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New Keynesian Economics through the Extensive Margin

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Abstract

This paper reformulates the New Keynesian model to incorporate output adjustments through the extensive margin. Shifting from adjustments through the intensive to the extensive employment margin, the model introduces predetermined output, altering key properties of the New Keynesian framework. First, the Taylor principle is inverted: stability is achieved when nominal rates respond less than one-for-one with inflation. Second, the model significantly alters the output responses to changes in monetary policy. We argue that this represents a challenge and an opportunity for the literature. Sticky information allows the model to correct the sign of impulse responses.

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1 Introduction

The New Keynesian (NK) framework has been a cornerstone of modern monetary policy analysis. However, its canonical version relies on the intensive margin adjustment of labor (hours), setting aside that the main margin of output adjustment is the extensive margin of hours (job flows). This omission leaves that model void of empirical predictions about the unemployment and vacancy rates and their statistical correlation with inflation and policy interest. Naturally, multiple attempts have been made to marry the New Keynesian model with search-and-matching (SAM) models within the Diamond-Mortensen-Pissarides tradition, the dominant modeling device to analyze labor flows. In this paper, we highlight that, like most marriages, this one faces its difficulties. We also highlight a path toward a permanent reconciliation.

The main challenge in the SAM+NK marriage attempts is an inherent feature of the SAM model: output is predetermined. This is because labor employment in SAM models is a stock predetermined by past hires. In turn, predetermined output is problematic for the NK model because it alters the model's transmission mechanism: A key property of the NK model that unleashes its transmission mechanism is the possibility of consumption jumps upon the realization of information. For example, an expansionary monetary policy shock that provokes persistent low rates induces a period of low consumption growth—dictated by the Euler equation. For monetary shocks to be expansionary, consumption must jump contemporaneously with the shock, but that jump is only possible if output can adjust immediately. The inherent short-run labor capacity constraint SAM models inhibit that jump.

To reconcile SAM and NK models, past attempts assume that hiring happens instantaneously: i.e., firms can add job postings, hire workers, and put them to work immediately. That assumption not only blurs the distinction between flows and stocks, a key motivation for setting up SAM models but, more importantly, poses several theoretical challenges. In particular, the common timing convention in SAM+NK models makes the resulting predictions non-invariant to the choice of the model period, i.e., whether the notion of a period is a year, quarter, month, or day. This is a problem: Just like we want to write models whose predictions are invariant to the frequency at which interest rates are compounded, we want to write models whose conclusions are invariant to whether it takes a month or a year to incorporate workers or whether the interest is compounded in advance. Put differently, should we be comfortable with a model whose predictions depend on our choice of model period?

The main insight of this paper is that once we revert to the consistent timing convention of SAM models, the key predictions of the NK+SAM model are flipped. We open the discussion by contrasting the timing convention in the difference equation that governs labor flows in textbook SAM models, e.g., Mortensen and Pissarides (1994) and Shimer (2005) in which hires take a period to be ready for production, and the convention in SAM+NK models, i.e. Blanchard and Gali (2010), in which hires are readily available for production. We then take the limit as time intervals go to zero and show that the job-flow equations collapse to the same differential equation. Moreover, under that common differential equation, consumption can jump only if job postings exhaust all the resources in the economy. Because consumption cannot jump as the time period shrinks in NK+SAM, there must be a time-period calibration at which the model's properties flip. However, predictions under the traditional SAM convention are consistent regardless of the model period, not under the SAM+NK convention.

After setting the stage, the paper presents a continuous-time formulation of the SAM+NK model that incorporates the continuous-time job-flow equation. Unlike other marriage attempts, our version maintains the tractability of the two-equation system that has made the canonical NK framework popular. That tractability is obtained only by setting a wage-setting protocol that delivers a constant labor share. Our continuous-time formulation renders analytic transparency, provides a clear distinction between job flows and stocks, and is amenable to analysis as the standard intensive margin NK model.

We formally show that the shift from the intensive to the extensive margin of employment adjustment dramatically changes the NK model's properties once the consistent SAM timing convention is adopted. Because the instantaneous jumps in output that characterize the standard model are impossible, the reformulation has profound implications for the NK literature.

The first implication is, in principle, a virtue. In our reformulation, the Taylor principle is inverted: determinacy is obtained when nominal rates respond less than one-for-one to inflation. This feature is desirable because it means that determinacy is not achieved through exuberant off-equilibrium threats (Atkeson, Chari and Kehoe, 2010). Rather, determinacy is achieved because, unlike the standard NK model, this version does not lack an initial condition. A second virtue is that this feature allows the model to generate hump-shaped impulse response functions (IRFs) to monetary shocks without additional frictions such as habit formation or capital adjustment costs, which have been criticized repeatedly. A third virtue is that, without the Taylor principle, we can explore shocks in isolation of endogenous monetary policy responses. This allows the model to isolate the effects of shocks that are otherwise confounded with endogenous policy responses (Angeletos and Lian, 2023).[1]

[1]Take the case of government expenditure shocks, their effects are confounded with the endogenous response of interest rates via the Taylor rule.

The second implication is problematic: the signs of the effects of monetary policy and demand shocks are flipped: reductions in policy rates provoke recessions. The key reason is the short-run labor capacity constraint implicit in the SAM literature. If output is predetermined, the consumption jump cannot occur. As a result, after reductions in policy rates, output follows a negative hump-shaped response. This is not true about every shock: for example, the IRF to TFP or layoff shocks feature the conventional sign. However, this feature leads to completely different optimal policy responses than in the standard NK model.

Because the marriage of SAM and NK models has virtues we should embrace, we conclude the paper by offering a possible avenue to resolve the issue with the sign of responses. We reformulate the Euler equation to correct the sign of the IRFs while maintaining the right virtues of the model. The key to this extension is that the responses to aggregate consumption are distributed over time. This extension relies on models that depart from full-information rational expectations in the Lucas-island models' original spirit—i.e., models like Mankiw and Reis (2002, 2007); Reis (2006).

References. Influential attempts to marry the SAM and NK models include Gertler, Sala and Trigari (2008), Blanchard and Gali (2010), Christiano, Eichenbaum and Trabandt (2016), Ravn and Sterk (2017), Kekre (2023) and more recent attempts include Benigno and Eggertsson (2023). Virtually all of these attempts allow for hiring within the period. Michaillat and Saez (2015) introduce a matching approach to the product and labor markets within the framework of Barro and Grossman (1971), examining how aggregate demand affects unemployment fluctuations. However, the analysis is restricted to the steady state, where unemployment jumps instantaneously in response to shocks. Our paper contributes to these attempts to integrate labor-market frictions into New Keynesian models.

A virtue is that our approach maintains the analytical tractability of the baseline NK model. Our reformulation allows for a two-equation system regarding unemployment deviations rather than the output gap. Few SAM+NK models achieve this tractability. An exception is found in Michaillat and Saez (2024) who introduces a directed search, a different tradition from the standard SAM, into the NK model and provides a two-equation system featuring the unemployment and inflation rates.[2]

The result that labor-search frictions alter the Taylor principle is also found in Kurozumi and Van Zandweghe (2010). However, that paper studies determinacy under the SAM+NK model

[2]Since labor flow in each sub-labor market is assumed to be balanced, the unemployment rate and, consequently, output—is a jump variable in their setting.

timing convention. The paper presents numerical conditions that depend on the firing rate and the Taylor coefficient. Here, we show that a sufficient condition for determinacy requires setting the Taylor coefficient to be less than one.

Other work has also shown that the transmission mechanism of the New Keynesian model is not robust but for different reasons. Dupor (2001) introduces investment into the standard continuous-time New Keynesian model and shows that an interest rate peg ensures determinacy.[3] In turn, Rupert and Šustek (2019) examines how in a NK model with capital consumption and investment feature the wrong co-movement after monetary policy shocks. These results occur because the investment share of output is a margin of adjustment in those models. Here, past hires inhibit consumption jumps as a margin of adjustment.

The remainder of the paper is organized as follows. Section 2 clarifies the timing conventions in the literature. Section 3 presents the continuous-time model and derives the equilibrium conditions. Section 4 examines the model's stability properties, the inversion of the Taylor principle, and the dynamic responses to various shocks. Finally, Section 5 concludes by offering a possible avenue for resolution.

2 Timing Conventions in the SAM and NK Literatures

To set the stage, let's explain why we need a consistent timing convention for job flows. Suppose we have already calibrated a discrete-time model with model period Δ relative to some calendar time notion for parameters. For example, we can have an annualized discount rate target of 4%, a 10% shock mean-reversion rate, and $\Delta = 1/12$ to think of the period as a month. We can keep our annualized notions fixed and vary the calibration of the model period.

