t, r_t, \psi_t) \tag{230}$$

$$\dot{b}_t = r_t - \rho + \psi_t \equiv g_x(w_t, r_t, \psi_t) \tag{231}$$

$$\dot{x}_t = r_t - \rho \equiv g_b(w_t, r_t, \psi_t), \tag{232}$$

given the initial condition b_0 and the process for the fiscal shock $\psi_t = e^{-\theta_\psi t}\psi_0$.

Denote the set of feasible w by $\mathcal{F}$. The optimal policy problem is then given by:

$$\max_{w \in \mathcal{F}} \int_0^\infty e^{-(\rho+\theta)t} f(w_t) dt, \tag{233}$$

where $f(w_t) \equiv -\frac{1}{2}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2\right]$.

Let w^* denote a candidate solution at the interior of the feasible set $\mathcal{F}$. Given a scalar $\epsilon > 0$, consider the perturbation $\hat{w}_t = w_t^* + \epsilon \eta_t$. We say that the deviation $\eta = \{\eta_t \in \mathbb{R} : t \geq 0\}$ is feasible if the path of $w_t^* + \epsilon \eta_t$ belongs to the feasible set.

Fixing a given deviation η and a candidate solution w^*, the value of a perturbed solution is a function of ϵ:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} f(w_t^* + \epsilon \eta_t) dt. \tag{234}$$

Given functions $(\mu_{\pi,t}, \mu_{x,t}, \mu_{b,t})$, we can write the value of the perturbation as follows:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} \left[f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} \left(g_z(\hat{w}_t, \hat{r}_t, \psi_t) - \dot{\hat{z}}_t \right) \right] dt, \tag{235}$$

where $\hat{r}$ corresponds to the path of real interest rates associated with the perturbed solution $\hat{w}$. Notice that $g_z(\hat{w}_t, \hat{r}_t, \psi_t) - \dot{z}_t = 0$ for all $t \geq 0$, as $\hat{w}$ is feasible, so the value of $\mathcal{W}(\epsilon)$ is independent of the functions $(\mu_{\pi,t}, \mu_{x,t}, \mu_{b,t})$.

We can use integration by parts to express the following integral in a more convenient form:

$$\int_0^\infty e^{-(\rho+\theta)t} \mu_{z,t} \dot{\hat{z}}_t dt = \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} \hat{z}_t - \mu_{z,0} \hat{z}_0 - \int_0^\infty e^{-(\rho+\theta)t} \left[\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} \right] \hat{z}_t dt, \tag{236}$$

for $z \in \{\pi, x, b\}$. Combining the previous two expressions, we obtain

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} \left[f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} g_z(\hat{w}_t, \hat{r}_t, \psi_t) + \sum_z \left(\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} \right) z_t \right] dt$$
$$+ \sum_z \left[\mu_{z,0} z_0 - \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} z_t \right], \tag{237}$$

Notice that b_0 is fixed, x_0 is free to be chosen by the planner, but π_0 is determined by the choice of r and x_0. It is useful to eliminate π_0 from the expression above. First, notice that π_0 can be written as

$$\pi_0 = \int_0^\infty e^{-(\rho+\theta)t} h_0(r_t, \psi_t; x_0, b_0) dt, \tag{238}$$

where $h_0(r_t, \psi_t; x_0, b_0) \equiv \kappa \left[x_0 + \lambda\Phi b_0 + \frac{1+\lambda\Phi}{\rho+\theta}(r_t - \rho) + \frac{\lambda\Phi}{\rho+\theta}\psi_t \right]$.

