A Q-Theory Of Banks

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October 2020

Abstract

We document five facts about banks: (1) market and book leverage diverged during the 2008 crisis, (2) Tobin's Q predicts future profitability, (3) neither book nor market leverage appears constrained, (4) banks maintain a market leverage target that is reached slowly, (5) pre-crisis, leverage was predominantly adjusted by liquidating assets. After the crisis, the adjustment shifted towards retaining earnings. We present a Q-theory where leverage notions differ because book accounting is slow to acknowledge loan losses. We estimate the model and show that it reproduces the facts. We examine counterfactuals: different accounting rules produce a novel policy tradeoff.

Keywords: Banks, Tobin's Q, Delayed Accounting, Adjustment Costs
JEL: G21, G32, G33, E44

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1 Introduction

Models of banks are important because they shape financial regulation and, through regulation, macroeconomic outcomes. However, developing suitable frameworks for policy analysis is a fine-tuning process in which regulators and academics alike are constantly reassessing their models. In fact, bank regulation was entirely redesigned in the aftermath of the Great Recession, only to meet a global wave of regulatory forbearance in the first major crisis that followed, the Covid-19 crisis. Hence, the continuous development of banking models is a macroeconomic priority and understanding the dynamics of bank leverage, in particular, is critical to that process.

This paper presents a novel theory of banks. Our theory is motivated by five stylized facts that inform us about the dynamics of bank leverage. A unifying theme among these facts are both cross-sectional and time series variations in Tobin's Q, i.e., variations in the market-to-book equity ratio. A novel aspect of our theory is the slow recognition of loan losses on banks' accounting statements. This slow loan loss recognition causes the fundamental value of loans, which does take into account loan losses, to differ from the book value of loans, which does not. We label the ratio of fundamental-to-book values as little q, which differs from big Q (Tobin's Q). In our theory, variations in Q are in part driven by variations in q. Quantitatively, slow loan loss recognition is essential to explain the five stylized facts. We argue that our Q-theory improves our understanding of bank leverage dynamics and illustrates an important policy tradeoff.

The five facts that motivate our Q-theory are:

1. Banks' book and market leverage ratios behaved very differently during the 2008-2009 crisis. Market leverage rose dramatically during the crisis whereas book leverage remained constant. In particular, between 2007 Q3 and 2014 Q4, bank holding companies lost 54% of their market capitalization. Book equity losses were only 7% and this loss was entirely made up by equity issuances.

2. Market values capture information that book values do not: in particular, information on future portfolio losses and profitability.

3. The cross-section of market leverage shows a large dispersion across banks. That dispersion is considerably smaller for book leverage and few banks were close to their regulatory constraints, even in the midst of the crisis.

4. Banks appear to operate with a target for the market leverage ratio. The adjustment to that target is slow: In response to an unexpected negative net-worth shock, proxied by a shock to stock returns, market-leverage increases on impact and take several years to return to the initial level. By contrast, book leverage does not respond on impact and the overall response is muted.

5. Prior to the crisis, in response to an unexpected negative net worth shock, banks would primarily sell assets to return to their market-leverage target. Post-crisis, banks intensified the use of retained earnings and equity issuances, and reduced the extent of adjustment through asset sales as means to return to their market leverage targets.

Fact 1 emphasizes the difference between the banks' book value of equity and their corresponding market value.[1] Understanding this difference is important because in models we must take a stance on whether book or market equity (or both) is the relevant state variable.[2] This stance matters for evaluating the quantitative performance of models: empirically, book-measured leverage and market-measured leverage lead to different inferences about the time series properties of leverage and the price of risk (see the debate between Adrian et al. 2014 and He et al. 2017), but typically both measures co-move in models. Fact 2 implies that market valuations are informative about bank losses much earlier than when losses are reported on the balance sheet.[3] Fact 3 points to a rich cross-section of market and book leverage ratios. Even before the 2008 crisis, the cross-sectional dispersion of market leverage was wide. During the crisis, both the average market-leverage and its cross-sectional dispersion increased dramatically.[4] In terms of book leverage, the pre-crisis distribution was more concentrated and most banks maintained a substantial equity buffer beyond the regulatory requirements. In the midst of the 2008-2009 crisis, the regulatory capital ratio of the vast majority of banks remained far above their regulatory limits, even among banks whose market valuations eroded significantly.[5] Taken together, these facts suggest that although market values of bank equity are good predictors of bank health, banks' market leverage does not appear to be constrained as it increased dramatically for many banks during the crisis. In turn, book equity values are not a timely predictor of bank health, but regulatory constraints are explicitly stated in terms of book equity. Since, book values take time to incorporate information on losses, regulatory constraints may not be binding even after a negative shock to banks' assets. In this paper, we argue that models should account for (i) the differences between book and market equity, and (ii) how the slow loan loss recognition mechanism affects the tightness of banks' regulatory constraints.

[1] This dichotomy between the banks' book and market values of equity has prompted policy discussions in earlier banking crisis (see post savings and loan crisis survery in Berger et al., 1995).

[2] Papers that study the asset pricing implications of intermediary net worth (e.g., He and Krishnamurthy, 2013 and Brunnermeier and Sannikov 2014) focus on market measures of equity, while papers focused on the effects of regulation focus on book measures of equity (e.g., Adrian and Boyarchenko 2013; Begenau 2020; Adrian and Shin 2013; Corbae and D'Erasmo 2019; Begenau and Landvoigt 2020).

[3] This observation is consistent with Blattner et al. (2019). This conclusion is also shared with the accounting literature (see Laux and Leuz, 2010) that explains how banks have flexibility in accounting for losses. In fact, this was an issue raised by the United States Congress after the Savings and Loans crisis (General Accounting Office, 1990).

[4] This suggests a countercyclical average leverage ratio. He et al. (2010) and He et al. (2017) document that market-based leverage for intermediaries is countercyclical. Using the book equity definition, Adrian and Shin (2013) show that broker dealer leverage is procyclical.

[5] An extreme example is Citibank, a bank that experienced market-based losses of up to 90% with only a minor changes in its book equity.

Facts 4 and 5 relate to the adjustment dynamics of banks' balance sheet after a negative shock to their assets. It is challenging to empirically identify such shocks: according to Fact 2, accounting measures of bank equity do not convey all available information about shocks to bank wealth. On the flip side, market valuations may capture additional information not contained in books. But market valuations are also affected by variation in risk premia unrelated to an individual bank's health. To tackle this identification challenge, we exploit the cross-sectional variation in banks' stock returns and estimate impulse-responses to innovations in individual bank stock returns. The strategy builds on the efficient-market hypothesis: The idea is that once we introduce adequate statistical controls, idiosyncratic deviations from the average market returns pick-up idiosyncratic information about banks' effective net worth that is not contained in their books. Thus, we contend that idiosyncratic return shocks proxy for net worth shocks. Once we construct a time series of return shocks for each bank, we estimate the average impulse-responses of market and book leverage, liabilities, dividends, equity, and other variables to a return shock. These impulse responses shed light on the adjustment process after shocks that affect bank net worth.

Fact 4 is obtained from this impulse-response analysis: The main observation is that the behavior of banks is consistent with a pattern where they maintain a target for market-based leverage, but the adjustment process to return to that target is gradual. Namely, in response to a negative return shock, which mechanically increases market-leverage on impact, banks take actions to slowly reduce market-leverage to their targets. This gradual response indicates that frictions prevent banks from immediately returning to their leverage targets.[6] Fact 5 describes the actual deleveraging process, both before and after the 2008 financial crisis. Pre-crisis, a bank that experienced negative return shocks relied predominantly on the reduction of liabilities to lower its leverage, with almost no change in book equity.[7] During and after the financial crisis, the deleveraging process was faster, but the reduction of bank debt slowed down. Instead, the increase in the deleveraging speed followed from an increase in retained earnings and equity issuance.[8] Facts 5 suggests that there was a regime shift in the frictions that govern the leverage dynamics. Taken together, Facts 4 and 5 call for a theory that can account for the slow balance-sheet adjustment process and how this process changed over time.

The paper presents a partial-equilibrium model of banks that meets the challenges brought by these facts. To reproduce these facts, the model features (1) meaningful differences between book and market values, (2) accounting valuations that are less responsive than market values to loan default shocks, (3) a rich cross-section of book and market leverage ratios, (4) an endogenous leverage ratio target and frictions that lower the speed of adjustment to that target, (5) a regime

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[6]Gropp and Heider (2010) document that banks capital ratios appear to follow a target leverage ratio and analyzes the empirical drivers of that target capital ratio.

[7]This is consistent with Figure 2 in Adrian and Shin (2013).

[8]One interpretation of this fact is that the cost of liquidating assets increased during the crisis, presumably because the adverse selection problem gets worse. This interpretation is consistent with models proposed by Gorton and Ordonez (2011) and Dang et al. (2017).

shift that can explain the switch in the deleveraging process.

In our model, banks are owned by diversified shareholders. Banks maximize the value of the discounted stream of dividend income, under risk neutrality. Banks issue deposits and loans, where loans are exposed to default shocks. The supply (demand) of deposits (loans) is perfectly elastic. The expected return spread between loans and deposits is positive and constant, a feature that makes the return on equity increasing in leverage. However, loan default shocks expose a bank to liquidation risk, if its leverage is too high. In particular, a bank is liquidated if it violates regulatory constraints or if it becomes insolvent. As in Leland and Pyle (1977a), there is a tradeoff between levered returns and liquidation risk. This tradeoff induces a notion of a target for fundamental leverage, which differs from book or market leverage.[9] This tradeoff is present even though banks are owned by diversified shareholders and thus, behave as risk-neutral firms. The model has several frictions that drive a wedge between book and market values, and produce a slow return to the market-leverage target. First, banks do not raise equity and have a preference for dividend smoothing. This friction prevents market leverage from adjusting immediately via equity finance. Furthermore, this friction drives a wedge between a dollar inside and outside the bank, one source of variation in Tobin's Q.[10] Second, the market value of loans reflects losses that book values do not. Specifically, only a fraction of loan defaults are recognized immediately in the books and the full recognition of losses takes time. Delayed accounting is a second source of variation in Tobin's Q. The combination of dividend smoothing and delayed accounting already produces smooth responses of market leverage, similar to those observed in the data. Finally, the theory features cost of reselling loans, modeled as price adjustment costs in the spirit of O'Hara (1983) or Shleifer and Vishny (1997).

We calibrate the parameters of the model except for three parameters that we estimate. Each of these three parameters corresponds to a financial frictions in the model. We estimate these parameters to gauge the importance of each friction to explain our facts. Concretely, we estimate the parameters that govern banks' dividend smoothing motive, delayed loan loss recognition, and the loan-adjustment cost. We estimate these parameters by matching the impulse responses to return shocks of market leverage, book leverage, and liabilities. Identification is obtained as follows: the difference between the responses of market and book leverage, are informative about the delayed recognition of losses. Once this parameter is obtained, the response of market leverage and liabilities, are informative about the degree of dividend smoothing and loan adjustment costs, respectively. A striking result is that to match the pre-crisis responses, the estimation requires virtually no loan adjustment costs. That is, to match the pre-crisis impulse responses, the model

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[9]In corporate finance, it is standard to explain a leverage target through either tradeoff theory (a tax advantage) or a risk-return tradeoff (e.g. Kraus and Litzenberger, 1973, Leland and Pyle, 1977a, Myers, 1984, Hennessy and Whited, 2005, Frank and Goyal, 2011). An early dynamic model of banks, O'Hara (1983), a leverage target follows from undiversified ownership that induces risk-averse behavior.

[10]Because of its importance, a slow adjustment of leverage was the topic of Darrell Duffie's presidential address to the American Finance Society in 2010 (Duffie, 2010).

only needs dividend smoothing and delayed loan loss recognition. The economic intuition behind this result is that, because banks do not recognize losses immediately and prefer to stay levered, they do not take immediate actions to delever when hit by a loan default shock. This reflects in an immediate responses of market leverage upon the shock, but only a negligible response of book leverage. Over time, banks delever gradually, as loan losses are slowly recognized on the books. As losses are slowly recognized, the regulatory constraints tighten over time. As a result, banks need to reduce debt, but only at the pace at which losses are recognized. Through this channel, delayed accounting is enough to explain the slow responses of market leverage through a slow reduction of liabilities during the pre-crisis sample.

To rationalize the change in the impulse responses after the 2008 financial crisis, we feed the model with a common aggregate loan loss shock of 2.5% and then re-estimate the model. For the post-crisis moments, the estimation needs to account for an effect that speeds up the adjustment of market leverage, but slows down the reduction in liabilities. Because the market-to-book ratio (on impact) is about the same for the pre- and post-crisis samples, the estimate of the parameter that governs the recognition of loan losses remains the same. Hence, to explain the regime change in the deleveraging pattern, the estimation calls for an increase in loan adjustment costs, from a negligible to a non-negligible value. This increase in adjustment cots, permits the model to explain the even slower response of liabilities in the post crisis.[11] In turn, the estimation of post-crisis parameters needs a reduction in the dividend smoothing motive, to explain the more intensive use of retained earnings as a deleveraging device and the overall faster decline in leverage. These larger estimate of adjustment costs speaks to a reduction in liquidity in the secondary market for loans, which is itself consistent with systemic delayed accounting after an aggregate shock. The reduction in dividend smoothing is interpreted as greater pressure to cut back on dividends upon a negative shock in the aftermath of the crisis, even for banks that were not pressured by policy. The model is not only able to reproduce the impulse responses of the data, but also goes a long way in explaining the cross-sectional variation in Tobin's Q and the predictive power of Tobin's Q, items that estimation procedure does not target. Furthermore, with an aggregate shock of 2.5%, the model explains about 50% of the observed decline in Tobin's Q during the crisis, exclusively attributed to changes in q, that is, excluding changes in investor risk-premia.[12]

Our Q-theory allows us to study the effects of changing the degree of slow loan loss recognition for bank equity growth and lending. This reveals a tradeoff: more lenient accounting rules allow banks to increase their fundamental leverage beyond regulatory limits. However, more lenient accounting rules also mitigate the effects of loan losses as banks are no longer forced to reduce their assets immediately in order to adjust their leverage back to target. This creates a tension between riskier banks and a milder lending contraction thanks to laxer accounting rules. All in all,

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[11]The aggravation of adjustment costs is consistent with adverse selection in the secondary loans market, which is in turn consistent with the idea that banks can delay the recognition of losses.

[12]Atkeson et al. (2018) explain the reduction in the aggregate market-to-book ratio of banks post-crisis since the financial crisis with a reduction in government guarantees.

the mechanism of delayed loan loss recognition acts like counter-cyclical regulation that allows the capital requirement constraint to remain slack during a crisis with the advantage of being bank specific.

Related Literature. The financial crisis of 2008 has renewed interest in banking models as banks are viewed to be critical in reallocating resources in the economy—e.g. Adrian and Shin 2010; Rampini and Viswanathan 2012; Jermann and Quadrini 2012; Gertler et al. 2012; Adrian and Shin 2013; He and Krishnamurthy 2012; Brunnermeier and Sannikov 2014; Gertler et al. 2016. Bank leverage is at the heart of these theories that can be organized into some where markets impose constraints on leverage and others where regulation limits leverage. From a theoretical angle, more equity and lower leverage relax financial constraints and allow a bank to expand its lending. Models in this category include Gertler and Kiyotaki (2010); Brunnermeier and Sannikov (2014); He and Krishnamurthy (2013); Gertler et al. (2016); Nuño and Thomas (2017).[13] For these models, a market based equity value is the natural empirical counterpart because it measures the value of equity that affects incentives. The second group of models takes regulation as a given institutional feature. Such models study the effects of declines in equity buffers (Adrian and Boyarchenko, 2013; Begenau, 2020; Bianchi and Bigio, 2017; Martinez-Miera and Suarez, 2011; Corbae and D'Erasmo, 2019). The present paper makes two contributions: First, it presents a set of facts that shed light on the relevant state variables and constraints that affect bank decisions. Second, the paper presents a Q-theory of banks that is consistent with these facts. A key contribution of this paper is to show how both market and equity based measures are relevant state variables. In particular, we put forth the idea that delayed accounting is important and quantify the tradeoffs involved in the design of different accounting regimes for financial stability and economic growth.

In terms of models with market based constraints, our empirical findings suggest that such constraints operate in richer ways than conceived by many of these models. Although some of these models can generate the counter-cyclical movements in market leverage that we see in the data, they typically cannot account for the cross-sectional changes in leverage: upon an aggregate shock, most of these models would predict a compression of the distribution of leverage at a higher level after a large shock as more banks get closer to their constraints.

In addition, these models cannot account for the lack of responses in book values because book and market values co-move in those models. Adrian et al. (2016) raises a similar point when they argue that "as for market leverage, we show that virtually all the cyclical variation of market

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[13]Financial frictions on banks matter for the provision of credit —and hence economic performance. Examples of these frictions include costly verification (Townsend, 1979; Bernanke and Gertler, 1989), lack of commitment (Hart and Moore, 1994), or moral hazard (Holmstrom and Tirole, 1997, 1998). Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) were the first to model the connection between firm equity and aggregate outcomes. A different perspective is taken by Diamond and Rajan (2000) who argue that deposits through bank runs (a là Diamond and Dybvig, 1983) act as a disciplining device in the presence of agency frictions.

leverage is driven by fluctuations in the book-to-market ratio, reflecting the valuation changes of free cash flows generated by the bank." To meet this challenge, models based on market-constraints would need to generate changes in time *and* in the cross section of leverage ratios. In our model, due to delayed loss recognition, regulatory constraints become more slack upon an aggregate shock, book values barely move, but the average market leverage and its dispersion increase.

Our model shares elements with He and Krishnamurthy (2013) and Brunnermeier and Sannikov (2014) in that banks equity accumulates slowly over time (see the survey by Gertler et al., 2016). Unlike these papers though, bankers are risk-neutral despite smoothing dividends. Like this papers, banks face market-based constraints. Regulation that limits book leverage also connects this paper to the work by Adrian and Boyarchenko (2013). Our paper is also connected with Milbradt (2012) because we distinguish between the fundamental value of assets and book assets where losses are not registered in real time. We differ from Milbradt (2012) in that we focus on aggregate and cross-sectional bank lending and leverage dynamics. Furthermore, we do not interpret delayed accounting as exclusively resulting from accounting practices, but also as following from deliberate evergreening practices as described in Caballero et al. (2008).

Our evidence on the gradual adjustments in market leverage relates to two other theories on adjustment costs (see Hayashi, 1982, for a neoclassical Tobin Q theory). In finance, adjustment costs are not thought of as stemming from physical constraints. Instead, one financial literature strand rationalizes the slow adjustment of leverage with equity issuance costs and asymmetric information. Early models of equity issuance costs based on agency problems are Myers (1977, a debt overhang model) and Myers and Majluf (1984, a private information model). Adjustment costs on assets arise naturally when banks hold informationally sensitive assets, typically viewed as a specialty of banks—e.g., Leland and Pyle (1977b); Diamond (1984a); Williamson (1986); Tirole (2011); Dang, Gorton, Holmström and Ordonez (2017); Shachar (2012); Hachem (2011). In a more recent strand of work, DeMarzo and He (2016) and Gomes et al. (2016), a leverage target and slow adjustment emerge due to long-term debt and default, i.e. as the result of debt dilution. A novel feature of our Q-theory is that slow moving leverage can exclusively result from the delayed loss recognition mechanism. Furthermore, to explain the pre-crisis patterns, our model only needs delayed loss accounting. However, for the post-crisis the model needs higher balance sheet adjustment costs which are consistent with these theories.

The closest papers to ours are Corbae and D'Erasmo (2019) and Rios-Rull et al. (2020). Like these papers we also try to match cross-sectional bank leverage data. The novelty of our work relative to these studies is that our focus is on the importance of book and market-value differences that followed from delayed accounting in shaping bank decisions. Finally, our paper also relates to the literature on macro-prudential bank regulation. Some authors, hold the view that marking assets to market can amplify crisis by worsening financial frictions (Shleifer and Vishny, 2011; Laux and Leuz, 2010; Plantin and Tirole, 2018). In the midst of the Covid-19 crisis, questions such as how and if to mark loans to market and how much regulatory forbearance is good for the economy

are once again at the center of the discussion (Blank, Hanson, Stein and Sunderam, 2020). Our focus on delayed accounting suggests policy should also evaluate the role of evergreening practices in the formation of zombie banks (Caballero et al., 2008). In the final part of the paper, we study the effect of delayed loan recognition and also highlight a policy-relevant tradeoff.

The rest of the paper is organized as follows. Section 2 presents our set of five facts while section 3 presents the model and its results. Section 4 concludes.

2 Five Stylized Facts

Data. We use panel level data on top-tier United States Bank Holding Companies (BHCs).[14] BHCs provide a comprehensive picture of the activities of a financial organization beyond the narrower accounts of their commercial bank subsidiaries. We take BHC accounting data (balance sheet and income statements) from the FR-Y-9C regulatory reports filed with the Federal Reserve. We merge the accounting data with market data from the Center for Research in Security Prices (CRSP). We focus on the sample period from 2000 Q1 to 2015 Q4.[15] BHCs file FR-Y-9C forms if they have assets above $500 million.[16] This sample is highly representative of the banking sector. Appendix Section A.2 presents the time series of key balance sheet variables for all BHCs in our sample and the four largest BHCs: Bank of America; J.P. Morgan, Citigroup, and Wells Fargo.

2.1 Characteristics of bank equity

Book equity versus market equity of banks. In most models of banks, net worth is a key state variable that puts a cap on leverage. However, bank net worth can be measured in terms of accounting measures (book equity) or market value (market equity). Figure 1 presents the time series of book equity and market equity aggregated across all BHCs in our sample (left panel) and the same time series for the four largest BHCs (right panel).[17] The figure shows a stark discrepancy between market and book equity, particularly during the crisis. This discrepancy raises the question of which empirical counterpart best captures the economic concept of net worth. Just looking at book equity, it is hard to detect that 2008-2009 where the years of a major financial crisis! The sharp drop in market equity, on the other hand, clearly indicates a crisis. The pattern for the

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[14]A bank holding company is an umbrella company that holds banks and other financial institutions. A commercial banks is a single bank that provides traditional banking services such as deposits and loans. For example, Citibank, a commercial bank, is held by Citigroup, a BHC that holds Citibank and other banks, including non-commercial banks.

[15]We extend the sample when we estimate impulse response functions. When constructing aggregate time series, we drop entrants to correct for the entry of major financial institutions such as Goldman Sachs and Morgan Stanley. Without this correction, aggregate bank assets increase due to the reclassification of large actors such as Morgan Stanley and Goldman Sachs into bank holding companies.

[16]Prior to 2006 Q1, this threshold was $100 million, and the threshold became $1 Billion in March 2015.

[17]Book equity for publicly traded BHCs is close to book equity for all BHCs, which shows that the publicly traded sample compromises most of the banking sector (weighted by equity).

largest banks is very similar.[18] Citigroup is an extreme example of the discrepancy between book and market values: Citigroup lost 90% of its market capitalization, but its book equity remained intact.[19]

This difference between market and book equity is not the result of the composition of public equity injections. Although public equity injections are counted as preferred equity in accounting books, and market equity is measured relative to common equity, Figure 1 shows that preferred equity cannot explain the discrepancy between market and book equity.[20]

To get a quantitative sense of how much book and market equity differed during the crisis, in Table 1 we present the percentage change in bank market equity valuations (top two rows) and book equity valuations (middle two rows) together with the change in the S&P 500 stock return index from the beginning of the crisis in 2007 Q3 to the end of 2008, 2009, and 2010 respectively. We report simple percentage changes in the real value (columns "real change") as well as the changes in fitted log-linear trend (columns "log-linear"). Between 2007 Q3 and 2008 Q4, the market capitalization of the banking sector dropped by 54% compared to a 42% drop in the S&P 500. By 2010 Q4, market equity was still down 30% from its value in 2007 Q3. Much of this rebound followed from new equity issuances. By contrast, book equity did not fall during the crisis and actually increased substantially post crisis. In fact, recorded book equity losses were entirely made up for by new equity issuance. This large discrepancy implies that banks' average Tobin's Q, defined as the ratio of the market–to–book equity ratio, drastically declined during the crisis and remained much lower thereafter.[21] To summarize, our first fact is:

Fact 1. *Book values and market values diverged substantially during the crisis.*

The divergence between book and market equity during the crisis is already known from other studies (see for example Adrian and Shin, 2010; He et al., 2017). However, this variation in Tobin's Q is a common thread in the paper and begets the question of what empirical counterpart of equity should be used in macro models? We next turn to some cross-sectional evidence that sheds some light on this question.

Information content in market equity and book equity. A natural question is whether differences between the market and book equity valuations reflect differences in the informational content of these measures. Conceptually, book measures are *backward looking* in that they register historical losses. By contrast, market equity measures are *forward looking* in that they price future expected cash flows. Still, this conceptual difference does not imply that both measures contain

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[18]The discontinuities in the individual bank series reflect mergers and acquisitions, e.g. the acquisition of Wachovia by Wells Fargo during the crisis.

[19]Citigroup suffered heavy losses during the crisis and did not undergo any major mergers or acquisitions, making it a particularly clean example case.

[20]Preferred equity rose temporarily during the crisis due to the Troubled Asset Relief Program (TARP). Note that preferred equity is included in book equity, but not in market equity.

[21]We are referring to the market-to-book *equity* ratio as Tobin's Q, as opposed to the market-to-book *assets* ratio.

different information: In principle, we can write a model where the history of events is encoded in bank balance sheets, and the information contained in the books is enough to predict future cash flows. In such a case, the informational content of market values would be the same while the time-series and cross-sectional variation of Tobin's Q would only respond to changes in risk premia.

However, we suspect that market and book value measures contain different information: one reason is that changes in the underlying market value of loans (see filing instructions for FR-Y-9C BHCs regulatory reports) reflect default expectations. This expectations are not updated in loan accounting books. During our sample period, a loan is only written off once the loss has occurred, in contrast to when the loss is expected.[22] This alone could produce difference in the informational content.[23] Another reason is the delayed acknowledgment of known losses. If banks can delay recognizing losses, or refinance non-performing loans to avoid registering losses (evergreening), book values will be over-optimistic. If market participants can update their valuations quickly, detecting these losses, differences in informational content will emerge. A casual indication that market values contain more information can be seen from Figure 2, which shows that loan charge-offs peaked in 2010 when the economy was no longer officially in a recession. The decomposition of net charge-offs shows that these losses were heavily driven by real estate consistent with the nature of the crisis.[24]

Next, we analyze the differences in informational content formally. Our strategy builds on the following idea: If market equity values contain more information about bank profitability than book equity values, then Tobin's Q, i.e. the market-to-book equity ratio, should predict future profitability once controlling for book equity. In addition, Tobin's Q should be correlated with contemporaneous predictors of future performance. In Figure 3, we show binned scatter plots of (logged) outcomes on the log market-to-book ratio (market capitalization over book equity), for a pre-crisis quarter (2006 Q1) in navy and a post-crisis quarter (2009 Q1) in maroon—the plots control for log book equity.[25] The top left panel shows the log return on equity over the past

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[22]At the time of writing, a new accounting standard has been developed. According to this standard, loan losses should be calculated according to "current expected credit losses" (CECL). Publicly traded banks began to follow CECL (expected loss accounting rules) since January 1st 2020. Smaller banks were supposed to follow over the next year. However, on March 27, 2020 (https://www.federalreserve.gov/newsevents/pressreleases/bcreg20200327a.htm), the Fed moved to provide an optional extension of the regulatory capital transition for the new credit loss accounting standard.

[23]In Appendix A.3 we survey important contributions from the accounting literature on the issue of delayed loss recognition. See also Bushman (2016) and Acharya and Ryan (2016) for useful discussions.

[24]When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as PLL. Later when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset later. Net charge-offs is charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLL because the FR Y-9C does not provide information on PLL by loan category.

[25]To control for log book equity, the left and right-hand side variables are residualized on log book equity, and then the mean of each variable is added back to maintain the centering. It is important to control for log book equity to prevent spurious results due to ratio bias (see Kronmal, 1993). Log book equity is part of the right hand side, and also appears on the left hand side of many of these regressions; it would have a mechanical effect if it were

year plotted against the log market-to-book ratio. Banks with higher Tobin's Q earn a higher return on equity. The top right panel examines the log return on equity over the next year: higher market-to-book ratios predict higher future profits. These correlations are especially strong in the post-crisis quarter. Banks with higher market-to-book ratios also have a lower share of delinquent loans (bottom left panel) and, in the post-crisis quarter, have a lower net charge-off rate on their loans over the next quarter (bottom right).

The results suggest that market participants have some ability to predict future profitability and incorporate this into their valuation, beyond what they see in accounting books.[26] This is consistent with the view that books are slow to reflect true conditions. Indeed, the results suggest that banks with lower profitability and more delinquencies have lower Tobin's Q, and Tobin's Q predicts future loan writedowns and future profitability. Our second fact is the take-away from this analysis.

Fact 2. *Tobin's Q predicts future cash flows in the cross-section of banks. That is, market values capture information that book values do not, and book values do not fully respond to shocks.*

The Time Series and Cross-Section of Book and Market Leverage. Models of banks typically impose constraints on either market or book leverage. In this section, we show the time series pattern of banks' market and book leverage ratios and their cross-sectional differences. In the left panel of Figure 4, we plot the aggregate market and book leverage for the entire sample of public BHCs. In the right, we show the corresponding series for the four largest banks. A common pattern is evident: Book leverage rose only moderately pre-crisis and actually fell during the crisis. Market leverage, by contrast, spiked dramatically during the crisis and remained almost twice as high for at least four years.

In terms of the cross-section, Figure 5 presents different sections of the distribution of market leverage over time. The figure plots the median market leverage (in maroon) and each 10th percentile (in blue). Market leverage increased across the board and took a long time to return to its pre-crisis levels. Strikingly, the distribution of market leverage fans out, with a substantial 10% of banks sustaining market leverage ratios of nearly 80. This suggests that there was no strictly binding ceiling on market leverage in the midst of a deep financial crisis.

Whereas constraints on market leverage are a theoretical possibility that has to be validated empirically, regulatory constraints are indisputable. Regulation is based on book equity ratios. Does the cross-section of book leverage show evidence of distress? How many banks were close to violating their regulatory constraints during the crisis? Figure 6 presents the share of BHCs

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not controlled for.

[26]Of course, an alternative explanation is that market equity over-reacts to information about future profitability. However, the fact that book equity did not decline during the crisis suggests that sluggishness of books plays a role. Also, to the extent that discount rate variation will affect most banks similarly it is unlikely to drive the results in this cross-sectional regression.

that have been below different levels of regulatory capital ratios.[27] We can observe that the vast majority of banks kept a capital buffer above the regulatory minimum. The distance to the regulatory constraints shortened for a significant number of banks during the crisis, but still, only a minority of banks were close to their regulatory constraints. We summarize these observations into:

Fact 3. *Most banks keep an equity buffer above the regulatory capital ratio minimum; fewer banks did so during the crisis. Market leverage and the dispersion of market leverage increased substantially during the crisis.*

Fact 3 poses a challenge for standard models of banks. We should expect systemic loan defaults during a crisis like 2008 and this should lead to an increase in market leverage for all banks, provided that liabilities are not liquidated immediately. At the same time, we should expect substantial reallocation of assets from highly levered banks toward banks with lower leverage. This means that we should expect a compression in the dispersion of leverage in the cross-section. Figure 5 suggests that there was no such compression. This observation suggests a delay in the reallocation of assets across banks. In terms of regulatory constraints, this third fact suggests that only a minority of banks hit their regulatory constraints. In light of fact 2, we argue that this is partially due to the slow response of book values. Interestingly, the share of banks near the regulatory limits rose to its peak in the first quarter of 2010, at least 2 years after the first symptoms of a mortgage crisis. From the perspective of models of banks, we contend that *neither regulatory nor market constraints bind operate in a static way.* Of course, just because a constraint is not actively binding we can conclude a constraint does not affect banks. On the contrary, banks may worry that it will bind in the future, and take steps to avoid hitting the constraint and that is the spirit of our *Q*-theory. The next section analyzes the dynamic responses of banks to shocks.

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[27]Under Basel II (the regulatory standard in place during the crisis), bank holding companies were subject to regulatory minimums on their total capital ratio and their tier 1 capital ratio. These capital ratios are computed as Qualifying Capital/Risk-Weighted Assets, and thus a bank with a higher capital ratio has lower leverage. Basel II required banks hold a minimum Tier 1 capital ratio of 4% and a minimum total capital ratio of 8%. In order to be categorized as "well-capitalized," banks had to meet minimum capital ratios that were two percentage points higher (6% and 10% respectively). Being categorized as well-capitalized is desirable because banks that are not well-capitalized are subject to additional regulatory scrutiny (Basel Committee on Banking Supervision, 1998, 2006). After the crisis, tighter capital requirements were phased in under Basel III. The minimum total capital ratio stayed at 8% throughout our sample period, but the Tier 1 capital ratio rose to 4.5% in 2013, 5.5% in 2014, and finally settled at 6% starting in 2015. Also under Basel III, additional capital ratios (e.g. tier 1 leverage and common equity capital ratio) began being monitored (however these ratios are quite similar to the preexisting Tier 1 and total capital ratios), and starting in 2016, a "capital conservation buffer" and special requirements for systemically important financial institutions were introduced (Basel Committee on Banking Supervision, 2011). Kisin and Manela (2016) study whether banks violate different regulatory constraints and find that typically banks do not fail multiple regulatory constraints.