In NK models, the time zero effects of monetary policy on output are given by a jump in consumption δC_0 and a consumption trajectory δC_t for all $t \in \{\Delta, 2\Delta, 3\Delta, ...\}$. The notation δ represents the difference relative to a steady state. Of course, the entire sequence can be indexed by our choice of Δ. Let's call that sequence $\delta C_t(\Delta)$. A model where the periodicity is immaterial is one in which the choice of Δ is quantitatively irrelevant in shaping the responses.[4] Critically, the conventional response to an *expansionary* and mean reverting policy shock is to have a jump $\delta C_0(\Delta) > 0$ and then a decreasing sequence $\delta C_t(\Delta) > \delta C_{t'}(\Delta)$ for any $t < t'$ dictated by the

[3] On the same vein, Carlstrom and Fuerst (2005) analyze a discrete-time NK model with investment demonstrates that current-looking interest rate rules ensure determinacy if the policy aggressively responds to current inflation.

[4] Mathematically, we would expect the step functions with values $\delta C_t(\Delta)$ to converge to some limit function, $\Delta \to 0$, but this is a mathematical detail we don't need to make the point here.

Euler equation.

Let's come to the issue at stake: In discrete time, equation (2.1) represents the labor flow equation commonly used in the conventional NK models with SAM frictions surveyed above. In contrast, equation (2.3) describes the labor flow equation employed in SAM models. In both equations, n_t is the available workforce, $m(\cdot,\cdot)$ is a matching function, $\mathcal{U}_t$ is the number of unemployed workers, and V_t is the number of vacancy postings. The function $\lambda\left(\Delta\right)$ and $\Xi\left(\Delta\right)$ and the efficiency of the matching function and the job separation rates, respectively. These satisfy $\lim_{\Delta\to 0}\frac{\lambda(\Delta)}{\Delta}=\overline{\lambda}$ and $\lim_{\Xi\to 0}\frac{\lambda(\Delta)}{\Delta}=\overline{\Xi}$. These limits are important to have consistent hiring and firing rates as we set the model period.

The key distinction between the discrete-time NK convention and the SAM convention is that matched workers are available for work within the period in the NK model. In SAM, however, employers must wait a period. Why is this an issue?

Suppose we want to compare a NK model with the said convention across different calibrations of the model period Δ. We can do so by analyzing what happens as $\Delta\to 0$. Both formulations will converge to the same ordinary-differential Equations: (2.2) and (2.4). Because n_0 is known, unless vacancies explode to infinity, the number of matches will be bounded. This means that n_t cannot jump unless vacancy postings $v_t\to\infty$. In any equilibrium, this will violate any optimality or goods-market clearing condition.[5] In other words, employment cannot jump in the continuous-time limit.

The above observation is a problem for discrete-time NK models with the hiring-within-the-period convention: Recall that their standard IRFs feature a jump in $\delta C_0(\Delta)>0$. The continuous time limit says that as we shrink the model period Δ, the IRFs will start to look more and more different. This insight indicates that if we want to favor the NK hiring-within-the-period convention, the specific choice of Δ should be a calibration target that must be justified. This inconsistency does not appear if we adopt the SAM convention. However, many things in the NK with model SAM frictions once we adopt the SAM convention. We investigate that version next.

[5]In our setting below, infinite vacancy postings imply infinite household consumption because the cost of posting vacancies is paid to households, while output—determined by employment—remains finite. Alternatively, if vacancy posting costs are drawn directly from aggregate output rather than paid to households, infinite vacancy postings imply negative consumption, which is not feasible in general equilibrium.

Labor Flows and Stocks

NK Convention

Labor flow in discrete time is given by:

$$n_t = m\left(\mathcal{U}_{t-\Delta}, V_t\right)\lambda\left(\Delta\right) - n_{t-\Delta}\Xi\Delta + n_{t-\Delta}. \tag{2.1}$$

Taking the limit $\Delta \to 0$, we obtain:

$$\dot{n}_t = \overline{\lambda} m\left(\mathcal{U}_t, V_t\right) - \Xi n_t. \tag{2.2}$$

SAM Convention

Labor flow in discrete time is given by:

$$n_t = m\left(\mathcal{U}_{t-\Delta}, V_{t-\Delta}\right)\lambda\left(\Delta\right) - n_{t-\Delta}\Xi\Delta + n_{t-\Delta}. \tag{2.3}$$

Taking the limit $\Delta \to 0$, we obtain:

$$\dot{n}_t = \overline{\lambda} m\left(\mathcal{U}_t, V_t\right) - \Xi n_t. \tag{2.4}$$

3 Model

Time is indexed by $t \in [0, \infty)$ and the economy is deterministic. There is a consumption bundle whose price index is P_t, and we denote the rate of inflation as $\pi_t \equiv \dot{P}_t / P_t$.

The economy features a representative household, monopolistically competing retailers, and competitive intermediate goods firms.

The Demand Side Block: Households. The representative household faces a standard consumption-saving problem with instantaneous utility $U\left(C_t\right) \equiv \left(C_t^{1-\sigma} - 1\right) / \left(1-\sigma\right)$ over a bundle of differentiated final goods differentiated, $\left\{y_t^j\right\}_{j\in[0,1]}$ where $C_t = \left(\int_0^1 \left(y_t^j\right)^{1-1/\varepsilon} dj\right)^{\frac{1}{1-1/\varepsilon}}$.

The individual demand for retail goods follows from the standard cost minimization assumptions:

$$y_t^i = \left(P_t / p_t^i\right)^{\varepsilon} C_t. \tag{3.1}$$

where p_t^j is the individual price of good j and $P_t = \left(\int_0^1 \left(p_t^j\right)^{1/\varepsilon} dj\right)^{\varepsilon}$ and where $\epsilon > 1$.

We assume a representative household that insures its members. Households feature many members that differ in their employment status, being either employed or unemployed. The transition rate from employment to unemployment is the exogenously given by Ξ_t. By contrast, the transition rate from unemployment to employment, ζ_t, is endogenous. The household pools the income of the employed and unemployed and distributes the same consumption to its members. The nominal wage of the employed is w_t. Aggregate demand satisfies the following standard Euler equation:

$$\frac{\dot{C}_t}{C_t} = \frac{i_t - \pi_t - \rho}{\sigma}.$$

where ρ is the discount rate and i_t is nominal interest rate.

The Supply Side Block: Employment, Production, and Prices. The supply-side block features nominal price rigidity combined with search frictions in the labor market. Intermediate goods firms use labor as their sole input and produce identical intermediate goods sold competitively to retailers. Retailers buy the input and sell the differentiated final goods to households in a monopolistically competitive setting. Intermediate goods firms and retailers are owned by households, and their profits are distributed back to the households. Nominal price adjustment costs arise at the retail level. The labor-search friction arises at the intermediate good sector.

Retailer j purchases intermediate inputs x_t^j to produce y_t^j:

$$y_t^j = x_t^j \qquad \forall t. \tag{3.2}$$

The retailer's problem is:

Problem 1 *[Retailer's Problem] Retailer j chooses the change in its individual price $\dot{p}_\tau^j$ to maximize*

$$q\left(p_t^j, t\right) = \max_{\left\{\dot{p}_\tau^j\right\}} \int_t^\infty \exp\left(-r_\tau \tau\right) \left[\frac{p_\tau^j y_\tau^j - p_\tau x_\tau^j}{P_\tau} - \frac{\Theta}{2}\left(\frac{\dot{p}_\tau^j}{p_\tau^j}\right)^2 Y_\tau\right] d\tau,$$

subject to its individual demand (3.1) and production function (3.2).

The retailer takes the price of the intermediate good, p_t, and the price of the aggregate final good bundle, P_t, as given. The retailer chooses $\dot{p}_{j,\tau}$ to maximize the present discounted value of real profits minus a Rotemberg price-adjustment cost with coefficient Θ where Y_t is aggregate output. We assume that price adjustment cost is paid as a transfer to the representative household. The retailer discounts future profits by using real interest rates $r_\tau = i_\tau - \pi_\tau$. The state variable of the retailer is p_t^j. The real retailer marginal cost is $mc_t \equiv p_t / P_t$, measured in terms of final goods. The marginal cost in real terms appears in the following standard Phillips curve:[6]

Lemma 1 *The path of prices satisfies the following Phillips curve:*

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\Theta}\left(mc_t - \frac{\epsilon - 1}{\epsilon}\right) + \dot{\pi}_t. \tag{3.3}$$

Intermediate goods firms are identical. An intermediate goods firm produces x_t employing n_t workers according to:

$$x_t = A_t n_t \qquad \forall t. \tag{3.4}$$

[6] See Supplemental Appendix B.1 for the proof.

where A_t is productivity. Labor flows at this firm evolve according to:

$$\dot{n}_t = J_t V_t - \Xi_t n_t, \tag{3.5}$$

where Ξ_t is the exogenous job-separation rate, J_t is the job-filling rate per vacancy, and V_t is the firm's vacancy postings. Vacancies cost μ in real terms, and the firm takes the hiring rate as given. We assume that the vacancy cost is paid as a transfer to the representative household.

The Intermediate goods firm's problem is:

Problem 2 *[Intermediate Producer's Problem] The intermediate goods firm chooses*

$$G(n,t) = \max_{\{V_\tau\}} \int_t^\infty \exp(-r_\tau \tau) \left[\frac{p_\tau}{P_\tau} x_\tau - \frac{w_\tau}{P_\tau} n_\tau - V_\tau \mu \right] d\tau,$$

subject to its production function (3.4) and its labor flows (3.5).