We can then write the $\mathcal{W}(\epsilon)$ in terms of a Hamiltonian, properly modified to incorporate the effect of the initial conditions:

$$\mathcal{W}(\epsilon) = \int_0^\infty e^{-(\rho+\theta)t} \left[H(\hat{w}_t, \hat{r}_t, \psi_t; b_0, \hat{x}_0) + \sum_{z\in\{\pi,x,b\}} \left(\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} \right) z_t \right] dt$$
$$- \sum_{z\in\{\pi,x,b\}} \lim_{t\to\infty} e^{-(\rho+\theta)t} \mu_{z,t} z_t, \tag{239}$$

where $H(\hat{w}_t, \hat{r}_t, \psi_t) \equiv f(\hat{w}_t) + \sum_{z\in\{\pi,x,b\}} \mu_{z,t} g_z(\hat{w}_t, \hat{r}_t, \psi_t) + (\rho+\theta)\left[\mu_{b,0} b_0 + \mu_{x,0} x_0\right] + \mu_{\pi,0} h_0(\hat{r}_t, \psi_t; b_0, \hat{x}_0)$.

A necessary condition for w^* to be an interior solution of the optimal policy problem is that

$$\mathcal{W}'(\epsilon) = 0. \tag{240}$$

Let $\hat{r} = r^* + \epsilon\eta_{r,t}$ and $\hat{x}_0 = x_0^* + \epsilon\eta_{x,0}$ denote the path of real interest rates and initial output gap associated with the perturbation $\hat{w}$. We can then write the derivative with respect to ϵ as follows:

$$\mathcal{W}'(0) = \int_0^\infty e^{-(\rho+\theta)t}\left[\sum_z \left(H_z(w_t^*, r_t^*, \psi_t) + \dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t}\right)\eta_{z,t}\right] dt - \sum_z \left[\lim_{t\to\infty} e^{-(\rho+\theta)t}\mu_{z,t}\eta_{z,t}\right]$$
$$+ \int_0^\infty e^{-(\rho+\theta)t} H_r(w_t^*, r_t^*, \psi_t)\eta_{r,t} dt. \tag{241}$$

The functions $\mu_{z,t}$ are arbitrary, so we can choose them to satisfy the condition:

$$\dot{\mu}_{z,t} - (\rho+\theta)\mu_{z,t} = -H_z(w_t^*, r_t^*, \psi_t), \tag{242}$$

subject to the boundary condition $\lim_{t\to\infty} e^{-(\rho+\theta)t}\mu_{z,t} = 0$. As any feasible perturbation is bounded, this ensures that the term $\lim_{t\to\infty} e^{-(\rho+\theta)t}\mu_{z,t}\eta_{z,t}$ is equal to zero.

As the perturbation $\eta_{r,t}$ is arbitrary, the following condition must be satisfied:

$$H_r(w_t^*, r_t^*, \psi_t) = 0. \tag{243}$$

Finally, the optimality condition for x_0 is given by

$$(\rho+\theta)\mu_{x,0} + \mu_{\pi,0}\kappa = 0. \tag{244}$$

The general case. Suppose that equilibrium variables evolve according to the more general dynamics:

$$\dot{\pi}_t = (\rho+\lambda_f)\pi_t - \kappa x_t - \lambda_f\kappa\Phi(b_t - b^n) \tag{245}$$
$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{246}$$
$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n), \tag{247}$$

We are interested in the effect of the initial conditions, so let's set $r_t = \rho$ and $\psi_t = 0$. In this case, the evolution of b_t and x_t is given by

$$\begin{bmatrix} \dot{\pi}_t \\ \dot{x}_t \\ \dot{b}_t \end{bmatrix} = \begin{bmatrix} \rho + \lambda_f & -\kappa & -\kappa\lambda_f\Phi \\ -1 & \theta_h & -\theta_h^* \\ -1 & 0 & -\gamma \end{bmatrix} \begin{bmatrix} \pi_t \\ x_t \\ b_t - b^n \end{bmatrix}. \tag{248}$$

The homogeneous solution is given by

$$b_t - b^n = e^{-\gamma t}(b_0 - b^n), \qquad x_t = \frac{\theta_h^*}{\gamma + \theta_h}(b_t - b^n), \qquad \pi_t = \left[\frac{\theta_h^*}{\gamma + \theta_h} + \lambda_f\Phi\right]\frac{\kappa}{\rho + \lambda_f + \gamma}(b_t - b^n). \tag{249}$$