2.2 Characterizing bank leverage and balance sheet dynamics

This section analyzes the dynamics of leverage and the balance sheet. This analysis is informative about the constraints faced by banks and their process of leverage adjustment, as we will argue.

Empirical framework. We empirically investigate whether and how a target leverage ratio and adjustment costs drive the leverage dynamics of banks. To test this hypothesis, we estimate the following panel regressions:

$$\Delta \log(y_{i,t}) = \alpha_t + \sum_{h=0}^{k} \beta_h \cdot \log(1 + \varepsilon_{i,t-h}) + \gamma_h \cdot Post_t \ \log(1 + \varepsilon_{i,t-h}) + \psi_{i,t}, \qquad (1)$$

where i indexes over banks, t indexes over quarters, $y_{i,t}$ is the outcome of interest, α_t is a time fixed effect, $\varepsilon_{i,t}$ denotes our measure of a cash flow shock to net-worth (i.e., the idiosyncratic excess stock return innovations over the past quarter for bank i in quarter t; see detailed description below), and $Post_t$ is an indicator variable equal to one if the current quarter is post-crisis (we treat 2007 Q4 as first quarter for which $Post_t = 1$).[28] These regressions allow us to construct impulse response functions for liabilities, market leverage, market equity, and book equity.[29] We include time-fixed effects α_t to absorb aggregate shocks, e.g. changes in investors' discount rates or the price of loans due to demand shocks. We thus recover a partial equilibrium supply-side impulse response, estimated off of the cross-sectional variation in return shocks. In all specifications, we use $k = 20$. Because of these many lags, we extend our data to 1990 Q3 in order to obtain precise pre-crisis estimates.[30] We cluster standard errors by bank. Finally, to report the impulse response function, we sum the coefficients: the pre-crisis contemporaneous response is β_0, the next period is $\beta_0 + \beta_1$, and so on. For the post-crisis, we also add the corresponding γ terms.

Before we show the results, we first discuss how we obtain the shock measure $\varepsilon_{i,t}$. We follow Gandhi and Lustig (2015) to adjust bank stock returns for aggregate risk factors. That is, we regress the excess stock returns $r_{i,t} - r_t^f$ of bank i on a bank fixed effect α_i and a matrix of factors

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[28]One might favor an alternative specification which includes lags of the dependent variable in addition to contemporaneous and lagged returns. This faces two issues: Nickell bias and bad control. Including the dependent variable as a lag will induce bias, as documented by Nickell (1981). Dealing with this bias is challenging, and may result in poor precision. Perhaps more importantly, the lagged dependent variable is a "bad control," in that it is endogenous to the regressor. We wish to back out the effect of a return shock in $t - 3$ on the change in liabilities in t: if we condition on liabilities in $t - 1$, which is itself also affected by the past return shock, then we will not identify our parameter of interest.

[29]Since market returns are changes in market equity valuations, taking first differences in logs provides a tight conceptual link between the outcome and the regressor. Using levels would mean that the outcome was highly correlated with bank size. This would raise concerns about stationarity. Using levels could also result in a regression that was heavily influenced by a few large banks, given the highly skewed bank size distribution. For the same reason we do not weight our regressions: the bank size distribution is highly skewed, and so a weighted regression would be equivalent to a regression with only the handful of the largest banks. If the variance of the residuals were lower for larger banks, then using weights would yield a more efficient estimator. Empirically however, the variance of the residuals does not appear to vary substantially by bank size.

[30]This is the first quarter in which we can identify which banks are top-tier BHCs from the FR Y-9C.

X_t as follows:

$$\underbrace{r_{it}}_{\text{Raw Return}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t}_{\text{factors}}\underbrace{\beta_i}_{\text{loadings}} + \underbrace{\varepsilon_{i,t}}_{\text{Idiosyncratic Component}} .$$

The vector of factor loadings β has dimension $K \times 1$ and the matrix of factors X_t has dimension $T \times K$. We include the same factors as in Gandhi and Lustig (2015), namely the three Fama-French factors (Fama and French, 1993), a credit factor calculated as the excess return on an index of investment-grade corporate bonds, and an interest rate factor calculated as the excess return on an index of 10-year U.S. Treasury bonds.[31] See Appendix Section B.1 for further details on the risk-adjustment process. The idea behind risk-adjusting returns and using their innovations is that we want to isolate information about banks' cash flows, as opposed to discount rates, which are driven by aggregate movements in the factors.[32] Crucially, we rely on the efficient-markets hypothesis according to which excess return variations should be unpredictable *ex ante* after adjusting for the risk-premium. By stripping out the predictable components of returns, the innovations $\varepsilon_{i,t}$ to the risk-adjusted returns are ex-ante unpredictable across banks. This forms the basis of our identification strategy: we treat cross-sectional variation in $\varepsilon_{i,t}$ as unanticipated shocks that perturb bank equity. In the Appendix (see Figure 4), we show for the largest four banks that the time series of $\varepsilon_{i,t}$ indeed resembles white noise. In Section 2.2, we conduct a variety of robustness check to validate our identification strategy and interpretation of $\varepsilon_{i,t}$ as cash flow shocks. For the rest of the paper, we refer to these innovations $\varepsilon_{i,t}$ as return shocks.

Impulse responses. How do shocks affect banks' balance sheet, financing, and payout choices? We estimate impulse response functions for liabilities, market capitalization, book equity, market leverage, and the common dividend rates, and show the results in Figure 7. To normalize the effect, we report the response to a negative one percent return shock. The y-axis of our plots shows the contemporaneous response ($-\beta_0$ for pre-crisis and $-\beta_0 - \gamma_0$ for post-crisis) as quarter 1, the cumulative response one quarter later ($-\beta_0 - \beta_1$ and $-\beta_0 - \beta_1 - \gamma_0 - \gamma_1$) as quarter 2, and so on. If banks maintain a target market-leverage ratio, we would expect banks to respond to a negative wealth shock (which mechanically increases market leverage) by moving back towards their target leverage. As we can see from the impulse response function of market leverage in Panel a) of Figure 7, the data is consistent with an adjustment back to target that appears to be slow, presumably due to adjustment costs. As discussed before, adjustment costs are a key ingredient of our Q-theory.

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[31]The three Fama-French factors are downloaded from Ken French's website. The credit factor is the excess return on the Dow Jones Corporate Bond Return Index that we download from Global Financial Data. The interest rate factor is the U.S. 10-year Government Bond Total Return Index (ltg) that we also download from Global Financial Data. We use the one-month risk-free rate from Ken French's website to calculate excess returns.

[32]The results for simply adjusting returns with a time-fixed effect are qualitatively and quantitatively similar, and are also reported in the Appendix Section B.2.

The impulse response of log market leverage, defined here as log(Liabilities/Market Capitalization), is simply the difference between the response of log market capitalization (Panel b) and log liabilities (Panel c).[33] The impulse response function for log market capitalization reveals that in the quarter of impact of a return shock, a mechanical effect on the denominator dominates and explains the jump in market leverage. The adjustment on the numerator is slow, and the effect of return shocks on market leverage does not vanish, even after five years. This yields our fourth fact:

Fact 4. *Banks appear to operate with a target leverage ratio to which they only return slowly after shocks, suggesting adjustment costs, consistent with the Q-theory.*

A few noteworthy differences in the pre-crisis and post crisis impulse responses emerge from Figure 7. First, banks adjust their leverage ratios more quickly during the post-crisis years compared to the pre-crisis. As a way to delever and return to the target leverage ratio, banks can increase equity and/or decrease liabilities. Second, Panel b) shows that during the pre-crisis period market equity did not change after a shock decreases market equity (a return shock mechanically lowers equity one-for-one). Third, by contrast, in the post-crisis period, half of the impact is reversed over 5 years. In particular, panel c) shows that banks decrease liabilities by around 0.6% during the pre-crisis period and but only by 0.2% in the post-crisis in response to a 1% returns shock. Panel d), in turn, shows that book equity values adjust slowly in the pre-crisis and post-crisis. This is consistent with Section 2.1, where we showed that book values respond only slowly to losses that are more quickly reflected in market returns. The final observation regards the behavior of dividends: In Panel e) of Figure 7, we estimate the impulse response of the common dividend rate.[34] The response of the common dividend rate to a negative return shock (top left panel) is surprisingly positive pre-crisis. This is driven by the mechanical initial effect on the denominator. Market equity falls in response to a negative return shock. Post-crisis, the initial positive mechanical effect is overtaken by the negative effect on dividends. All in all, these impulse responses show that banks switched from responding exclusively by decreasing assets and liabilities in the pre-crisis period to a combination of balance sheet and equity adjustment in the post-crisis period, with equity adjustments being more important. To summarize, our final stylized fact is:

Fact 5. *Prior to the crisis, banks adjusted leverage primarily by reducing debt keeping equity unchanged. Post-crisis leverage adjustments appear more gradual. During this period, banks also adjusted leverage by raising equity.*

Identification and Robustness.   Our interpretation of the estimates relies on the assumption that bank specific variations in risk-adjusted bank stock returns identify cash flow shocks on the

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[33]The impulse responses of leverage instead defined as log(Liabilities + Market Capitalization)/Market Capitalization) are nearly identical.

[34]For this ratio, we use $\Delta \log(1 + y_{i,t})$ as the outcome variable in our specification 1, since $\log(1 + y_{i,t})$ provides an approximation to percentage points, and since flow variables such as dividends can be equal to zero.

existing portfolio, such as specific default shocks, as opposed to shocks to the profitability of future business opportunities. We conduct various analyses to alleviate identification concerns including a narrative approach to validate our interpretation of the return shocks as unanticipated and specific cash flow shocks.

As stated above, one could be concerned that the return shocks capture idiosyncratic information about the relative profitability of the banks' future portfolio (e.g. the default rate on future mortgages of a bank) and thus affect the bank's problem through channels other than perturbing equity. If a bank's expected return on its future assets falls, the bank would want to reduce equity or lower its scale for a reason that would be unrelated to a target leverage ratio and adjustment costs.

To investigate this concern, we study how banks' liquid asset ratios respond to a negative return shock. If negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. We test this notion by looking at the impulse response function of banks' liquidity ratio, calculated as (cash + treasury bills) / total assets. The impulse response function in Appendix Figure 9 shows no statistically significant response pre-crisis. There is a small temporary response post crisis that is reversed within a few quarters (recall that we show cumulative responses).[35] In sum, banks do not tilt their portfolios towards safe and liquid assets in response to our return shock, which pushes against a story of worsening investment opportunities.

The lack of response of the liquid asset ratio is suggestive for our interpretation of the return shocks. However, we cannot fully rule out that the shock picks up information about the profitability of future assets. To provide additional corroborating evidence for our identification strategy, we use a narrative approach, detailed in Appendix B.3. To this end, we take the largest positive and largest negative values of the return shocks $\varepsilon_{i,t}$ over the sample period for each of the four largest banks (J.P. Morgan Chase, Bank of America, Citigroup, and Wells Fargo). We then search various newspapers for articles that mention the name of any of the four banks in the quarter for which the absolute value of $\varepsilon_{i,t}$ was high. Table 5 in the Appendix lists the results of our newspaper article search. In most cases, we can find supporting evidence for our $\varepsilon_{i,t}$ estimates. For example, in the second quarter of 2009 Bank of America had a high and positive value of $\varepsilon_{i,t}$. Our article search revealed that Bank of America fared better in the stress test and exceeded expectations. In 1999 Q1 Citigroup had a large positive $\varepsilon_{i,t}$. This coincided with a Wall Street journal article that stated that Citigroup had exceeded profit expectations even though profits fell. In 2001 Q1, a negative shock at Wells Fargo coincided with news reports that stated that Wells Fargo's venture capital portfolios had incurred significant losses.

In Appendix Section B.3, we provide additional robustness checks. We verify that our results

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[35]This is not a perfect test as perhaps banks would also want to raise liquidity in response to a cash flow shock on their current portfolio. A hypothetical example is the following. Suppose the bank was caught in unfair lending practices that causes a lawsuit. Banks might respond by increasing their cash holdings to prepare for the upcoming lawsuit.

are not driven by mergers or by specific events during the crisis by excluding mergers and the crisis years 2008 and 2009 from our sample. For more details refer to Appendix Section B.3.[36]

2.3 Taking stock

In this section we presented five facts about bank leverage, a key variable in models of banks. The denominator of leverage, net worth, can have two different empirical counterparts, book or market equity. However, these two measures diverge during the Great Recession (fact 1) and in ways related to their informational content (fact 2). Importantly, leverage constraints based on market equity measures, did not seem to put a cap on market-leverage during the crisis (fact 3). Furthermore, regulatory constraints, which depend on book leverage, seemed to be circumvented by delaying losses. We also argued that banks operate as if they have a market leverage target, but the adjustment to that target is slow (fact 4). Furthermore, pre-crisis, banks would predominantly reduce liabilities as means to delever, but the deleveraging pattern changed in the post-crisis and retained earnings gained a predominant role to the point that leverage adjusted faster (fact 5).

What do these facts mean for macro-finance oriented banking models? They call for a theory that can explain book and market-value differences and can account for slow responses of leverage. They also call for thinking of leverage constraints as not binding directly, but rather affecting banks in a dynamic sense. In the neo-classical Q-theory of investment (Hayashi, 1982) the discrepancy between market and book measures follows physical adjustment costs which create a wedge between the average and the marginal cost of capital. Since bank assets consists mainly of financial contracts, it is difficult to interpret this difference as the result of decreasing returns to technology. However, there is a tradition in finance that explains financial adjustment costs through various financial frictions. For example, the illiquidity of loans can result from asymmetric information—Leland and Pyle, 1977b; Diamond, 1984b; Dang, Gorton, Holmström and Ordonez, 2017— or from fire sale costs as in Shleifer and Vishny (1997). Thus, taking the insights from the neoclassical Q-theory, one route is to explain the slow adjustments and variation in Q through asset adjustment costs.

A model with only loan adjustment costs would not explain why book equity reacted so little during the crisis (upon a large shock) and would not explain the predictive power of Tobin's Q. For that reason, the Q-theory we develop in the next section, is motivated by the idea of delayed accounting. We introduce loan adjustments costs, for comparison and because they are needed to explain the fifth fact, but we will show that delayed accounting can explain the discrepancies between book and market values, the predictability of Tobin's Q,the lack of binding constraints and also, the delayed responses in market leverage. In particular, delayed accounting can explain the slow adjustment of market leverage because upon a shock, leverage does not have to be adjusted

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[36]We also test for heterogeneous impulse responses by type of bank. We do not find evidence of sizable heterogeneity, but we have limited statistical power to detect these differences.

immediately to satisfy regulatory constraints. Rather, liabilities are reduced gradually as losses are slowly recognized.

There is substantial work on delayed accounting in the accounting literature. In particular, like us, Laux and Leuz (2010) also argue that delayed accounting was a prevalent phenomenon during the Great Recession. In Appendix A.3, we further detail the bank accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 also offer useful discussions). In macro-finance, delayed accounting is a topic that has been overlooked, with some exceptions. Among these exceptions, Milbradt (2012) distorted incentives brought about by Level 3 fair-value accounting and Caballero et al. (2008) note that regulatory constraints may be a factor contributing to evergreening. Motivated by the facts presented in this paper, we argue that delayed accounting is furthermore important to explain the dynamics of book and market leverage. We also argue that delayed accounting has important policy implications. We will argue that with delayed accounting, banks can weather loan losses and hope for recovery of some of the loans. On the flip side, delaying expected losses allows banks to maintain greater leverage, or continue to extend loans to businesses that should fail (Caballero et al., 2008; Blattner et al., 2019).

3 Q-Theory

We now present our Q-theory of banks. The theory is inspired by the facts presented in Section 2. The main innovation is that the dynamics of leverage are affected by accounting rules. The model has three frictions: first, equity financing frictions prevent banks from offsetting loan losses with equity issuances. Second, delayed loan-loss recognition induces a dynamic tradeoff between loan origination and the tightening of future regulatory constraints. Finally, we introduce loan adjustment costs that, although not central to our theory, allow us to contrast our Q-theory with models of adjustment costs and to fit the post-crisis responses. Later in the section, we match the model with the data and discuss the model's ability to replicate the facts we highlight above. Proofs and derivations are contained in Appendix C.

3.1 The Model

Environment. Time is continuous, infinite, and indexed by t.[37] There is a continuum of banks. Loan defaults are the only source of risk. Each bank maximizes the expected discounted value of dividends, C_t. Banks are risk-neutral, but prefer to smooth dividends across time. Their objective function is defined recursively as

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[37]We choose a continuous-time setup for computational reasons: An earlier version of this paper, presented the same model in discrete time. Whereas the cross-sectional properties of both models are quantitatively practically identical, the speed of computation is substantially faster in continuous-time, something that facilitates the estimation step.

$$V_t = \mathbb{E}_t \left[\int_t^\infty f\left(C_s, V_s\right) ds \right],$$

where:

$$f\left(C, V\right) \equiv \frac{\rho}{1-\theta} \left[\frac{C^{1-\theta} - \left\{1 + (1-\psi) V\right\}^{\frac{1-\theta}{1-\psi}}}{\left\{1 + (1-\psi) V\right\}^{\frac{1-\theta}{1-\psi} - 1}} \right].$$

C_t denotes dividend payouts at time t and f is a Duffie-Epstein aggregator with a time discount rate $\rho > 0$, an intertemporal elasticity of substitution (IES) of $1/\theta$, and risk aversion of ψ.[38] The Duffie-Epstein aggregator is the continuous-time counterpart of Epstein-Zin preferences. The recursive formulation allows us to characterize banks as risk-neutral (assuming $\psi \to 0$), as is standard in the theory of the firm.[39] At the same time, we retain flexibility to introduce dividend smoothing motives that deliver smooth dividend payout patterns that are consistent with the empirical evidence, e.g., Lintner (1956); Dickens et al. (2002); Leary and Michaely (2011).

Bank balance sheet. Banks hold long term loans that are funded with deposits and equity. At each instant, a fraction δ of loans matures. Maturity is innocuous, but is introduced to distinguish between loan flows and stocks. Loan default shocks are governed by a Poisson process N_t: a default event occurs with instantaneous probability σ, and a constant fraction ε of loans defaults during said event.

Book accounting and fundamental values differ.[40] Defaults are recognized slowly in the accounting values, whereas they are immediately captured in the fundamental values. We denote the fundamental value of loans by L_t, and the book value of loans by $\bar{L}_t$. Deposits, denoted D_t, are risk-free, and thus their accounting and fundamental values coincide. The bank state variables are $\left\{L, \bar{L}, D\right\}$.

Financing frictions Banks face an equity financing friction: banks cannot issue equity and must rely on retained earnings to grow equity. As we noted, the bankers' utility embeds a dividend smoothing motive that violates the assumptions of the Modigliani-Miller theorem. The assumption of no equity finance can be relaxed, by defining utility over net dividends.

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[38] With other forms of dividend adjustment costs, dividend smoothing and risk-aversion are coupled together. In the model, however, the bank can be risk-averse (due to its franchise value) or risk-loving (due to the option to default), as we explain later.

[39] Similar objective functions for banks are found in Bianchi and Bigio (2017) and Di Tella and Kurlat (Forthcoming). We are the first to model dividend smoothing in isolation from risk-aversion.

[40] This is a novel feature of our model and critical to capture banks' ability to engage in evergreening and to avoid the recognition of losses immediately (see empirical evidence in Blattner, Farinha and Rebelo (2019)). Evergreening as described in Caballero, Hoshi and Kashyap (2008) occurs when banks roll over a loan that won't be paid. The objective is to avoid registering losses. Since rolling over a loan does not require new funds, evergreening allows the bank to reduce its accounting equity without a cost.

Laws of motion. The law of motion for the fundamental value of loans is

$$dL = (-\delta L + I)\, dt - \varepsilon L dN. \tag{2}$$

The change in the fundamental value equals new issuance I, net of maturing loans δL, minus the defaulting fraction of loans, εL in the events of default $dN = 1$.

The law of motion for book loans is

$$d\bar{L} = (-\delta L + I)\, dt - \alpha \left(\bar{L} - L\right) dt - \tau \varepsilon L dN. \tag{3}$$

This law of motion is similar to Equation (2). It fully captures the flow of repaid principal and new loans, $(-\delta L + I)\, dt$. Differently from Equation (2), the loan default shock affects the book value with a delay. When the fraction ε of loans L defaults, only a fraction $\tau \in [0, 1]$ is recognized immediately.[41] The term $\alpha \left(\bar{L} - L\right)$ reflects the speed of loss recognition: α is the rate at which the gap between $\bar{L}$ and L closes. This partial recognition of losses in the model is motivated by the discussion of evergreening and internal-valuation accounting above.

The law of motion for deposits is:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + \Phi(I, L) + C\right] dt. \tag{4}$$

The bank issues deposits to pay out dividends C, to fund the cost of new lending, $\Phi(I, L)$, and to pay the interest r^D on deposits. The bank receives an inflow of deposits from the interest r^L earned on loans and from the repayment of maturing loans δL. The interest rates r^L and r^D are exogenous and constant.[42]

Loan market friction. The loan market is simplified for tractability. Each instant, banks choose a flow of new loans I_t. The cost of issuing loans (in deposits) is

$$\Phi(I, L) = I + \frac{\gamma}{2} (I/L - \delta)^2 L,$$

where $\gamma \geq 0$. Given L, $\Phi(I, L)$ determines the increase (or decrease) in bank funds that stem from issuing (selling) new loans.[43]

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[41]If $\tau = 0$, books do not acknowledge any fraction of the loss on impact. If $\tau = 1$, the book and fundamental values are equal since losses are fully accounted for immediately. An initial condition for any bank is that $\bar{L}_0 \geq L_0$, which guarantees that $\bar{L}_t \geq L_t$.

[42]A constant interest margin is consistent with the empirical evidence in Atkeson, d'Avernas, Eisfeldt and Weill (2018) or Wang (2018). It is important to note that constant net-interest rate margins are not informative about bank interest rate risk exposure, as shown by Begenau et al. (2015) and Begenau and Stafford (2019). Gomez et al. (2020) and Haddad and Sraer (2019) present empirical evaluations of banks' interest rate exposures.

[43]This function has the following properties: First, $\Phi(\delta L, L) = \delta L$, that is, the bank does not incur a higher issuance cost when it replaces maturing loans. Second, $\Phi(I, L)$ is increasing in I ($\Phi_I(I, L) > 0$) for any $I > \bar{I} \equiv (\delta - 1/\gamma) L$. Note that the bank would never choose to sell assets (negative issuance) below $\bar{I}$, as the bank would

The function Φ induces an exogenous portfolio readjustment cost. For $I > \delta L$ the convexity in I in $\Phi(I, L)$ can be interpreted as representing decreasing returns to lending activities. For $I < \delta L$ the marginal cost of selling can represent fire sale costs, as in Shleifer and Vishny (2011), different areas of lending expertise, or adverse selection in the secondary loans market.

Notation. Before we proceed, we define the ratio of the fundamental value to the book value of loans by $q_t \equiv L_t/\bar{L}_t \in [0, 1]$. The fundamental value of bank equity is $W_t \equiv L_t - D_t$. In addition, we define leverage as $\lambda_t \equiv D_t/W_t$. Our notion of q_t is purposely chosen to relate to Tobin's Q, which measures the ratio of market-to-book values: In our model, market values differ from fundamental values due to the external funding frictions and the accounting rules. Hence, q_t and Tobin's Q_t are different but related concepts.

Since the laws of motion feature drifts and jumps produced by defaults, we introduce the following notation: We use $\mu^x W$ to denote the drift of a variable x; μ^x is the drift scaled by wealth W. Similarly, we use $J^x W$ to refer to the jump in x, in proportion to W.

Liquidation. Banks continue to operate as long as they satisfy two constraints. First, banks are subject to a regulatory requirement that stipulates that deposits cannot exceed a fraction ξ of their book loans $D \le \xi \bar{L}$, for $\xi < 1$. We can express this regulatory constraint more conveniently:

$$\lambda \le \xi/(q - \xi). \tag{5}$$

Notice that the higher the q, the tighter the constraint. This constraint is the main constraint of interest in our theory. Second, banks are subject to a market-based leverage constraint. The constraint requires bank leverage to be below $\bar{\lambda}$:

$$\lambda \le \bar{\lambda} \equiv (1 - \varepsilon)/\varepsilon. \tag{6}$$

The value of $\bar{\lambda}$ guarantees solvency in all states, $(1 - \varepsilon) L - D > 0$.

If either the market or regulatory conditions are violated, the bank is liquidated. The value after liquidation is an exogenous value proportional to equity $V_o \equiv v_o W^{1-\psi}$. Because of its franchise value, the bank has incentives to avoid defaults: when active, it earns a positive intermediation spread. The problem is that a bank cannot fully control its leverage since loans are subject to random default shocks and adjusting leverage takes time. Let Γ^r be the states where regulatory liquidations occur and Γ^m the states where market liquidations occur. The overall states that trigger liquidations are $\Gamma \equiv \Gamma^r \cup \Gamma^m$.

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actually have to pay another bank to take its loans. Third, $\Phi(I, L)$ is convex in I, $\Phi_{II}(I, L) \ge 0$. The latter property captures that the greater the loan issuance, the costlier each additional unit of I becomes, i.e. the bank has to issue more deposits on the margin. When a loan is sold, the bank receives less deposits in return for each additional unit $I < \delta L$ that is sold.

Bank problem. At each t, banks choose a dividend payout C and a flow of new loans I to solve the following problem:

Problem 1 *[Bank's Problem] The bank's policy functions are the solutions to:*

$$0 = \max_{\{C,I\}} f\left(C, V\left(L, \bar{L}, D\right)\right) + V_L\left(L, \bar{L}, D\right)\mu^L W + V_{\bar{L}}\left(L, \bar{L}, D\right)\mu^{\bar{L}} W + V_D\left(L, \bar{L}, D\right)\mu^D W$$
$$+\sigma\left[V\left((1-\varepsilon)L, (1-\tau\varepsilon)\bar{L}, D\right) - V\left(L, \bar{L}, D\right)\right]$$

subject to: (2), (3), (4) and liquidation $V\left(L, \bar{L}, D\right) = v_o W^{1-\psi}$ *if* $\{L, \bar{L}, D\} \in \Gamma$.

This problem is a standard Hamilton-Jacobi-Bellman (HJB) equation, associated with the Duffie-Epstein preferences. The last term is the change in value that results from liquidation.

Market value of equity. To construct market returns, we need an asset pricing model of banks. For that, we assume that banks are owned by outside investors. Investors can hold bank shares, but cannot lend or issue deposits directly. Because of this friction, the market value of equity diverges from W, the fundamental value of bank equity. Hence, we distinguish between three concepts of bank equity: market value, accounting value, and the fundamental value. Investors price bank shares according to the net-present value of discounted dividends. We assume investors are diversified so that a bank's idiosyncratic risk does not affect their discount factor. Because we are interested in the cross-sectional behavior of banks, we abstract from expected aggregate shocks and shocks to investor risk premia. For that reason, we endow the investor with a constant discount rate ρ^I.[44]

We construct a pricing equation for the bank's equity and map the underlying default shocks to the return shocks, to be able to build the analogue impulse responses to those presented in Section 2. The market value of a bank, $S\left(L, \bar{L}, D\right)$, satisfies the following recursive representation:

$$\rho^I S\left(L, \bar{L}, D\right) = C\left(L, \bar{L}, D\right) + S_L\left(L, \bar{L}, D\right)\mu^L W + S_{\bar{L}}\left(L, \bar{L}, D\right)\mu^{\bar{L}} W + S_D\left(L, \bar{L}, D\right)\mu^D W \quad (7)$$
$$+\sigma\left[S\left((1-\varepsilon)L, (1-\tau\varepsilon)\bar{L}, D\right) - S\left(L, \bar{L}, D\right)\right].$$

This captures the dividend flow $C\left(L, \bar{L}, D\right)$ determined by the payout policy. Naturally, this recursive representation reflects the law of motion of the bank's state variables. The valuation takes into account how changes in the state variables will affect future valuations, considering the effect of loan defaults. We assume that upon liquidation, the investor receives zero for her equity.

An important point is that, implicitly, Equation (7) assumes that investors observe dN. Thus, information about losses is contained in market prices, but not in book values. This is important

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[44]In principle, we could allow discount rates to vary with time, ρ_t^I, which would not change the cross-sectional implications of the model. For simplicity, we set investors' discount rate to a constant.

for our analysis, because it implies that market values are indeed informative about loan default shocks, beyond the information content in books, as we argue in Section 2.

The ratio of market to accounting book values, Tobin's Q, is given by:

$$Q\left(L, \bar{L}, D\right) \equiv \frac{S\left(L, \bar{L}, D\right)}{\bar{L} - D}.$$

We exploit this expression in Section 3.4. In the model, variation in Q will result from changes in the value of loans that are not immediately recognized on the books.

The optimization problem of banks. The bank's problem is scale invariant, as we show formally in Appendix 1. Banks with the same λ and q but different W are scaled replicas. We express the dividend rate as $c \equiv C/W$ and the loan growth rate by $\iota \equiv I/L - \delta$. The growth rate of bank equity satisfies the following stochastic differential equation:

$$\frac{dW}{W} = \Bigg[\underbrace{r^{L}\left(\lambda + 1\right) - r^{D}\lambda}_{\text{net interest income}} - \underbrace{\frac{\gamma}{2}\iota^{2}\left(\lambda + 1\right)}_{\text{capital loss from adjustment}} - \underbrace{c}_{\text{dividend rate}} \Bigg] dt + \underbrace{\left(-\varepsilon\left(\lambda + 1\right)\right)}_{\substack{\text{loss after default} \\ \equiv J^{W}}} dN, \quad (8)$$
$$\underbrace{\hphantom{r^{L}\left(\lambda + 1\right) - r^{D}\lambda - \frac{\gamma}{2}\iota^{2}\left(\lambda + 1\right) - c}}_{\equiv \mu^{W}}$$

where μ^{W} denotes the drift and J^{W} the jump in wealth after a default. The first term of Equation (8) captures the interest income on loans per unit of wealth, $\lambda + 1$, net of the interest on leverage, λ. The second term is the net capital loss from adjusting the loan portfolio: when the issuance rate equals the maturing fraction of loans, loan issuances do not eat up bank capital. The greater the deviation, the greater the cost. The third term is the dividend rate. The final term is the jump in wealth after a default: the jump scales with leverage, λ. Hence, the expression captures how leverage increases risk. In turn, the law of motion for leverage is

$$d\lambda = \underbrace{\left(\iota - \mu^{W}\right)\left(\lambda + 1\right)}_{\equiv \mu^{\lambda}} dt + \underbrace{\frac{\varepsilon\left(\lambda + 1\right)}{1 - \varepsilon\left(\lambda + 1\right)}\lambda}_{\equiv J^{\lambda}} dN. \quad (9)$$

The drift in leverage is the difference between the growth rate of loans, ι, and the growth rate of equity, μ^{W}, scaled by leverage $\lambda + 1$. The jump in leverage J^{λ} reveals how leverage changes with defaults. The jump in leverage increases with leverage. Finally, q has the following law of motion

$$dq = \underbrace{\left(\iota + \alpha\right)\left(1 - q\right)q}_{\equiv \mu^{q}} dt + \underbrace{\left(-\left(\frac{\varepsilon - \tau\varepsilon q}{1 - \tau\varepsilon q}\right)q\right)}_{\equiv J^{q}} dN, \quad (10)$$

where μ^{q} denotes the drift and J^{q} its jump. Notice that q drifts to one with the loan growth and loss recognition rates, $\{\iota, \alpha\}$. The term $\left(1 - q\right)q$ is intuitive: when q is close to zero, the ratio does

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not move because the numerator is very small relative to the denominator; when q is 1 accounting loans already reflect the fundamental value and issuances do not change this aspect. The following proposition is a characterization of the banks' problem and market values:

Proposition 1 *[Bank's Problem] Given $\{\lambda, q\}$, $V\left(L, \bar{L}, D\right) = (1 + v(\lambda, q)) W - 1$, $C\left(L, \bar{L}, D\right) = c(\lambda, q) \cdot W$ and $I\left(L, \bar{L}, D\right) = (\iota(\lambda, q) + \delta) \cdot L$, where $\{v, c, \iota\}$ solve*

$$0 = \max_{\{c, \iota\}} f(c, v) + \underbrace{v_\lambda \mu^\lambda + v_q \mu^q}_{\text{change in financial ratios}} + \underbrace{(1 + v) \mu^W}_{\text{equity growth}} \tag{11}$$

$$+ \sigma \underbrace{\left[\left(1 + v\left(\lambda + J^\lambda, q + J^q\right)\right)\left(1 + J^W\right) - (1 + v)\right]}_{\text{default jump in wealth}} \quad \text{in } (\lambda, q) \notin \Gamma$$

and $v = v_o$ for $\{\lambda, q\} \in \Gamma$. The bank's market value is $S\left(L, \bar{L}, D\right) \equiv s(\lambda, q) \cdot W$, where s solves a version of (11) where $f(c(\lambda, q), s)$ is substituted with $c(\lambda, q) - \rho^I s$. Finally, Tobin's Q is $Q(\lambda, q) = s(\lambda, q) \times \left(\left(q^{-1} - 1\right)\lambda + 1\right)^{-1}$.