The intermediate goods firm chooses vacancies to maximize its real operational profits net of hiring costs. It discounts future profits using real interest rates, given by $r_\tau = i_\tau - \pi_\tau$. To maximize revenues, the firm must post vacancies to offset the loss of workers. The firm's state variable is its current number of workers.

In SAM models it is customary to model wage-setting through ex-ante bargaining. Thus, the value of workers is typically a state variable that pins down wages. This complicates the analysis because it adds dynamic equations to the system. We can reduce the complexity by modeling a setting where wages are bargained ex-post. Here, as in the microfoundation in Caballero and Hammour (1998), workers can abscond the fraction $(1-\eta)$ of the intermediate-good output. If they do so, the firm loses its entire production. To avoid losing its goods, wages are renegotiated after production, resulting in a labor share of $1-\eta$. Thus, $w_t = (1-\eta)\, p_t$. Neither workers nor the firm can credibly commit to a compensation package or separations.

The following Lemma characterizes the intermediate goods firm's problem:[7]

Lemma 2 *The value of the intermediate goods firm is $G(n,t) = g_t \cdot n_t$ where g_t is the value of a worker:*

$$g_t = \eta \times \int_t^\infty \exp\left(-(\tau - t)(r_\tau + \Xi_\tau)\right) \left(\frac{p_\tau}{P_\tau} A_\tau - \frac{w_\tau}{P_\tau} \right) d\tau. \tag{3.6}$$

The wage-setting protocol here does not alter the main message of the paper; we could adopt the wage setting in SAM models without changing the essence. However, that formulation would require us also to keep track of an additional variable, the value of unemployment.

[7]See Supplemental Appendix B.2 for the proof.

The flow of job matches follows a homogeneous-of-degree-1 matching function, $m(\mathcal{U}_t, V_t)$. It is convenient to define two auxiliary functions:

$$\Phi(x) \equiv m(1, x) \quad \text{and} \quad \mathcal{J}(x) \equiv m\left(x^{-1}, 1\right).$$

Then, the job-filling rates and job-finding rates are given by:

$$J_t = \mathcal{J}\left(\frac{V_t}{\mathcal{U}_t}\right), \qquad \zeta_t = \Phi\left(\frac{V_t}{\mathcal{U}_t}\right). \tag{3.7}$$

The job-finding rate is increasing in the vacancy-to-unemployment ratio. In turn, the job-filling rate decreases in v_t. Given these rates, aggregate unemployment evolves according to:

$$\dot{\mathcal{U}}_t = \Xi_t(1 - \mathcal{U}_t) - \zeta_t \cdot \mathcal{U}_t. \tag{3.8}$$

Since labor is indivisible, output is $Y_t \equiv A_t(1 - \mathcal{U}_t)$. From the intermediate goods firms, we obtain a relationship between the vacancy-to-unemployment ratio and the value of a worker.

Equilibrium. The monetary authority sets the nominal interest rate according to a Taylor rule:

$$i_t = i_{ss} + \phi_\pi \pi_t + z_t,$$

where i_{ss} is the steady-state nominal interest rate and z_t is a monetary policy disturbance.

Aggregate output is given by $Y_t = A_t n_t$, because all retailers and intermediate goods firms are symmetric. Goods market equilibrium implies: $Y_t = C_t$.

4 Characterization

Next, we investigate the properties of the baseline model. The model's linearized dynamics are described by a two-equation system akin to those used in the standard New Keynesian model. This expression allows us to analytically derive a condition for the model's determinacy and evaluate the IRFs.

To simplify our analysis, we make the following assumptions: households have log-utility, $\sigma = 1$, symmetric matching function, $m(\mathcal{U}_t, V_t) = \mathcal{U}_t^{0.5} V_t^{0.5}$, and the monetary authority targets zero inflation at the steady state, $\pi_{ss} = 0$. We set $A_{ss} = 1$ and $\Xi_{ss} = \Xi$ at the steady state. The auxiliary variables are defined in Table 1. The full dynamic system is described in Appendix A.1. We solve the model by log-linearizing it and express all variables, except for inflation, in log deviations from a steady state. These deviations are denoted with small letters: for example, $u_t = \log \mathcal{U}_t - \log \mathcal{U}_{ss}$ is the log deviation of the unemployment rate and $y_t = \log Y_t - \log Y_{ss}$ is the log deviation of output.

Parameters			ODE Representation		
Variable		Definition	Variable		Definition
κ	$\equiv$	$\frac{\epsilon}{\Theta}\frac{\mu}{1-\eta}$	a_1	$\equiv$	$\frac{1}{\iota} - \kappa$
J_{ss}	$\equiv$	$\frac{\frac{\mu}{1-\eta}(\rho+\Xi)}{\frac{\epsilon-1}{\epsilon}}$	a_2	$\equiv$	$\frac{\varphi-\iota\kappa\left(\rho-\frac{1}{J_{ss}}\right)}{1-\iota\kappa}$
$\mathcal{U}_{ss}$	$\equiv$	$\frac{\Xi J_{ss}}{1+\Xi J_{ss}}$	a_3	$\equiv$	$\frac{\iota\kappa\gamma(\rho+\Xi)}{1-\iota\kappa}$
γ	$\equiv$	$\Xi + \frac{1}{J_{ss}}$	λ_1	$\equiv$	$\frac{a_2+\sqrt{a_2^2-4a_3}}{2}$
ι	$\equiv$	$\frac{\phi_\pi-1}{\Xi J_{ss}}$	λ_2	$\equiv$	$\frac{a_2-\sqrt{a_2^2-4a_3}}{2}$
φ	$\equiv$	$\rho - \kappa\frac{\phi_\pi-1}{J_{ss}}$	$a_{2,t}$	$\equiv$	$\frac{\frac{\Xi J_{ss}}{1-\phi\pi}F_t(2f_t+\varphi F_t)+\kappa(\rho+\Xi-\gamma)F_t^3}{\left(\frac{\Xi J_{ss}}{\phi\pi-1}-\kappa F_t\right)F_t^2}$
			$a_{3,t}$	$\equiv$	$\frac{\frac{\Xi J_{ss}}{1-\phi\pi}\left(\dot{f}_t F_t-2f_t^2-\varphi f_t F_t\right)+\kappa\gamma(\rho+\xi)F_t^3}{\left(\frac{\Xi J_{ss}}{\phi\pi-1}-\kappa F_t\right)F_t^2}$

Table 1: Auxiliary Variables

The equilibrium system. Assuming there are no shocks, the equilibrium system of linearized equations is given by the following equations (4.1) and (4.2).

Equation (4.1) is the NK Phillips curve that links inflation and unemployment, while equation (4.2) is the linearized Euler equation coupled with the goods clearing conditions and the job-flows equation. The equilibrium system, which involves a two-equation first-order ordinary-differential equation system, can be represented as the second-order homogeneous linear differential equation given by equation (4.3).[8]

For comparison, we also summarize the equilibrium system of the standard formulation. Equation (4.4) is the NK Phillips curve and equation (4.5) is the linearized Euler equation from the conventional NK model.[9] The equilibrium system similarly forms a second-order homogeneous linear differential equation given by (4.6). The contrast highlights the virtue of the NK+SAM limit: unemployment is a predetermined state variable, fixing the initial condition at $u_0 = 0$. In contrast, the conventional NK model leaves the initial condition indeterminate since inflation is a jump variable. Consequently, determinacy in the conventional NK model relies on off-equilibrium threats, as discussed by Atkeson et al. (2010) and Angeletos and Lian (2023).

[8]See Appendix A.1 for details on the derivation.

[9]$\overline{\kappa}$ is the slope of the Phillips curve.

Characteristic ODE Systems

NK+SAM first-order ODE system representation:

$$\dot{\pi}_t = \left(\rho + \frac{\kappa}{J_{ss}} \frac{1 - \Xi J_{ss}}{\frac{\Xi J_{ss}}{\phi_\pi - 1} - \kappa}\right) \pi_t + \left(\frac{\kappa\gamma\left(\rho + \Xi\right)}{1 - \kappa \frac{\phi_\pi - 1}{\Xi J_{ss}}}\right) u_t, \tag{4.1}$$

$$\dot{u}_t = \left(\frac{1 - \phi_\pi}{\Xi J_{ss}}\right) \pi_t. \tag{4.2}$$

Second-order representation:

$$\ddot{u}_t - a_2 \dot{u}_t + a_3 u_t = 0. \tag{4.3}$$

Boundary conditions: $\{u_0, u_\infty\} = \{0, 0\}$.

Conventional NK first-order ODE system representation:

$$\dot{\pi}_t = \rho\pi_t - \overline{\kappa} c_t, \tag{4.4}$$

$$\dot{c}_t = \left(\phi_\pi - 1\right) \pi_t. \tag{4.5}$$

Second-order representation:

$$\ddot{\pi}_t - \rho\dot{\pi}_t + \overline{\kappa}\left(\phi_\pi - 1\right) \pi_t = 0. \tag{4.6}$$

Boundary condition: $\{\pi_\infty\} = \{0\}$ and one free boundary.