D.4 Optimal policy with discretion

Optimal policy with finite planning horizon. Consider a planning with a finite planning horizon. We assume that a new planner takes over with a Poisson intensity $\bar{\lambda}$. The current planner takes the actions of future decision-makers as given. This ensures that the Euler equation is satisfied even after a new planner takes over. Let $\mathcal{P}_t(b_t)$ denote the value of a planner at period t with a given level of government debt, and $\mathcal{P}^*(b^*)$ denotes the value of a planner in the inflationary-finance phase. The planner's objective is given by

$$\mathcal{P}_0(b_0) = \mathbb{E}_0\left[-\frac{1}{2}\int_0^\tau e^{-\rho t}\left[\alpha x_t^2 + \beta\pi_t^2\right]dt + e^{-\rho\tau}\tilde{P}_\tau(b_\tau)\right], \tag{250}$$

where τ denotes the random time the economy switches to either the inflationary-finance phase, so the planner's value becomes $\tilde{P}_\tau(b_\tau) = \mathcal{P}^*(b_\tau)$, or a new planner's take over, so the planner's value is $\tilde{\mathcal{P}}_\tau(b_\tau) = \mathcal{P}_\tau(b_\tau)$. The density of τ is given by $(\lambda + \bar{\lambda})e^{-(\lambda+\bar{\lambda})t}$ and, conditional on switching, the probability of moving to the inflationary-finance phase is $\frac{\lambda}{\lambda+\bar{\lambda}}$, while the probability of a new planner taking over is given by $\frac{\bar{\lambda}}{\lambda+\bar{\lambda}}$ (see e.g. Cox and Miller (1977) for a derivation).

Using the density of τ, we can then express $\mathcal{P}_0(x_0, b_0)$ as follows:

$$\mathcal{P}_0(b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\bar{\lambda})t}\left[\alpha x_t^2 + \beta\pi_t^2 + \lambda\Upsilon(b_t - b^n)^2\right]dt + \int_0^\infty e^{-(\rho+\lambda+\bar{\lambda})t}\bar{\lambda}\mathcal{P}_t(b_t)dt. \tag{251}$$

The planner's problem consists of maximizing the objective above subject to the constraints

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n), \qquad \dot{b}_t = r_t - \rho + \psi_t, \qquad \dot{x}_t = r_t - \rho.$$

We also include a penalty on π_0 and x_0, as in the case with full commitment.

Optimality conditions The optimality conditions are given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta\pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{252}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda\Upsilon(b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{253}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa\mu_{\pi,t}, \tag{254}$$

where $\mathcal{P}_{b,t}(b_t)$ denotes the partial derivative of $\mathcal{P}_t(b_t)$ with respect to debt.

The optimality condition for the interest rate is given by

$$\mu_{x,t} + \mu_{b,t} = -\xi, \tag{255}$$

where $\xi \equiv \frac{\kappa(1+\lambda\Phi)}{\rho+\theta}\xi_\pi$.

The optimality condition for x_0 is given by

$$\mu_{x,0} = 0. \tag{256}$$

Standard envelope arguments imply that

$$\mu_{b,t} = \mathcal{P}_{b,t}(b_t). \tag{257}$$

The discretion limit. Consider the limit as $\overline{\lambda} \to \infty$, so each planner has commitment only over an infinitesimal amount of time. In the limit, the co-states on π_t and x_t are given by

$$\mu_{\pi,t} = 0, \qquad \mu_{x,t} = 0. \tag{258}$$

Integrating the expression for $\mu_{x,t}$ forward, we obtain

$$\mu_{x,t} = -\int_t^\infty e^{-(\rho+\lambda+\overline{\lambda})(s-t)}\left[\alpha x_s + \kappa\mu_{\pi,s}\right] ds \Rightarrow \lim_{\overline{\lambda}\to\infty} \overline{\lambda}\mu_{x,t} = -\alpha x_t, \tag{259}$$

using the fact that $\lim_{\overline{\lambda}\to\infty} \mu_{\pi,t} = 0$. Hence, from the optimality condition for x_0, we obtain