A takeaway from this proposition is that the model is scale invariant. Also, a key object of interest is the equity multiplier function, v: the term $1 + v$ represents how a unit of bank net worth is transformed into a unit of certainty-equivalent net-present value of dividends. Itself, v, is the solution to the (HJB) Equation (38). The novelty of this HJB is that it takes into account the growth of equity and the evolution of bank ratios. Notice that although the model is not stationary in levels, it is stationary in ratios. For common $\{\lambda, q\}$, the dividend and lending policies scale with W. Since the problem is scale invariant, the market capitalization is also proportional to W, where the price per unit of wealth s depends only on $\{\lambda, q\}$. Below, we describe how v governs the dynamics of leverage, through its influence on the dividend and issuance policies, given q.

3.2 Inspecting the Mechanism

We next proceed to explain the model's mechanics, turning on one friction at a time. Formal results are relegated to Appendix C.

Immediate accounting without loan adjustment costs. To shed light on the mechanics of our Q-theory, we first solve for the case where default shocks are instantaneously recognized ($\tau = 1$) and there are no adjustment costs ($\gamma = 0$). In this case, $q = 1$ at all times so α plays no role. Since $q = 1$, only the regulatory liquidation matters and the liquidation boundary is given by a threshold leverage, $\Gamma = \{\lambda | \lambda > \xi / (1 - \xi)\}$. Also, since $\gamma = 0$, the bank can choose a jump in the stock of loans and deposits, and hence, controls a jump in λ.[45] We refer by $\bar{J}^x$ to the controlled (endogenous) jump for any variable x.

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[45] In this version, the path of loans does not have to be continuous even in the absence of shocks.

A key object for the dynamics is the shadow liquidation boundary, Λ. The shadow boundary is the leverage rate such that upon a loan default shock, leverage jumps exactly to the boundary of the liquidation set. Thus, Λ solves:

$$\Lambda + J^\lambda = \xi / (1 - \xi),$$

where J^λ is the jump in leverage after a default shock, starting from a leverage Λ.

For this limiting case of the model, the multiplier v is a constant and leverage solves:

$$\max_{\lambda \in [0, \xi/(1-\xi)]} \underbrace{(1+v)\left(r^L - r^D\right)}_{\text{value of levered returns}} \lambda + \sigma \left\{ (1+v) \underbrace{\left(1 + J^W\right) \mathbb{I}\left[\lambda \leq \Lambda\right]}_{\text{wealth upon default shock}} + \underbrace{v_o \mathbb{I}\left[\lambda > \Lambda\right]}_{\text{liquidation value}} \right\}. \quad (12)$$

This problem clarifies that leverage results from a tradeoff between intermediation profits and liquidation risk. The first term in the objective describes how leverage increases levered returns: intermediation profits, $\left(r^L - r^D\right)$, increase equity at the margin. In turn, the marginal value of equity is the multiplier $1 + v$. The second term is the value after a loan default. Defaults occur with intensity σ and lead to two possibilities. The bank can avoid liquidation, if $\lambda \leq \Lambda$, and in that case, the cost of default is only the reduction in the bank's scale by $1 + J^W$. Otherwise, the bank is liquidated and recovers v_o per unit of equity. Because the objective is piece-wise linear, the target leverage is a corner solution. The interesting parameter combination satisfies $\left(r^L - r^D\right) \in [\epsilon\sigma(1+v), \sigma v_o]$.[46] Under this combination, the expected return of increasing leverage is positive up to $\lambda = \Lambda$. Past the shadow boundary, the hazard of default makes the benefit of increasing leverage negative. Hence, under those parameters, the bank sets its target leverage to $\lambda^* = \Lambda$.[47]

To guarantee that $\lambda^* = \Lambda$ always, the endogenous jump must neutralize the jump caused by defaults, $\bar{J}^\lambda = -J^\lambda(\Lambda)$. With neither delayed accounting nor loan adjustment costs, the bank can do that adjustment at no cost. When $dN = 0$, the dividend and the loan issuance rates are constant. In particular, c^* is given by a formula that captures wealth and substitution effects[48]

$$c^*(v) = \rho^{1/\theta}(1+v)^{1-1/\theta}. \quad (13)$$

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[46]Otherwise, leverage is set to zero or the bank is liquidated upon its first default: If $\left(r^L - r^D\right) < \epsilon\sigma[1 + \bar{v}]$, the expected return of increasing leverage is negative and $\lambda^* = 0$. If $\left(r^L - r^D\right) > \epsilon\sigma[1 + \bar{v}]$, the bank would like to lever up as much as possible. If $v_o > \left(r^L - r^D\right) > \sigma$ the return spread exceeds the bank's expected liquidation value σv_o. In that case, the bank operates at the fringe of solvency, setting λ^* to its permissible limit. As soon as it suffers a loan default, the bank is liquidated. Generically, leverage can be set to either of three values depending on parameters: to zero, to the shadow boundary value or the liquidation boundary value.

[47]Critical to this result is that risk-aversion is zero. Otherwise, leverage would involve an additional tradeoff between return and equity growth risk.

[48]Here, $1 + v$ acts like a total return on wealth. When $\theta > 1$ ($\theta < 1$), the substitution (wealth) effect dominates and the bank retains (pays out) more dividends as v increases.

Given c^*, the loan issuance rate ι^* is such that leverage is constant, $\mu_\lambda = 0$. Finally, the equity multiplier solves:

$$0 = f\left(c^*\left(v\right), v\right) + \left(1 + v\right)\left(\mu^W + \sigma\left(J^W - 1\right)\right).$$

Turning back to our motivating facts, this limit case produces a leverage target, as desired. However, the impulse response of leverage to a return shock looks like a blip with no persistence. In turn, the response of total liabilities looks like a one time shock. Clearly, other aspects of the model are needed to match all the facts highlighted above.

Delayed accounting without loan adjustment costs. We now study the case with only delayed accounting ($\tau < 1$ and $\gamma = 0$). This case explains most of the intuition of the general model. In this case, since losses are not recognized immediately, in general $q < 1$. The liquidation region therefore depends on q, and $\Gamma(q) = \left\{\lambda | \lambda > \min\left\{\xi/\left(q - \xi\right), \bar{\lambda}\right\}\right\}$. As a result, the shadow boundary is no longer a scalar, but a function of q, which we now label $\Lambda(q)$. To construct the shadow boundary, we must consider the jumps J^q and J^λ. The shadow boundary $\Lambda(q)$ solves:

$$\Lambda(q) + J^\lambda\left(\Lambda(q)\right) = \min\left\{\frac{\xi}{q + J^q(q) - \xi}, \bar{\lambda}\right\}. \tag{14}$$

As with immediate loss accounting, a loan default takes the bank from a point $\{\lambda, q\}$ in the shadow boundary to a point in the boundary of the liquidation set Γ. Again, for suitable parameter conditions, the dynamics are such that a bank will immediately delever until it reaches another point in the shadow liquidation boundary $\Lambda(q)$. The necessary loan sale to return to the shadow boundary induces a jump $\bar{J}^\lambda$, but it also alters q, inducing a jump $\bar{J}^q$. Hence, the bank no longer returns to the same leverage, but rather a loan default takes the bank from a point $\{q, \lambda\}$ in the shadow boundary to another point, $\{q', \lambda'\} = \left\{q + J^q + \bar{J}^q, \lambda + J^\lambda + \bar{J}^\lambda\right\}$, in the shadow boundary.

It is important to understand the choices of $\{\lambda, q\}$ and $\{c, \iota\}$ along the boundary to understand the ability to match the data impulse-responses. Figure 1 allows us to sketch these dynamics: the y-axis represents values of λ and the x-axis values of q. Starting from an arbitrary point $\{q, \lambda\}$ in the shadow boundary a loan default event takes the bank to a point on the boundary of Γ. Since the bank survives, but wishes to avoid a future liquidation, it immediately sells assets to delever. This takes the bank to another point in the shadow boundary, the point $\Lambda(q')$. Absent defaults, when $dN = 0$, the state variable $\{\lambda, q\}$ drifts along the shadow boundary. This is because to the right of the boundary, liquidation risk is positive. To the left of the boundary, the bank prefers to lever up to increase its equity return. Hence, the choice of issuances and dividends must guarantee a smooth drift along the shadow boundary, taking the bank slowly to $q = 1$, as shown in the figure. Because the shadow boundary has a negative slope, $\Lambda_q(q)$, the bank must guarantee a decline in leverage as q increases with the pace of loan loss recognition.

Turning to the impulse responses: on impact, q falls whereas λ increases. As a result, market

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leverage features a jump by more than book leverage, a feature of our impulse responses. After the period of impact, λ will fall along the boundary whereas q approaches 1. This is consistent with a mean reversion in market leverage, with only a modest response in book leverage, as in the data. We can also describe in detail the dynamics of liabilities in the model. With delayed accounting, each point in the boundary is associated with a value $v(q)$. The dividend policy is modified to:

$$c^* = \rho^{1/\theta} \left[\frac{(1 + v(q))}{\left(1 + v(q) + v_q \frac{(1+\Lambda(q))}{\Lambda_q(q)}\right)^{1/\theta}} \right]. \tag{15}$$

The expression reveals that even under delayed accounting, the dividend choice is still governed by a race between wealth (in the numerator) and substitution effects (in the denominator). The substitution effect is modified because on the margin, the choice of dividends affects q indirectly.[49] Once dividends are determined, to stay at the shadow boundary, issuances $\{\iota\}$ must satisfy the restriction that $\Lambda_q(q) = \mu^\lambda/\mu^q$. We can obtain a formula for issuances that guarantees that the bank stays in the shadow boundary:

$$\iota(q) = \mu^W(q)\,\omega(q) - \alpha(1 - \omega(q)) \tag{16}$$

with the weight given by

$$\omega(q) \equiv \frac{(1 + \Lambda(q))}{q(1-q)} \Big/ \left(\frac{(1 + \Lambda(q))}{q(1-q)} - \Lambda_q(q) \right).$$

The intuition for the slow adjustment of liabilities that we find in the impulse responses is also found in the formula. Upon a default shock, losses are recognized slowly on the books, at the rate α. As losses are recognized, and q increases, banks must take gradual actions to delever because their fundamental leverage is too high. The expression for issuances captures how issuances are chosen to reduce leverage. Leverage has a tendency to fall because retained earnings increase equity, $\mu^W(q)$; recall that $\Lambda_q(q)$ has a negative slope. Hence, the higher the growth in equity, the less the need to reduce loan issuances to reduce leverage. By contrast, a higher α implies that losses are recognized faster. Hence, with a higher α, the bank requires lower (and possibly negative) issuances. Since liabilities increase one for one with issuances, the expression captures how liabilities fall after a default shock. This formula captures the pattern for issuances, which fall with the rate of loan loss recognition. However, notice that the issuance rate only affects liabilities slowly, which unlike the version without delayed accounting, is capable of explaining the slow response of liabilities.

To summarize, the model with delayed accounting is consistent with dynamics where, upon a

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[49]Because leverage must stay in the boundary, $(1 + \Lambda)/\Lambda_q(q)$ measures the change in q corresponding to a change in leverage that keeps the bank along the boundary. The change in q effectively has a price of v_q.
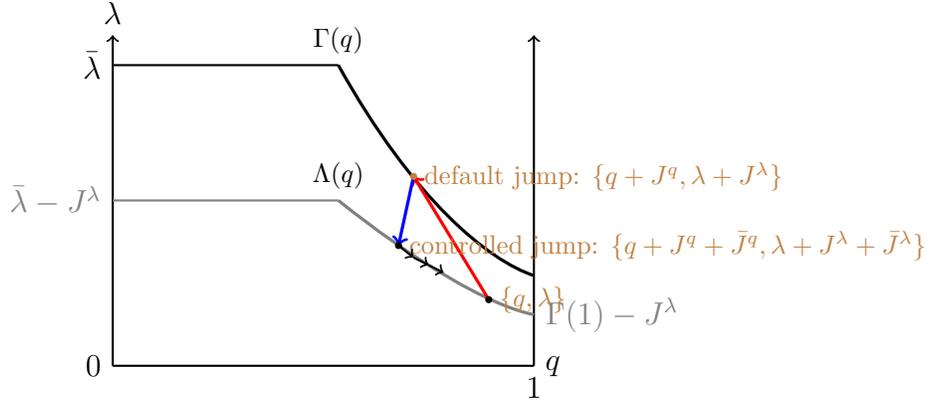
Figure 1: Illustration of $\{\lambda, q\}$ dynamics.

default shock, the bank takes quick actions to return to the shadow boundary. Once at the shadow boundary, the bank continues a slow deleveraging, which induces a slow response of market leverage and total liabilities, as we observed in the data. Thus, even though there are no loan adjustment costs, the adjustment of leverage is gradual along the shadow boundary as losses are recognized slowly in the balance sheet. This slow adjustment forms the basis of our Q-theory.

Delayed accounting and loan adjustment costs.   We now discuss the dynamics allowing for loan adjustment costs ($\tau < 1$ and $\gamma > 0$). Loan adjustment costs only introduce inertia to any attempt to deviate from $\iota = 0$, but the mechanics are very similar to those of the earlier case. From the previous limit cases, we know that banks have a target for leverage that trades off returns for liquidation risk. We also know that without adjustment costs, upon receiving a default shock banks delever rapidly to return to the shadow boundary and then drift slowly along the shadow boundary. Loan adjustment costs slow down the endogenous response on impact and slightly perturb the dynamics along the shadow boundary.

Unfortunately, this case must be solved numerically. Figure 8 presents a numerical solution to v, the drift of λ and the policy functions $\{\iota, c\}$, for different values of λ in the x-axis and for various cross sections of q. The figure is constructed under the calibration detailed in the following section. Panel (a) depicts v. The tradeoff between returns and liquidation risk is evident from the shape of v: for a fixed q, the value v is first increasing, but becomes flatter as leverage approaches the shadow liquidation boundary. Past the liquidation region, the value function jumps to a value equal to its liquidation value. From the figures, we can also observe that v is decreasing in q, a reflection that a bank that seems better capitalized in its books faces less regulatory constraints.

Panel (b) presents the drift of leverage for various values of q. The figure shows how, in this complete version of the model, leverage is again mean reverting. The target leverage is the point where the expected change in λ is zero: when leverage is low, the drift is very high, reflecting the effort to increase leverage. To the right of the shadow boundary, the drift is negative and,

furthermore, very steep, reflecting a slightly smoother version of the jumps that occur without adjustment costs.

As before, the behavior of leverage is governed by the bank's payout and lending policies. The dividend rate in this case solves:

$$c^* = \rho^{1/\theta} \left[\frac{(1+v)}{((1+v) - v_\lambda (\lambda+1))^{1/\theta}} \right]. \tag{17}$$

Relative to the dividend policy in Equation (13), the substitution effect is also corrected by the change in the value function as a result of the change in leverage produced by dividends.

The loan growth policy solves:

$$\underbrace{\gamma \left((1+v) - v_\lambda\right) \cdot \iota^*}_{\text{marginal cost of change in assets}} = \underbrace{v_q \frac{(1-q)\, q}{(\lambda+1)} + v_\lambda}_{\text{marginal benefit of change in financial ratios}}. \tag{18}$$

The left-hand side is the marginal cost of changing loans, deviations from ι^*. These deviations cost γ in equity, have a marginal value of $(1+v)$, but carry an effect by increasing leverage v_λ. The right-hand side is the marginal benefit of changing the bank's ratios. Panels (c) and (d) of Figure 8 show how the desire to delever is reflected in the banks loan growth and payout policies. When leverage is low, the bank has incentives to lever up; it cuts back on dividends and lending. Close to the liquidation boundary, the bank is eager to delever. It cuts back the growth of loans, and slashes dividends. The effects are more dramatic the more realistic its books.

The desire to remain in the neighborhood of the shadow boundary is still present. This is evident from the invariant distribution depicted in Figure 10, but not from the Equations (17) and (18), because this desire is encoded in the shape of v which is not evident from the formula. This is why it is important to describe the mechanics of the model in layers. For completeness, Figure 9, is the analogue of Figure 8, along the q-dimension.

3.3 Calibration and Estimation

We now describe the calibration and estimation procedures and then investigate the model's ability to reproduce the five facts. We use quarterly data from 1990 Q3 to 2015 Q4, as described in Section 2, to produce target moments. Thus, all corresponding model moments are also at the quarterly frequency. To keep the parametrization tractable, we calibrate $\left\{r^L, r^D, \delta, \xi, \rho, \rho^I, \varepsilon, \sigma, \alpha\right\}$ independently, matching model moments to target moments in the data. Then, conditional on these calibrated parameters, we jointly estimate $\{\gamma, \theta, \tau\}$, the parameters that govern the delay in the balance sheet responses. As in Section 2, we break the sample into two periods, corresponding to the pre- and post-crisis periods, and estimate pre- and post-crisis values for $\{\gamma, \theta, \tau\}$ to match the impulse response functions to return shocks as discussed in Section 2. The parameter values

are listed in Table 2. Table 3 presents both targeted and untargeted moments in the data and the corresponding model moment.

Calibrated parameters .   The exogenous returns on loans and deposits, r^L and r^D, are respectively set to 1.01% and 0.51%, consistent with the quarterly yield on loans (total interest income on loans divided by total loans) and the rate banks pay on their debt (total interest expenses divided by interest bearing liabilities) in bank call reports. These values are consistent with the calibration in Corbae and D'Erasmo (2019).

We set the capital requirement parameter ξ to 92.6% in order to reflect a Tier 1 risk-based capital ratio requirement of 8%, a value for which a bank is considered well capitalized.[50] This means that banks' book leverage ratio (debt to book equity) cannot exceed 14.

The parameters $\{\alpha, \tau\}$ directly speak to our Q-theory. Whereas τ accounts for loan loss recognition on impact, α governs the speed of loan-loss recognition over time. We estimate τ, but calibrate α because we have a direct counterpart for α. Namely, we set α to 4%. With this value for α, 65% of unrecognized losses are recognized within 10 quarters.[51] This delay is consistent with Figure 2, panel (d), where the net charge-offs taper off by the end of 2010, about two and a half years after the trough in bank market values.

In our model, banks choose a book leverage ratio on the shadow boundary. The distance between the shadow boundary and the liquidation set is determined by the size of the idiosyncratic loan default shock ε. Thus, once ξ is fixed, we pick $\varepsilon = 0.25\%$ to match the ergodic mean of banks' book leverage ratio to the average pre-crisis book leverage ratio of 10.6. We choose the pre-crisis period to calibrate this idiosyncratic shock parameter as the post-crisis sample is more likely to capture the effects of the financial crisis, an aggregate event.

Market leverage is a function of the market price of equity, which in turn is increasing in the bank's discount rate, ρ. Therefore, given a value for book leverage which is pinned by the regulatory parameter, ρ affects market leverage by moving dividends and, therefore, the market price of equity. We set ρ to 0.25% to target an average market leverage of 6.8 for the pre-crisis period. Note that while most of the parameters discussed in this section are calibrated as if they influenced only their target moment, ρ and ε alone do not pin down book leverage and market leverage. We would ideally estimate ρ and ε jointly with τ, θ, and γ, if we were not limited by computational tractability. For this reason, the targets are not matched exactly.

Finally, given the value for default shocks ε, the default intensity σ is set to match a mean net charge-off rate of 0.48% per year in the full sample. We use the full sample of net charge-off rates to allow for a larger time series as credit events are rare. Also, because investors are risk-neutral, their discount factor ρ^I approximately equals the average market return on bank shares. Thus, we set ρ^I to 3.5% to approximately produce that value for bank market equity returns.

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[50]See the publication by the Federal Reserve.
[51]See Appendix C.7 for the derivation.

Estimated parameters. We estimate $\{\tau, \theta, \gamma\}$, the parameters that govern the speed of the responses of the bank's balance sheet, using simulated method of moments. Each parameter in this subset is directly associated with a different friction that impacts the balance sheet adjustment process after a loan default shock. Thus, these parameters determine the model's ability to replicate facts 4 and 5.

We estimate these parameters to match the impulse response functions of market leverage, book leverage, and bank liabilities to a return shock in the data. These impulse responses render a transparent identification: the sensitivity of loan adjustment costs, γ, governs the cost of deleveraging through asset sales. Thus, γ impacts the speed at which banks can delever by selling liabilities along the shadow boundary. A higher γ translates into a slower response of liabilities to a return shock. In turn, the desire to smooth dividends is governed by θ—see Equation (17). Thus, θ governs the speed at which leverage falls through the effect of dividends on retained earnings. A lower value of θ turns the bank's objective closer to linear and makes the bank deleverage faster by cutting back dividends and increasing retained earnings. Hence, the response of market leverage is informative about θ. Finally, recall that τ governs how much of a loan default shock is recognized on impact. Therefore, τ directly maps into the response of book leverage, on impact. For that reason, because τ governs the response only on impact, we only use the first period response of book leverage as a target to identify τ.

To produce analogue estimated impulse responses to return shocks in the model, we solve and simulate the model. We run the same specification for the impulse responses of Section (2). Given that the impulse responses of liabilities and market leverage pre–crisis and post-crisis differ, we estimate values for θ and γ for both periods; we label these values as $\{\theta^{pre}, \gamma^{pre}\}$ and $\{\theta^{post}, \gamma^{post}\}$, correspondingly for these estimates. We construct excess return shocks by first calculating the realized equity returns between adjacent quarters and the cross-sectional equity return. The bank specific return shock is then just the difference between a bank's individual realized equity return and the cross-sectional average of the realized equity returns. The latter absorbs any potentially time varying aggregate effects on banks' equity returns, similar to only using a time fixed-effect specification. Formally, the model is overidentified because each impulse response in the data contains effectively 21 moments, one for each β_h in Equation (1). However, model generated moments such as these are highly correlated so the effective degree of over-identification is lower.[52]

To generate a panel of banks as in the data, we first compute the stationary distribution by simulating 10,000 banks until the cross-sectional mean and standard deviation of λ and q are approximately constant. We then take this cross-section as initial condition and simulate 66 quarters that represent the pre-crisis period. With this sample, we run the same pre-crisis cross-sectional regressions on model simulated data, as we did with the actual data, to estimate θ^{pre} and γ^{pre}. To simulate the post-crisis, we hit the stationary pre-crisis economy with an aggregate shock and continue simulating for an additional 33 quarters. That is, to estimate the post-crisis

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[52]Each impulse response is well approximated by only two moments, a jump on impact and a persistence.

parameters, θ^{post} and γ^{post}, we start banks from their pre-crisis stationary distribution and hit all banks at once with a 2.5% aggregate loan default shock in one quarter. We chose a 2.5% default shock as this number is in line with the accumulated loan default shocks from 2008 through 2010, consistent with our assumed length of loan default recognition. Once again, we estimate the same regression and construct impulse response functions on the model simulated data as we did with actual bank data. We discuss the values from this estimation and the resulting model fit in the next paragraph.

Estimated Values, Model Fit, and Interpretation. To match the initial impulse response of book leverage, τ needs to be small, about 1%. The impact response of book leverage to the return shocks does not change across periods. For this reason, we estimate τ only for the pre-crisis period. The pre-crisis period estimates of the other parameters are $\theta^{pre} = 2.3$ and $\gamma^{pre} = 0.01$. For this low adjustment cost value, the effect of γ on the model is negligible in the pre-crisis in the sense that the dynamics of the model are almost identical to not having adjustment costs. This is interesting because it means that delayed accounting—and dividend smoothing—can account for all of the slow balance sheet adjustment after shocks.

The post-crisis impulse responses of market leverage and bank liabilities produce values of $\theta^{post} = 1.7$ and $\gamma^{post} = 4.0$. This means that unlike in the pre-crisis period, the post-crisis period model estimation requires loan adjustment costs to fit the data. The estimation thus reflects some of the narratives around the financial crisis: a higher value of γ implies that banks face higher costs to selling loans. This feature fits the narrative of aggravated fire sale externalities during the post-crisis period. In turn, a lower value for θ makes the banks' objective closer to being linear. This means that banks accept larger adjustments to dividend payouts and, therefore, can rely more on internal equity accumulation as a mechanism to delever. This estimate is in line with the fact that banks cut dividends during the period.[53] We return to these results when we discuss the model's ability to reproduce the impulse responses again in Section 3.4.

To conclude the evaluation of the calibration, Table 3 compares the moments generated by the model and those obtained from the data: our model fits most data moments well, with the exception of pre-crisis leverage and (untargeted) dividends. The model fits the charge-off rates both in the pre-crisis as in the post-crisis. Average market-equity returns in the model match the data during the pre-crisis sample, but differ for the crisis sample. This is because in both cases, we calibrate parameters to their pre-crisis values. With a value of 9.4, the model overshoots its 6.8 data target for market leverage during the pre-crisis period. As noted above, market leverage depends on more parameters than just ρ so matching market leverage closely is more difficult without a computationally intensive joint estimation. The model fits the untargeted market leverage moment during the post crisis period perfectly. The model hits book leverage in the targeted pre-crisis period, but does not generate the same reduction in book leverage as in the data. This is not

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[53]See for example, https://www.ft.com/content/7a4aa5bc-0a85-11de-95ed-0000779fd2ac.

surprising because our model abstracts from an increase in bank capital requirements during the post-crisis period that forced banks to decrease book leverage. Finally, our model generates a similar level in the market-to-book equity ratio during the pre-crisis (1.53 in the data vs. 1.32 in the model) and a reduction in the market-to-book equity ratio in the post crisis (0.90 in the data vs. 1.108 in the model). Thus, our simple model explains about one-third of the reduction in the market-to-book ratio of banks from the pre- to the post-crisis.

The bottom panel of Table 3 shows the cross-sectional average and standard deviations of key model variables. It also includes the two unobservable state variables of our model: fundamental leverage λ and q, the discrepancy between banks' fundamental value of assets and their corresponding accounting value. Fundamental leverage is 18 in the pre-crisis period and jumps to 22 in response to the 2.5% aggregate default shock. The average value for q is 0.977, implying that fundamental values and accounting values differ by 2.3%. The post-crisis q is slightly lower compared to the pre-crisis q. The loan growth rate is about 2.4% and the growth rate of equity is around 1.2%. Our model also produces values of equity (equity price) of 1.91 in the pre-crisis and 1.97 in the post-crisis. The valuation is roughly constant across periods. While bank dividends fall slightly during the post-crisis period but banks operate with higher leverage (and therefore higher equity growth).

The distribution of the state variables and market leverage are reported in Figure 10. Panel (a) shows how the distribution of $\{\lambda, q\}$ traces out a shadow boundary. Panel (b) contrasts the distribution of market leverage with the distribution in the data. Although book leverage has virtually no dispersion, market leverage captures substantial variation driven entirely by variations in q.

3.4 Taking Stock: Matching facts 1 to 5

Fact 1. Market and book equity value divergence. The first fact we highlight in the paper is the divergence between book and market bank equity values during the financial crisis of 2008. We now discuss the extent to which variations in Tobin's Q can be explained with our model. In our Q-theory, Tobin's Q can diverge through changes in the price per share s and the discrepancy between book and fundamental values, captured by q. To that end, we produce an approximation of the law of motion of q in our model (see Equation 10). We denote by $\boldsymbol{x}$ the aggregate version of a variable x. The law of motion for aggregate $\boldsymbol{q}$ is approximately

$$\frac{d\boldsymbol{q}}{dt} \approx (\boldsymbol{\iota} + \alpha)\left(1 - \boldsymbol{q}\right)\boldsymbol{q} - \sigma\varepsilon\left(1 - \tau\boldsymbol{q}\right)\boldsymbol{q}.$$

Setting $d\boldsymbol{q}/dt = 0$, we obtain an approximate value of $\boldsymbol{q} = 0.977$ using our pre-crisis model values (see Tables 2 and 3). This approximated value is identical to the average simulated value of q reported in Table 3. With an aggregate loan default shock ϵ of 2.5%, common to all banks, the

jump in the aggregate $\boldsymbol{q}$ is approximately,

$$J^q = -\epsilon \frac{(1 - \tau \boldsymbol{q})}{(1 - \tau \epsilon \boldsymbol{q})} \boldsymbol{q} \approx -2.4\%.$$

Thus, since the initial q is close to one and τ is close to zero, the jump in q is almost equal to the size of the aggregate default shock.

We can decompose that jump in Tobin's Q into the effect that is exclusively attributed to the jump in q as opposed to a change in banks' stock price. This approximate response is given by

$$\Delta Q \text{ due to } q \approx J^q \cdot (\lambda + 1) = -45\%.$$

The approximation states that the jump in Tobin's Q attributed to q is the jump in J^q times fundamental leverage. The value of this approximation is rather large. A 2.5% shock in loan losses gets amplified almost twenty times. Recall that although book and market leverage are around 10, fundamental leverage is about 20, due to the initial accounting differences. Hence, although q is close to 98% in normal times, implying that books are an almost accurate description of reality, small unrecognized losses get magnified by fundamental leverage, accounting for a decline of almost 45% in Tobin's Q.

The quality of this approximation is good and telling of the contribution of delayed accounting in rationalizing the changes in Tobin's Q in the data. In the simulations used to construct Table 3, we feed the model with a 2.5% loan default shock. With this shock, on impact the induced reduction in average Tobin's Q at the moment of the shock is 28%. In the simulations, the isolated effect of little q on the drop in big Q is 53%, close to the analytic approximation above. The overall effect is offset by an increase in the market value per unit of fundamental wealth, s, which increases by 25%; recall that the marginal value s increases with the jump in λ. Thus, the 28% drop in Q that our model can generate accounts for two thirds of the change in the Tobin's Q during the 2008 financial crisis which fell by 42%. Our Q-theory explains this large portion of the decline without changes in aggregate risk-premia, any changes in the regulatory landscape, or any decline in implicit government guarantees.

Fact 2. Predictive Power. Our second fact of interest is the predictive power of Tobin's Q in terms of book-equity returns and loan charge-off rates up to even two years. Our model captures this effect because market values capture losses that are unrecognized in books. We replicate Figure 3 with data generated by the model (see Figure 13 in the Appendix). This figure shows that market equity contains predictive power for return on equity and loan losses over and above the information contained in book equity. To see why, recall that without considering the changes in the price per unit of real equity, upon a default shock, returns fall approximately by J^W. A bank's charge-off rate per unit of equity is approximately $\alpha \cdot (1/q - 1)(\lambda + 1)$. Since, the jump J^q

is correlated with the jump J^W, the predictability follows by construction.

Fact 3. Equity Buffer.   The third fact highlights that banks keep a buffer of book equity away from the maximal leverage allowed by regulation. Earlier we discussed that liquidation is not desirable because of liquidation costs. For this reason, banks stay at a shadow boundary, which guarantees solvency in case of a default shock event. Note that in our model, while the default event is random, the size of the default shock is not. In our model, average book leverage is 12.4, almost 2 points below the maximal admissible value given the regulatory constraint. The model also predicts that in a recession, the fraction of banks near the regulatory constraint increases. While our model features little cross-sectional variation in book leverage, it captures some of the cross-sectional dispersion in market leverage as can be seen in Figure 11b. All cross-sectional dispersion in market leverage must come from dispersion in equity valuation and cross-sectional differences in fundamental value to accounting value of bank assets as our model features negligible dispersion in book leverage. Thus, even though we abstract from features in the data that could generate dispersion in book leverage, for example different business models, the accounting mechanism alone generates dispersion in market leverage.

Fact 4. Target leverage and slow adjustments.   Banks appear to have a target leverage ratio but respond only slowly to deviations from it (see the impulse response estimation described in Section 2). Figure 12a compares the pre-crisis impulse responses to return shocks of total liabilities (left panel) and market leverage (right panel) of the model with the data. The black lines represent the average estimated response from the data—the shaded area their 95% confidence interval. The blue lines represent the analogue impulse response in the model. The red line represents the model responses corresponding to the model without balance sheet adjustment costs and accounting frictions.

The figure shows that the model generates an initial (mechanical) jump and slow adjustment of market leverage as well as the slow adjustment of liabilities as in the data. Note that in the pre-crisis period, our Q–theory does not rely on loan adjustment costs to reproduce the data as γ is near zero. Once we allow for accounting values to differ from fundamental values, i.e., $\tau < 1$, our model does not require further adjustment costs to generate slow adjustments of the liabilities. The red lines in Figure 12a also show that a dividend smoothing motive cannot account for the impulse response function of the data without accounting frictions. In fact, without accounting or balance sheet frictions (red lines), banks immediately reduce assets in response to a negative wealth shock leaving leverage unchanged.