Determinacy. The second-order equation representation allows for a simple determinacy analysis.[10] The following proposition summarizes the main result:

Proposition 1 (Sufficient Condition)**.** *Determinacy is achieved if $\phi_\pi < 1$ or $\phi_\pi > 1 + \frac{\Xi J_{ss}}{\kappa}$.*

Proof. See Supplemental Appendix B.3. □

Unlike the standard New Keynesian model, a *sufficient* condition for determinacy is that the Taylor coefficient be *less* than one. For any $\phi_\pi > 1$, there is another region of parameters that guarantees determinacy and one that does not. Figure 1 numerically illustrates this. For any $\phi_\pi > 1$, the system becomes explosive if the separation rate Ξ is above a threshold.

Unsuccessful attempts to construct self-fulfilling equilibria provide intuition for Proposition 1. Assume households form expectations such that $\pi_0 > \pi_{ss} = 0$. With $\phi_\pi < 1$, this results in a

[10]The characteristic equation associated with this differential equation is $\lambda^2 - a_2\lambda + a_3 = 0$. The roots of this equation are given by λ_1 and λ_2. Since u_t is a state variable, the dynamic system requires one positive and one negative characteristic root to ensure a unique and stable optimal trajectory. If no negative characteristic roots exist, the system becomes explosive.

negative real interest rate at $t = 0$, $r_0 < 0$. Given that consumption is predetermined, this leads to lower future consumption through the household's Euler equation. The contraction in future aggregate demand lowers future inflation—through reduced vacancy postings and wages. As inflation is forward-looking, the realized inflation at $t = 0$ falls below zero. Therefore, the case where $\phi_\pi < 1$ precludes self-fulfilling equilibria and secures determinacy.

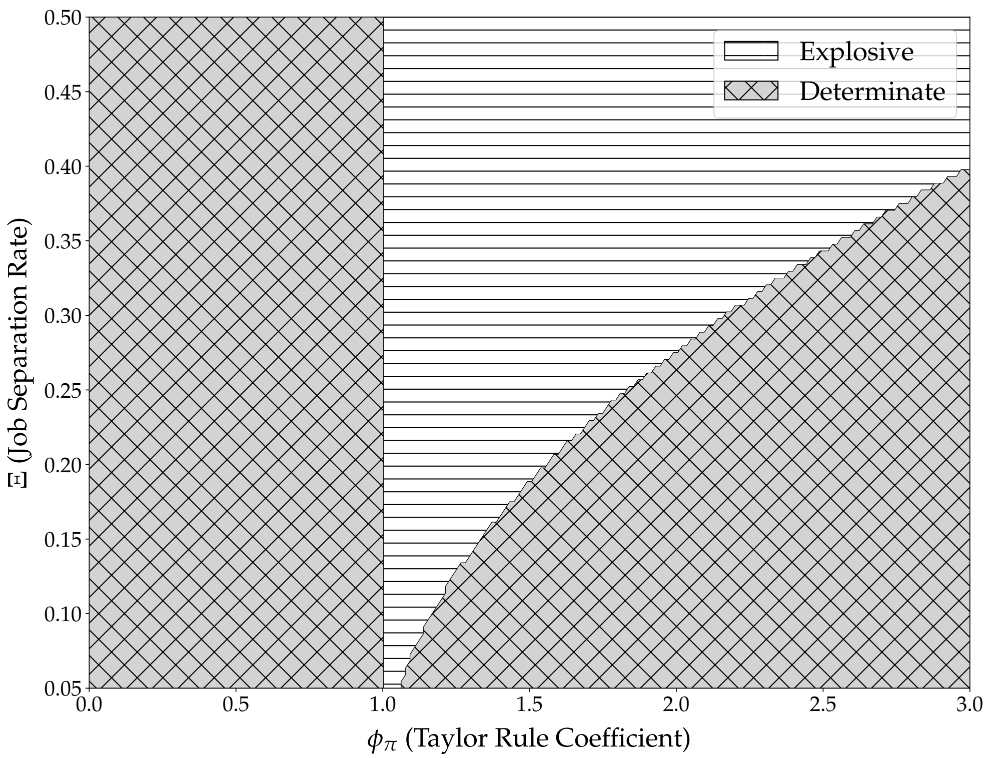

Figure 1: Determinacy Regions

Note: We set $\rho = 0.05$, $\epsilon = 10$, $\Theta = 100$ and $\frac{\mu}{1-\eta} = \frac{1}{3.7}$. The value of $\frac{\mu}{1-\eta}$ is chosen so that the steady-state unemployment rate is 5.7% when $\Xi = 0.42$, which corresponds to an annual job separation rate of 34.4%. The x-axis represents ϕ_π (Taylor rule coefficient) ranging from 0 to 3, while the y-axis represents Ξ ranging from 0.05 to 0.5.

To understand why the transition dynamics become explosive when $\phi_\pi > 1$ and Ξ is high, let's again consider that initial inflation deviates, $\pi_0 > \pi_{ss} = 0$. With $\phi_\pi > 1$, the real interest rate increases at $t = 0$. Given that consumption is predetermined, this leads to consumption growth through the household's Euler equation. The subsequent rise in aggregate demand drives up future inflation. Since inflation is forward-looking, this feeds back into higher inflation at $t = 0$, amplifying the initial deviation and causing the economy to become explosive. Determinacy can be achieved for values $\phi_\pi > 1$ only for low separation rates Ξ. Again, if $\pi_0 > \pi_{ss}$ due to some shock, consumption increases with real rates. This results in a burst of employment, but without the offsetting effect of separations, future wages decrease, causing inflation at $t = 0$ to decline. This downward pressure on inflation acts as a stabilizing force, and if it is sufficiently strong, the system achieves determinacy.

Impulse Responses. A virtue of the two-equation representation is its tractability which renders paper-and-pencil formulas for the IRFs to a monetary policy shock, firing shock ($\xi_t \equiv \log \Xi_t - \log \Xi_{ss}$), and productivity shock ($\alpha_t \equiv \log A_t - \log A_{ss}$).[11] When $\phi_\pi < 1$ the IRF to a productivity shock has the conventional sign, so we only report all responses for that case. All shocks are assumed to be mean-reverting and follow the ordinary differential equations: $\dot{\varsigma}_t = -\delta \varsigma_t$, $\dot{\xi}_t = -\delta \xi_t$, and $\dot{\alpha}_t = -\delta \alpha_t$ where $\delta > 0$ captures its persistence.

Proposition 2. *In response to a monetary policy, firing, and productivity shocks, the path of unemployment is given by:*

$$u_t = \Lambda_t(\delta)\theta_x(x_0, \delta),$$

where $\Lambda_t(\delta) \equiv \frac{e^{-\delta t} - e^{\lambda_2 t}}{a_1} \frac{1}{\delta^2 + \delta a_2 + a_3} \geq 0$*, and* $x \in \{\varsigma, \xi, \alpha\}$ *is an index representing a monetary policy, firing, and productivity shocks respectively.* x_0 *represents the initial value for the shock.*
The functions $\theta_x(x_0, \delta)$ *are defined as:*

$$\theta_\varsigma(\varsigma_0, \delta) = -\varsigma_0 \rho (\delta + \rho), \quad sign(\theta_\varsigma) = -sign(\varsigma_0),$$

$$\theta_\xi(\xi_0, \delta) = \xi_0 \frac{\kappa}{J_{ss}} (\rho + 2\Xi + \delta), \quad sign(\theta_\xi) = sign(\xi_0),$$

$$\theta_\alpha(\alpha_0, \delta) = \alpha_0 \left(\frac{\delta}{\phi_\pi - 1} (\delta + \varphi) - \frac{\epsilon - 1}{\Theta} \frac{1}{1 - \eta} \right), \quad sign(\theta_\alpha) = -sign(\alpha_0).$$

The paths of output, inflation, and vacancy postings are:

$$y_t = -\Xi J_{ss} u_t + \alpha_t,$$

$$\pi_t = \frac{1}{\phi_\pi - 1} \left(-\Xi J_{ss} \dot{u}_t + \dot{\alpha}_t - z_t \right),$$

$$v_t = -(1 + 2\Xi J_{ss}) u_t - 2 J_{ss} \dot{u}_t + 2\xi_t.$$

Proof. See Supplemental Appendix B.4. □

The derived unemployment path allows us to compute the paths of other macroeconomic variables, such as output, inflation, and vacancy postings, since these depend on unemployment and shocks. We discuss and illustrate the proposition with the aid of Figure 2. The figure

[11] For a monetary policy shock, given the path of nominal interest rate $\{i_t = i_{ss} e^{\varsigma_t}\}_{t=0}^{\infty}$, we find the equilibrium unemployment and inflation and subsequently construct the underlying policy innovation (z_t). The dynamics of aggregate variables under this shock are recovered by setting $\phi_t = 0$ and $z_t = i_{ss} \varsigma_t$.

displays the impulse responses of output, inflation rate, vacancy posting, and unemployment rate in response to three types of shocks: a loosening monetary policy shock, a positive firing shock and a positive productivity shock. In the simulation, we set $\phi_\pi = 0$, meaning the nominal interest rate does not respond to changes in the inflation rate. For comparison, we also present the IRFs within the context of a standard two-equation New Keynesian model.