$x_0 = 0$. Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda \Upsilon (b_t - b^n) - \overline{\lambda}\mu_{b,t} + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{260}$$

where we used the envelope condition for b_t

Given $\mu_{b,t} = -\xi - \mu_{x,t}$, and combining the previous two expressions, we obtain

$$(\rho + \lambda)\xi = \lambda \Upsilon (b_t - b^n). \tag{261}$$

Therefore, the interest rate is given by

$$r_t - \rho = -\psi_t. \tag{262}$$

The case of partial commitment. In the case of discretion, planner's do not take into account promises made by prior planners. Hence, each planner sets a new value of x_t as they take control, and promise that output gap will evolve according to the Euler equation in the future. As we reduce the planning horizon to zero, each planner chooses the value of the output gap regardless of the path of interest rates. We consider next the case of partial commitment, where the planner has to respect past promises made about the output gap. In this case, the output gap must satisfy the Euler equation at every point in time, except at $t = 0$ when news about the shock arrives.

In this case, the planner's objective is given by

$$\mathcal{P}_0(x_0, b_0) = -\frac{1}{2}\int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\left[\alpha x_t^2 + \beta \pi_t^2 + \lambda \Upsilon (b_t - b^n)^2\right] dt + \int_0^\infty e^{-(\rho+\lambda+\overline{\lambda})t}\overline{\lambda}\mathcal{P}_t(x_t, b_t) dt, \tag{263}$$

and we impose a penalty on π_0, but not on x_0, as the initial output gap is not free.

The optmality conditions are now given by

$$\dot{\mu}_{\pi,t} - (\rho + \lambda + \overline{\lambda})\mu_{\pi,t} = \beta \pi_t - (\rho + \lambda)\mu_{\pi,t} \tag{264}$$

$$\dot{\mu}_{b,t} - (\rho + \lambda + \overline{\lambda})\mu_{b,t} = \lambda \Upsilon (b_t - b^n) - \overline{\lambda}\mathcal{P}_{b,t}(x_t, b_t) + \lambda\kappa\Phi\mu_{\pi,t} \tag{265}$$

$$\dot{\mu}_{x,t} - (\rho + \lambda + \overline{\lambda})\mu_{x,t} = \alpha x_t + \kappa \mu_{\pi,t} - \overline{\lambda}\mathcal{P}_{x,t}(x_t, b_t). \tag{266}$$

The optimality condition for the interest rate is the same as under discretion, and the envelope conditions for output gap and debt are given by

$$\mu_{x,t} = \mathcal{P}_{x,t}(x_t, b_t), \qquad \mu_{b,t} = \mathcal{P}_{b,t}(x_t, b_t). \tag{267}$$

Differentiating the optimality condition for the interest rate with respect to time, we obtain

$$(\rho + \lambda + \overline{\lambda})\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) - \overline{\lambda}(\mu_{b,t} + \mu_{x,t}) + \kappa(1 + \lambda\Phi)\mu_{\pi,t}, \tag{268}$$

where we used the envelope conditions.

Taking the limit as $\overline{\lambda} \to \infty$, we obtain

$$(\rho + \lambda)\xi = \alpha x_t + \lambda\Upsilon(b_t - b^n) \Rightarrow r_t - \rho = -\frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t. \tag{269}$$

In period $t = 0$, the planner is allowed to choose x_0, which must satisfy the condition:

$$\mu_{x,0} = 0 \Rightarrow 0 = \int_0^\infty e^{-(\rho+\lambda)t}\alpha x_t dt = 0, \tag{270}$$

where we used the fact that $\mu_{\pi,t} = 0$ as $\overline{\lambda} \to \infty$. Therefore, optimal policy with partial commitment coincides with the optimal policy with commitment for a dovish central bank, that is, when $\beta = 0$.