Fact 5. Post-crisis leverage adjustment.   During the crisis and immediate post-crisis period, bank leverage adjusted faster compared to the pre-crisis period. Unlike the pre-crisis, adjustments in leverage were driven by increases in equity. The response of assets as captured by the response

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in liabilities was slower during this period. To capture these features, our model needs higher balance sheet adjustment costs and a lower dividend smoothing motive as captured by a higher estimated value for γ and lower estimated value for θ, respectively. A higher estimated value for γ in the post-crisis means that selling assets is costlier. These post-crisis estimates are consistent with several narratives regarding the aggravation of frictions during the crisis. The reduction in θ during the crisis/post crisis period means that the model responds to shocks with an increase in retained earnings (and therefore a reduction in dividends) that leads to more equity. The estimated lower value for θ could reflect pressures by regulators to cut dividends during the crisis to bolster banks' equity position. All in all, our Q-theory generates all five facts fairly well.

3.5 Effects of Accounting Rules

In the previous sections, we argued that delayed accounting is key to explain the five facts this paper highlights. But why should we care about accounting rules? We argue that a reform toward more transparent accounting rules, accounting that leads to faster loss recognition, involves a tradeoff that policy makers should be aware off. Namely, faster accounting involves a tradeoff between the scale of bank losses and the speed of adjustment after these losses.

To highlight that tradeoff, we solve the model for different values of α. Recall that a lower α means that losses are more slowly recognized. Lower values of α have the interpretation of accounting rules that are less transparent. In the model, a lowering α has effects on the distribution of state variables: on one hand it lowers the average q and on the other, it increases the fundamental leverage, λ. Indeed, lower values of α provide banks with more slack to circumvent regulatory constraints and this manifests in a higher fundamental leverage and a greater discrepancy between the fundamental value and the book value of loans. Figure 13a reports cross-sectional means of q and λ, as we vary α keeping other parameters in the pre-crisis values. The negative relationship between q and λ is evident from the figure. Strikingly, although the fundamental leverage ratio λ differs for different accounting rules, average book leverage is constant in each of these equilibria. According to the model, to a regulator that uses accounting leverage to gauge the health of the financial system, all of these economies will look the same. Yet, laxer accounting rules increases the potential equity losses and will produce greater assets sales in response to those losses. This feature highlights one aspect of the tradeoff: laxer accounting rules can lead to greater potential losses.

The other aspect of the tradeoff is that laxer accounting rules allow for a smoother adjustment process to those equity losses. To see this, Figure 13b reports how bank loans respond to a negative net worth shock, depending on α. The lower the α, the smaller the decline in bank loans in response to a negative wealth shock. Given a 10% negative wealth shock, our baseline calibration of $\alpha = 4\%$ implies a 6% decline in loans after 5 years. Lowering α to 3% reduces the decline to 3.7%, while with a value of α of 5%, loans fall by 7%. The reason for this pattern is that by delaying losses,

banks no longer need to sell additional loans to keep their book leverage constant. In that sense, delayed accounting acts like an automatic bank-specific countercyclical buffer.

We can transparently represent this tradeoff from the model's equilibrium equations. One side of the tradeoff, the increased effect on risk (equity losses), can be read from how a lower α leads to a higher λ that increases $-J^W$ in Equation 8. The other side of the tradeoff, the slower adjustment of liabilities after a default shock, can be read from how a lower α leads to a smaller reduction in ι in Equation 16.

The result connects our model with the debate on whether accounting rules should incorporate market based information to improve macro-prudential regulation. In the paper, we argued that market values incorporate information on losses much faster than book values. Thus we can envision how incorporating market values to accounting would increase α. We also discussed how delayed accounting can follow from either the backward looking nature of book values and or evergreening, the continuous debt rollover of non-performing loans in order to avoid booking equity losses. The literature has identified another tradeoff: On the one hand, marking assets to market can exacerbate fire sale dynamics (Laux and Leuz, 2010; Ellul et al., 2011; Shleifer and Vishny, 2011). If in particular market values are driven by risk premia, regulation should insulate banks from changes in risk-premia that are not germane to the health of the banking industry. On the other hand, the discretion in accounting rules opens the door to evergreening which contributes to the creation of zombie loans and drags economic efficiency, Caballero, Hoshi and Kashyap (2008); Huizinga and Laeven (2012); Blattner, Farinha and Rebelo (2019). While a welfare analysis that quantifies this additional tradeoff is beyond the scope of this paper, we believe that the race between the size and smoothing out of equity losses that we illustrate here should also be part of that active tradeoff.

4   Conclusion

This paper summarizes five empirical facts about the dynamics of bank leverage. We use these facts to explore what features banking models need to explain the data. Our empirical findings suggest a theory wherein banks target market leverage, but where adjustments to that target are gradual. A comparison between pre- and post-crisis responses suggests that, in contrast to the pre-crisis period, in the post-crisis banks relied more on retained earnings than on asset sales to readjust market leverage back to target.

The paper presents a heterogeneous-bank model that distinguishes book from market values. In our model, both measures of equity matter for banking decisions. A novel feature is that banks have the ability to delay the recognition of losses on their books. The model produces an endogenous target for leverage and features adjustment costs to the resale of assets. The model reproduces the impulse responses that we estimated from the cross-sectional data. Strikingly, the estimation highlights the exclusive role of delayed loss accounting to explain the data, and

puts little weight on standard adjustment costs. We also demonstrate that regulatory reforms designed to accelerate loss recognition introduce a tradeoff between the scale of equity losses and the posterior adjustment process.

The model highlights the essential frictions that are necessary to reproduce the five empirical facts. As part of the continuous fine-tuning process of banking models, future work can use our model as a building block to study these frictions in general equilibrium.

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Figure 1: Book Equity and Market Equity for Bank Holding Companies.
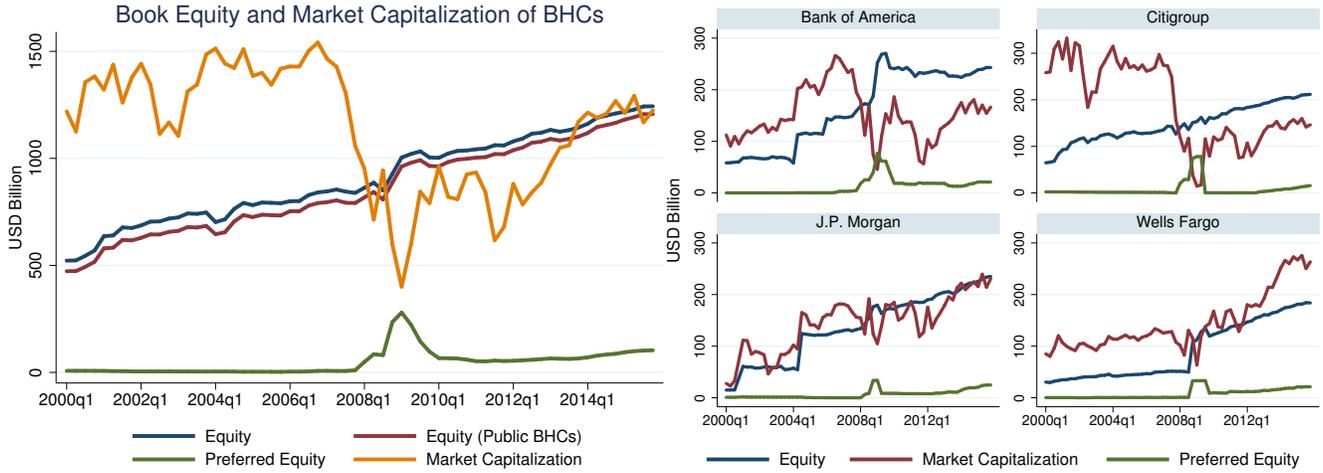
Notes: These figures show data on book equity, market capitalization, and preferred equity for BHCs. Book equity and preferred equity data come from the FR Y-9C, and market capitalization data is based on CRSP data. All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. The left panel shows aggregate series, excluding new entrants such as Goldman Sachs and Morgan Stanely. "Equity" refers to book equity for all BHCs in the sample and "Equity (Public BHCs)" refers to only publicly-traded BHCs that we can match to CRSP data. We also show preferred equity for all banks and the aggregate market capitalization, i.e. shares outstanding times the share price, of the publically traded BHCs in our sample. The right panel shows the equivalent time series for the four largest BHCs.

Figure 2: Decomposition of Net Charge-offs
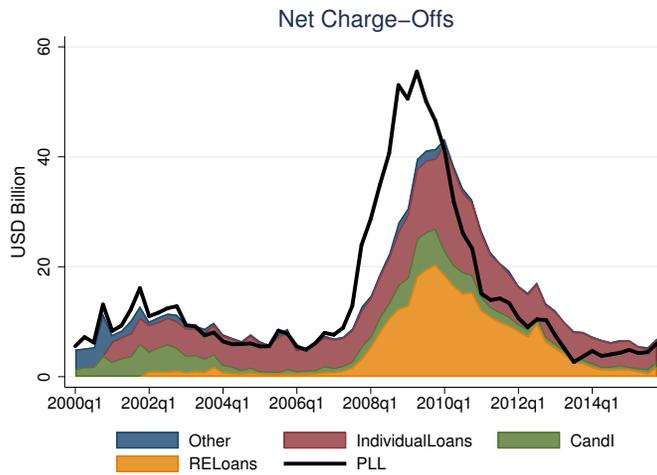
Notes: This figure shows aggregate net charge-offs for different categories (area chart) and aggregate loan loss provisions (solid black line). The data source are FR Y-9C reports. Net charge-offs for loans are defined as charge-offs minus recoveries. We decompose the net charge-offs into loans backed by real estate, commercial and industrial (C&I) loans, loans to individuals (e.g., such as credit card loans), and all other loans (e.g. interbank loans, agricultural loans, and loans to foreign governments).

Figure 3: Market equity contains more cash-flow relevant information than book equity
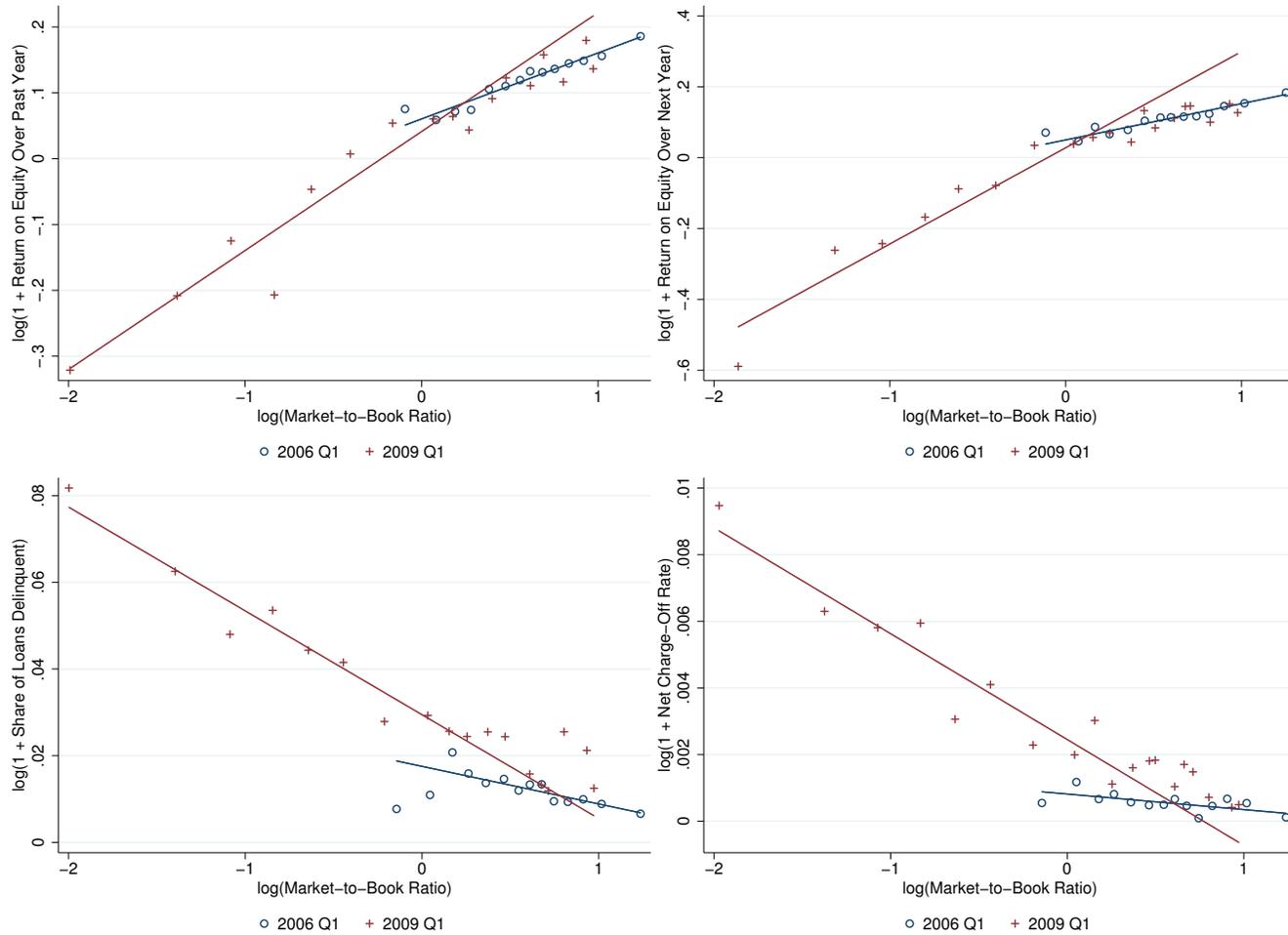
Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log market-to-book equity ratio for BHCs, in 2006 Q1 and 2009 Q1. All plots control for log book equity by residualizing the variables on log book equity, and then adding back the mean of each variable to maintain centering. Data on market capitalization are from CRSP, and all other data are from the FR Y-9C. ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; ROE over the next year is defined the one lead of this variable (i.e. profits over the next four quarters divided by current book equity). The share of loans delinquent is the ratio of loans past due 30 days or more plus loans in non-accrual, over total loans. The net charge-off rate is loan charge-offs over the next quarter minus loan recoveries over the next quarter, divided by total loans this quarter. In all plots, Irwin Financial Corporation is dropped from the sample in both time periods, because it is an extreme outlier in 2009 Q1 (a few months before its failure in September 2009).

Figure 4: Book and Market Leverage of Bank Holding Companies
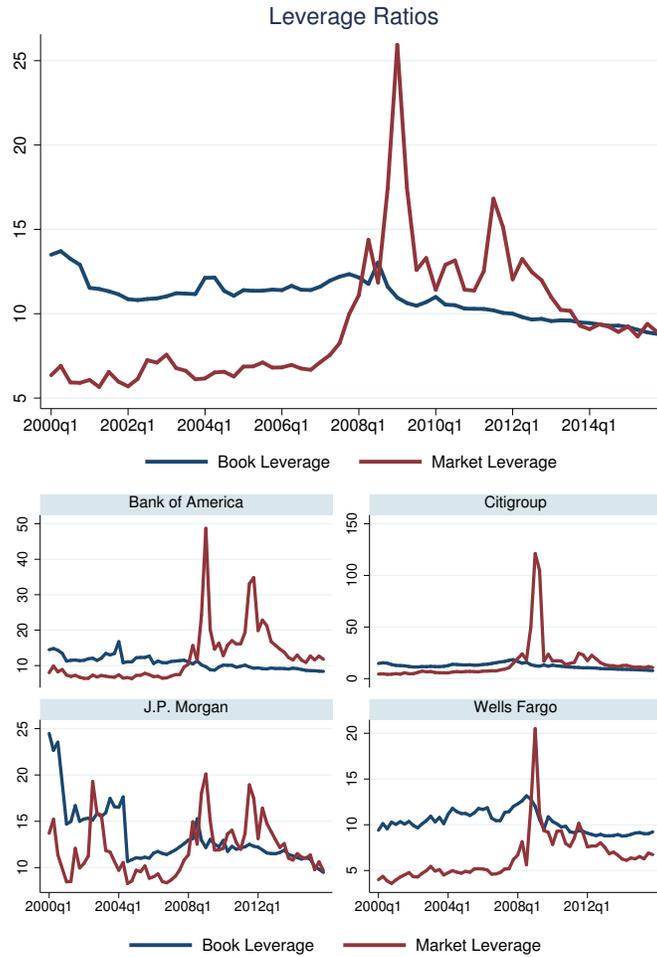
Notes: These figures show data on book and market leverage for BHCs. Book data (book equity and liabilities) come from the FR Y-9C, and market equity data is from CRSP data. The left panel shows aggregates from BHC balance sheets (excluding new entrants such as Goldman Sachs and Morgan Stanley). The right panel shows data for the four largest BHCs. Book leverage is computed as assets/book equity, and market leverage is computed as (liabilities + market equity)/market equity. The aggregate leverage ratios are computed as (aggregate liabilities + aggregate book equity)/aggregate book equity.

Figure 5: Quantiles of Market Leverage of Bank Holding Companies


Market Leverage Quantiles
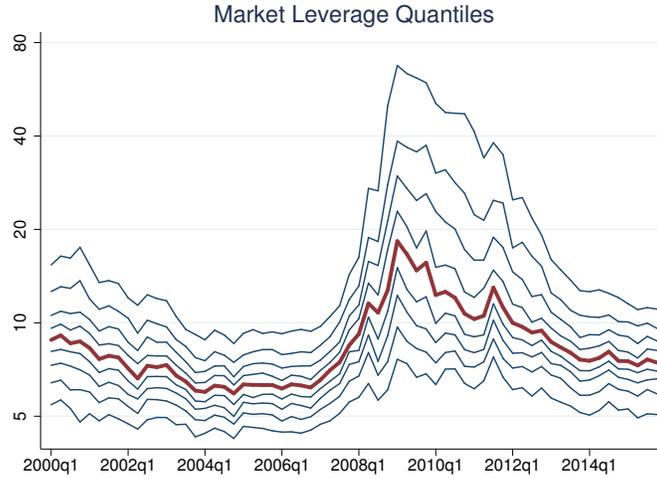
Notes: This figure shows data on the quantiles of market leverage for BHCs. Book data (liabilities) comes from the FR Y-9C, and market equity data is from CRSP data. Market leverage is computed as (liabilities + market equity)/market equity. The median market leverage is plotted in maroon, and each tenth percentile is plotted in blue. To improve visibility, the vertical axis uses a log scale.

Figure 6: Regulatory Capital Ratios for Bank Holding Companies

Distribution of Tier 1 Capital Ratio

Distribution of Total Capital Ratio
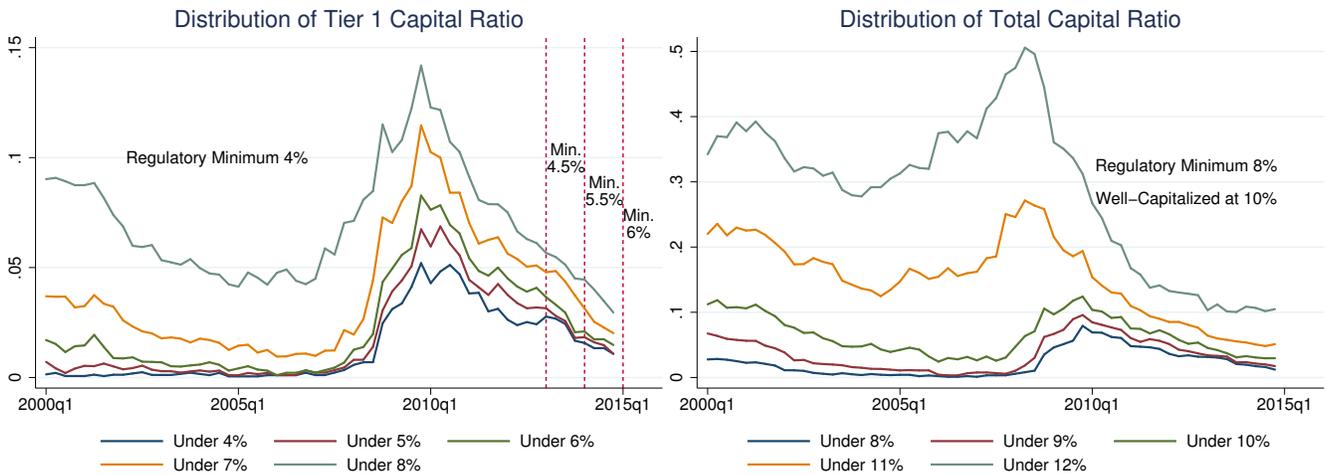
Notes: These figures show data on regulatory capital ratios for BHCs from the FR Y-9C. The left panel shows data on the distribution of the Tier-1 capital ratio, defined as (Tier-1 capital)/(Risk-weighted assets), and the right panel shows data on the distribution of the total capital ratio defined as (Total Capital / Risk weighted assets). The figures plot the share of banks whose regulatory capital ratio falls below a given level, computed using the full, unweighted sample. The regulatory capital requirements are shown on the graph and described in the text.

Figure 7: Estimated Impulse Responses
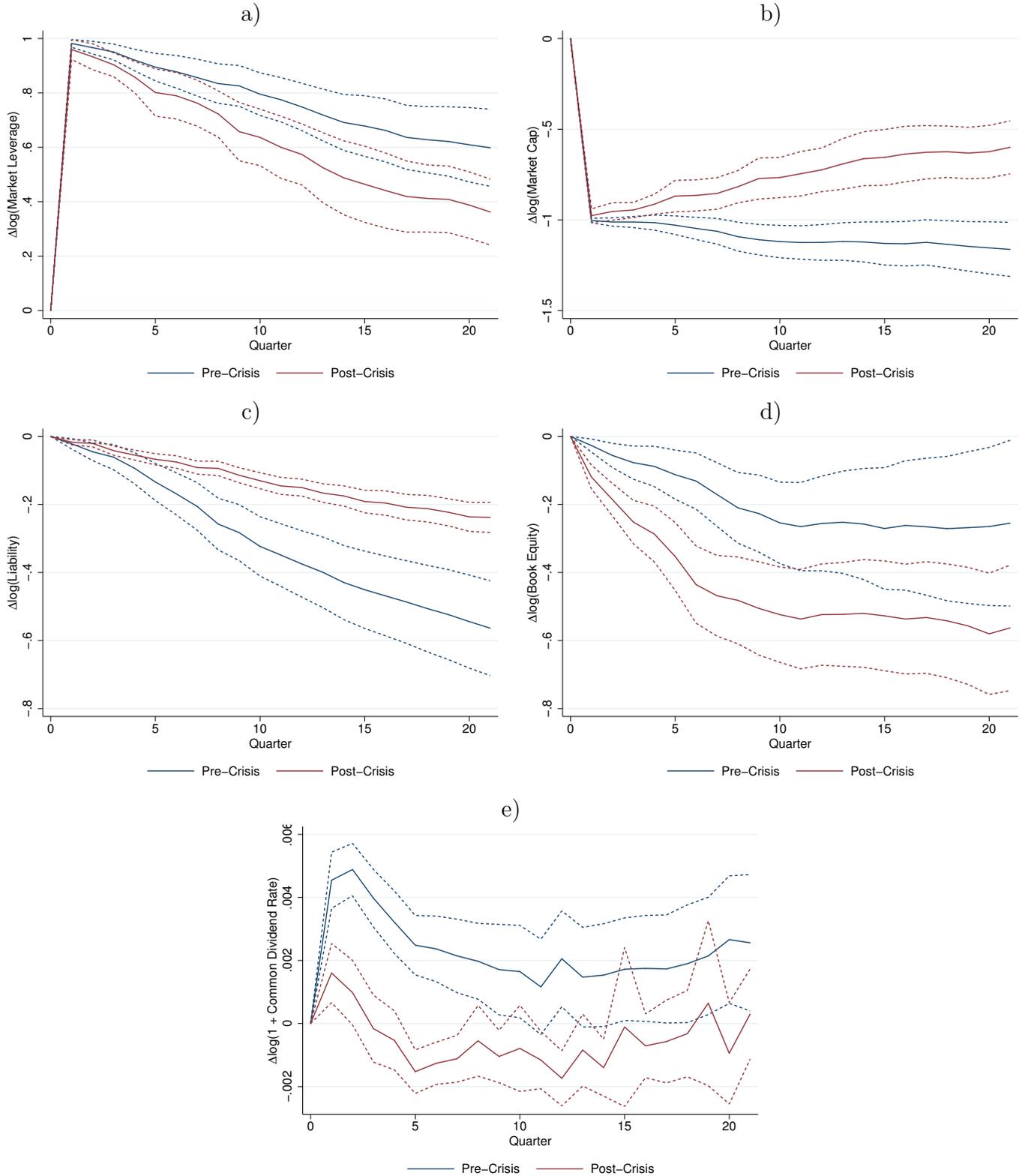
a)

b)

c)

d)

e)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated percent impulse response to a 1% negative return shock. For example, in Panel b) we show that market capitalization decreases by roughly 1% in response to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log market leverage (Panel a), log market capitalization (Panel b), log liabilities (Panel c), log book equity (Panel d), and the common dividend rate (Panel e). Market leverage is defined as (Liabilities/Market Capitalization). The logged common dividend rate is defined as log(1 + Common Dividends/Market Capitalization).

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Figure 8: Value and Policy Functions for given q's
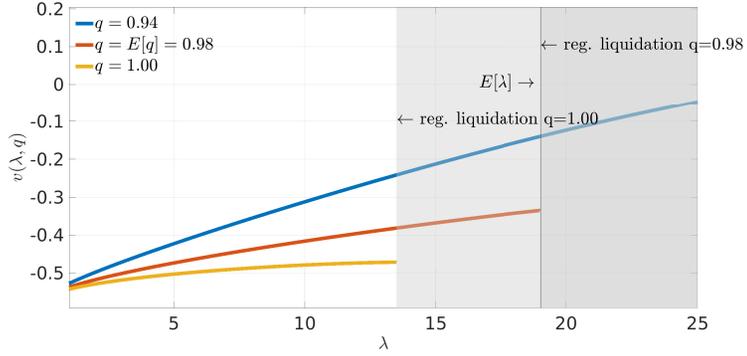
(a) Value Function
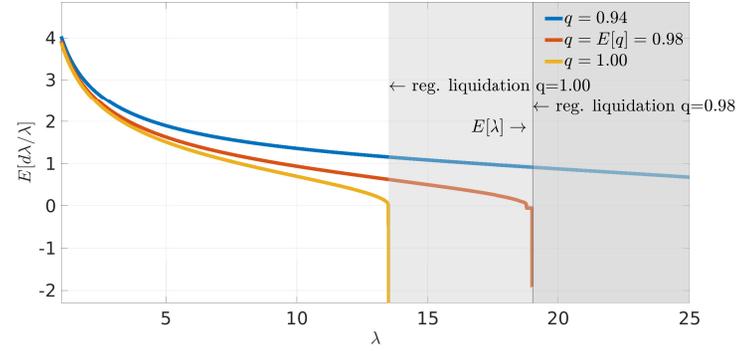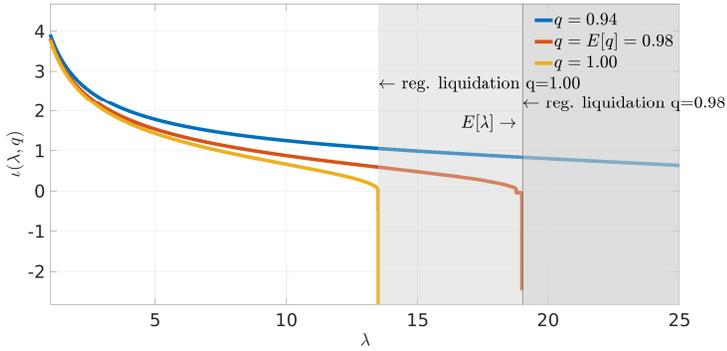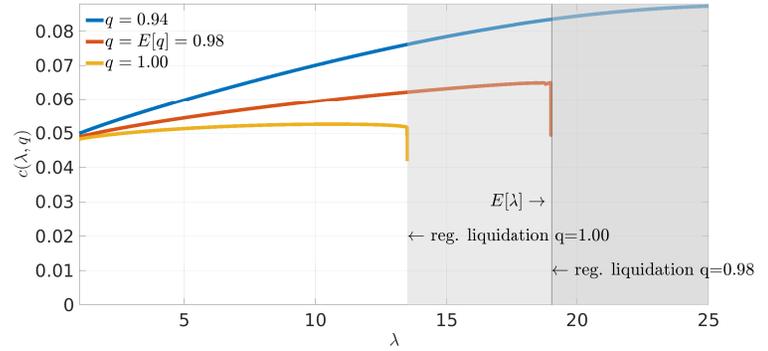
(b) Drift of Leverage

(c) Issuance Rate

(d) Dividend Rate

Notes: These figures show the value and policy functions generated under the pre-crisis parametrization of the model, across the λ dimension, for three particular values of q.

Figure 9: Value and Policy Functions for given λ's
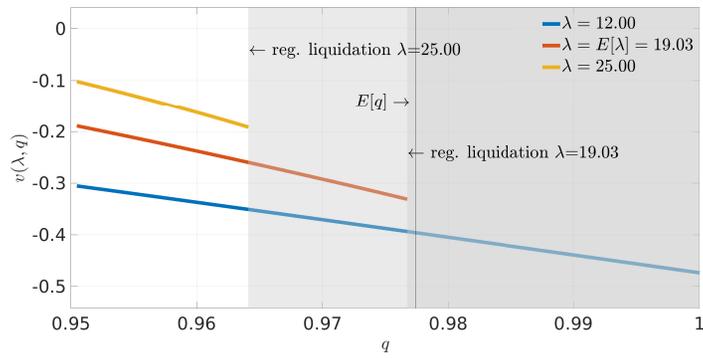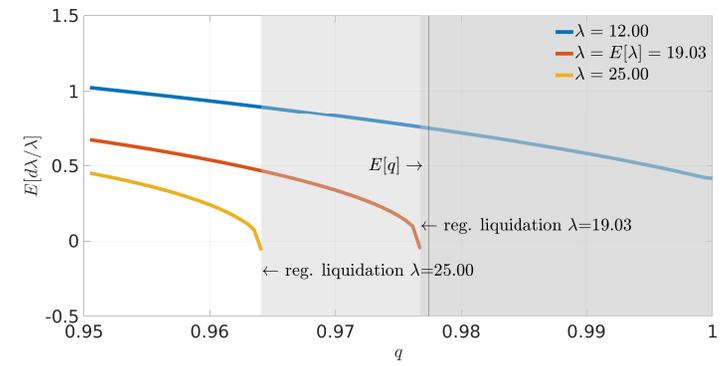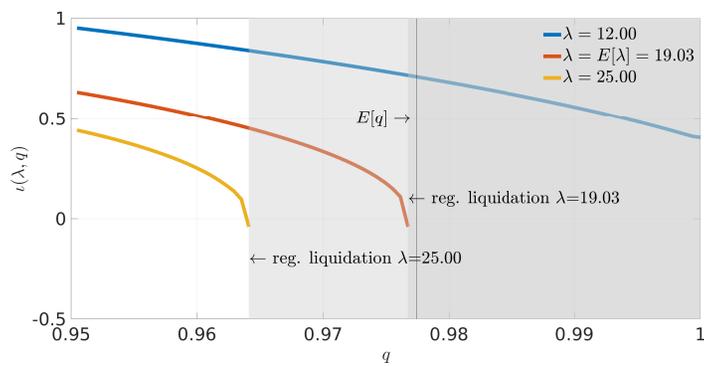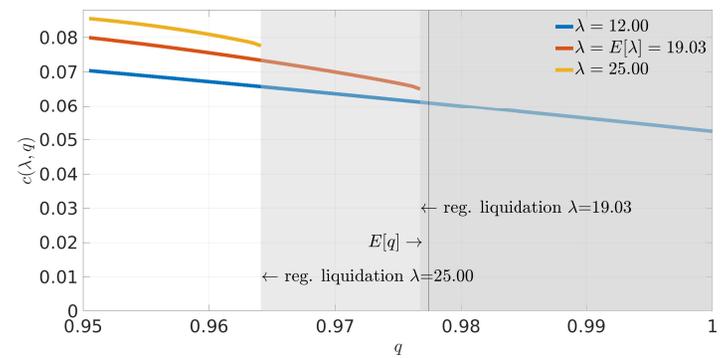
(a) Value Function

(b) Drift of Leverage

(c) Issuance Rate

(d) Dividend Rate

Notes: These figures show the value and policy functions generated under the pre-crisis parametrization of the model, across the q dimension, for three particular values of λ.