The output responses to a loosening monetary policy is notably different from those in the standard New Keynesian model. When monetary policy loosens—and the monetary authority responds passively, $\phi_\pi < 1$—the real interest rate decreases. According to the household's Euler equation, this decrease in the real interest rate reduces the growth rate of consumption. Since consumption is predetermined, it cannot adjust instantaneously at time 0, leading to decreased consumption, output, and inflation in subsequent periods. The reduction in aggregate demand lowers vacancy postings, which increases the unemployment rate. As time passes, the real interest rate starts to rise, which increases consumption growth and draws the economy back to its steady state.

A positive firing shock, reflected in an increase in the job separation rate, raises the unemployment rate and reduces output. To clear the goods market, consumption decreases, driven by a decline in the real interest rate caused by rising inflation under a fixed nominal interest rate. As the shock dissipates, the economy returns to the steady state, requiring consumption to increase. This is achieved through a rise in the real interest rate, driven by a decline in the inflation rate.

In response to a positive productivity shock, the value of workers rises, prompting intermediate goods firms to increase vacancy postings. This reduces unemployment and raises output through the direct effect of higher productivity and the indirect effect of increased employment. The rise in productivity lowers the marginal cost of production; however, it simultaneously increases the value of workers and wages. The inflation rate rises as the latter effect dominates.

Taking Stock. We have seen that adopting the SAM timing convention inverts the Taylor principle. This avoids the reliance on exuberant off-equilibrium threats to obtain determinancy and, furthermore, enables the analysis of shocks in isolation of monetary policy feedbacks. These are virtues.

However, the main drawback of adopting the SAM convention is that the conventional effects of monetary policy are reversed: lower rates can induce recessions due to the predetermined nature of output. These shifts bring different implications for optimal monetary policy. We could discard the model altogether because of its unconventional sign predictions, but we

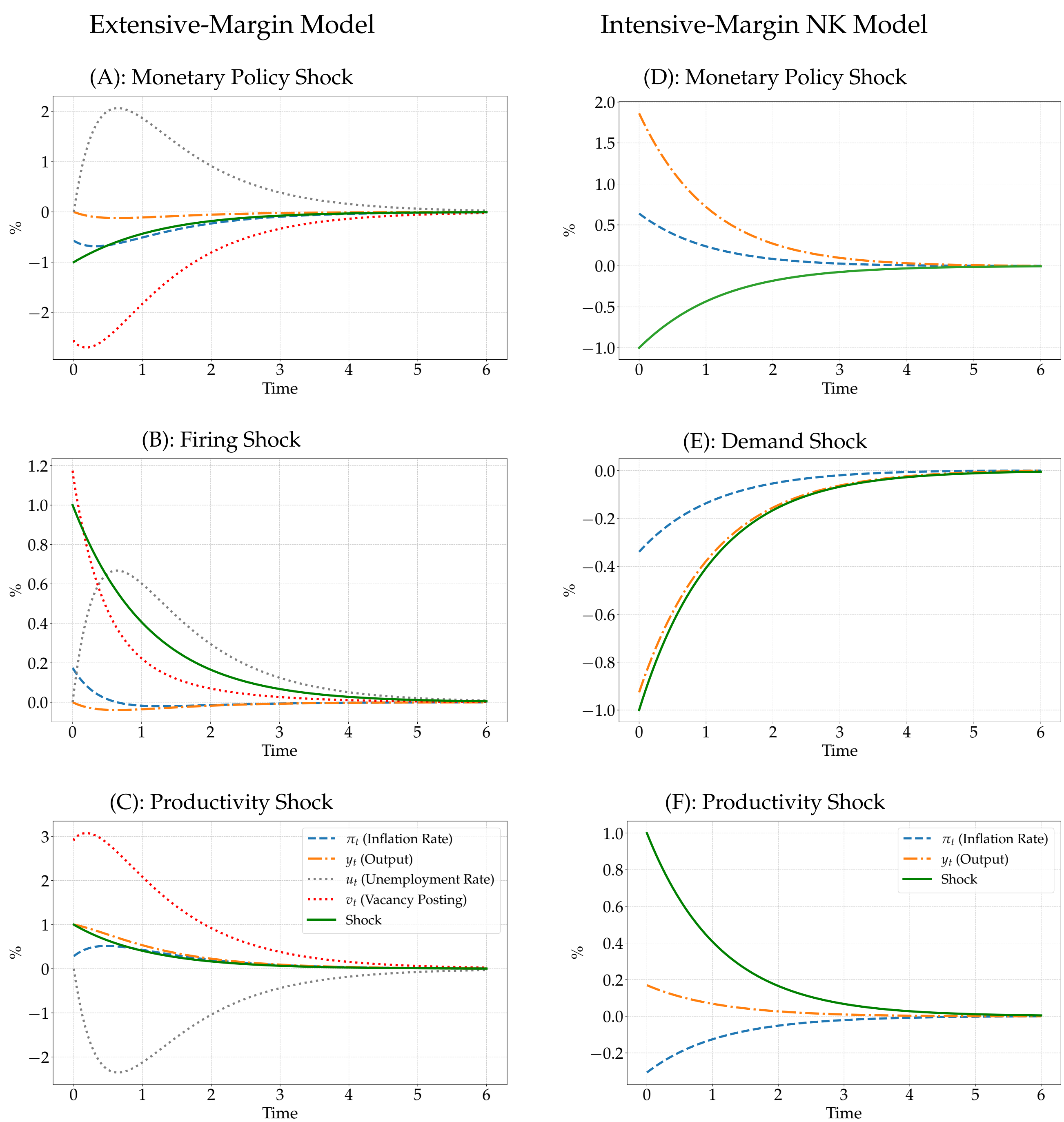

Figure 2: Impulse Response Functions

Note: Panels (A), (B), and (C) show the impulse response functions from our model, while Panels (D), (E), and (F) show the impulse response functions from the standard New Keynesian model. We set $\rho = 0.05$, $\epsilon = 10$, $\Theta = 100$, $\frac{\mu}{1-\eta} = \frac{1}{3.7}$, $\Xi = 0.42$, $\eta = 0.5$, $\overline{\kappa} = 0.35$ and $\delta = 0.9$. In our model, we set $\phi_\pi = 0$, while in the standard New Keynesian model, $\phi_\pi = 1.5$ to achieve determinacy. Output, unemployment rate, vacancy posting, firing shock and productivity shock are expressed as percentage deviations from the steady state, while inflation, demand shock (shock to discount rate), and monetary policy shock are expressed as percentage point differences from the steady state.

would miss the opportunity to build on its virtues. Instead, we conclude by offering an avenue for possible resolutions.

5 Conclusion: Avenues for Problem Resolutions

Limited attention. Next, we consider an unanticipated loosening monetary policy shock under rational inattention.[12] The goal is to correct the problematic signs of the IRFs of monetary policy shocks. To that end, we follow Mankiw and Reis (2007)[13] and let F_t denote the fraction of households aware of the shock by time t, with $f_t \equiv \frac{dF_t}{dt}$ as the probability density function. We assume that all firms are aware of the shock at $t = 0$. The IS equation is modified to:

$$\dot{u}_t = \frac{1 - \phi_\pi}{\Xi J_{ss}} \pi_t F_t + \frac{f_t}{F_t} u_t. \tag{5.1}$$

The equilibrium system is characterized by a modified IS equation (5.1) and the NK Phillips curve (4.1) and also yields a second-order ODE for unemployment:

$$\ddot{u}_t - a_{2,t}\dot{u}_t + a_{3,t}u_t = 0, \tag{5.2}$$

where the coefficients b_t and c_t are now time-varying. When all households recognize the shock, $F_t = 1$ and $f_t = 0$, the equation collapses to the original IS equation (4.2). The boundary conditions are again $u_0 = 0$ and $u_\infty = 0$. Because in this version, output is still pre-determined, we still obtain determinacy without the Taylor principle.

We report the IRFs (numerically) in response to a loosening monetary policy shock in Figure 3: Under rational inattention, the effects of monetary policy have a conventional shape: in response to a loosening monetary policy shock, output rises gradually, while inflation and vacancy postings exhibit upward jumps. The reason for this suecces is that when households acquire information slowly, their individual consumption IRFs feature jumps and mean-reverting decaying paths. However, since the fraction aware of the shock is initially zero and growing, aggregate demand features a positive hump shape in response to a loosening monetary policy shock (Auclert, Rognlie and Straub, 2020, see also). This path is internally consistent in general equilibrium because firms anticipate the rise in consumption and have already increased vacancy postings at $t = 0$.

Importantly, the version with rational inattention preserves the virtues of the setting: it generates hump-shaped responses and achieves determinacy without a Taylor rule, which allows the isolation of shocks without policy feedback and avoids equilibrium selection based on off-equilibrium threats.