Taking the limit of a discrete-time economy. Welfare is measured by

$$\sum_{t=0}^{\infty} \left(e^{-\rho\Delta t}\right)^t \left[\alpha x_t^2 + \beta\pi_t^2\right]\Delta t. \tag{271}$$

The NKPC is given by

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + \left(\kappa x_t + u_t\right)\Delta t. \tag{272}$$

Under discretion, the planner's problem is given by

$$\max_{x_t, \pi_t} -\frac{1}{2}\left[\alpha x_t^2 + \beta\pi_t^2\right]\Delta t, \tag{273}$$

subject to

$$\pi_t = e^{-\rho\Delta t}\mathbb{E}_t\left[\pi_{t+\Delta t}\right] + \left(\kappa x_t + u_t\right)\Delta t, \tag{274}$$

taking as given $\mathbb{E}_t\pi_{t+\Delta t}$.

The optimal solution is given by

$$x_t = -\frac{\kappa\beta}{\alpha}\pi_t \Delta t. \tag{275}$$

D.5 Optimal policy under the timeless perspective

The dynamics under the optimal policy are characterized by the conditions:

$$\dot{\pi}_t = (\rho + \lambda)\pi_t - \kappa x_t - \lambda\kappa\Phi(b_t - b^n) \tag{276}$$

$$\dot{b}_t = r_t - \rho - \gamma(b_t - b^n) + \psi_t \tag{277}$$

$$\dot{x}_t = r_t - \rho + \theta_h x_t - \theta_h^*(b_t - b^n) \tag{278}$$

$$\dot{\mu}_{\pi,t} = \beta\pi_t \tag{279}$$

$$\dot{\mu}_{b,t} = (\rho + \lambda)\mu_{b,t} + \lambda\Upsilon(b_t - b^n) + \kappa\lambda\Phi\mu_{\pi,t} \tag{280}$$

$$\dot{\mu}_{x,t} = (\rho + \lambda)\mu_{x,t} + \alpha x_t + \kappa\mu_{\pi,t}, \tag{281}$$

where the real rate is given by

$$r_t - \rho = -\beta\frac{\kappa(1 + \lambda\Phi)}{\lambda\Upsilon + \alpha}\pi_t - \frac{\lambda\Upsilon}{\lambda\Upsilon + \alpha}\psi_t, \tag{282}$$

given the initial value of debt, b_0, and the boundary conditions $\mu_{x,0} = \mu_{\pi,0} = 0$.

Consider the case without a fiscal shock, $\psi_t = 0$, and denote the co-states in this case with no shocks by $\mu^{ns}_{x,t}$ and $\mu^{ns}_{\pi,t}$. The optimal policy under the timeless perspective corresponds to the solution to the system above when we replace the initial conditions by the long-run values of these multipliers: $\mu_{x,0} = \lim_{t\to\infty}\mu^{ns}_{x,t}$ and $\mu_{\pi,0} = \lim_{t\to\infty}\mu^{ns}_{\pi,t}$ (see Giannoni and Woodford (2017) for a discussion in the context a general model). This is equivalent to the problem of a planner who started its planning in a distant past, so the multipliers had time to converge to their long-run values.

Even without shocks, the limits $\lim_{t\to\infty}\mu^{ns}_{x,t}$ and $\lim_{t\to\infty}\mu^{ns}_{\pi,t}$ will not be equal to zero, provided that $b_0 \neq b^n$. However, in the case $b_0 = b^n$, the solution to the system above in the absence of shocks is simply $\pi_t = x_t = b_t = \mu_{\pi,t} = \mu_{x,t} = \mu_{b,t} = 0$. Hence, we have that $\lim_{t\to\infty}\mu^{ns}_{x,t} = 0$ and $\lim_{t\to\infty}\mu^{ns}_{\pi,t} = 0$, so the boundary conditions for the problem under the timeless perspective coincide with the time-zero commitment solution.