Figure 10: Invariant Distributions
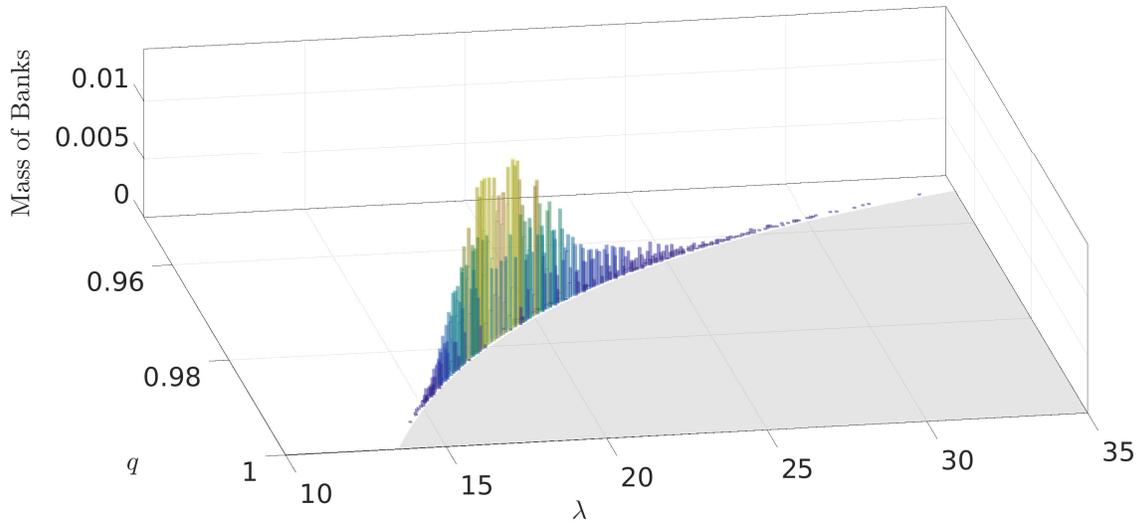
(a) Model Stationary Distribution of Banks Across the q and λ State Space
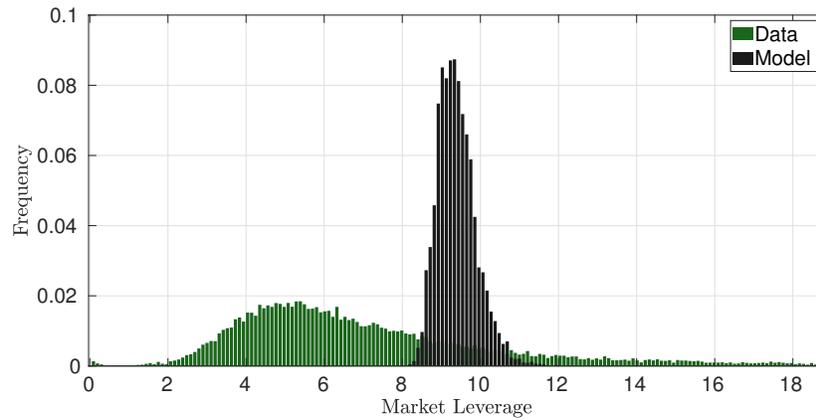
(b) Histogram for Market Leverage

Notes: Panel (a) shows a two dimensional histogram of the stationary distribution of banks across the (λ, q) space. The grey area shows the regulatory liquidation region. Panel (b) compares the distribution of market leverage generated by the model against the data.

Figure 11: Model and Data Impulse Responses to a Return Shock
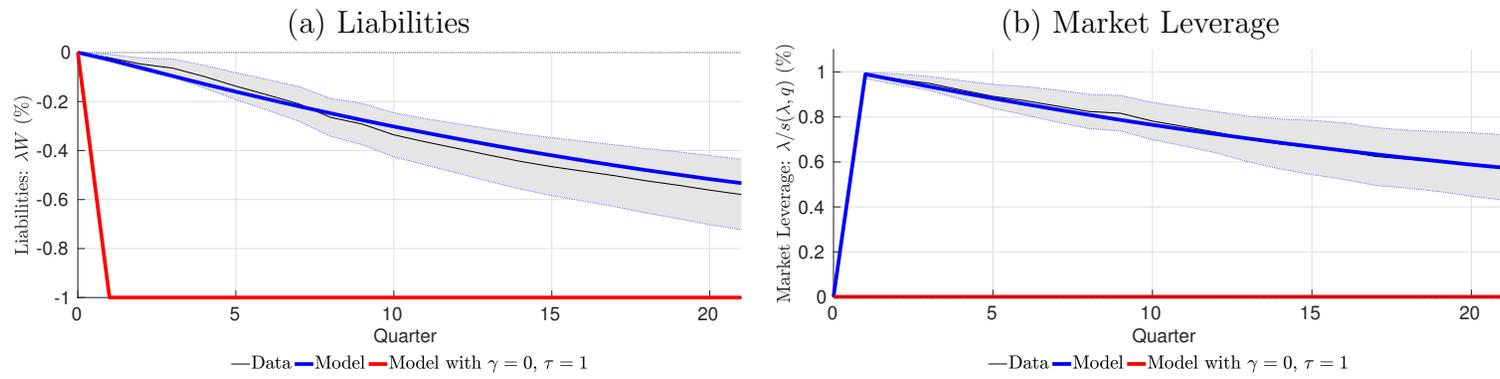
(a) Pre-crisis IRFs
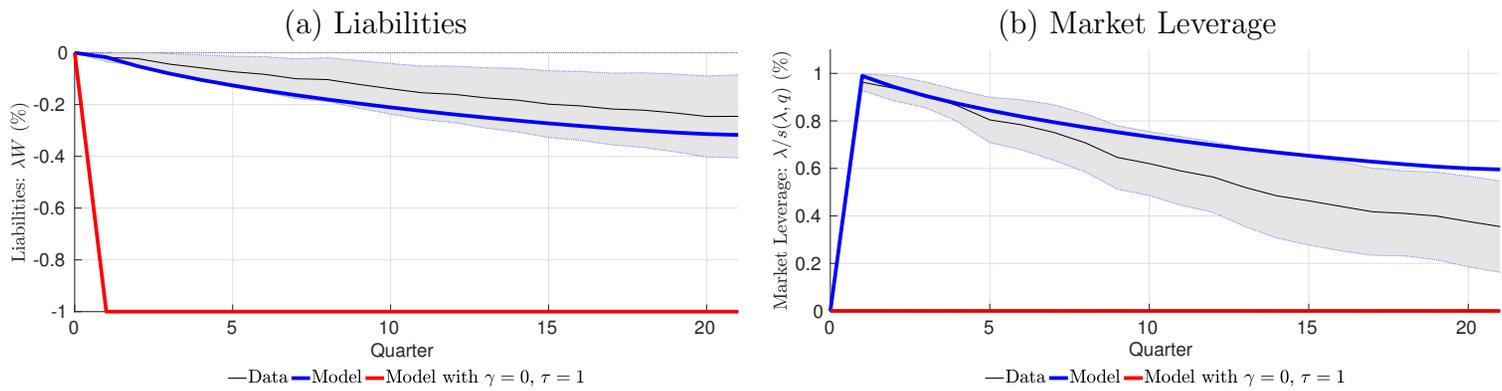
(b) Post-crisis IRFs

Notes: The figure shows the impulse response functions of market leverage and liabilities to return shocks. The blue line corresponds to the model generated IRFs whereas the black line is from the empirical section, with the shaded region corresponding to the confidence band for the estimated responses. The shock occurs in quarter 1.

Figure 12: Counterfactual Exercise
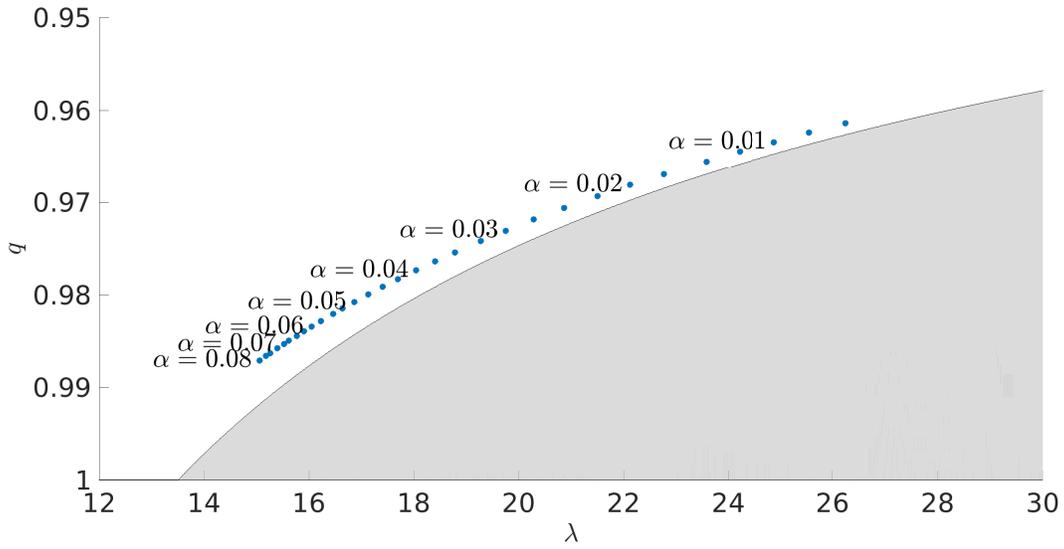
(a) Effect of Delayed Loss Recognition on q and λ
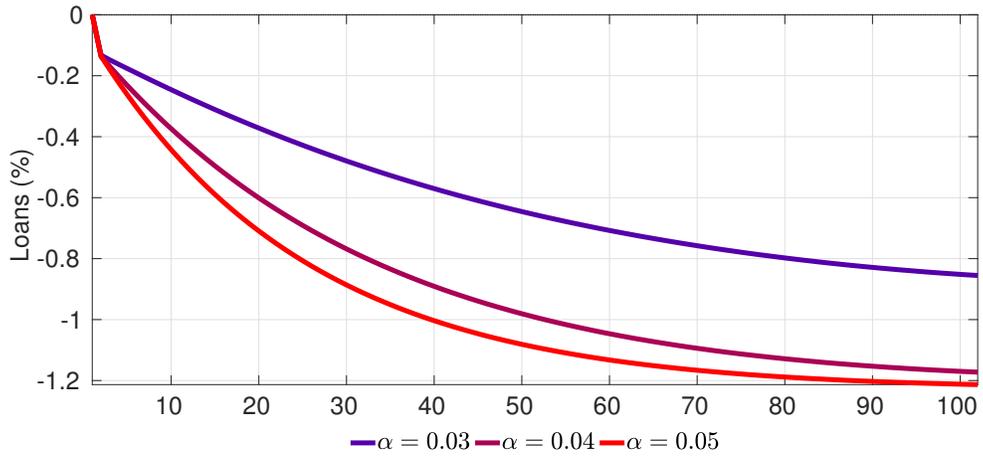
(b) Impulse Response Function of Loans

Notes: The figures show the results from our counterfactual exercise. Each dot in the top figure is a pair of cross-sectional means of λ and q in the pre-crisis stationary allocation, for a given value of α. The gray area represents the regulatory liquidation set. Each of these stationary allocations is characterized by approximately the same mean book leverage of 12.4. The bottom panel shows the IRF to a 1% return shock for aggregate loans, for three different values of α.

Table 1: Aggregate Descriptive Statistics

	Real Change			Log-Linear		
	2008	2009	2010	2008	2009	2010
Market Equity	-54.08%	-39.35%	-29.03%	-61.21%	-49.98%	-42.86%
	(-$705B)	(-$513B)	(-$378B)	(-$945B)	(-$790B)	(-$694B)
Book Equity	11.83%	21.70%	25.97%	-3.46%	-1.50%	-4.41%
	($94B)	($172B)	($206B)	(-$32B)	(-$15B)	(-$46B)
S&P 500	-42.08%	-28.83%	-21.20%	-25.55%	-7.01%	4.63%

Notes: The columns headed with the "Real Change" label show the percentage change from the raw variables. The columns headed with the "Log-Linear" label show the cyclical deviations from a log-linear trend in percentage points since 2007 Q3. Market Equity refers to shares outstanding times share price aggregated across all publicly traded BHC. Book equity is the book equity of publicly traded BHCs. All variables deflated using the seasonally-adjusted GDP deflator and converted to 2012 Q1 dollars. The dollar values are obtained by multiplying the cumulative percentage point deviation by real market capitalization and real book equity at the end of 2007 Q3, respectively. The last row shows the percentage change in the return on the S&P 500 in the first three columsn, while the last three columns show the change relative to a linear log-linear trend.

Table 2: PARAMETRIZATION

Parameter	Description	Target
Independently calibrated		
$r^L = 1.01\%$	Loan yield	BHC data: interest income / loans
$r^D = 0.51\%$	Bank debt yield	BHC data: interest expense / debt
$\delta = 7.69\%$	Loan maturity	FFIEC 031/041: average maturity of loans and securities
$\xi = 0.926$	Capital requirement	Capital requirement of 8% to be well-capitalized
$\varepsilon = 0.25\%$	Average default shocks	Accumulated bank losses
$\sigma = 0.4791$	Arrival rate of Poisson process	Match loan charge-off rate
$\alpha = 4\%$	Recognition rate of books	Peak of charge-off rate after fin. crisis
$\rho = 0.25\%$	Banker's discount rate	CRSP: Mean market leverage
$\rho^I = 3.51\%$	Investor's discount rate	CRSP: Bank equity returns
Jointly calibrated		
$\tau = 1\%$	Initial fraction of loan loss recognition	Initial jump of book leverage IRF
$\theta^{pre} = 2.30$	Inverse IES pre-crisis	Match market leverage IRF pre-crisis
$\theta^{post} = 1.71$	Inverse IES post-crisis	Match market leverage IRF post-crisis
$\gamma^{pre} = 0.01$	Balance sheet adj. costs pre-crisis	Match liabilities IRF pre-crisis
$\gamma^{post} = 3.96$	Balance sheet adj. costs post-crisis	Match liabilities IRF post-crisis

Table 3: MODEL AND DATA MOMENTS

	Pre-crisis		Post-crisis	
	Data	Model	Data	Model
Log Market Returns	0.035	0.045	-0.012	0.046
	(0.149)	(0.015)	(0.230)	(0.017)
Market leverage	6.799	9.388	10.025	11.056
	(0.598)	(0.052)	(0.815)	(0.070)
Book Leverage	10.619	12.409	9.870	11.967
	(0.366)	(0.001)	(0.502)	(0.010)
Market to Book Equity	1.532	1.322	0.897	1.082
	(0.469)	(0.052)	(0.584)	(0.076)
Log Common Dividend Rate	0.006	0.033	0.005	0.028
	(0.006)	(0.000)	(0.006)	(0.002)
Log Net Charge-Off Rate	0.001	0.001	0.002	0.001
	(0.003)	(0.002)	(0.004)	(0.002)

	Pre-crisis	Post-crisis
λ	18.048	22.029
	(2.114)	(4.142)
q	0.977	0.966
	(0.006)	(0.008)
c	0.065	0.055
	(0.005)	(0.005)
ι	0.024	0.024
	(0.002)	(0.004)
dW/W	0.012	0.029
	(0.033)	(0.042)
$s(\lambda, q)$	1.914	1.972
	(0.119)	(0.218)

Notes: The columns labeled "Data Pre-crisis" refer to the period 1990 Q3 to 2007 Q3, whereas "Data Post-crisis" refers to the period 2007 Q4 to 2015 Q4. The moments from the model are generated from a panel of 10,000 banks with the same number of quarters as in the respective periods for data. For column "Pre-crisis model", we calculate moments using the stationary distribution of banks. We compute the stationary distribution by first simulating enough quarters so that the mean and standard deviation of the state variables (λ, q) are approximately constant, and then keeping the last one as the initial quarter of the simulated sample. For the post-crisis model moments, banks suffer an aggregate default shock of 2.5% in the first quarter. The first row for each variable shows the mean. The second row shows standard deviations in parenthesis. For market leverage, book leverage and market-to-book equity, the mean and standard deviation are computed on the logs, but when reporting the mean we apply exponential to show the mean in levels.

For Online Publication

Online Appendix for
"A Q-Theory of Banks"

Juliane Begenau   Saki Bigio

Jeremy Majerovitz   Matias Vieyra

A   Data Appendix

A.1   Sample Selection

We analyze bank holding companies (BHCs), drawing data from multiple sources. We focus on top-tier bank holding companies that are headquartered in the 50 states or in Washington D.C. In most of our analyses, we analyze data from 2000 Q1 to 2015 Q4. For the analysis of impulse response functions, we extend the sample back to 1990 Q3 (this is the first year for which we can identify whether a BHC is top-tier). For book variables, we use data from the FR Y-9C, downloaded through Wharton Research Data Services (WRDS). We match this to data on market capitalization and returns from the Center for Research in Securities Prices (CRSP) using the PERMCO-RSSD links data set provided by the New York Fed (https://www.newyorkfed.org/research/banking_research/datasets.html). For analyses that use solely book data, we use data for those BHCs that we find in our sample in the FR Y-9C; for analyses that use market data, we use only the observations which we observe in both FR Y-9C and CRSP. In one robustness check, we use information on the dates of, and participants in, bank mergers and acquistions; we obtain data on bank mergers from the Chicago Fed (https://www.chicagofed.org/banking/financial-institution-reports/merger-data). In an additional robustness check, we drop all banks that were ever stress-tested (CCAR and DFAST). We obtain information on whether banks were ever stress tested from the Federal Reserve (The main website is https://www.federalreserve.gov/supervisionreg/stress-tests-capital-planning.htm, and the specific data sets can be found at https://www.federalreserve.gov/supervisionreg/ccar.htm and https://www.federalreserve.gov/supervisionreg/dfa-stress-tests.htm).

A.2   Evolution of main balance sheet variables

To get a sense of how the crisis affected banks, we report the evolution of key balance sheet components in Figure 1. This figure shows total assets, liabilities, and loans—not netting out the allowance for loan losses–for the aggregate banking sector (left panel) and the four largest BHCs in terms of assets. The banking industry is highly concentrated: the "Big Four" largest BHCs account for roughly 50 percent of aggregate assets. At the onset of the crisis, the growth of bank assets, loans, and liabilities slowed down, but never dropping as dramatically as bank equity market valuations (see below). The amount of outstanding loans, the largest component of bank assets, stagnated during the crisis and eventually fell. By 2009 Q4, loans net of the allowance for loan losses had fallen by \$361 billion, a drop of only 6.84%.[54] This number is driven only in part by losses as banks also slowed down the issuances of new loans.

A.3   Difference between Market and Book Data

Difference between market and book data.   To get a sense about how the crisis affected banks, we report the changes in select aggregate balance sheet components and aggregate bank equity return data since the beginning of the Great Recession in 2007 Q3 in Table 4. We do so in two ways. We first fit a linear trend to the logged real series and report deviations from that trend in the first three columns.[55] We estimate the trend using the data through 2007 Q3 and

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[54]The allowance for loan losses is an estimate of likely loan losses for the outstanding loans on the balance sheet. The next subsection provides more detail on how bank accountants calculate this number.

[55]We use the seasonally-adjusted GDP deflator to adjust for inflation, and report all values in 2012 Q1 dollars.

Figure 1: Balance Sheets of BHCs
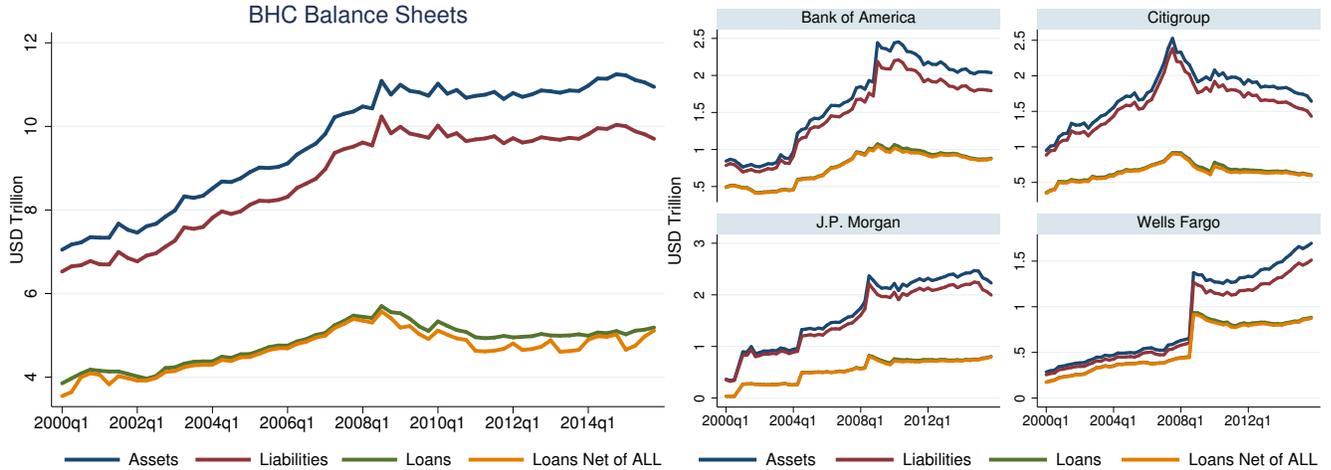
Notes: These figures show data on assets, liabilities, loans, and loans net of ALL for BHCs. Data come from the FR Y-9C. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out "probable and estimable" future losses on the current stock of loans). All variables converted to 2012 Q1 dollars using the seasonally-adjusted GDP deflator. The left panel shows aggregate time series, excluding new entrants to the sample BHC such as Goldman Sachs. The right panel shows the same data for the "Big Four", i.e., the four largest BHCs. Note that the spike in the data of Wells Fargo is due to its acquisition of Wachovia. Likewise, JP Morgan took on Bear Stearns and WaMu, while Bank of America took on Merrill Lynch and what was left of CountryWide.

report changes since at that trend.[56] Second, we report simply the real changes since 2007 Q3 in the last three columns. Each column computes the change until the fourth quarter of the year indicated by the column. For aggregate bank balance sheet quantities, we focus our attention on the aggregate series of loans and different measures of equity since these are the quantities that are at the hear of macro-finance models. We also report the changes in bank equity return data to provide a summary of shareholder losses and in the S&P stock market index for comparison.

Bank Accounting Practices. The discrepancy between book and market equity reflects bank accounting practices. Banks can delay acknowledging losses on their books (e.g. Laux and Leuz 2010), because banks are not required to mark-to-market the majority of their assets. There are many incentives to delay book losses. In practice, a key metric for measuring success of a bank is the book return on equity (ROE).[57] Given that ROE is a measure of success, manager compensation is linked to book value performance. Moreover, shareholders and other stakeholders may base their valuations on information from book data. Finally, banks are required to meet capital standards based on book values.

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[56]Since market return and book ROE are flows rather than levels, we detrend by simply subtracting the pre-crisis average. Also, since flows can be negative, we use $\log(1 + r)$ instead of $\log(r)$. A concern with log-linear detrending is that it could be based on an unsustainable boom, yielding an overestimate of the size of the cyclical deviation. Simply looking at raw changes in this series sidesteps these concerns, but only by not dealing with the trend altogether. We report both estimates for completeness, but we acknowledge that each of these estimates is imperfect. We also computed (available upon request) estimates from HP-filtered data. The HP-filtered residuals were typically of substantially smaller magnitude then the residuals estimated with a log-linear trend. The HP-filter seemed to be overfitting the data and treating as trend what is really just a persistent cyclical component.

[57]For example, JP Morgan's 2016 annual report states "the Firm will continue to establish internal ROE targets for its business segments, against which they will be measured" (on page 83 of the report).

Table 4: Aggregate Descriptive Statistics .

	Log-Linear			Real Change		
	2008	2009	2010	2008	2009	2010
Market Cap.	-61.21%	-49.98%	-42.86%	-54.08%	-39.35%	-29.03%
	(-$945B)	(-$790B)	(-$694B)	(-$705B)	(-$513B)	(-$378B)
Book Equity	-3.46%	-1.50%	-4.41%	11.83%	21.70%	25.97%
	(-$32B)	(-$15B)	(-$46B)	($94B)	($172B)	($206B)
Common Equity	-28.44%	-11.69%	-10.42%	-17.35%	8.29%	16.64%
	(-$275B)	(-$120B)	(-$114B)	(-$145B)	($69B)	($139B)
Loans Net of ALL	2.68%	-10.41%	-14.33%	2.58%	-6.84%	-7.27%
	($141B)	(-$571B)	(-$819B)	($136B)	(-$361B)	(-$384B)
S&P 500	-25.55%	-7.01%	4.63%	-42.08%	-28.83%	-21.20%
Bank Market Return	-57.87%	-61.42%	-60.23%	-54.26%	-55.28%	-50.78%
	(-$755B)	(-$801B)	(-$785B)	(-$708B)	(-$721B)	(-$662B)
Book Return on Equity	-20.30%	-27.89%	-33.58%	-7.84%	-6.34%	-3.11%
	(-$171B)	(-$236B)	(-$284B)	(-$66B)	(-$54B)	(-$26B)

Notes: Top row shows cyclical deviations in percentage points since 2007 Q3; bottom row shows deviations converted into raw values. Book equity refers to book equity of publicly traded BHCs. Loans net of ALL refers to loans minus the allowance for loan losses (this subtracts out "probable and estimable" future losses on the current stock of loans). All variables deflated using the seasonally-adjusted GDP deflator. Level variables are converted to 2012 Q1 dollars, flow variables are deflated by subtracting inflation. Bank market return deviations and book return on equity are cumulated since the end of 2007 Q3, and dollar values are obtained by multiplying the cumulative percentage point deviation by real market capitalization and real book equity at the end of 2007 Q3, respectively.

The flexibility of accounting their accounts is studied extensively in the accounting literature (Bushman, 2016 and Acharya and Ryan, 2016 review the literature on this issue, Francis et al., 1996 studies the same issue for non-financial firms). In practice, banks can record securities on the books using two methodologies: either amortized historical cost (the security is worth what it cost the bank to buy it with appropriate amortization) or fair value accounting.[58] In addition to mis-pricing securities, another degree of freedom is the extent to which banks can acknowledge impairments: banks have the right to delay acknowledging impairments on assets held at historical cost, if they deem those impairments as temporary (i.e. they believe the asset will return to its previous price). This gives banks substantial leeway, and led banks to overvalue assets on the books during the crisis. Huizinga and Laeven (2012) find that banks used discretion to hold real-estate related assets at values higher than their market value. (Laux and Leuz, 2010) note some notable cases of inflated books during the crisis: Merrill Lynch sold $30.6 billion dollars of CDOs for 22 cents on the dollar while the book value was 65 percent higher than its sale price. Similarly, Lehman Brothers wrote down its portfolio of commercial MBS by only three percent, even when an index of commercial MBS was falling by ten percent in the first quarter of 2008. Laux and Leuz (2010) also document substantial underestimation of loan losses in comparison to external estimates.

This shows up in our own analysis as well: Figure 2 shows that provisions for loan losses and net charge-offs only reached their peak in 2009 and 2010 respectively, and remained quite elevated at least through 2011, well after the recession had ended. The decomposition of net charge-offs shows that these losses were heavily driven by real estate, suggesting they were associated with the housing crisis.[59] Banks' books were only acknowledging in 2011 losses that the market had already predicted when the crisis hit.

Harris et al. (2013) construct an index, based on information available in the given time period, that predicts future losses substantially better than the allowance for loan losses.[60] This implies that the allowance for loan losses is not capturing all of the available information to estimate losses. This may in part be strategic manipulation, but there may also be a required delay in acknowledging loan losses. Under the "incurred loss model" that was the regulatory standard during the crisis, banks are only allowed to provision for loan losses when a loss is "estimable and probable" (Harris et al., 2013). Thus, even if banks know that many of their loans will eventually suffer losses, they were not supposed to update their books until the loss was imminent.

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[58]Fair value accounting can be done at three levels: Level 1 accounting uses quoted prices in active markets. Level 2 uses prices of similar assets as a benchmark to value assets that trade infrequently. Level 3 is based on models that do not involve market prices (e.g. a discounted cash flow model). Banks are required to use the lowest level possible for each asset. In practice, most assets are recorded at historical cost. The majority of fair value measurements are Level 2 (Goh et al. 2015; Laux and Leuz 2010). Recent work has shown that the stock market values fair value assets less if they are measured using a higher level of fair value accounting. This leaves room to mis-price assets on books. Particularly during 2008, Level 2 and Level 3 measures of assets were valued substantially below one (Goh et al. 2015; Kolev 2009; Song et al. 2010). Laux and Leuz (2010) document sizable reclassifications from Levels 1 and 2 to Level 3 during this period. They highlight the case of Citigroup, which moved $53 billion into Level 3 between the fourth quarter of 2007 and the first quarter of 2008 and reclassified $60 billion in securities as held-to-maturity which enabled Citi to use historical costs.

[59]When a bank has a loss that is estimable and probable, it first provisions for loan losses, which shows up as PLL. Later when the loss occurs, the asset is charged off and thus taken off the books, which shows up as charge-offs, although occasionally the bank can recover the asset later. Net charge-offs is charge-offs minus recoveries. We show a decomposition by category for net charge-offs but not for PLL because the FR Y-9C does not provide information on PLL by loan category.

[60]The ALL is the stock variable corresponding to the PLL.

Information content We test whether book equity captures all available information about bank cash flows using cross-sectional regressions of market equity on book equity and several other profitability measures. We are motivated by the efficient markets hypothesis that suggests that market values reflect all available information about future dividends, and, by extension, about banks future profits and net worth today: If market equity indeed contains additional information about bank profitability not captured by their book values, then market equity will be correlated with variables capturing profitability, even after conditioning on book equity. To help us visualize the additional information content of market values over and above book values, consider the following cross-sectional regression

$$\log\left(\text{Market Equity}_i\right) = \alpha + \beta \log\left(\text{Book Equity}_i\right) + f(X_i) + \epsilon_i,$$

where $f(X)$ represents polynomials in our variables of interest, and i indexes banks.[61] We then construct the partial residual $\log\left(\text{Market Equity}\right) - \alpha - f(X)$ and plot this on the vertical axis of Figure 2. We plot the regressor of interest, X, on the horizontal axis. By construction, the polynomial $f(X)$ that best fits the outcome variable $\log\left(\text{Market Equity}\right)$ will also be the polynomial that best fits the partial residual. Thus, Figure 2 allows us to plot $f(X)$ and assess the goodness of its fit. We consider a quartic in log RoE over the past year (controlling for log book equity) and a quartic in log RoE over the next year (controlling for log book equity and a quartic in log RoE over the past year) as $f(X)$.[62] These graphs confirm that market capitalization, controlling for book equity, is increasing in both RoE over the past year and in RoE over the next year. Hence even after controlling for book equity, market capitalization captures information content of net-income from the past and upcoming year. Note that the non-linear regression specification is important. For example, in the post-crisis period, there is a left tail of banks with very negative RoE. In this region the marginal effect of RoE on market capitalization is much smaller.

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[61]Appendix Section A.3 shows the partial R^2 for this regression for a range of variables capturing profitability measure such as loan charge offs and ROE over various time horizons.

[62]For improved visibility, we exclude outliers from the graph window by limiting the graph's horizontal axis to values within ± 3 standard deviations from the mean.