[12] The detailed model setup is provided in Appendix A.2

[13] Also see Gabaix and Laibson (2001), Mankiw and Reis (2002), and Reis (2006).

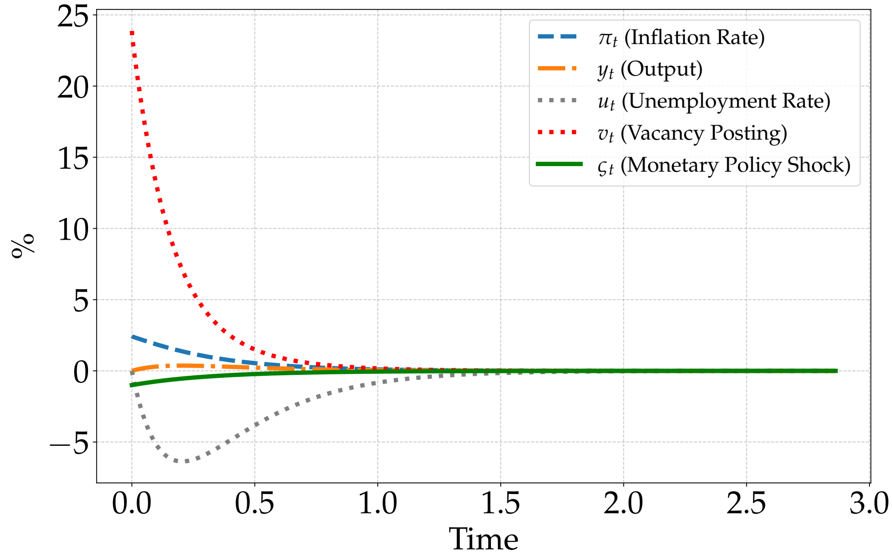

Figure 3: Impulse Response Functions

Note: We set $\rho = 0.05$, $\epsilon = 10$, $\Theta = 100$, $\frac{\mu}{1-\eta} = \frac{1}{3.7}$, $\Xi = 0.42$, and $F_t = 1 - e^{-4t}$. Output, unemployment rate, and vacancy posting are expressed as percentage deviations from the steady state. Inflation and monetary policy shock are expressed as percentage point differences from the steady state.

Intensive Margin. We deliberately chose to study a New Keynesian model that operates exclusively through the extensive margin of labor adjustment. The key feature that alters the behavior of the New Keynesian model is that output is predetermined. A natural reaction to this paper is to contend that some adjustments must also occur through the intensive margin. Yet, even if hours can adjust in the short run, how quickly and by how much the intensive margin can respond to demand conditions is a matter of degree. After all, many labor contracts specify fixed hours, and management has to program hours even if these are flexible. Moreover, even if hours can be adjusted immediately, there's a limit to how much current workers can expand their working hours. There are likely sectoral differences along adjustment margins, leading to predictions regarding relative prices. This aspect highlights the importance of understanding short-run and labor capacity constraints.

To conclude. If hours are hard to adjust, the jump in consumption that characterizes the New Keynesian model will not occur in equilibrium, leading to very different predictions. The lessons in the paper are useful for continuing efforts to marry labor-market flows with New Keynesian models. We think that the marriage offered here is better than others. As in all marriages, it is better not to shove problems under the table. We are confident that the resolutions offered here will someday produce many offspring.

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A Appendix A

A.1 The Dynamic System

The dynamic system is characterized by:

$$\dot{\mathcal{U}}_t = \Xi_t \left(1 - \mathcal{U}_t\right) - \frac{\mathcal{U}_t}{J_t}, \tag{A.1}$$

$$Y_t = A_t \left(1 - \mathcal{U}_t\right), \tag{A.2}$$

$$g_t = \frac{\mu}{J_t}, \tag{A.3}$$

$$\frac{p_t}{P_t} = \frac{\mu}{A_t - \eta} \left(\left(i_t - \pi_t + \Xi_t\right) \frac{1}{J_t} + \frac{\dot{J}_t}{J_t^2} \right), \tag{A.4}$$

$$\rho \pi_t = \frac{\epsilon}{\Theta} \left(\frac{p_t}{P_t} - \frac{\epsilon - 1}{\epsilon} \right) + \dot{\pi}_t, \tag{A.5}$$

$$\frac{\dot{C}_t}{C_t} = i_t - \pi_t - \rho, \tag{A.6}$$

$$i_t = i_{ss} + \phi_\pi \pi_t + z_t, \tag{A.7}$$

$$Y_t = C_t. \tag{A.8}$$

Equation (A.1) describes the transition of the unemployment rate, equation (A.2) is the production function, equation (A.3) provides the expression for the value per worker, equation (A.4) arises from the vacancy posting problem of the intermediate goods firm, equation (A.5) represents the New Keynesian Phillips curve, equation (A.6) is the household's Euler equation, equation (A.7) describes monetary policy, and equation (A.8) is the goods market clearing condition.

The steady states can be calculated as follows: From equation (A.5) and the assumption of $\pi_{ss} = 0$, we obtain $\frac{p_{ss}}{P_{ss}} = \frac{\epsilon-1}{\epsilon}$. Using equation (A.6), we find $i_{ss} = \rho$. Combining these expressions with equation (A.4), we derive $J_{ss} = \frac{\frac{\mu}{1-\eta}(\rho+\Xi)}{\frac{\epsilon-1}{\epsilon}}$. From equation (A.1), we calculate $\mathcal{U}_{ss} = \frac{\Xi J_{ss}}{1+\Xi J_{ss}}$. Finally, using equations (A.2) and (A.8), we find $Y_{ss} = C_{ss} = (1 - \mathcal{U}_{ss})$.

By substituting equations (A.4) and (A.7) into equation (A.5), we obtain:

$$\rho \pi_t = \frac{\epsilon}{\Theta} \frac{\mu}{A_t - \eta} \left(\left(\rho + \left(\phi_\pi - 1\right) \pi_t + z_t + \Xi_t\right) \frac{1}{J_t} + \frac{\dot{J}_t}{J_t^2} \right) - \frac{\epsilon}{\Theta} \frac{\epsilon - 1}{\epsilon} + \dot{\pi}_t. \tag{A.9}$$

By substituting equations (A.2), (A.7), and (A.8) into equation (A.6), we obtain:

$$\frac{\dot{A}_t}{A_t} - \frac{\dot{\mathcal{U}}_t}{1 - \mathcal{U}_t} = \left(\phi_\pi - 1\right) \pi_t + z_t. \tag{A.10}$$

The system is now described by equations (A.1), (A.9), and (A.10). Log-linearization around the steady state yields the following equations:

$$j_t = (1 + \Xi J_{ss})\, u_t + J_{ss}\dot{u}_t - \xi_t, \tag{A.11}$$

$$\dot{\pi}_t = \varphi\pi_t + \frac{\kappa}{J_{ss}}\left((\rho + \Xi)\, j_t - \dot{j}_t\right) - \frac{\kappa}{J_{ss}}\left(z_t + \Xi\xi_t\right) + \frac{\epsilon - 1}{\Theta}\frac{\alpha_t}{1-\eta}, \tag{A.12}$$

$$\pi_t = \frac{\Xi J_{ss}}{1 - \phi_\pi}\dot{u}_t + \frac{1}{1-\phi_\pi}\left(z_t - \dot{\alpha}_t\right). \tag{A.13}$$

In the absence of shocks ($z_t = \xi_t = \alpha_t = 0$), equation (A.13) simplifies to equation (4.2). Equation (A.11) demonstrates that j_t depends on u_t and $\dot{u}_t$, while $\dot{j}_t$ depends on $\dot{u}_t$ and $\ddot{u}_t$. By substituting these relationships into equation (A.12), we obtain:

$$\begin{aligned}\dot{\pi}_t = \varphi\pi_t + \kappa\left\{\left((\rho+\Xi)\,\dot{u}_t - \ddot{u}_t\right) + \gamma\left((\rho+\Xi)\, u_t - \dot{u}_t\right)\right\} \\ + \frac{\kappa}{J_{ss}}\left(-z_t - (\rho + 2\Xi)\,\xi_t + \dot{\xi}_t\right) + \frac{\epsilon-1}{\Theta}\frac{\alpha_t}{1-\eta}.\end{aligned} \tag{A.14}$$

Equation (A.13) shows that π_t depends on $\dot{u}_t$, z_t, and $\dot{\alpha}_t$, and that $\dot{\pi}_t$ depends on $\ddot{u}_t$, $\dot{z}_t$, and $\ddot{\alpha}_t$. By substituting these relationships into equation (A.14) and assuming the AR(1) shock processes, we obtain:

$$\ddot{u}_t - a_2\dot{u}_t + a_3 u_t = \frac{z_t}{a_1}\frac{1}{\phi_\pi - 1}(\delta+\rho) + \frac{\xi_t}{a_1}\frac{\kappa}{J_{ss}}(\rho + 2\Xi + \delta) + \frac{\alpha_t}{a_1}\left(\frac{\varphi+\delta}{\phi_\pi - 1}\delta - \frac{\epsilon-1}{\Theta}\frac{1}{1-\eta}\right). \tag{A.15}$$

In the absence of shocks, equation (A.15) simplifies to equation (4.3). By substituting equation (4.2) and its time derivative into equation (4.3), we obtain equation (4.1).