E. Historical Shock Decomposition and Taylor Counterfactual

For the historical shock decomposition, we build the discrete version analogue of the model in Section 2. Moreover, we add three more shocks besides the fiscal shock already analyzed. First, we include a monetary shock that allows the monetary authority to deviate from the prescriptions of the interest rate rule. We calibrate the Taylor coefficient on the lower end of its plausible range to minimize the importance of these shocks in the conclusion. Second, we include a standard-cost push shock, to capture movements in the inflation rate that are orthogonal to the evolution of debt. It represents sectoral reallocations and supply bottlenecks, as experienced during the pandemic. This shock is identified through the Phillips curve implied by the model. Third, we add a "term premium" shock to the return on government debt. This shock is meant to capture the effect on the one-period holding return on government debt of revaluation effects, risk premium movements, or changes in the maturity structure. The shock is directly extracted from the government debt path, given the fiscal rule, the primary deficits and the path of nominal rates. This type shock appears, e.g., in Bianchi and Melosi (2017). Since shocks are inferred directly from the data series, the Kalman filter optimizes the initial conditions to best fit the shock decomposition—the initial conditions' quantitative contribution is minor.

E.1 The Model

The model can be characterized by the following equations:

1. IS curve

$$x_t = x_{t+1} - \sigma \left(i_t - \pi_{t+1} - \rho \right)$$

2. New Keynesian Phillips Curve (NKPC)

$$\pi_t = \beta \left[\left(1 - \theta_f\right) \pi_{t+1} + \lambda_f \kappa \Phi b_{t+1} \right] + \kappa x_t + \mu_t$$

3. Interest rate rule

$$i_t = \rho + \phi_\pi \pi_t + u_t^m$$

4. Government debt evolution

$$b_t = (1-\gamma)\, b_{t-1} + r_t b_n + u_t^f$$

5. Primary surplus

$$\psi_t = u_t^f - (\rho + \gamma)\, b_{t-1}$$

6. One-period holding return on government debt

$$r_t = (i_t - \pi_t - \rho) + u_t^r,$$

where μ_t denotes the cost-push shock, u_t^m denotes the monetary shock, u_t^f denotes the fiscal shock, and u_t^r denotes the term-premium shock. Note that in the data we do not observe the fiscal shock directly but through its effect on the primary surplus. Thus, we denote ψ_t the primary surplus, which includes the fiscal shock and the automatic adjustment of transfers to changes in the stock of debt. Finally, we assume that all disturbances are white noise processes with standard deviation of 1% (annualized).

E.2 The Data

For the exercise, we take the dynamics of inflation, the primary surplus, the stock of debt, and the nominal rate as observables. The inflation rate is measured as the growth rate of the GDP deflator (NIPA Table 1.1.7 line 1). The primary surplus is the difference between government receipts (NIPA Table 3.1 line 1) and total expenditures (NIPA Table 3.1 line 20) net of interest payments (NIPA Table 3.1 line 12 - NIPA Table 3.1 line 27), divided by nominal GDP (NIPA Table 1.1.5 line 1). The stock of debt is the market value of government debt held by the private sector from Hall, Payne and Sargent (2018) plus reserves of depository institutions (Fred TOTRESNS). Finally, the nominal rate is the federal funds effective rate (Fred DFF).

Since our model does not incorporate growth, we need to make an adjustment to the evolution of the fiscal shock. In the data, the primary surplus over GDP moves both because of the fiscal shock and GDP growth. To account for this, we combine the growth component of GDP into the purely fiscal component. Thus, our fiscal shock is a composite of an exogenous transfer shock and the contribution of growth to the debt-to-GDP ratio.