Figure 2: Information content in books
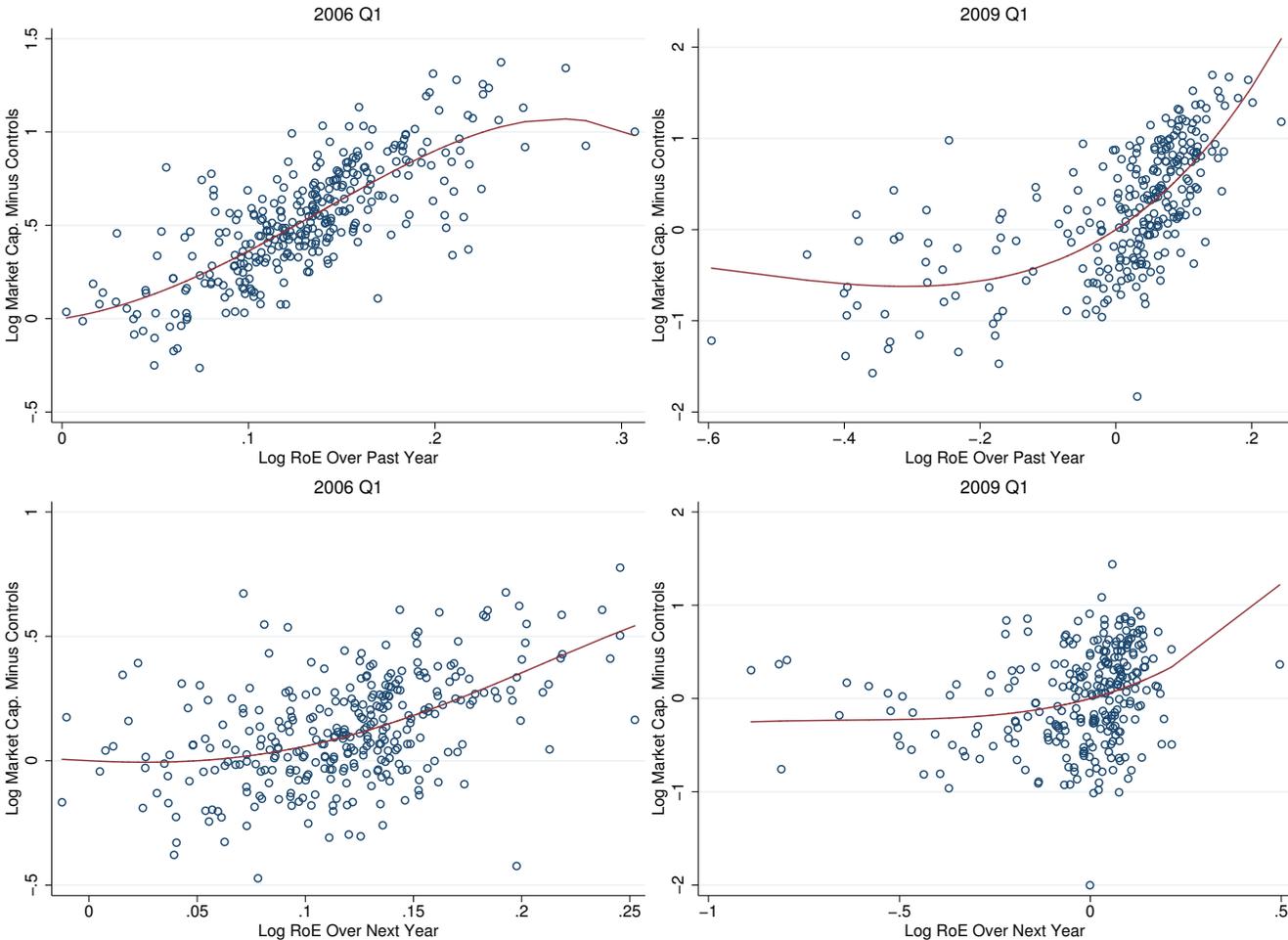
Notes: These figures show results from a cross-sectional regression of log market equity on assorted variables. The top row shows results from a regression of log market capitalization on log book equity and a quartic in log RoE over the past year. The bottom row shows results from a regression of log market capitalization on log book equity, a quartic in log RoE over the past year, and a quartic in log RoE over the next year. The horizontal axis shows the regressor of interest, and the vertical axis shows the outcome minus the effect of the controls (for the top row, the controls are a constant and log book equity, for the bottom row, the controls are a constant, log book equity, and a quartic in log RoE over the past year). The left column shows results for 2006 Q1, the right column shows results for 2009 Q1. Regressions are run on the cross-section of banks with all variables available, but the horizaontal axis of the graph window is restricted to ±3 standard deviations from the mean to improve visibility. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. Log RoE is defined as $\log(1 + \text{RoE})$. RoE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; RoE over the next year is defined as the one year lead of this variable.

B Additional Impulse Responses

B.1 Risk Adjustment

For our main impulse response results, we wish to use risk-adjusted returns, rather than raw returns. More formally, we assume that the market returns of bank i at time t are given by

$$\underbrace{r_{it}}_{\text{Raw Return}} - \underbrace{r_t^f}_{\text{Risk-Free Rate}} = \alpha_i + \underbrace{X_t}_{\text{factors}} \underbrace{\beta_i}_{\text{loadings}} + \underbrace{\varepsilon_{i,t}}_{\text{Idiosyncratic Component}}$$

All returns are logged, e.g. r_{it} refers to $\log\left(1 + \text{Raw Bank Return}\right)$. We wish to isolate variation in the idiosyncratic shocks, $\varepsilon_{i,t}$, and use this variation to estimate the impulse responses.

A natural, but naive, approach would be to estimate the above model for each bank i using OLS, and then use the estimated residuals, $\hat{\varepsilon}_{it}$, as the regressors in the impulse response estimation. The problem here is that it induces bias: $\hat{\varepsilon}_{it}$ is a noisy measure of the true regressor ε_{it}, which leads to bias as long as T is finite (the bias will shrink as T grows large, because $\hat{\varepsilon}_{it}$ will converge to the true ε_{it}).

Fortunately, there is a simple solution: we estimate $\hat{\varepsilon}_{it}$ using OLS, and then we use $\hat{\varepsilon}_{it}$ as an instrument for the unadjusted return. Since our main regressions use contemporaneous returns, twenty lags, and their interaction with a post-crisis dummy, this means we use contemporaneous $\hat{\varepsilon}_{it}$, twenty lags of $\hat{\varepsilon}_{it}$, and their interaction with a post-crisis dummy as instruments. Instrumental variables does not suffer from the same problem of bias under classical measurement error. Instead, to get identification under the assumed model for returns, we need our instrument to be correlated with the "good variation", ε_{it}, and uncorrelated with the "bad variation," $\alpha_i + X_t\beta_i$. This is mechanically what we are doing when we run OLS at the bank level, and if the assumed model for returns is correct, then we have $\mathbb{E}\left[\hat{\eta}_{it}\left(\alpha_i + X_t\beta_i\right)\right] = \alpha_i\mathbb{E}\left[\hat{\eta}_{it}\right] + \mathbb{E}\left[\hat{\eta}_{it}X_t\right]\beta_i = 0 + 0$. Thus, our instrumental variables strategy will give us a consistent estimator of the true impulse response, under the assumption that we have the correct model of returns. Since the OLS regression estimating $\hat{\varepsilon}_{it}$ is conducted at the bank level, we cluster our standard errors at the bank level (clustering at the bank level is already a good idea).

B.2 Results without factor risk adjustment

While we favor the risk-adjusted results, we also have computed "unadjusted results" for the impulse responses, which we report here for completeness. The results are qualitatively and quantitatively similar across the methods. Compared to the risk-adjusted results, howver, the unadjusted results, suggest a smaller response of liabilities in the pre-crisis period, and thus also suggest a slower pre-crisis adjustment of leverage.

Figure 3: Estimated Impulse Responses for Stock Variables (No Risk Adjustment)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).

Figure 4: Idiosyncratic Shock Series of Big Four Bank Holding Companies
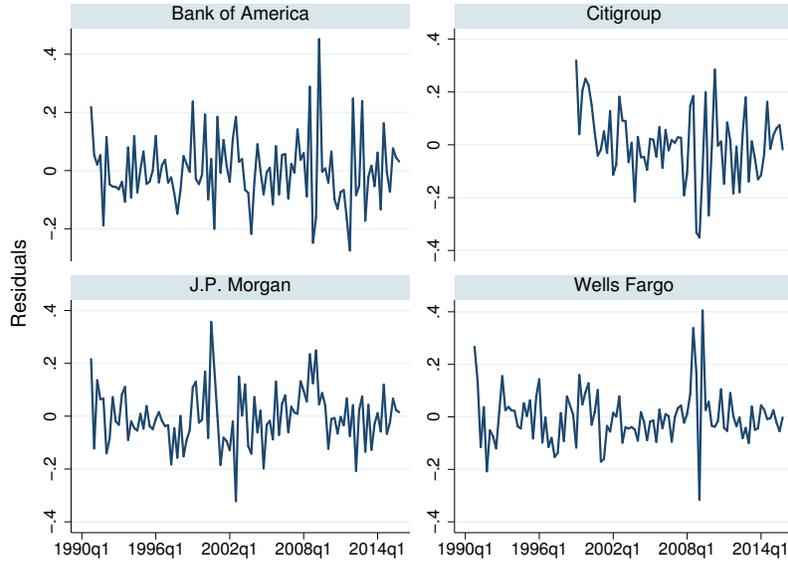
Notes: This figure plots the idiosyncratic shocks (for the Big Four BHCs) used to estimate the impulse response functions. First, we isolate the idiosyncratic component of returns using the factor model, and then we residualize this on time fixed effects.

B.3   Robustness and Validity of Identification Strategy

In this section, we conduct various tests to check the validity of our identification strategy and robustness of our results.

A narrative approach to corroborate the idiosyncratic shocks   To provide corroberating evidence of the validity of our identification strategy, we first show that the estimated return shocks do indeed look like idiosyncratic shocks for the four largest banks (Bank of America, J.P. Morgan Chase, Wells Fargo, Citigroup). To construct the idiosyncratic shocks, we regress each bank's market return on the Fama-French three-factor returns and regress the residual further on time fixed effects. The residuals from this regression represent our idiosyncratic shocks.[63] Figure 4 presents our estimates of the idiosyncratic shocks. They indeed look like white noise and do not seem to be substantially autocorrelated. Note that the time series for Citigroup starts a little later because Citigroup did not exist until 1998 when Traveler's merged with Citicorp.

We also provide narrative support for the idiosyncratic nature of our estimated shocks using an extensive search of newspaper articles for large idiosyncratic shock value estimates.

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[63]We are controlling for the time fixed effects, because they are included in the regression we actually run to get the impulse response function.

Table 5: Narrative Support for Idiosyncratic Shocks

Bank Name	Year-Qtr	idiosyncratic shock	Bank specific events
Bank of America	2000q4	-0.200	Sunbeam (which BofA lended to) posted $86M loss. BofA said net charge-offs in Q4 will double. BofA issues warning on $1B uncollectible debt, may miss the December quarter profit forecast by as much as 27%.
	2003q4	-0.218	BofA agrees to pay $47 to buy FIeetBoston Financial "hefty premium" & "could dilute earnings."
	2008q3	0.288	BofA to buy Merrill for $50B (Sept 15)
	2009q2	0.452	Stress test: BofA needs to address $34B capital shortfall, better than expectation.
	2011q4	-0.275	Merrill Lynch has agreed to pay $315 million to end a mortgage-securities lawsuit (Dec 7)
	2012q4	0.248	BofA considered better buy after increase in house prices that (given its portfolio composition) particularly benefited BofA.
Citigroup	1999q1	0.319	Citigroup Profit Fell 53% in 4th period, but still topped analysts' expectations
	1999q3	0.205	Citigroup posts an unexpected increase of 9.3% in net income for second quarter (July 20)
	1999q4	0.250	Citigroup's citibank unit is marketing credit card for the internet to millions
	2000q1	0.226	Citi Intelligent Technology Receives Investment; Dividends increase from $1.05 to $1.20
	2003q4	-0.215	Citi to repay certain funds $16 mln plus interest; Citigroup Asset Management faces federal probe.
	2009q1	-0.351	Citigroup had $2B in direct gross exposure to LyondellBasell Industries, who filed for bankruptcy protection last week. Fitch cuts Citi preferred to junk
	2009q3	0.199	Citi reports profit after gain from Smith Barney. Citigroup's mortgage mitigation rises 29% in second quarter.
	2009q4	-0.267	Citi fined in tax crackdown. Abu Dhabi's sovereign wealth fund is demanding that Citigroup scraps a deal that would see the fund make a heavy loss on a $7.5 billion investment in the bank.
	2010q2	0.285	Citi reported quarterly earnings of $4.4B exceeding expectations
J.P. Morgan Chase	1997q2	-0.182	J.P. Morgan particularly large exposure to 1997 Asian Financial Crisis. https://www.imf.org/external/pubs/ft/wp/1999/wp99138.pdf
	2000q1	0.169	J.P. Morgan told investors on Monday that January and February had topped performance levels seen in the fourth quarter. Dividends increase from $0.2733 to $0.3200 on March 21.
	2000q3	0.357	Chase buying J.P. Morgan.
	2001q2	-0.185	J.P. Morgan Chase disclosed this week that their venture capital portfolios had incurred significant losses.
	2002q3	-0.322	JPMorgan Partners Reports $165M Operating Loss for Q2. J.P. Morgan sees third-quarter shortfall.
	2004q4	-0.198	JPMorgan Chase profit falls 13%.
	2008q3	0.234	J.P. Morgan profit falls 53%, but tops Wall Street target.
	2009q1	0.249	J.P. Morgan net falls sharply, but tops Wall Street view. J.P. Morgan to sell Bear Wagner to Barclays Capital: WSJ
	2012q2	-0.207	J.P. Morgan: London Whales $2 Billion Losses. Two Shareholder Suits Filed Against J.P. Morgan
Wells Fargo	2001q2	-0.161	Wells Fargo disclosed that their venture capital portfolios had incurred significant losses. Wells Fargo to take $1.1 billion charge
	2008q3	0.338	Wells Fargo's net dropped 21% as it set aside $3 billion for loan losses, better than expected. Earnings declined but beat estimates.
	2009q1	-0.315	Wells Fargo posted a surprise $2.55B Q1 loss, later revised to $2.77B. Wells Fargo added a pretax $328.4M impairment of perpetual preferred securities to its fourth-quarter loss.
	2009q2	0.405	Wells Fargo sees record Q1 profit, projections easily exceed expectations (expects earnings of $3 billion).

Table 5 shows that large absolute idiosyncratic shock values are consistent with good or bad bank specific events, such as "Wells Fargo sees record Q1 profit, projections easily exceed expectations," or "Citi fined in tax crackdown." The table shows that large positive or negative idiosyncratic shocks can be corroborated with specific events that appear bank specific, which supports the validity of our identification strategy.

Placebo Tests   To test the validity of our identification strategy, we conduct placebo tests where we include ten leads of returns (in addition to the contemporary value and twenty lags as before). If the returns really are unanticipated shocks, then the leading values should not affect current behavior. This is similar to testing for pre-trends. We are testing whether the banks that will experience higher returns in the future are already acting differently today. Overall, the placebo test are encouraging, and suggest that our results are not driven by prior differences in the behavior of banks which experience return shocks.

Identification robustness   We provide a few additional pieces of evidence that corroborate the validity and robustness of our identification strategy.

First, we verify that our results are robust to excluding the crisis years 2008 and 2009 from our sample. The idea is to rule out a lot of stories related to specific events during the crisis (e.g. the realization that the government might not guarantee that a bank wouldn't fail, or that this was somehow about exposure to Lehman). The results are below (for our main outcomes: Liabilities, Market Cap, and Market Leverage). It makes no noticeable difference to the results.

Second, we check whether bank mergers drive the results. To this end, we drop the quarter of the merger as well as the quarter before and after the merger. The results for our main outcomes: liabilities, market equity, and market leverage are in Figure 7. Again, it makes no noticeable difference to the results.

Similarly, we check whether the results are driven by the stress tests performed by banks: these stress tests were implemented after the onset of the crisis, and encouraged or mandated that banks raise additional capital. To show that the stress tests do not drive the results, we drop all banks that ever participated in a stress test (e.g. Bank of America participated in the stress tests, and so we drop Bank of America from our sample in all periods). The results for our main outcomes are in Figure 8. Again, it makes no noticeable difference to the results.

Another potential concern is that the return shocks could be picking up shocks to future investment opportunities, rather than default shocks. To test this concern, we check the response of the liquid assets ratio: if negative return shocks indeed predict lower future investment opportunities rather than current cash flows, we would expect banks to respond to these shocks by moving their portfolio into liquid assets. The results, shown in Figure 9, show no statistically significant response of liquid assets pre-crisis, and a small temporary response post crisis that is reversed within a few quarters. We take this as evidence against the hypothesis that return shocks reflect shocks to investment opportunities.

Am alternative, broader version of the liquidity ratio test calculates the liquidity ratio as the ratio of (Cash + Federal Funds Sold + Securities Purchased Under Agreement to Resell + Securities)/Total Assets. We display the impulse response function for this version of the liquidity ratio in Figure 10. The impulse response function has no significant response pre-crisis, and a significant but quantitatively small response post-crisis.

To put the size of the post-crisis response in perspective, the graph is saying that if there is a 10% negative shock to market returns, then the liquid asset ratio would rise by 0.02 over the

Figure 5: Estimated Impulse Responses for Stock Variables (Risk-Adjusted, with Placebo)

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).

Figure 6: Estimated Impulse Responses: Dropping 2007 and 2008
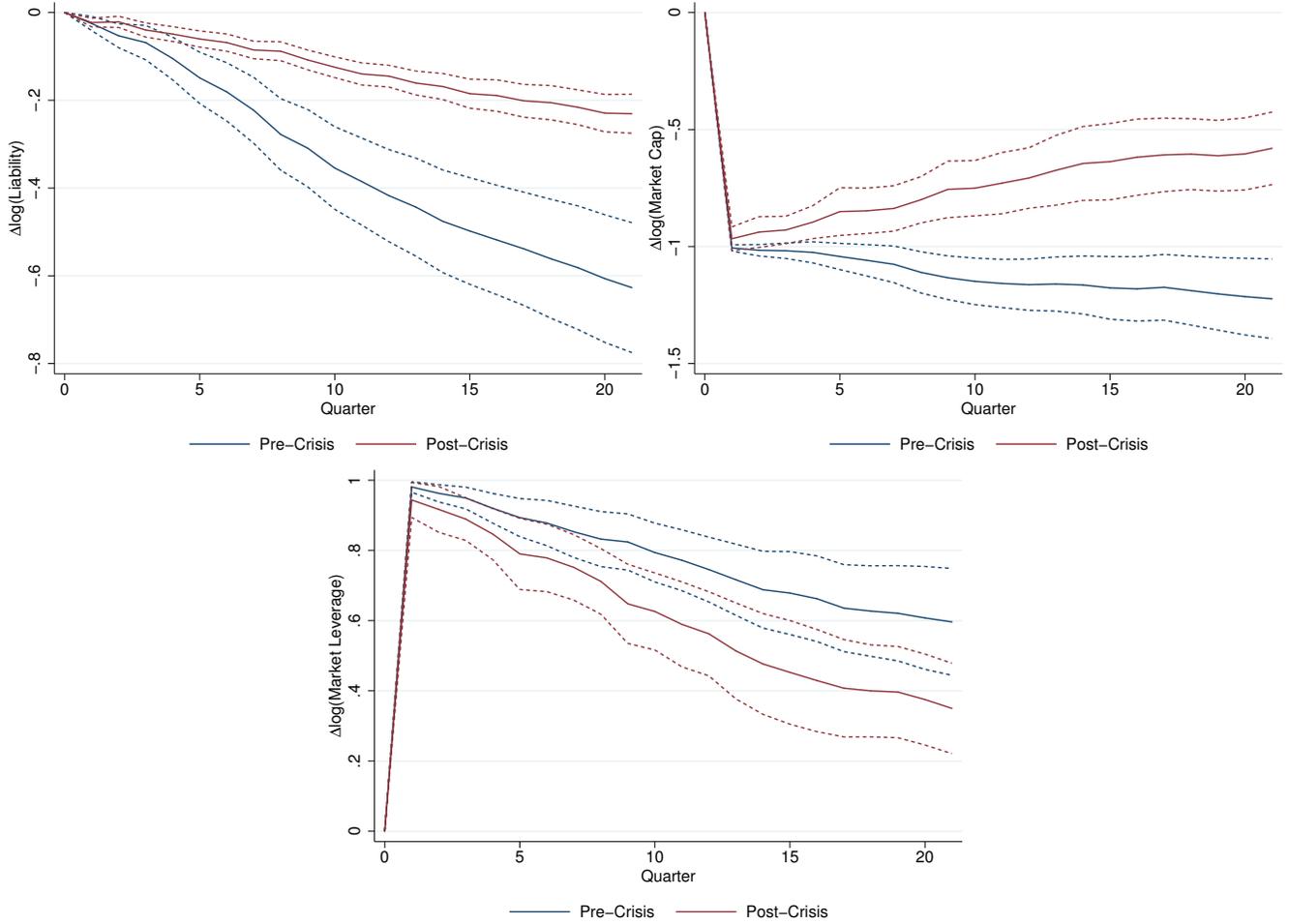
Notes: These figures show estimated impulse response functions for BHCs, dropping observations from the years 2007 and 2008. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.

Figure 7: Estimated Impulse Responses: Excluding Mergers

Notes: These figures show estimated impulse response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.

Figure 8: Estimated Impulse Responses: Excluding Mergers

Notes: These figures show estimated impulse response functions for BHCs, dropping observations from quarters in which the bank is recorded as taking part in a merger, as well as dropping the quarter before and the quarter after. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization.

Figure 9: Estimated Impulse Responses of the Liquidity Ratio
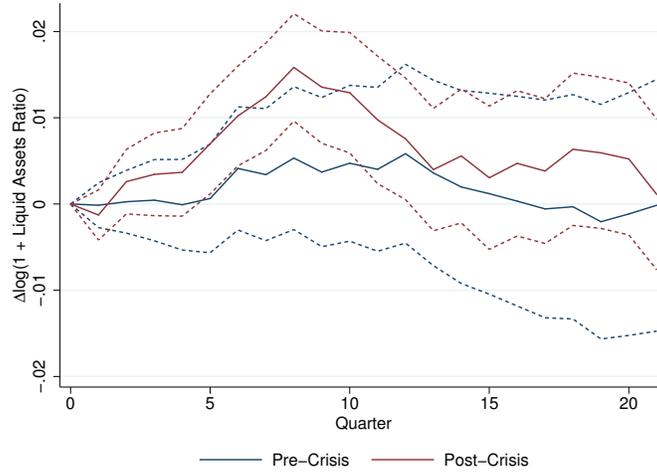
Notes: This figure shows the estimated impulse response function for BHCs to a 1% negative return shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The liquid assets ratio is defined as $\log((\text{Cash} + \text{Treasury Bills})/\text{Total Assets})$. Within the regression sample, the average liquid assets ratio is 0.057.

## Figure 10: Estimated Impulse Responses of Liquidity Ratios (Alternative Formula)
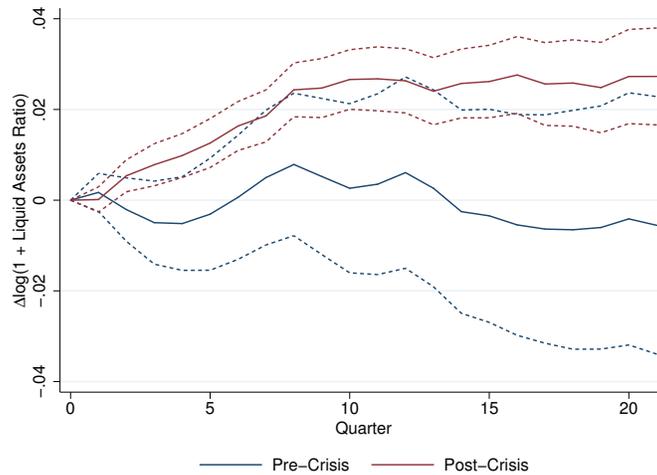
Notes: This figure shows estimated impulse response functions for BHCs. The figure shows the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The liquid assets ratio is defined as $\log((\text{Cash} + \text{Fed Funds Sold} + \text{Securities Purchased Under Agreement to Resell} + \text{Securities})/\text{Total Assets})$.

course of two years. This is off of a base of 0.25-0.30, depending on whether we are taking the mean of log(1+ratio) or of the raw liquid assets ratio.

B.4 Heterogeneity

We explore heterogeneity in impulse response functions by dividing banks into two groups based on a variable, and estimating impulse responses separately for each group. We divide banks by size (total assets), by trading assets ratio (trading assets as a share of total assets), by the risk-weighted asset ratio (risk-weighted assets as a share of total assets), and by the mortgage ratio (real estate loans as a share of total assets). We use the value of the variable in 2000 Q1 to sort banks into two groups: above-median and below-median. We report the results in this section. Broadly, we do not find strong evidence of differential responses, but we lack statistical power to rule out some meaningful differences.

Since bank size is among the most important differences across different banks, we begin by discussing the results for heterogeneity by size. The results are shown in Figures 11 and 12. Visually, these impulse responses look remarkably similar to each other. However, the standard errors are sufficiently large that we cannot rule out meaningful differences in the impulse responses.

We summarize the results of these impulse responses, as well as of the other potential groupings (by trading assets ratio, risk-weighted assets ratio, and mortgage ratio) in Tables 6, 7, 8, and 9 below. For each grouping, we report the cumulative impulse response for the high and low groups after 10 quarters and after 20 quarters, and we also report the p-value of a test of equality between the impulse responses of the two groups. In a table of 64 tests, only one of the tests rejects the null at the 5% level. As before, we take this to suggest that there is not strong evidence in favor of sizable heterogeneity, but we caution that the standard errors are too large to rule out meaningful heterogeneity.

Figure 11: Impulse Responses for Small Banks

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).

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Figure 12: Impulse Responses for Large Banks

Notes: These figures show estimated impulse response functions for BHCs. The figures show the estimated impulse response to a one unit negative returns shock. Dashed lines denote the 95% confidence interval. Standard errors are clustered by bank. The "post-crisis" period begins in 2007 Q4. Data on market capitalization and returns are from CRSP, and all other data are from the FR Y-9C. The panels display the impulse responses of log liabilities, log market capitalization, log market leverage, and log book equity. Market leverage is defined as log(Liabilities/Market Capitalization), so that it represents the difference between the response of log liabilities and log market capitalization (results using log(Liabilities+Market Capitalization)/Market Capitalization) are extremely similar).

Table 6: Heterogeneity in Impulse Responses: Small vs. Large Banks

		Response After 10 Quarters			Response After 20 Quarters		
		Small	Large	p-value on Equality	Small	Large	p-value on Equality
Market Equity	Pre-Crisis	-1.13	-1.09	0.75	-1.22	-1.14	0.61
		(0.08)	(0.07)		(0.12)	(0.11)	
	Post-Crisis	-0.71	-0.76	0.71	-0.58	-0.61	0.84
		(0.14)	(0.06)		(0.14)	(0.07)	
Liabilities	Pre-Crisis	-0.42	-0.33	0.39	-0.65	-0.54	0.52
		(0.07)	(0.07)		(0.13)	(0.10)	
	Post-Crisis	-0.13	-0.15	0.49	-0.23	-0.25	0.67
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.71	0.76	0.59	0.57	0.60	0.87
		(0.08)	(0.05)		(0.11)	(0.09)	
	Post-Crisis	0.58	0.61	0.81	0.35	0.35	0.95
		(0.13)	(0.06)		(0.11)	(0.06)	
Book Equity	Pre-Crisis	-0.25	-0.31	0.68	-0.29	-0.49	0.44
		(0.12)	(0.08)		(0.23)	(0.13)	
	Post-Crisis	-0.78	-0.44	0.04	-0.73	-0.50	0.32
		(0.15)	(0.09)		(0.20)	(0.12)	

Notes: The table compares impulse responses of small vs. large BHCs. BHCs are categorized into the small vs. large group based on their total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for small banks. The second column shows the same results, but for large banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for small banks vs. large banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 7: Heterogeneity in Impulse Responses: Low vs. High Trading Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.10	-1.20	0.60	-1.15	-1.32	0.54
		(0.05)	(0.19)		(0.07)	(0.28)	
	Post-Crisis	-0.76	-0.57	0.11	-0.60	-0.47	0.33
		(0.09)	(0.08)		(0.10)	(0.09)	
Liabilities	Pre-Crisis	-0.36	-0.36	0.99	-0.56	-0.63	0.78
		(0.05)	(0.14)		(0.08)	(0.22)	
	Post-Crisis	-0.14	-0.15	0.73	-0.24	-0.25	0.91
		(0.02)	(0.04)		(0.03)	(0.05)	
Market Leverage	Pre-Crisis	0.74	0.84	0.35	0.59	0.70	0.50
		(0.05)	(0.10)		(0.07)	(0.14)	
	Post-Crisis	0.63	0.42	0.08	0.37	0.22	0.25
		(0.08)	(0.09)		(0.08)	(0.09)	
Book Equity	Pre-Crisis	-0.25	-0.42	0.35	-0.28	-0.75	0.17
		(0.07)	(0.17)		(0.13)	(0.32)	
	Post-Crisis	-0.62	-0.45	0.39	-0.66	-0.54	0.64
		(0.10)	(0.16)		(0.13)	(0.20)	

Notes: The table compares impulse responses of low vs. high trading asset ratio BHCs. BHCs are categorized into the low vs. high group based on their trading assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 8: Heterogeneity in Impulse Responses: Low vs. High Risk-Weighted Asset Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.09	-1.15	0.59	-1.10	-1.22	0.43
		(0.07)	(0.08)		(0.11)	(0.12)	
	Post-Crisis	-0.72	-0.79	0.64	-0.52	-0.66	0.44
		(0.12)	(0.09)		(0.14)	(0.11)	
Liabilities	Pre-Crisis	-0.29	-0.41	0.21	-0.47	-0.66	0.20
		(0.06)	(0.07)		(0.10)	(0.11)	
	Post-Crisis	-0.13	-0.17	0.17	-0.25	-0.25	0.97
		(0.03)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.80	0.73	0.47	0.63	0.56	0.59
		(0.07)	(0.06)		(0.09)	(0.10)	
	Post-Crisis	0.59	0.62	0.82	0.27	0.41	0.31
		(0.11)	(0.09)		(0.11)	(0.09)	
Book Equity	Pre-Crisis	-0.19	-0.35	0.23	-0.24	-0.45	0.40
		(0.10)	(0.09)		(0.16)	(0.18)	
	Post-Crisis	-0.49	-0.74	0.17	-0.51	-0.81	0.23
		(0.09)	(0.16)		(0.11)	(0.23)	

Notes: The table compares impulse responses of low vs. high risk-weighted asset ratio BHCs. BHCs are categorized into the low vs. high group based on their risk-weighted assets as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

Table 9: Heterogeneity in Impulse Responses: Low vs. High Mortgage Ratio

		Response After 10 Quarters			Response After 20 Quarters		
		Low	High	p-value on Equality	Low	High	p-value on Equality
Market Equity	Pre-Crisis	-1.04	-1.21	0.17	-1.07	-1.27	0.26
		(0.05)	(0.11)		(0.07)	(0.17)	
	Post-Crisis	-0.75	-0.75	0.98	-0.61	-0.56	0.79
		(0.13)	(0.08)		(0.17)	(0.09)	
Liabilities	Pre-Crisis	-0.28	-0.46	0.11	-0.45	-0.73	0.09
		(0.05)	(0.10)		(0.08)	(0.15)	
	Post-Crisis	-0.17	-0.11	0.09	-0.28	-0.19	0.13
		(0.02)	(0.02)		(0.05)	(0.03)	
Market Leverage	Pre-Crisis	0.76	0.75	0.92	0.62	0.54	0.55
		(0.06)	(0.07)		(0.08)	(0.11)	
	Post-Crisis	0.59	0.64	0.72	0.34	0.37	0.81
		(0.11)	(0.09)		(0.13)	(0.07)	
Book Equity	Pre-Crisis	-0.20	-0.36	0.32	-0.27	-0.42	0.63
		(0.07)	(0.15)		(0.09)	(0.30)	
	Post-Crisis	-0.66	-0.56	0.57	-0.70	-0.59	0.65
		(0.10)	(0.14)		(0.13)	(0.19)	

Notes: The table compares impulse responses of low vs. high mortgage ratio BHCs. BHCs are categorized into the low vs. high group based on their real estate loans as a share of total assets in 2000 Q1, relative to the median for all banks in the IRF sample. The first column shows the cumulative impulse response after 10 quarters of each variable, pre-crisis and post-crisis, to a one unit negative return shock, for low banks. The second column shows the same results, but for high banks. Standard errors, clustered at the bank level, are in parentheses. The third column shows the p-value of a test of equality between the impulse response for low vs. high banks. The fourth through sixth columns mirror the first three columns, but examining the cumulative impulse response after 20 quarters.

C Model Appendix: Derivations and Proofs

C.1 Derivation of Laws of motion

Summary Table. The summary table of the drift and jump objects as functions of the ratios $\{\lambda, q\}$ is the following:

Table 10: DRIFT AND JUMP VARIABLES IN TERMS OF $\{\lambda, q\}$

	Formula	Interpretation
μ^W	$r^L(\lambda+1) - r^D\lambda + (\iota - \Phi(\iota,1))(\lambda+1) - c$	Levered returns
μ^L	$\iota(\lambda+1)$	Drift of loans
$\mu^{\bar{L}}$	$\left(\iota - \alpha\left(\frac{1}{q}-1\right)\right)(\lambda+1)$	Drift of book loans
μ^D	$\left(r^D\lambda - \left(r^L+\delta\right)(\lambda+1) + (\Phi(\iota,1)+\delta)(\lambda+1) + c\right)$	Drift of deposits
μ^q	$(\iota + \alpha)(1-q)q$	Drift of q
μ^λ	$\left(\iota - \mu^W\right)(\lambda+1)$	Drift of leverage
μ^W_c	-1	Dividend effect on wealth growth
μ^W_ι	$(1 - \Phi_\iota(\iota,1))(\lambda+1)$	Issuance effect on wealth growth
μ^λ_c	$(\lambda+1)$	Dividend effect on leverage growth
μ^λ_ι	$(1 - (1 - \Phi_\iota(\iota,1))(\lambda+1))(\lambda+1)$	Issuance effect on leverage growth
μ^q_ι	$(1-q)q$	Issuance effect on q growth
J^W	$-\varepsilon(\lambda+1)$	Jump in wealth
J^L	$-\varepsilon(\lambda+1)$	Jump in loans
J^L	$-\tau\varepsilon\frac{1}{q}(\lambda+1)$	Jump in book loans
J^D	0	Jump in deposits
J^q	$-\frac{(\varepsilon-\tau\varepsilon q)}{(1-\tau\varepsilon q)}q$	Jump in q
J^λ	$\frac{\varepsilon(\lambda+1)}{1-\varepsilon(\lambda+1)}\lambda$	Jump in leverage

The rest of the appendix, derives the terms of this table and the proofs of the theoretical results.