A.2 Extension: Limited Attention

This extension introduces limited attention on the household side, while the supply side of the model remains unchanged from the benchmark. Households are classified into two groups: inattentive households, who are unaware of the shock, and attentive households, who have recognized the shock.

For inattentive households, consumption remains fixed at the steady-state level, as we are considering an unanticipated shock:

$$C_{t,inattentive} = C_{ss}. \tag{A.16}$$

Attentive households adjust their consumption based on the Euler equation and the intertemporal budget constraint:

$$\frac{\dot{C}_{t,attentive}}{C_{t,attentive}} = i_t - \pi_t - \rho, \tag{A.17}$$

$$\int_t^\infty C_{\tau,attentive} e^{-\int_t^\tau r_s ds} d\tau \leq a_t + h_t, \tag{A.18}$$

where a_t represents financial assets, and h_t is human wealth, defined as the discounted sum of future income:

$$h_t = \int_t^\infty e^{-\int_t^\tau r_s ds} Y_\tau d\tau. \tag{A.19}$$

Following Mankiw and Reis (2007), an insurance contract ensures equal financial wealth across households at the beginning of each period. As a result, attentive households share identical financial assets and consumption levels, regardless of when they recognize the shock:

$$C_{t,attentive} = \rho \left(a_t + h_t \right). \tag{A.20}$$

Aggregate consumption is given by:

$$C_t = F_t C_{t,attentive} + \left(1 - F_t\right) C_{t,inattentive}. \tag{A.21}$$

Log-linearizing this expression yields:

$$c_t = F_t c_{t,attentative}. \tag{A.22}$$

Taking the time derivative and using the log-linearized Euler equation (A.17) for attentive households, we obtain:

$$\dot{c}_t = \frac{f_t}{F_t} c_t + F_t \left(i_t - \pi_t - \rho \right) \tag{A.23}$$

By combining this equation with the log-linearized versions of equations (A.2) and (A.8), equation (A.7), and assuming no shock, we recover equation (5.1) in the main text.

B Appendix B (Supplemental Appendix)

B.1 Proof of Lemma 1

The maximization problem is

$$q\left(p_t^j, t\right) = \max_{\left\{\dot{p}_\tau^j\right\}} \int_t^\infty \frac{\exp\left(-r_\tau \tau\right)}{P_\tau} \left(\left(p_\tau^j - p_t\right) y_{j,\tau} - P_\tau \frac{\Theta}{2} \left(\frac{\dot{p}_\tau^j}{p_\tau^j}\right)^2 Y_\tau\right) d\tau,$$

$$s.t. \quad y_\tau^j = \left(\frac{p_\tau^j}{P_\tau}\right)^{-\epsilon} Y_\tau.$$

Then, we use the fact that:

$$P_\tau = P_t \exp\left(\int_t^\tau \pi_s ds\right),$$

to express the objective as:

$$q\left(p_t^j, t\right) P_t = \max_{\left\{\dot{p}_\tau^j\right\}} \int_t^\infty \exp\left(\int_t^\tau \left(-\pi_s - r_\tau\right) ds\right) \left(\left(p_\tau^j - p_t\right) y_{j,\tau} - P_\tau \frac{\Theta}{2} \left(\frac{\dot{p}_\tau^j}{p_\tau^j}\right)^2 Y_\tau\right) d\tau,$$

$$s.t. \quad y_\tau^j = \left(\frac{p_\tau^j}{P_\tau}\right)^{-\epsilon} Y_\tau.$$

This expression leads to the HJB equation for the nominal value, $Q\left(p_t^j, t\right) \equiv q\left(p_t^j, t\right) P_t$. The corresponding HJB equation is:

$$\left(r_t + \pi_t\right) Q\left(p_t^j, t\right) = \max_{\left\{\dot{p}_\tau^j\right\}} \left(p_t^j - p_t\right) \left(\frac{p_t^j}{P_t}\right)^{-\epsilon} Y_t - P_t \frac{\Theta}{2} \left(\frac{\dot{p}_\tau^j}{p_\tau^j}\right)^2 Y_t + Q_p \dot{p}_t^j + \dot{Q}.$$

Next, we obtain the first-order condition:

$$Q_p - \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t = 0. \tag{B.1}$$

Differentiating this condition with respect to time yields:

$$Q_{pp} \dot{p}_t^j + \dot{Q}_p = \Theta \left(\frac{\ddot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t - 2\left(\frac{\dot{p}_t^j}{p_t^j}\right)^2 \frac{P_t}{p_t^j} Y_t\right) + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} \dot{P}_t Y_t + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t \dot{Y}_t. \tag{B.2}$$

Next, we produce the envelope condition. For that, we take the derivative of $Q\left(p_t^j, t\right)$ with respect to the individual price p_t^j. We obtain:

$$(r_t + \pi_t)\, Q_p = -\epsilon \left(p_t^j - p_t\right)\left(p_t^j\right)^{-\epsilon-1} P_t^{\epsilon} Y_t + \left(p_t^j\right)^{-\epsilon} P_t^{\epsilon} Y_t + \Theta \left(\frac{\dot{p}_t^j}{p_t^j}\right)^2 \frac{P_t}{p_t^j} Y_t + Q_{pp}\dot{p}_t^j + \dot{Q}_p. \tag{B.3}$$

Substituting (B.2) and (B.1) into (B.3), we obtain:

$$(r_t + \pi_t)\, \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t = -\epsilon \left(p_t^j - p_t\right) \frac{y_t^j}{p_t^j} + y_t^j + \Theta \left(\frac{\dot{p}_t^j}{p_t^j}\right)^2 \frac{P_t}{p_t^j} Y_t$$
$$+ \Theta \left(\frac{\ddot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t - 2 \left(\frac{\dot{p}_t^j}{p_t^j}\right)^2 \frac{P_t}{p_t^j} Y_t \right) + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} \dot{P}_t Y_t + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t \dot{Y}_t.$$

Simplifying terms, we obtain:

$$(r_t + \pi_t)\, \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t = -\epsilon \left(p_t^j - p_t\right) \frac{y_t^j}{p_t^j} + y_t^j$$
$$+ \Theta \left(\frac{\ddot{p}_t^j}{\left(p_t^j\right)^2} P_t Y_t - \left(\frac{\dot{p}_t^j}{p_t^j}\right)^2 \frac{P_t}{p_t^j} Y_t \right) + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} \dot{P}_t Y_t + \Theta \frac{\dot{p}_t^j}{\left(p_t^j\right)^2} P_t \dot{Y}_t.$$

Now, using that all retailers act identically, we substitute, $p_t^j = P_t$ and $y_t^j = Y_t$, and obtain:

$$(r_t + \pi_t)\, \Theta \frac{\dot{P}_t}{P_t^2} P_t Y_t = -\epsilon \left(P_t - p_t\right) P_t^{-1} Y_t + Y_t + \Theta \left(\frac{\ddot{P}_t}{P_t^2} P_t Y_t - \left(\frac{\dot{P}_t}{P_t}\right)^2 Y_t \right) + \Theta \frac{\dot{P}_t}{P_t^2} \dot{P}_t Y_t + \Theta \frac{\dot{P}_t}{P_t^2} P_t \dot{Y}_t.$$

Recall that inflation and the price acceleration are:

$$\pi_t = \frac{\dot{P}_t}{P_t},$$

and

$$\dot{\pi}_t = \frac{\ddot{P}_t}{P_t} - \left(\frac{\dot{P}_t}{P_t}\right)^2.$$

Replacing these conditions, in the condition above, we arrive that the following condition:

$$r_t \Theta \pi_t Y_t + \Theta \pi_t^2 Y_t \;=\; \left\{ 1 - \epsilon \left(1 - \frac{p_t}{P_t}\right) \right\} Y_t + \Theta \dot{\pi}_t Y_t + \Theta \pi_t^2 Y_t + \Theta \pi_t \dot{Y}_t.$$

Equivalently

$$r_t \pi_t \;=\; \frac{1 - \epsilon \left(1 - \frac{p_t}{P_t}\right)}{\Theta} + \dot{\pi}_t + \pi_t \frac{\dot{Y}_t}{Y_t}.$$

Denote $mc_t \equiv \frac{p_t}{P_t}$ which is real marginal cost for retailers. Then, we arrive at the NK Phillips curve:

$$\left(r_t - \frac{\dot{Y}_t}{Y_t}\right)\pi_t = \frac{\epsilon}{\Theta}\left(mc_t - \frac{\epsilon-1}{\epsilon}\right) + \dot{\pi}_t.$$