Formally, we have that the government's budget constraint is given by

$$\dot{B}_t = (i_t - \pi_t)\, B_t - (\rho + \gamma)\, B_t + \Psi_t.$$

Dividing by *quarterly* output, we get

$$\underbrace{\frac{\dot{B}_t}{X_t} - \frac{B_t}{X_t}\frac{\dot{X}_t}{X_t}}_{=\dot{b}_t} + \frac{B_t}{X_t}\frac{\dot{X}_t}{X_t} = (i_t - \pi_t)\,\frac{B_t}{X_t} - (\rho + \gamma)\,\frac{B_t}{X_t} + \frac{\Psi_t}{X_t}.$$

Let $b_t \equiv \frac{B_t}{X_t}$ and $g_t \equiv \frac{\dot{X}_t}{X_t}$, to get

$$\dot{b}_t = (i_t - \pi_t)\, b_t - (\rho + \gamma)\, b_t + \underbrace{\left(\frac{\Psi_t}{X_t} - b_t g_t\right)}_{\equiv \psi_t}.$$

Let $\hat{b}_t \equiv b_t - b_n$ and $\hat{\psi}_t \equiv \psi_t - \overline{\psi}$. We assume that γ is such that $\gamma = \frac{\overline{\psi}}{b_n}$. Then

$$\dot{\hat{b}}_t = \underbrace{(i_t - \pi_t - \rho)\,\hat{b}_t}_{=\mathcal{O}(||\psi_t||^2)} + (i_t - \pi_t - \rho)\, b_n - \gamma \hat{b}_t + \hat{\psi}_t,$$

or

$$\dot{\hat{b}}_t \approx (i_t - \pi_t - \rho)\, b_n - \gamma \hat{b}_t + \hat{\psi}_t.$$

E.3 The Shocks

Figure 9 shows the identified time series of the shocks.

Panel (a) depicts the fiscal shock. The fiscal shock presents a massive increase in spending in 2020Q2 and then minor increases in 2021. Interestingly, and despite being short-lived, the shock has a persistent effect on debt and inflation, as shown in Figure 8. As explained, this is because our model's dynamics is governed by the *stock* of debt, rather than the *flow* of government spending. While the spending shock is short-lived, there was almost no time in this period where the government run surpluses to pay for the increase in debt (the negative value of the shock in 2020Q3 is driven by the rebound in economic activity). Thus, absent inflation, debt and inflation would have remained higher for longer.

The model also identifies an important role for the exogenous cost-push shock, as shown in Panel (b). The model interprets the low inflation of 2020 as a negative cost-push shock in this period, and the high inflation of 2022 as a positive shock. These dynamics can be attributed first to the lock-down in 2020, and then to the reopening in 2022, which affected the services sector in particular. Guerrieri, Lorenzoni, Straub and Werning (2022) show that shocks that affect different sector asymmetrically can show as cost-push shocks in the New Keynesian model.

Finally, we have the monetary shock and the term-premium shock. The monetary shock exhibits the pattern we identified in Figure 7 Panel (d), that is, the under-reaction relative to a Taylor rule in 2021 and 2022. As shown in Figures 8 and 10, this under-reaction was crucial to keep the inflation rate lower, since, absent primary surplus, it was the low real rates the financed the increase in government debt. The term-premium shock is a amplified version of the monetary shock, which suggests that the term-premium amplified the effect of short-rates on yields.