Notation and Definitions. We begin by presenting some definitions and deriving the laws of motion of the state variables. We use μ^x and J^x to refer to the drift and jump components of the path of a variable x scaled by wealth W, respectively.

Define the net investment rate of the bank as:

$$\iota \equiv I/L - \delta$$

and express the dividend-to-equity ratio as:

$$c \equiv C/W.$$

Note that the following identities allow us to recover the original state variables $\{L, \bar{L}, D\}$ from the triplet $\{\lambda, q, W\}$:

$$L = (\lambda + 1) W \tag{19}$$
$$D = \lambda W \tag{20}$$
$$\bar{L} = q^{-1} (\lambda + 1) W. \tag{21}$$

We present some observations that aid the proof of the proposition.

Observation 1: homogeneity of Φ in W. We prove results for a more general class of adjustment costs,

$$\Phi (I, L) = I + \frac{\gamma}{2} \left| \frac{I}{L} - \delta \right|^{\kappa} L.$$

Recall that in the body of the paper $\kappa = 2$.

We can factor out L and employing the definition of ι to obtain:

$$\begin{aligned} \Phi (I, L) &= \left(\iota + \delta + \frac{\gamma}{2} |\iota|^{\kappa} \right) L \\ &= \Phi (\iota, 1) L + \delta L. \end{aligned}$$

Thus, we can express the funding cost relative to equity as:

$$\Phi (I, L) / W = (\Phi (\iota, 1) + \delta) (\lambda + 1), \tag{22}$$

which is a function independent of the bank's size and depends on leverage and the investment rate.

Observation 2: homogeneity of regulatory constraint in W. We want to express the regulatory capital requirement in terms of the end-of-period choices (λ, q). The regulatory constraint is

$$D \leq \xi \bar{L}, \tag{23}$$

as we noted in the main body of the text. By dividing both sides by bank net worth, we obtain:

$$\frac{D}{W} \leq \xi \frac{\bar{L}}{L} \frac{L}{W}$$

Using the definitions of λ and q:

$$\lambda \leq \xi \frac{1}{q} (\lambda + 1)$$

and clearing out λ, we obtain:

$$\lambda \leq \frac{1}{\frac{q}{\xi} - 1}. \tag{24}$$

Note that the constraint is independent of W and only depends on (λ, q). The solvency constraint is expressed in terms of leverage:

$$\lambda \leq \bar{\lambda} \equiv (1 - \varepsilon)/\varepsilon. \tag{25}$$

Hence, we summarize the set of states where the bank is not liquidated by:

$$\lambda = \min\left\{\frac{1}{\frac{q}{\xi} - 1}, \bar{\lambda}\right\}. \tag{26}$$

Observation 3: Derivations of Laws of Motion. With probability σ over interval Δ, the bank receives deterministic default shock $\varepsilon < 1$. Let:

$$dN = \begin{cases} 0 & \text{with prob } 1 - \sigma dt \\ 1 & \text{with prob } \sigma dt \end{cases}$$

denote a default event process. Recall that dN is a Poisson process.

Now consider a time interval of length Δ. The law of motion for fundamental loans satisfies:

$$L_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \varepsilon L_t (N_{t+\Delta} - N_t),$$

with the interpretation that the first term is the non-maturing fraction of loans, the second are loan issuances, and the third are losses in a time interval. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dL = (I - \delta L) dt - \varepsilon L dN.$$

We express this law of motion in terms of net worth, replacing (19), to obtain:

$$dL = \iota (\lambda + 1) W dt - \varepsilon (\lambda + 1) W dN. \tag{27}$$

To ease the notation, we define the growth rate of fundamental loans and the jump relative to net worth:

$$\mu^L \equiv \iota (\lambda + 1) \text{ and } J^L \equiv -\varepsilon (\lambda + 1).$$

Similarly, for deposits we have that:

$$D_{t+\Delta} = \left(1 + r^D\Delta\right) D_t - \left(r^L\Delta + \delta\Delta\right) L_t + \Phi(I_t, L_t)\Delta + C_t\Delta$$

with the interpretation that the first term is the increase in deposits that results from paying interest with deposits; the second term is the reduction in deposits by the interest and principal payments on outstanding loans; the third term is the increase in deposits as a result of loan issuances; and the final term is dividend payments, all paid with deposits. Taking $\Delta \to 0$, we obtain the following law of motion:

$$dD = \left[r^D D - \left(r^L + \delta\right) L + \Phi(I, L) + C\right] dt.$$

We express this law of motion in terms of wealth, by using (20), to obtain:

$$dD = \left[r^D\lambda - \left(r^L + \delta\right)(\lambda + 1) + (\Phi(\iota, 1) + \delta)(\lambda + 1) + c\right] W dt. \tag{28}$$

We define the growth rate of deposits relative to net worth:

$$\mu^D \equiv r^D\lambda - \left(r^L + \delta\right)(\lambda + 1) + (\Phi(\iota, 1) + \delta)(\lambda + 1) + c.$$

Finally, the law of motion for book loans satisfies:

$$\bar{L}_{t+\Delta} = (1 - \delta\Delta) L_t + I_t\Delta - \alpha\Delta \left(\bar{L}_t - L_t\right) - \tau\varepsilon L_t \left(N_{t+\Delta} - N_t\right),$$

with the interpretation that the first term represents how book loans fall as the principal of fundamental loans gets repaid; the second term increases book loans by newly issued loans; the third term decreases book loans at the speed of loan loss recognition α times the gap in the book versus fundamental loans; and the final term is the fraction of losses recognized in books upon receiving a default shock. Taking $\Delta \to 0$, we obtain the following law of motion:

$$d\bar{L} = (-\delta L + I) \, dt - \alpha \left(\bar{L} - L\right) dt - \tau\varepsilon L dN.$$

We express this law of motion, by using (21), in terms of wealth to obtain:

$$d\bar{L} = \left[\iota - \alpha \left(\frac{1}{q} - 1\right)\right] (\lambda + 1) W dt - \tau\varepsilon\frac{1}{q} (\lambda + 1) W dN. \tag{29}$$

We define the growth rate of book loans and the jump relative to net worth accordingly:

$$\mu^{\bar{L}} \equiv \left[\iota - \alpha \left(\frac{1}{q} - 1\right)\right] (\lambda + 1) \text{ and } J^{\bar{L}} \equiv -\tau\varepsilon\frac{1}{q} (\lambda + 1).$$

Observation 3: growth independence. Next, we present the evolution of net worth, which evolves according to:

$$
\begin{aligned}
dW &= dL - dD \\
&= \left[\underbrace{\left(r^L + \delta\right) (\lambda + 1) - r^D\lambda}_{\text{levered returns}} + \underbrace{(\iota - (\Phi(\iota, 1) + \delta)) (\lambda + 1)}_{\text{capital loss from adjustment}} - \underbrace{c}_{\text{dividend rate}}\right] W dt \\
&= \underbrace{(-\varepsilon (\lambda + 1))}_{\text{loss rate}} W dN. \tag{30}
\end{aligned}
$$

where the second line uses the laws of motion in (27) and (28), and employed observation 1. The interpretation of this expression is natural: the terms multiplying rates represent the net interest margin on the bank, which are the banks levered return; the second term are the capital gains that are accounted immediately as the bank creates an asset that can be worth more or less than a liability; the third term is the banks' dividend rate; and the final term is the loss rate, which scales with leverage. To aid the calculations, define the drift of the growth rate of bank equity as:

$$\mu^W \equiv \left[r^L (\lambda + 1) - r^D\lambda + (\iota - \Phi(\iota, 1)) (\lambda + 1) - c\right] W$$

and denote the jump component of wealth as:

$$J^W \equiv -\varepsilon (\lambda + 1) W = J^L W.$$

We also note that:

$$\mu^W = \mu^L - \mu^D.$$

28

Observation 4: law of motion for leverage. Next, we derive the law of motion for leverage λ given any choice of ι and c. Employing the formula for the differential of a ratio we get:

$$
\begin{aligned}
\mu^\lambda &= \left(\mu^D W - \frac{D}{W} \mu^W W \right) \frac{1}{W} &(31)\\
&= \mu^D - \lambda \mu^W \\
&= \mu^L - (\lambda + 1) \mu^W.
\end{aligned}
$$

Upon a default shock, the discontinuous jump in leverage is given by:

$$
J^\lambda = \frac{D}{(W - \varepsilon (\lambda + 1) W)} - \frac{D}{W} = \left(\frac{1}{1 - \varepsilon (\lambda + 1)} - 1 \right) \lambda.
$$

Therefore, combining the drift and jump portions of the law of motion, we obtain:

$$
d\lambda = \left(\iota - \mu^W \right) (\lambda + 1) \, dt + \frac{\varepsilon (\lambda + 1)}{1 - \varepsilon (\lambda + 1)} \lambda dN. \qquad (32)
$$

The interpretation of this law of motion is that leverage increases with the issuance rate, falls as loans mature and falls as the bank makes earns income on its current portfolio, μ^W. We thus have:

$$
\mu^\lambda = \left(\iota - \mu^W \right) (\lambda + 1).
$$

Naturally, leverage jumps with defaults, and more so the more levered the bank is.

Observation 5: law of motion for q. Next, we produce the law of motion for leverage λ given any choice of ι and c. We first describe the continuous portion of the law of motion dq^c. Employing the formula for the ratio:

$$
dq^c = \left(\frac{dL^c}{L} - \frac{d\bar{L}^c}{\bar{L}} \right) q dt.
$$

The first term is

$$
\frac{dL^c}{L} = \iota
$$

and the second term is

$$
\frac{d\bar{L}^c}{\bar{L}} = \frac{\left(\iota - \alpha \left(\frac{1}{q} - 1 \right) \right) (\lambda + 1) W}{\frac{1}{q} (\lambda + 1) W} = \iota q - \alpha (1 - q).
$$

Consequently:

$$
dq^c = (\iota + \alpha) (1 - q) q dt. \qquad (33)
$$

Upon a default shock, the discontinuous jump in leverage is given by:

$$
J^q = \frac{L - \varepsilon L}{\bar{L} - \tau \varepsilon L} - q = \frac{(1 - \varepsilon) L}{(1 - \tau \varepsilon q) \bar{L}} - q = -\frac{(\varepsilon - \tau \varepsilon q)}{(1 - \tau \varepsilon q)} q.
$$

Therefore, combining the continuous and discrete portions of the law of motion, we obtain:

$$dq = (\iota + \alpha)(1 - q)qdt - \left(\frac{\varepsilon - \tau\varepsilon q}{1 - \tau\varepsilon q}\right)qdt. \tag{34}$$

Finally, note the relationship:

$$dq^c = \mu^q = \left[\mu^L - \mu^{\bar{L}}q\right]\frac{q}{(\lambda + 1)}dt.$$

Duffie-Epstein. The value function of the Duffie-Epstein satisfies:

$$V_t = E_t \int_t^\infty f(C_s, V_s)\, ds,$$

where the f is given by:

$$f(C,V) \equiv \frac{\rho}{1 - \theta}\left[\frac{C^{1-\theta} - \{1 + (1-\psi)V\}^{\frac{1-\theta}{1-\psi}}}{\{1 + (1-\psi)V\}^{\frac{1-\theta}{1-\psi} - 1}}\right]$$

$$= \frac{\rho}{1 - \theta}\{1 + (1-\psi)V\}\left[\frac{C^{1-\theta}}{\{1 + (1-\psi)V\}^{\frac{1-\theta}{1-\psi}}} - 1\right].$$

A useful calculation is the derivative with respect to dividends:

$$f_c(C,V) = \rho\frac{C^{-\theta}}{\{1 + (1-\psi)V\}^{\frac{1-\theta}{1-\psi} - 1}}.$$

We have some limits of interest. First, the limit as risk-aversion vanishes:

$$\lim_{\psi \to 0} f(C,V) = \frac{\rho}{1 - \theta}(1 + V)\left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right].$$

and

$$\lim_{\psi \to 0} f_c(C,V) = \rho C^{-\theta}(1 + V)^\theta.$$

Second, the limit as the IES goes to 1:

$$\lim_{\theta \to 1}\frac{\rho}{1 - \theta}\left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right] = \lim_{\theta \to 0}\frac{\rho}{1 - \theta}\left[\frac{C^{1-\theta}}{(1+V)^{1-\theta}} - 1\right] - \rho V = \log C - \rho V.$$

and

$$\lim_{\theta \to 1} = f_c(C,V) = \rho C^{-\theta}(1 + V)^\theta = \rho\frac{(1+V)}{C}.$$

C.2 Proof of Proposition 1

In this Appendix we prove the following detailed version of Proposition 1:

Proposition 2 *[Bank's Problem] Given $\{\lambda, q\}$, $V\left(L, \bar{L}, D\right) = \left(1 + v\left(\lambda, q\right)\right)W - 1$, where v is the solution to the following HJB equation:*

$$0 = \max_{\{c, \iota\}} f\left(c, v\right) + \underbrace{v_\lambda \mu^\lambda + v_q \mu^q}_{\text{change in financial ratios}} + \underbrace{\left(1 + v\right)\mu^W}_{\text{equity growth}} \ldots \tag{35}$$

$$+ \sigma \underbrace{\left[\left(1 + v\left(\lambda + J^\lambda, q + J^q\right)\right)\left(1 + J^W\right) - \left(1 + v\right)\right]}_{\text{default jump in wealth}} \quad in \ \left(\lambda, q\right) \notin \Gamma$$

and $v = v_o$ for $\{\lambda, q\} \in \Gamma$. Policy functions can be recovered through the following relationships: $C\left(L, \bar{L}, D\right) = c\left(\lambda, q\right) \cdot W$ and $I\left(L, \bar{L}, D\right) = \left(\iota\left(\lambda, q\right) + \delta\right) \cdot L$. The bank's market value satisfies $S\left(L, \bar{L}, D\right) \equiv s\left(\lambda, q\right) \cdot W$, where s solves:

$$\rho^I s = c\left(\lambda, q\right) + s_\lambda \mu^\lambda + s_q \mu^q + s\mu^W + \sigma\left[s\left(\lambda + J^\lambda, q + J^q\right)\left(1 + J^W\right) - s\right], \tag{36}$$

and $s = 0$ for $\{\lambda, q\} \in \Gamma$. Finally, Tobin's Q is given by:

$$Q\left(\lambda, q\right) = s\left(\lambda, q\right) \times \left(\left(q^{-1} - 1\right)\lambda + 1\right)^{-1}. \tag{37}$$

Formulation. We next prove Proposition 2. The primitive bank value HJB equation is given by:

$$0 = \max_{\{C, I\}} f\left(C, V\left(L, \bar{L}, D\right)\right) + \frac{E\left[dV\left(L, \bar{L}, D\right)\right]}{dt} \tag{38}$$

subject to the laws of motion (27), (28), (29), and the boundary $V = V_o$ when (23) and (25) are not satisfied. In the objective, the differential form is:

$$\frac{E\left[dV\left(L, \bar{L}, D\right)\right]}{dt} = V_L\left(L, \bar{L}, D\right)\mu^L W + V_{\bar{L}}\left(L, \bar{L}, D\right)\mu^{\bar{L}}W + V_D\left(L, \bar{L}, D\right)\mu^D W$$

$$+ \sigma\left[V\left(\left(1 - \varepsilon\right)L, \left(1 - \tau\varepsilon\right)\bar{L}, D\right) - V\left(L, \bar{L}, D\right)\right].$$

Conjecture. We conjecture a solution to the value function and verify that it satisfies the HJB equation. The conjecture is:

$$V\left(L, \bar{L}, D\right) = \frac{\left[1 + \left(1 - \psi\right)v\left(\lambda, q\right)\right]W^{1-\psi} - 1}{1 - \psi}, \tag{39}$$

for a suitable candidate $v\left(\lambda, q\right)$. Under this conjecture, we verify that $C\left(L, \bar{L}, D\right) = c\left(\lambda, q\right)W$ and $I = \left(\iota\left(\lambda, q\right) + \delta\right)\left(\lambda + 1\right)W$.

Factorization. We perform some useful calculations on the guess (39). In particular, we factorize equity from every term in the HJB equation. Under the conjecture, we have that:

$$\left(1 + \left(1 - \psi\right)V\right) = \left[1 + \left(1 - \psi\right)v\right]W^{1-\psi}.$$

Therefore,

$$
\begin{aligned}
f\left(C,V\right) &= f\left(c\left(\lambda,q\right)W,\frac{\left[1+\left(1-\psi\right)v\left(\lambda,q\right)\right]W^{1-\psi}-1}{1-\psi}\right)\\
&= \frac{\rho}{1-\theta}\left[1+\left(1-\psi\right)v\right]W^{1-\psi}\left[\frac{c\left(\lambda,q\right)^{1-\theta}W^{1-\theta}}{\left(\left[1+\left(1-\psi\right)v\right]W^{1-\psi}\right)^{\frac{1-\theta}{1-\psi}}}-1\right]\\
&= \frac{\rho}{1-\theta}\left[1+\left(1-\psi\right)v\right]W^{1-\psi}\left[\frac{c\left(\lambda,q\right)^{1-\theta}}{\left(\left[1+\left(1-\psi\right)v\right]\right)^{\frac{1-\theta}{1-\psi}}}-1\right]\\
&= f\left(c\left(\lambda,q\right),v\right)W^{1-\psi}.
\end{aligned}
\tag{40}
$$

The change in the value function with respect to loans is:

$$
\begin{aligned}
V_L &= \partial\left[\frac{\left[1+\left(1-\psi\right)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right]\left(L-D\right)^{1-\psi}\left[\frac{v_q\left(1-q\right)q+v_\lambda\left(\lambda+1\right)}{\gamma\kappa\left(v_\lambda\left(\lambda+1\right)-\left(1+v\right)\right)\left(\lambda+1\right)}\right]^{\frac{1}{\kappa-1}}-1}{1-\psi}\right]/\partial L\\
&= -v_\lambda\frac{D}{\left(L-D\right)^2}W^{1-\psi}+v_q\frac{1}{\bar{L}}W^{1-\psi}+\left[1+\left(1-\psi\right)v\right]W^{-\psi}\\
&= -v_\lambda\frac{\lambda W}{W^2}W^{1-\psi}+v_q\frac{1}{\frac{1}{q}\left(\lambda+1\right)W}W^{1-\psi}+\left[1+\left(1-\psi\right)v\right]W^{-\psi}\\
&= \left(-v_\lambda\lambda+v_q\frac{q}{\left(\lambda+1\right)}+\left[1+\left(1-\psi\right)v\right]\right)W^{-\psi}.
\end{aligned}
\tag{41}
$$

The change in the value function with respect to deposits is:

$$
\begin{aligned}
V_D &= \partial\left[\frac{\left[1+\left(1-\psi\right)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right]\left(L-D\right)^{1-\psi}-1}{1-\psi}\right]/\partial D\\
&= v_\lambda\frac{1}{W}W^{1-\psi}+v_\lambda\frac{D}{\left(L-D\right)^2}W^{1-\psi}-\left[1+\left(1-\psi\right)v\right]W^{-\psi}\\
&= v_\lambda\frac{1}{W^2}W^{1-\psi}+v_\lambda\frac{\lambda W}{W^2}W^{1-\psi}-\left[1+\left(1-\psi\right)v\right]W^{-\psi}\\
&= \left(v_\lambda\left(1+\lambda\right)-\left[1+\left(1-\psi\right)v\right]\right)W^{-\psi}.
\end{aligned}
\tag{42}
$$

Finally, the derivative of the value function with respect with respect to $\bar{L}$ is given by:

$$
\begin{aligned}
V_{\bar{L}} &= \partial\left[\frac{\left[1+\left(1-\psi\right)v\left(\frac{D}{L-D},\frac{L}{\bar{L}}\right)\right]\left(L-D\right)^{1-\psi}-1}{1-\psi}\right]/\partial\bar{L}\\
&= -v_q\frac{L}{\bar{L}^2}W^{1-\psi}\\
&= -v_q\frac{\left(\lambda+1\right)W}{\left(\frac{1}{q}\left(\lambda+1\right)W\right)^2}W^{1-\psi}\\
&= -v_q\frac{q^2}{\left(\lambda+1\right)}W^{-\psi}.
\end{aligned}
\tag{43}
$$

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Finally, the jump in the value function after a default even is:

$$J^V = \left[V\left((1-\varepsilon)L, (1-\tau\varepsilon)\bar{L}, D\right) - V\left(L, \bar{L}, D\right) \right]$$

which according to our guess can be written as:

$$
\begin{aligned}
J^V &= \left[\frac{\left[1 + (1-\psi)v\left(\lambda + J^\lambda, q + J^q\right)\right]\left(\left(1 + J^W\right)W\right)^{1-\psi} - 1}{1 - \psi} - \frac{\left[1 + (1-\psi)v\left(\lambda, q\right)\right]W^{1-\psi} - 1}{1 - \psi} \right] \\
&= \left[\frac{\left[1 + (1-\psi)v\left(\lambda + J^\lambda, q + J^q\right)\right]}{1 - \psi}\left(1 + J^W\right)^{1-\psi} - \frac{\left[1 + (1-\psi)v\left(\lambda, q\right)\right]}{1 - \psi} \right]W^{1-\psi} \\
&= \left[\frac{\left[1 + (1-\psi)v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right]}{1 - \psi}\left(1 + J^W\right)^{1-\psi} - \frac{\left[1 + (1-\psi)v\left(\lambda, q\right)\right]}{1 - \psi} \right]W^{1-\psi}. \quad (44)
\end{aligned}
$$

Verification. We verify that the conjecture satisfies its HJB equation. With the factorizations above, equations (40-44), we have that (38) can be written as:

$$
\begin{aligned}
0 = {}& \max_{\{c,\iota\}} f\left(c, v\right)W^{1-\psi}\ldots \\
&+ \underbrace{\begin{bmatrix} v_\lambda & v_q & \left(1 + (1-\psi)v\right) \end{bmatrix} \times \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^D \\ \mu^{\bar{L}} \end{bmatrix} W^{1-\psi}}_{\equiv \mu^V}\ldots \\
&+ \sigma\underbrace{\left[\frac{\left[1 + (1-\psi)v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)\right)\right]}{1 - \psi}\left(1 + J^W\right)^{1-\psi} - \frac{\left[1 + (1-\psi)v\left(\lambda, q\right)\right]}{1 - \psi} \right]W^{1-\psi}}_{\equiv J^v}.
\end{aligned}
$$

where we used the fact that any choice of C and I, can be expressed as a choice of $c\left(\lambda, q\right)W$ as there is a one to one map from the $\{\lambda, q, W\}$ space to the original space—by change of coordinates. Then, we can factor wealth from this HJB equation to express it as:

$$0 = W^{1-\psi}\left[\max_{\{c,\iota\}} f\left(c, v\right) + \mu^V + J^V\right],$$

and since the maximization is independent of net worth, this verifies the linearity of the controls. To verify the conjecture, we need to express the drifts and jumps, $\{\mu^V, J^V\}$ exclusively in terms of $\{\lambda, q\}$. To do so, observe that:

$$\mu^V = \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^D \\ \mu^{\bar{L}} \end{bmatrix}$$

$$= \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^L - \mu^W \\ \mu^{\bar{L}} \end{bmatrix} \dots$$

$$= \begin{bmatrix} -\lambda\mu^L + (1+\lambda)\left(\mu^L - \mu^W\right) \\ \frac{q}{(\lambda+1)}\mu^L - \frac{q}{(\lambda+1)}q\mu^{\bar{L}} \\ \mu^L - \left(\mu^L - \mu^W\right) \end{bmatrix}$$

$$= \begin{bmatrix} \mu^\lambda \\ \mu^q \\ \mu^W \end{bmatrix}.$$

Thus, we have that,

$$\mu^V(\lambda, q) = v_\lambda \mu^\lambda + v_q \mu^q + (1 + (1-\psi)v)\mu^W,$$

where all the terms are functions of the state variables $\{\lambda, q\}$.

Consequently, the HJB solution to the HJB equation:

$$0 = \left[\max_{\{c,\iota\}} f(c,v) + v_\lambda \mu^\lambda + v_q \mu^q + (1 + (1-\psi)v)\mu^W + \sigma J^V\right], \tag{45}$$

subject to the solvency conditions (24) and (25 and the liquidiation value v_o, and the laws of motion (32-34). Since the choice is independent of wealth, and only depends on λ and q, this verifies that $\bar{v}$ is only a function of $\{\lambda, q\}$, and is not indexed by W. Thus, we verify the conjecture that the formula (39) satisfies the HJB equation (35), for v, a solution to .

Applying the result to the special case with $\psi = 0$, yields:

$$\begin{aligned} 0 = \; & \max_{\{c,\iota\}} f(c,v) + v_\lambda \mu^\lambda + v_q \mu^q + (1+v)\mu^W \\ & + \sigma\left[\left(1 + v\left(\frac{\lambda}{1 - \varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right)(1 - \varepsilon(\lambda+1)) - (1 + v(\lambda,q))\right] \end{aligned}$$

subject to the boundary conditions given by (24) and (25)—taking the value v_o, and the laws of motion (32), (34).

Limits of Interest. We now let $\psi \to 0$ to obtain:

$$
0 = \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^D \\ \mu^{\bar{L}} \end{bmatrix}
$$

$$
= \begin{bmatrix} -\lambda & (1+\lambda) & 0 \\ \frac{q}{(\lambda+1)} & 0 & -\frac{q^2}{(\lambda+1)} \\ 1 & -1 & 0 \end{bmatrix} \times \begin{bmatrix} \mu^L \\ \mu^L - \mu^W \\ \mu^{\bar{L}} \end{bmatrix} \cdots
$$

$$
= \begin{bmatrix} -\lambda\mu^L + (1+\lambda)\left(\mu^L - \mu^W\right) \\ \frac{q}{(\lambda+1)}\mu^L - \frac{q}{(\lambda+1)}q\mu^{\bar{L}} \\ \mu^L - \left(\mu^L - \mu^W\right) \end{bmatrix}
$$

$$
= \begin{bmatrix} \mu^\lambda \\ \mu^q \\ \mu^W \end{bmatrix}.
$$

Hence, we obtain:

$$
0 = \max_{\{c,\iota\}} \frac{\rho}{1-\theta}(1+v)\left[\frac{c^{1-\theta}}{(1+v)^{1-\theta}} - 1\right] + v_\lambda\mu^\lambda + v_q\mu^q + (1+v)\mu^W
$$
$$
+ \sigma\left[\left(1 + v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right)(1-\varepsilon(\lambda+1)) - (1+v(\lambda,q))\right].
$$

Now consider the limite where $\theta \to 1$. We recover:

$$
\rho v = \max_{\{c,\iota\}} \rho \log c + v_\lambda\mu^\lambda + v_q\mu^q + (1+v)\mu^W
$$
$$
+ \sigma\left[\left(1 + v\left(\frac{\lambda}{1-\varepsilon(\lambda+1)}, \left(\frac{1-\varepsilon}{1-\tau\varepsilon q}\right)q\right)\right)(1-\varepsilon(\lambda+1)) - (1+v(\lambda,q))\right].
$$

C.3 Policy Functions

We derive the first-order conditions of this problem.

Optimal Dividend. The first-order condition for dividends is given by:

$$
f_c(c,v) + v_\lambda\mu_c^\lambda + (1+(1-\psi)v)\mu_c^W = 0,
$$

and arranging terms, we obtain:

$$
f_c(c,v) + v_\lambda(\lambda+1) = (1+(1-\psi)v). \tag{46}
$$

In the special case of risk neutrality:

$$
\rho\frac{c^{-\theta}}{(1+v)^{-\theta}} + v_\lambda(\lambda+1) = (1+v),
$$

which we can solve to obtain:

$$c = \rho^{1/\theta} \left[\frac{(1 + (1 - \psi) v)}{(((1 + (1 - \psi) v) - v_\lambda (\lambda + 1)))^{1/\theta}} \right]. \tag{47}$$

Take the risk-neutral limit, $\psi \to 0$ and we obtain:

$$c = \rho^{1/\theta} \left[\frac{(1 + v)}{(((1 + v) - v_\lambda (\lambda + 1)))^{1/\theta}} \right].$$

In the special case of $v_\lambda = 0$ we obtain:

$$c = \rho^{1/\theta} (1 + v)^{1 - 1/\theta}.$$

In the special case of $\theta \to 1$, we have that:

$$c = \frac{\rho}{\left(1 - v_\lambda \frac{(\lambda + 1)}{(1 + v)} \right)}. \tag{48}$$

The solution resembles closely the solution to a portfolio problem where the dividend rate is exactly the discount rate ρ. However, in this problem, because leverage is a slow moving object, there's a correction term given by, $v_\lambda \frac{(\lambda + 1)}{(1 + v)}$, which measures the additional advantage of impacting on leverage through the dividend decision. If leverage is too high, such that it reaches beyond the point of zero, then dividend rate is distorted downwards.

The elasticity of dividends with respect to leverage in this special case is given by:

$$
\begin{aligned}
dc &= \left(\frac{v_{\lambda\lambda} \frac{(\lambda+1)}{(1+v)} - \frac{(v_\lambda)^2}{(1+v)} \frac{(\lambda+1)}{(1+v)}}{1 - v_\lambda \frac{(\lambda+1)}{(1+v)}} \right) c d\lambda \\
&= \left(\frac{\left(\frac{v_{\lambda\lambda}}{v_\lambda} (\lambda + 1) - \frac{v_\lambda}{(1+v)} (\lambda + 1) \right) v_\lambda}{(1 + v) - v_\lambda (\lambda + 1)} \right) c d\lambda
\end{aligned}
$$

Optimal Issuance. Next, we discuss the first-order condition in issuances, which yields:

$$v_\lambda \mu_\iota^\lambda + v_q \mu_\iota^q + (1 + (1 - \psi) v) \mu_\iota^W = 0.$$

Using the expressions in Table 10, we obtain:

$$v_q (1 - q) q + v_\lambda (1 - (1 - \Phi_\iota (\iota, 1)) (\lambda + 1)) (\lambda + 1) + (1 + (1 - \psi) v) (1 - \Phi_\iota (\iota, 1)) (\lambda + 1) = 0,$$

and collecting terms yields:

$$v_q (1 - q) q + v_\lambda (\lambda + 1) + ((1 + (1 - \psi) v) - v_\lambda (\lambda + 1)) (1 - \Phi_\iota (\iota, 1)) (\lambda + 1) = 0. \tag{49}$$

Using the first-order condition yields:

$$(1 - \Phi_\iota (\iota, 1)) = \frac{v_q (1 - q) q + v_\lambda (\lambda + 1)}{(v_\lambda (\lambda + 1) - (1 + (1 - \psi) v)) (\lambda + 1)}.$$

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The right hand side equals:

$$\left(1 - \Phi_\iota\left(\iota, 1\right)\right) = sign\left(\iota\right)\frac{\gamma}{2}\kappa\left(sign\left(\iota\right)\left(\iota\right)\right)^{\kappa-1}$$

and thus:

$$\left(sign\left(\left(\iota\right)\right)\iota\right) = \left[sign\left(\iota\right)\frac{v_q\left(1-q\right)q + v_\lambda\left(\lambda+1\right)}{\frac{\gamma}{2}\kappa\left(v_\lambda\left(\lambda+1\right) - \left(1+\left(1-\psi\right)v\right)\right)\left(\lambda+1\right)}\right]^{\frac{1}{\kappa-1}}.$$

Thus, we have that issuances are given by

$$\iota = \begin{cases} \left(\dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)} > 0 \\[3em] \left(-\dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+(1-\psi)v))(\lambda+1)} < 0. \end{cases}$$

In the limit as $\psi \to 0$, we obtain:

$$\iota = \begin{cases} \left(\dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)} > 0 \\[3em] \left(-\dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)}\right)^{\frac{1}{\kappa-1}} & \text{if } \dfrac{v_q(1-q)q+v_\lambda(\lambda+1)}{\frac{\gamma}{2}\kappa(v_\lambda(\lambda+1)-(1+v))(\lambda+1)} < 0. \end{cases}$$

In the special case considered in the paper, $\kappa = 2$:

$$1 - \Phi_\iota\left(\iota, 1\right) = \gamma\iota.$$

Thus,

$$\iota = \frac{1}{\gamma}\cdot\left(\frac{v_q\left(1-q\right)q + v_\lambda\left(\lambda+1\right)}{\left(v_\lambda\left(\lambda+1\right) - \left(1+v\right)\right)\left(\lambda+1\right)}\right).$$

C.4 Proofs for $\gamma = 0$ and $\tau = 1$ limit

Before we proceed to derive the main result, we first derive a some preliminary observations.