B.2 Proof of Lemma 2

The maximization problem is

$$G(n,t) = \max_{\{V_\tau\}} \int_t^\infty \exp(-r_\tau \tau)\left[\frac{p_\tau A_\tau}{P_\tau} n_\tau - \frac{w_\tau}{P_\tau} n_\tau - V_\tau \mu\right] d\tau,$$
$$s.t. \quad \dot{n}_\tau = J_\tau V_\tau - \Xi_\tau n_\tau.$$

This expression leads to the following HJB equation for the intermediate good firm. The corresponding equation is:

$$r_t G(n,t) = \max_{\{V_t\}} \frac{p_t}{P_t} A_t n_t - \frac{w_t}{P_t} n_t - V_t \mu + G_n \dot{n}_t + \dot{G},$$
$$s.t. \quad \dot{n}_t = J_t V_t - \Xi_t n_t.$$

And substituting the law of motion obtains:

$$r_t G(n,t) = \max_{\{V_t\}} \frac{p_t}{P_t} A_t n_t - \frac{w_t}{P_t} n_t - V_t \mu + G_n (J_t V_t - \Xi_t n_t) + \dot{G}.$$

The first-order condition with respect to V_t is:

$$G_n = \frac{\mu}{J_t}.$$

We conjecture the value function as follows:

$$G(n,t) = g_t n_t.$$

We verify this guess later. Under this assumption, we have

$$g_t = \frac{\mu}{J_t}.$$

This relationship which must hold in a solution with finite vacancies. Then, we replace the condition into the HJB equation and using our guess obtain:

$$(r_t + \Xi_t) g_t n_t = \left(\frac{p_t}{P_t} A_t - \frac{w_t}{P_t}\right) n_t + \dot{g}_t n_t.$$

Hence, we obtain:

$$(r_t + \Xi_t) g_t = \left(\frac{p_t}{P_t} A_t - \frac{w_t}{P_t}\right) + \dot{g}_t,$$

which verifies our conjecture. Then, we obtain its integral solution.

$$g_t = \int_t^\infty \exp\left(-(\tau - t)(r_t + \Xi_t)\right)\left(\frac{p_\tau}{P_\tau} A_\tau - \frac{w_\tau}{P_\tau}\right) d\tau.$$

B.3 Proof of Proposition 1

If $\phi_\pi < 1$ or $\phi_\pi > 1 + \frac{\Xi J_{ss}}{\kappa}$, then $4a_3 < 0$ because $\kappa\gamma(\rho + \Xi) > 0$. This ensures that $a_2 < \sqrt{a_2^2 - 4a_3}$, leading to one positive and one negative characteristic root.

B.4 Proof of Proposition 2

Loosening Monetary Policy Shock. We consider a loosening monetary policy shock $\varsigma_0 < 0$. The dynamics of aggregate variables under this shock are recovered by setting $\phi_\pi = 0$ and $z_t = i_{ss}\varsigma_t$. Substituting these relationship into equation (A.15) yields the following second-order non-homogeneous differential equation:

$$\ddot{u}_t - a_2\dot{u}_t + a_3 u_t = -\tau_t,$$

where $\tau_t = -\frac{1}{a_1}(-\rho)(\delta + \rho)\varsigma_t$.

The general solution of the differential equation consists of homogeneous and particular solutions. The homogeneous solution is:

$$u_t^{(h)} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t},$$

where $\lambda_1 = \frac{a_2 + \sqrt{a_2^2 - 4a_3}}{2} > 0$ and $\lambda_2 = \frac{a_2 - \sqrt{a_2^2 - 4a_3}}{2} < 0$. The particular solution, derived from τ_t, is:

$$u_t^{(p)} = \frac{1}{\lambda_1 - \lambda_2}\left(\int_{s=t}^{\infty} e^{\lambda_1(t-s)}\tau_s\, ds + \int_{s=-\infty}^{t} e^{\lambda_2(t-s)}\tau_s\, ds\right).$$

The general solution is the sum of the homogeneous solution and the particular solution:

$$u_t = u_t^{(h)} + u_t^{(p)} = C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} + \frac{1}{\lambda_1 - \lambda_2}\left(\int_{s=t}^{\infty} e^{\lambda_1(t-s)}\tau_s\, ds + \int_{s=-\infty}^{t} e^{\lambda_2(t-s)}\tau_s\, ds\right).$$

Using $\dot{\varsigma}_s = -\delta\varsigma_s$, we have $\varsigma_s = e^{-\delta s}\varsigma_0$. Substituting τ_s into the particular solution yields:

$$\begin{aligned} u_t &= C_1 e^{\lambda_1 t} + C_2 e^{\lambda_2 t} \\ &\quad - \frac{1}{\lambda_1 - \lambda_2}\frac{1}{a_1}(-\rho)(\delta + \rho)\varsigma_0\left(e^{\lambda_1 t}\int_{s=t}^{\infty} e^{-(\lambda_1+\delta)s}\, ds + e^{\lambda_2 t}\int_{s=-\infty}^{t} e^{-(\lambda_2+\delta)s}\, ds\right). \end{aligned} \tag{B.4}$$

The constants C_1 and C_2 are determined by the initial condition $u_0 = 0$ and the terminal condition $u_\infty = 0$. The terminal condition implies $C_1 = 0$ because $\lambda_1 > 0 > \lambda_2$. C_2 is determined by satisfying $u_0 = 0$:

$$C_2 = \frac{1}{\lambda_1 - \lambda_2}\frac{1}{a_1}(-\rho)(\eta_\epsilon + \rho)\varsigma_0\left(\int_{s=0}^{\infty} e^{-(\lambda_1+\delta)s} ds + \int_{s=-\infty}^{0} e^{-(\lambda_2+\delta)s} ds\right).$$

Substituting, we obtain:

$$u_t = -\varsigma_0\rho(\delta + \rho)\frac{1}{\lambda_1 - \lambda_2}\frac{1}{a_1}\left(\frac{1}{\lambda_1 + \delta} - \frac{1}{\lambda_2 + \delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right).$$

We consider two cases to show $\left(\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right) \leq 0$ where equality holds at $t = 0$ or $e^{\lambda_2 t} - e^{-\delta t} = 0$.

Case (i): $-\delta < \lambda_2 < 0$. In this case, $e^{-\delta t} \leq e^{\lambda_2 t}$, implying $e^{\lambda_2 t} - e^{-\delta t} \geq 0$. Additionally, $0 < \lambda_2 + \delta < \lambda_1 + \delta$, so $\frac{1}{(\lambda_1+\delta)} - \frac{1}{(\lambda_2+\delta)} < 0$. Therefore, $\left(\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right) \leq 0$.

Case (ii): $\lambda_2 < -\delta < 0$. In this case, $e^{\lambda_2 t} \leq e^{-\delta t}$, which implies $e^{\lambda_2 t} - e^{-\delta t} \leq 0$. Also, $\lambda_2 + \delta < 0 < \lambda_1 + \delta$, so $\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta} > 0$. Therefore, $\left(\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right) \leq 0$.

Thus, we have

$$\begin{aligned} u_t &= \underbrace{-\varsigma_0}_{>0} \underbrace{\rho}_{>0} \underbrace{(\delta+\rho)}_{>0} \underbrace{\frac{1}{\lambda_1 - \lambda_2}}_{>0} \underbrace{\frac{1}{a_1}}_{<0} \underbrace{\left(\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right)}_{\leq 0} \\ &= \underbrace{-\varsigma_0 \rho (\delta+\rho)}_{>0} \underbrace{\Lambda_t}_{\geq 0} \geq 0. \end{aligned}$$

where $\Lambda_t = \frac{1}{\lambda_1-\lambda_2}\frac{1}{a_1}\left(\frac{1}{\lambda_1+\delta} - \frac{1}{\lambda_2+\delta}\right)\left(e^{\lambda_2 t} - e^{-\delta t}\right)$. By using the definitions of λ_1 and λ_2, Λ_t can be simplified as: $\Lambda_t = \frac{e^{-\delta t} - e^{\lambda_2 t}}{a_1} \frac{1}{\delta^2 + \delta a_2 + a_3}$.

The dynamics of output (y_t) are derived by log-linearizing equation (A.2). The dynamics of inflation (π_t) follow from the linearized IS equation (A.13). The dynamics of vacancy postings (v_t) are obtained from the definition of the job finding rate combined with equation (A.11), which is the linearized job flow equation.

Positive Firing Shock and Positive Productivity Shock. The derivation is analogous to the case of a loosening monetary policy shock. From equation (A.15), for a positive firing shock, we use the following non-homogeneous second-order differential equation:

$$\ddot{u}_t - a_2 \dot{u}_t + a_3 u_t = \frac{\xi_t}{a_1} \frac{\kappa}{J_{ss}} (\rho + 2\Xi + \delta). \tag{B.5}$$

Similarly, for a positive productivity shock, the non-homogeneous second-order differential equation is:

$$\ddot{u}_t - a_2 \dot{u}_t + a_3 u_t = \frac{\alpha_t}{a_1} \left(\frac{\varphi + \delta}{\phi_\pi - 1} \delta - \frac{\epsilon - 1}{\Theta} \frac{1}{1 - \eta} \right). \tag{B.6}$$