E.4 Connection to the Textbook Model

The system of equations characterizing the equilibrium of our economy is isomorphic to the system in the textbook model, with the difference that the expectations include the possibility of a monetary-fiscal reform. To see this, assume that the economy starts in Phase I. The system of equations characterizing the equilibrium is given by

$$
\begin{aligned}
x_t^I &= E_t^h[x_{t+\Delta t}] - (i_t - E_t^h[\pi_{t+1}] - \rho)\Delta t \\
\pi_t^I &= \beta E_t^f[\pi_{t+\Delta t}] + \kappa x_t^I \Delta t \\
i_t &= \rho + \phi_\pi \pi_t^I + u_t \\
b_t &= b_{t-\Delta t} + (i_{t-\Delta t} - \pi_t^I - \rho) b^n - \gamma b_{t-\Delta t}\Delta t + \psi_t,
\end{aligned}
$$

where $\{x_t^I, \pi_t^I\}$ denote the output gap and inflation in Phase I, respectively, $\{E_t^h, E_t^f\}$ denote the households' and firms' expectation operator, respectively, $\{x_{t+\Delta t}, \pi_{t+\Delta t}\}$ are random variables representing the output gap and inflation in period $t + \Delta t$, and the time period is of length Δt.

If $\lambda_h = \lambda_f = 0$, then $x_{t+\Delta t} = x^I_{t+\Delta t}$ and $\pi_{t+\Delta t} = \pi^I_{t+\Delta t}$, and the system becomes the textbook system of difference equations when $\Delta t = 1$. In contrast, with $\lambda_h, \lambda_f > 0$ we have

$$E_t^h[x_{t+\Delta t}] = (1 - \lambda_h \Delta t) x_{t+\Delta t}^I + \lambda_h \Delta t x_{t+\Delta t}^{II},$$
$$E_t^j[\pi_{t+\Delta t}] = (1 - \lambda_j \Delta t) \pi_{t+\Delta t}^I + \lambda_j \pi_{t+\Delta t}^{II}, \quad \text{for } j \in \{h, f\},$$

where $\{x_{t+\Delta t}^{II}, \pi_{t+\Delta t}^{II}\}$ denote the output gap and inflation in Phase II, respectively. Removing the superscript I, and using that $x_{t+\Delta t}^{II} = b_{t+\Delta t} - b^n$ and $\pi_{t+\Delta t}^{II} = \kappa \Phi(b_{t+\Delta t} - b^n)$, the system becomes

$$x_t = (1 - \lambda_h \Delta t) x_{t+\Delta t} - (i_t - ((1 - \lambda_h \Delta t) \pi_{t+\Delta t} + \lambda_h \Delta t \kappa \Phi)(b_{t+\Delta t} - b^n)) - \rho) \Delta t + \lambda_h \Delta t (b_{t+\Delta t} - b^n)$$
$$\pi_t = \beta (1 - \lambda_f \Delta t) \pi_{t+\Delta t} + [\kappa x_t + \beta \lambda_f \kappa \Phi (b_{t+\Delta t} - b^n)] \Delta t$$
$$i_t = \rho + \phi_\pi \pi_t + u_t$$
$$b_t = b_{t-\Delta t} + [(i_{t-\Delta t} - \pi_t - \rho) b^n - \gamma b_{t-\Delta t} + \psi_t] \Delta t,$$

In the limit as $\Delta t \to 0$, it simplifies to

$$\dot{x}_t = i_t - \pi_t - r_t^n + \lambda_h x_t$$
$$\dot{\pi}_t = (\rho + \lambda_f) \pi_t - \kappa x_t - \mu_t$$
$$i_t = \rho + \phi_\pi \pi_t + u_t$$
$$\dot{b}_t = (i_t - \pi_t - \rho) b^n - \gamma b_t + \psi_t,$$

where $r_t^n \equiv \rho + \lambda_h (b_t - b^n)$ and $\mu_t \equiv \beta \lambda_f \kappa \Phi (b_t - b^n)$, and we used that $\beta = \frac{1}{1+\rho \Delta t}$. These expressions make it clear that, through the expectation of a reform, the system of equations characterizing equilibrium changes in the following ways: $i)$ the Euler equation features "discounting," $ii)$ the natural rate is endogenous and depends on the level of debt, $iii)$ the NKPC also features "discounting," $iv)$ the NKPC features a cost-push shock that depends on the stock of debt.