No adjustment costs. Without adjustment costs on ι, the bank can reset leverage at will. It can do so through an issuance intensity ι that controls the drift of λ and through the endogenous jump $\bar{J}^\lambda$. In particular, the bank can choose a discrete jump in loans, $\bar{J}^L$. Thus, the process for loans is given by:

$$dL = (\iota - \delta)(\lambda + 1) W dt + J^L W dN + \bar{J}^L W d\tilde{N}$$

where $d\tilde{N}$ is the event of a controlled jump. Likewise, the evolution of deposits satisfies:

$$dD = (c - \mu^W)(\iota - \delta)(\lambda + 1) W dt + J^L W dN + \bar{J}^L W d\tilde{N}.$$

Next, we can define the controlled jump in leverage given $\bar{J}^L$ which yields:

$$\bar{J}^\lambda = \frac{D + \bar{J}^L W}{L + \bar{J}^L W - D - \bar{J}^L W} - \lambda = \left(\lambda + \bar{J}^L - \lambda\right) = \bar{J}^L.$$

Shadow Boundary. Let Λ be the value of leverage such that a loan default shock takes leverage to the regulatory limit:

$$\Lambda = \frac{(1 - \varepsilon)\Xi}{1 + \varepsilon\Xi} = \frac{(1 - \varepsilon)\xi}{1 - (1 - \varepsilon)\xi},$$

As in the text, we label this leverage, the shadow liquidation boundary.

Main Result. In this Appendix we prove the following result:

Proposition 3 *[Bank's Problem] Let $\tau = 1$ and $\gamma = 0$. Then, leverage λ^* is constant. For suitable parameter conditions, $\lambda^* = \Lambda$. In that case, the equity multiplier is the constant that solves*

$$0 \quad = \quad \max_{\{c,\lambda\}} f(c, v) \quad + \quad (1 + v)\left[r^L + \left(r^L - r^D\right)\Lambda - c\right] \quad + \quad \sigma(1 + v)\left(\left(1 + J^W\right) - 1\right).$$

Then, for $dN = 0$ the dividend rate is the constant, c^* that solves

$$c^* = \rho^{1/\theta}(1 + v)^{1 - 1/\theta}, \tag{50}$$

the issuance rate is ι^* such that $\mu^\lambda = 0$.

Furthremore, $d\tilde{N} = dN$ and for $dN = 1$, λ is reflected back to λ^* $(\bar{J}^\lambda = -J^\lambda)$. Finally, the multiplier v is the constant that solves (35) given the constant values c and λ. The necessary parameter condition for this dynamics are:

$$\frac{\left(r^L - r^D\right)}{\sigma} \in [v_0, 1 + v].$$

Derivation of the Main Result. For this proof we work directly with the risk-neutral case, as we showed already the effect of risk-aversion in the earlier proof. Consider the case where $\gamma = 0$ such that $\Phi(\iota, 1) = \iota$ and $\tau = 1$. In this case, we have that each issued loans increases deposits one

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by one. Hence, loan issuances do not impact wealth. The evolution of net worth evolves according to:

$$
\begin{aligned}
dW &= dL - dD \\
&= \left[r^L + \underbrace{\left(r^L - r^D \right)}_{\text{levered returns}} \lambda - \underbrace{c}_{\text{dividend rate}} \right] W\,dt - \underbrace{\varepsilon\left(\lambda + 1\right)}_{\text{loss rate}} W\,dN
\end{aligned}
\tag{51}
$$

where the second line uses the laws of motion in (27) and (28), and using observation 1. Since there are no adjustment costs, leverage can be adjusted to any level immediately. It is done so with a combination of an issuance intensity ι and an endogenous discrete jump in issuances that produces a jump in leverage, $\bar{J}^\lambda$, as we show next.

The jump in wealth upon a default shock is:

$$
J^W \equiv -\varepsilon\left(\lambda + 1\right),
$$

and the drift of wealth is given by levered returns minus dividends:

$$
\mu^W \equiv \left[r^L + \underbrace{\left(r^L - r^D \right)}_{\text{levered returns}} \lambda - \underbrace{c}_{\text{dividend rate}} \right].
$$

Recall that the regulatory constraint (5):

$$
1 = \xi\left(1 + \lambda\right)/\lambda \rightarrow \lambda \leq \frac{\xi}{1 - \xi}.
$$

In this case, λ is chosen every period and ι is defined to be consistent with the drift in that choice. Since $q = 1$, the relevant constraint is the regulatory constraint, and no longer the market-based constraint. Then, the HJB equation in (45) becomes:

$$
0 = \left[\max_{\{c,\lambda\}} f\left(c, v\right) + v_\lambda \mu_\lambda + \left(1 + v\right) \mu^W v + \sigma J^V \right].
\tag{52}
$$

Different from (45), we conjecture that in this case, $\bar{v}$ is a scalar rather than a function of λ—or q which in this case is constant. Under this guess, the jump term upon a default event is:

$$
J^V = \left(1 + v\right)\left(\left(1 + J^W\right)^{1-\psi} \mathbb{I}\left[\lambda + J^\lambda < \frac{1}{\frac{1}{\rho} - 1} \right] + v_o \mathbb{I}\left[\lambda + J^\lambda > \frac{1}{\frac{1}{\rho} - 1} \right] - 1 \right),
$$

where as before,

$$
J^\lambda = \frac{\lambda}{1 - \varepsilon\left(\lambda + 1\right)}.
$$

Substituting for the drift μ^W, $v_\lambda = 0$, and the jump in wealth J^W in (52) we obtain:

$$0 = \max_{\{c,\lambda\}} f(c,v) + (1+v)\left[r^L + \left(r^L - r^D\right)\lambda - c\right]$$

$$+ \sigma(1+v)\left(\left(1+J^W\right)\left[\lambda + J^\lambda < \frac{1}{\frac{1}{\rho}-1}\right] + v_0\mathbb{I}\left[\lambda + J^\lambda > \frac{1}{\frac{1}{\rho}-1}\right] - 1\right).$$

Let's first solve for consumption. In this case, the first-order condition for consumption is:

$$\rho\frac{c^{-\theta}}{(1+\bar{v})^{-\theta}} = (1+v).$$

We rearrange terms to obtain:

$$c = \rho^{1/\theta}(1+v)^{1-1/\theta}. \tag{53}$$

Next, we obtain the solution for leverage. Then, the leverage choice maximizes

$$\max_{\lambda\in\left[0,\frac{\xi}{1-\xi}\right]} (1+v)\left(r^L - r^D\right)\lambda$$

$$+ \sigma(1+v)(1-\varepsilon(\lambda+1))\mathbb{I}\left[\frac{\lambda}{1-\varepsilon(\lambda+1)} < \frac{\xi}{1-\xi}\right] + \sigma\frac{v_0}{(1+\bar{v})}\mathbb{I}\left[\frac{\lambda}{1-\varepsilon(\lambda+1)} > \frac{\xi}{1-\xi}\right].$$

This is a linear program. The solution thus generically fall in a corner:

$$\lambda^* \in \begin{cases} 0 & \text{if } \left(r^L - r^D\right) < \epsilon\sigma\left[1+\bar{v}\right] \\ \frac{1}{\frac{1}{\xi}-1} & \text{if } \left(r^L - r^D\right) > \sigma U(\eta) \\ \Lambda & \text{otherwise.} \end{cases} \tag{54}$$

Considering the interesting case where $\lambda = \Lambda$, substituting (50) we have that $\bar{v}$ solves:

$$0 = \frac{\rho}{1-\theta}\left[\frac{\left(\rho^{1/\theta}(1+\bar{v})^{1-1/\theta}\right)^{1-\theta}}{(1+\bar{v})^{1-\theta}} - (1+\bar{v})\right]$$

$$+ (1+\bar{v})\left[r^L + \left(r^L - r^D\right)\Lambda + \left(1+J^W(\Lambda)\right) - \rho^{1/\theta}(1+\bar{v})^{1-1/\theta}\right]. \tag{55}$$

This equation verifies that indeed the value function is independent of the level of leverage. Then, since leverage is a constant, ι solves $\mu_\lambda = 0$ and thus:

$$\iota = \mu^W = r^L + \left(r^L - r^D\right)\Lambda - \rho^{1/\theta}(1+\bar{v})^{1-1/\theta}. \tag{56}$$

Finally, $\bar{J}^\lambda = -J^\lambda(\Lambda)$ whenever $dN = 1$.

Log-Limit. Consider now the limit $\theta \to 1$. Therefore, this case produces the usual formula:

$$c^* = \rho.$$

Thus, the value per unit of wealth is:

$$\bar{v} = \log \rho + \frac{\left[r^L + \left(r^L - r^D\right)\lambda^* - \rho\right] + \sigma\left(1 - \varepsilon\left(\lambda^* + 1\right)\right)}{\rho}.$$

C.5 Proofs for $\gamma = 0$ and $\tau \neq 1$ limit

We begin with a preliminary set of observations.

Endogenous Jump in q. As in the previous case, in this version, banks can control leverage immediately through a combination of an issuance intensity ι and a jump in leverage. The novelty in that with delayed accounting, the jump in loans produces a controlled jump in q. We can define the jump in q given $\bar{J}^L$ as

$$\bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right) = \frac{L + \bar{J}^L W}{\bar{L} + \bar{J}^L W} - q = \frac{(\lambda + 1) + \bar{J}^L}{q^{-1}(\lambda + 1) + \bar{J}^L} - q = q\left(\frac{(\lambda + 1) + \bar{J}^\lambda}{(\lambda + 1) + q\bar{J}^\lambda} - 1\right).$$

Thus, we can think of the banker as controlling a jump in leverage, that produces a corresponding jump in $\bar{J}^q$.

Liquidation and Shadow Boundary. With abuse of notation, we now define the following function that characterizes the liquidation boundary $\partial\Gamma$ by the function:

$$\Gamma(q) \equiv \min\left\{\frac{\xi}{q - \xi}, \bar{\lambda}\right\},$$

and a corresponding "shadow boundary" as given by the following function:

$$\Lambda(q) \equiv \Gamma(q + J^q) - J^\lambda = \min\left\{\frac{\xi}{\frac{q - \varepsilon q}{1 - \tau \varepsilon q} - \xi}, \bar{\lambda}\right\} - \frac{\varepsilon\lambda(\lambda + 1)}{1 - \varepsilon(\lambda + 1)}.$$

The graph of the shadow boundary is the set of points were $\{q, \lambda\}$ such that given a default event, the bank ends at the boundary of the liquidation region. Finally, note that:

$$\Gamma(q + J^q) = \min\left\{\frac{\xi}{\frac{q - \varepsilon q}{1 - \tau \varepsilon q} - \xi}, \bar{\lambda}\right\}.$$

From Liquidation to Shadow Boundary. We want to solve for a the position where the bank ends, if it jumps from the liquidation to the shadow boundary. We solve for the jump size as $\bar{J}^L$ that solves:

$$\Lambda\left(q + \bar{J}^q\right) \equiv \Gamma(q) + \bar{J}^L.$$

Then, considering an initial position $\{q, \lambda\}$ in the shadow boundary, the terminal jump is given by:

$$\Lambda\left(q + J^q + \bar{J}^q\left(q + J^q, \bar{J}^L\right)\right) = \Gamma(q + J^q) - J^\lambda + \bar{J}^L.$$

The critical value of q^m below which market liquidation occurs is:

$$\frac{\xi}{q^m - \xi} = \bar{\lambda}.$$

Hence, the critical point is

$$q^m = \xi\frac{1 + \bar{\lambda}}{\bar{\lambda}}.$$

Thus, the slope of the liquidation boundary is:

$$\Gamma_q(q) = \begin{cases} 0 & \text{if } q < q^m \\ -\frac{\xi}{(q-\xi)^2} & \text{if } q \geq q^m. \end{cases}$$

Likewise, the slope of the shadow boundary:

$$\Lambda_q(q) = \begin{cases} 0 & \text{if } \frac{1-\varepsilon}{1/q-\tau\varepsilon q} < q^m + \xi \\ -\frac{\xi}{\left(\frac{1-\varepsilon}{1/q-\tau\varepsilon} - \xi\right)^2} \cdot \frac{1}{(1/q-\tau\varepsilon)^2} \cdot \frac{1}{q} & \text{otherwise.} \end{cases}$$

This observation, that the slope of the shadow boundary is negative is key for the proof.

Characteristic Curves. Consider any pair $\{q, \lambda\}$. Then, consider two jumps, $\bar{J}^\lambda$ and $-\bar{J}^\lambda$ such that leverage remains constant. After the first jump, λ jumps to $\lambda' = \lambda + J^\lambda$. Next, observe that after these jumps, q remains the same. After the first jump:

$$q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right) = q\frac{(\lambda+1)+\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda} \equiv q'.$$

Then, after the second jump, we obtain that:

$$\begin{aligned} q' + \bar{J}^q\left(q', \lambda', -\bar{J}^\lambda\right) &= q'\frac{(\lambda'+1)-\bar{J}^\lambda}{(\lambda'+1)-q'\bar{J}^\lambda} \\ &= \left[q\frac{(\lambda+1)+\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}\right] \cdot \left[(\lambda+\bar{J}^\lambda+1) - \frac{q(\lambda+1)+q\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}\bar{J}^\lambda\right] \\ &= q. \end{aligned}$$

Consequently, for any $\{q, \lambda\}$ we define a characteristic curve as the set of points

$$\left\{q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right), \lambda + \bar{J}^\lambda\right\}$$

for any $\bar{J}^\lambda \in [-\lambda, \infty]$. A characteristic curve is a set of points for λ and q that are connected through a controlled jump in leverage. Thus, we can think of a characteristic curve as a function $q^c(\lambda; \lambda_o)$ such given a parameter q_o maps a value of leverage to a value of q—with one particular point given by $(\lambda; q_o)$.

Consider now an infinitesimal increase in leverage, $d\lambda$, then, we obtain the following limit

$$\frac{dq}{d\lambda} \equiv \lim_{\bar{J}^\lambda \to 0} \frac{J^q}{\bar{J}^\lambda} = \lim_{\bar{J}^\lambda \to 0} \frac{\left(\frac{q(\lambda+1)+q\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda} - q\right)}{\bar{J}^\lambda} = \frac{\frac{q(1-q)\bar{J}^\lambda}{(\lambda+1)+q\bar{J}^\lambda}}{\bar{J}^\lambda} = \frac{\mu_\iota^q}{\mu_\iota^\lambda}\frac{q(1-q)}{(\lambda+1)} = \frac{\mu_\iota^q}{\mu_\iota^\lambda}.$$

This is the slope of the characteristic curve at q and λ. The slope is positive and strictly so for any $q \in [0, 1]$. Since the curves of the characteristics are positive, but the slope of the shadow boundaries are negative, each characteristic crosses one point of the shadow boundary, for any q_o. Thus, for each $q^0 \in [0, 1]$ in the shadow boundary, we can associate one characteristic curve. This

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definition is important for the main result.

Main Result. We prove the following result:

Proposition 4 *[Bank's Problem] Let $\tau \neq 1$ and $\gamma = 0$. Then, for any q, given suitable parameter conditions, leverage is set to the shadow boundary $\lambda = \Lambda(q)$. Then, the equity multiplier v is a function of q that solves:*

$$0 = \left[\max_{\{c\}} f(c, v) + v_q \mu^q + (1 + v) \mu^W + \sigma \left(\left(1 + v\left(q + J^q + \bar{J}^q\right)\right)\left[1 + J^W\right] - (1 + v(q))\right)\right],$$

subject to:

$$\Lambda_q(q) = \frac{\mu^\lambda}{\mu^q}$$

For $dN = 0$ the (constant) dividend rate, c^*, is given by:

$$c^* = \rho^{1/\theta} \frac{(1 + v(q))}{\left(1 + v(q) - v_q \frac{(1+\lambda)}{\Lambda_q(q)}\right)}, \tag{57}$$

and the issuance rate $\iota^*(q)$ is such that for given $\{c^*(q), \lambda(q)\}$

$$\Lambda_q(q) = \frac{\mu^\lambda}{\mu^q}. \tag{58}$$

Thus,

$$\iota(q) = \mu^W(q) \frac{1}{1 - R(q)} - \alpha \frac{1}{1 - R(q)^{-1}}.$$

where

$$R(q) = \Lambda_q(q) \frac{q(1 - q)}{(1 + \Lambda(q))} < 0$$

Hence, for $dN = 0$, the state $\{\lambda, q\}$ moves continuously along the shadow boundary. Furthermore $d\tilde{N} = dN$. When $dN = 1$, q jumps by $J^q + \bar{J}^q(q, \bar{J}^\lambda)$ where

$$\bar{J}^q(q, \bar{J}^\lambda) \equiv q\left(\frac{(\lambda + 1) + \bar{J}^\lambda}{(\lambda + 1) + q\bar{J}^\lambda} - 1\right).$$

and λ jumps by $J^\lambda + \bar{J}^\lambda$ where $\bar{J}^\lambda$ solves:

$$\Lambda\left(q + J^q + \bar{J}^q\left(q + J^q, \bar{J}^\lambda\right)\right) = \Gamma\left(q + J^q\right) - J^\lambda + \bar{J}^\lambda.$$

The necessary parameter condition for this dynamics are:

$$\frac{\left(r^L - r^D\right)}{\sigma} \in [U(\eta), 1 + v(q)] \text{ for any } q.$$

Derivation of the Main Result. To derive the main result, we proceed through a sequence of observations. The proof follows similar steps as the proof with immediate accounting but now

considering the changes in q. We lever on the method of characteristics for the proof. Let the value function be given by some $v(\lambda, q)$, as derived earlier, for the general case with convex adjustment costs:

$$0 = \max_{\{c, \lambda\}} f(c, v) + v_\lambda \mu^\lambda + v_q \mu^q + (1 + v) \mu^W + \sigma J^v \tag{59}$$

where the jump in the value after the shock is given by:

$$
\begin{aligned}
J^V &= \left[1 + v\left(\lambda + J^\lambda, q + J^q\right)(1 - \varepsilon(\lambda + 1)) - (1 + v(\lambda, q)) \right] \mathbb{I}\left[\lambda + J^\lambda \leq \Gamma(q + J^q)\right] \\
&\quad + (v_o - (1 + v(\lambda, q))) \mathbb{I}\left[\frac{\lambda}{1 - \varepsilon(\lambda + 1)} > \frac{1}{\frac{1}{\rho} - 1} \right]
\end{aligned}
$$

subject to the boundary conditions given by (24) and (25), the laws of motion (32), (34) and the definitions of the exogenous jumps J^q and J^λ.

Conjecture: Constant Value Along Characteristics. Next we guess and verify that $v(\lambda, q)$ is constant along the characteristic curve where (q, λ) belongs too. That is:

$$v(\lambda, q) = v\left(\lambda + \bar{J}^\lambda, q + \bar{J}^q\left(q, \lambda, \bar{J}^\lambda\right)\right) \text{ for any } \bar{J}^\lambda.$$

Under this conjecture, it the value will equal $v(\lambda, q) = v(\Lambda(q_o), q_o)$ where q_o is the parameter of the characteristic curve of $\{q, \lambda\}$. Under this guess, taking total differentials with respect to λ, we obtain that:

$$v_\lambda - v_q \frac{dq}{d\lambda} = v_\lambda + v_q \frac{q(1 - q)}{\lambda + 1} = v_\lambda + v_q \frac{\mu_\iota^q}{\mu_\iota^\lambda} = 0.$$

This expression is the PDE representation of the condition that the value function is linear along the characteristic function. Thus, we obtain a relationship between the derivatives of the original value function:

$$v_\lambda = -v_q \frac{\mu_\iota^q}{\mu_\iota^\lambda}. \tag{60}$$

Verification: Constant Value Along Characteristics. Consider the optimal choice of λ. Under our guess, we have that the value does not change along the characteristic curve of $\{q, \lambda\}$. Hence, leverage can be chosen, without considering the change in the marginal value of equity $\bar{v}(\lambda, q)$ in (59). Thus, leverage solves

$$\max_{\lambda \in \left[0, \frac{\xi}{1 - \xi}\right]} (1 + v)\left(r^L - r^D\right)\lambda$$

$$+ \sigma(1 - \varepsilon(\lambda + 1)) \mathbb{I}\left[\lambda + J^\lambda \leq \Gamma\left(q + J^q\left(q, \lambda, J^\lambda\right)\right)\right] + \sigma v_o \mathbb{I}\left[\frac{\lambda}{1 - \varepsilon(\lambda + 1)} > \frac{1}{\frac{1}{\rho} - 1} \right].$$

This again is clearly a linear program. The solution is:

$$\lambda \in \begin{cases} 0 & \text{if } \left(r^L - r^D\right) < \sigma\epsilon(1 + v(q, \lambda)) \\ \Gamma(q) & \text{if } \left(r^L - r^D\right) > \sigma v_o \\ \Lambda(q) & \text{otherwise.} \end{cases} \tag{61}$$

In the paper, we consider only the interesting cases where leverage is not zero nor at the liquidation boundary. Rather, cases where leverage is set to the shadow boundary. Then, clearly when $dN = 1$, after any shock, λ must return to the shadow boundary,

$$\Lambda\left(q + J^q + \bar{J}^q\left(q + J^q, \bar{J}^\lambda\right)\right) = \Gamma\left(q + J^q\right) - J^\lambda + \bar{J}^\lambda.$$

In this case, we verify that leverage remains in the shadow boundary. Thus, for any $\{\lambda, q\}$ outside of the shadow boundary, the state variables jump to a point in the shadow boundary. Hence, under this guess, we obtain that $\bar{v}$ is constant along a characteristic curve and equal to the value at the shadow boundary,

Whenever there is no shock, it must be the case that

$$\mu^q \Lambda_q\left(q\right) = \mu^\lambda, \tag{62}$$

which guarantees that leverage and q remain at the shadow boundary.

Auxiliary Value Function. For any q, we can define a function q to its value at the point in the shadow boundary corresponding to q:

$$\tilde{v}\left(q\right) = v\left(\Lambda\left(q\right), q\right).$$

Therefore, if we differentiate this expression with respect to q we obtain:

$$\tilde{v}_q\left(q\right) = \left(v_\lambda \Lambda_q + v_q\right). \tag{63}$$

Multiplying both sides by μ^q, and using that (62) must hold along the optimal path, we obtain that:

$$\tilde{v}_q\left(q\right)\mu^q = \left(v_\lambda \Lambda_q + v_q\right)\mu^q = v_\lambda \mu^\lambda + v_q \mu^q.$$

Then, substituting v for $\tilde{v}$ in (59), using the above result, and the optimal policies for $\left\{\bar{J}^\lambda, \bar{J}^q\right\}$ we obtain an auxiliary HJB representation:

$$
\begin{aligned}
0 & = \max_{\{c\}} f\left(c, \tilde{v}\right) + \tilde{v}_q\left(q\right)\mu^q + \left(1 + \tilde{v}\right)\left(r^L + \left(r^L - r^D\right)\Lambda\left(q\right) - c - \sigma\right) \\
& \quad + \sigma\left(1 + \tilde{v}\left(q + \bar{J}^q\left(q, \Lambda\left(q\right), \bar{J}^\lambda\left(\Lambda\left(q\right)\right)\right) + J^q\right)\right)\left(1 - \varepsilon\left(\lambda + 1\right)\right).
\end{aligned}
$$

subject to:

$$\mu^q \Lambda_q\left(q\right) = \mu^\lambda.$$

Substituting the constraint:

$$
\begin{aligned}
0 & = \max_{\{c\}} f\left(c, \tilde{v}\right) + \tilde{v}_q\frac{\mu^\lambda}{\Lambda_q} + \left(1 + \tilde{v}\right)\left(r^L + \left(r^L - r^D\right)\Lambda\left(q\right) - c - \sigma\right) \\
& \quad + \sigma\left(1 + \tilde{v}\left(q + \bar{J}^q\left(q, \Lambda\left(q\right), \bar{J}^\lambda\left(\Lambda\left(q\right)\right)\right) + J^q\right)\right)\left(1 - \varepsilon\left(\lambda + 1\right)\right).
\end{aligned}
$$

Optimal dividend. We take the first-order condition with respect to dividends and obtain:

$$f_c(c, \tilde{v}) = \left(1 + \tilde{v} - \tilde{v}_q \frac{\mu_c^\lambda}{\Lambda_q}\right).$$

and therefore:

$$c^* = \rho^{1/\theta} \frac{(1 + \tilde{v}(q))}{\left(1 + \tilde{v}(q) - \tilde{v}_q(q) \frac{\mu_c^\lambda}{\Lambda_q}\right)^{-1/\theta}}.$$

Solution to the value function. In this case, using the expression for c^*, we obtain that $v(q)$ solves the equation

$$
\begin{aligned}
0 \;=\; & f\left(\rho^{1/\theta} \frac{(1 + \tilde{v}(q))}{\left(1 + \tilde{v}(q) + \tilde{v}_q(q) \frac{1 + \Lambda(q)}{\Lambda_q}\right)^{-1/\theta}}, \tilde{v}(q)\right) \\
& + (1 + \tilde{v}) \left(r^L + \left(r^L - r^D\right) \Lambda(q) - \rho^{1/\theta} \frac{(1 + \tilde{v}(q))}{\left(1 + \tilde{v}(q) + \tilde{v}_q(q) \frac{\mu_c^\lambda}{\Lambda_q}\right)^{-1/\theta}} - c\right) \\
& + \sigma\left(1 + \tilde{v}\left(q + \bar{J}^q(\lambda) + J^q\right)\right)(1 - \varepsilon(\lambda + 1)).
\end{aligned}
$$

With this value, we obtain:

$$c(q) = \frac{(1 + \tilde{v}(q))}{\left(1 + \tilde{v}(q) + \tilde{v}_q(q) \frac{1 + \Lambda(q)}{\Lambda_q}\right)^{-1/\theta}}$$

and a drift of wealth of:

$$\mu^W(q) = r^L + \left(r^L - r^D\right) \Lambda(q) - \rho^{1/\theta} (1 + \tilde{v}(q))^{1 - 1/\theta}$$

Finally, we obtain $\iota(q)$ from the condition:

$$\Lambda_q(q) = \frac{\left(\iota(q) - \mu^W(q)\right)(\lambda + 1)}{(\iota(q) + \alpha)(1 - q) q}.$$

We can obtain an interpretation for this formula using an expression for the ratio of the slope of the shadow boundary and the direction of change of the vector $\{q, \lambda\}$ along an infinitesimal issuance ι:

$$R(q) = \frac{\Lambda_q(q)}{\mu_\iota^\lambda / \mu_\iota^q} < 0.$$

With the aid of this expression, the rule for issuances is given by:

$$\iota(q) = \mu^W(q) \frac{1}{1 - R(q)} - \alpha \frac{1}{1 - R(q)^{-1}}.$$

Since the ratio of slopes $R(q)$ is negative, then issuances are increasing in the return on equity. This is because as equity increases, leverage falls so issuances can be maintained high to keep

leverage at the shadow boundary. In turn, the second terms states that the faster the accounting of past losses, the issuances have to be cutback by more.

C.6   Approximate jump in Q after an aggregate shock

Approximation to the law of motion for aggregate q.   To obtain an approximation to the aggregate behavior of q and analyze the impact of an aggregate shock, we approximate the law of motion of aggregate loans and book loans around an unconditional mean for λ and q. The approximation is exact for a representative bank. We denote by $\bar{\lambda}$, $\bar{\iota}$ and $\bar{c}$ are the population averages of λ and q, consider that every bank has approximately the same issuance rate ι and the same leverage λ. Then, the law of motion of the aggregate volume of loans, $\mathcal{L}$ will be given by:

$$\frac{d\mathcal{L}}{\mathcal{L}} = \bar{\iota} - \chi$$

where $\chi = \sigma\varepsilon$ are the the unconditional expectation of bank defaults per instant of time. Then, let $\boldsymbol{q}$ denote the aggregate version of q.

The law of motion for book loans, the law of motion would be approximately:

$$\frac{d\bar{\mathcal{L}}^c}{\bar{\mathcal{L}}} = \frac{\left(\bar{\iota} - \alpha\left(\frac{1}{\boldsymbol{q}} - 1\right)\right)(\bar{\lambda} + 1)W - \tau\chi(\bar{\lambda} + 1)W}{\frac{1}{\boldsymbol{q}}(\bar{\lambda} + 1)W} = \bar{\iota}\boldsymbol{q} - \alpha(1 - \boldsymbol{q}) - \tau\chi\boldsymbol{q}.$$

By differential of ratios:

$$d\boldsymbol{q} = \left(\frac{d\mathcal{L}}{\mathcal{L}} - \frac{d\bar{\mathcal{L}}^c}{\bar{\mathcal{L}}}\right)\boldsymbol{q} \approx (\iota + \alpha)(1 - \boldsymbol{q})\boldsymbol{q} - \chi(1 - \tau\boldsymbol{q})\boldsymbol{q}.$$

In a stationary equilibrium, $\mathbb{E}[dq]$, hence:

$$(\iota + \alpha)(1 - \boldsymbol{q}) \approx \chi(1 - \tau\boldsymbol{q})$$

and thus:

$$(\iota + \alpha - \chi) \approx (\iota + \alpha - \tau\chi)\boldsymbol{q}.$$

As a result:

$$\mathbb{E}[q] \approx \boldsymbol{q} = \frac{(\iota + \alpha - \chi)}{(\iota + \alpha - \tau\chi)} < 1.$$

Now consider an aggregate shock ϵ. We obtain an aggregate jump:

$$J^{\boldsymbol{q}} = -\epsilon\frac{(1 - \tau\boldsymbol{q})}{(1 - \tau\epsilon\boldsymbol{q})}\boldsymbol{q}.$$

The jump in

$$J^\lambda = \frac{\epsilon(\lambda + 1)}{1 - \epsilon(\lambda + 1)}.$$

Now consider the aggregate Tobin's Q, call it $\boldsymbol{Q}$. We have that:

$$\boldsymbol{Q} = s(\boldsymbol{q}, \lambda)\frac{\lambda}{\boldsymbol{q}(1 + \lambda) - \lambda}.$$

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Therefore, the jump in $\boldsymbol{Q}$ is given by:

$$d\boldsymbol{Q} = s\left(\boldsymbol{q}+J^q, \lambda+J^\lambda\right)\frac{\lambda+J^\lambda}{\left(\boldsymbol{q}+J^q\right)\left(1+\lambda+J^\lambda\right)-\lambda+J^\lambda} - s\left(\boldsymbol{q},\lambda\right)\frac{\lambda}{\boldsymbol{q}\left(1+\lambda\right)-\lambda}$$

C.7   Calibration of α

We derive the fraction of losses that are recognized in the books after T quarters for a default shock of size ε. To this end, consider a sequence of default shocks such that $dN_0 = 1$ and $dN_{t>0} = 0$. Normalize $L_0 = 1$ and set $\bar{L}_0 = 1/q_0$. Then,

$$L_t = 1 - \varepsilon \qquad t > 0$$

$$d\bar{L}_t = -\alpha\left(\bar{L}_t - L_t\right)dt \qquad t > 0$$

with $\bar{L}_0 = 1/q_0 - \tau\varepsilon$. Fundamental loans drop from the initial value 1 to the after-default value $1 - \varepsilon$ immediately. Book loans, on the other hand, only fall by $\tau\varepsilon$ on impact, and recognize the remaining fraction $(1 - \tau)\varepsilon$ at a rate governed by α and the gap in book and fundamental loans. We can guess and verify that:

$$\bar{L}_t = (1 - \varepsilon) + \left[\left(\frac{1}{q_0} - 1\right) + (1 - \tau)\varepsilon\right]e^{-\alpha t}$$

Define the fraction of the loss generated by this single default shock that is recognized by time t as the fall in book loans over the size of the loss:

$$f_t \equiv \frac{1/q_0 - \bar{L}_t}{\varepsilon}$$

Substituting in the formula for $\bar{L}_t$ and the parameters in the calibration, and starting from $q_0 = 0.977$, as in the pre-crisis mean value of q, and using a default shock of $\varepsilon = 2.5\%$ like the aggregate shock we use in the estimation of $\{\gamma, \theta\}$ for the post-crisis period, yields that after 10 quarters the fraction of losses recognized is 64.68%.

C.8   Replication of Fact 2

To demonstrate that the model delivers Fact 2, we take a straightforward approach and replicate Figure 3, using simulated data from the model. We construct variables in exactly the same way as in the original figure; we do not construct a panel relating the loan delinquency rate to the market-to-book ratio because we do not have the concept of "delinquent" loans in the model. We construct the quarters such that 2007 Q3 is the last pre-crisis quarter, and thus 2007 Q4 is the start of the post-crisis period.

Figure 13 shows the results. Astute readers will note that the magnitudes are somewhat different from the original figure. However, the model-generated figure matches the qualitative results of the original. Thus the model delivers the Fact: Tobin's Q predicts future cash flows in the cross-section, and market values capture information that book values do not.

Figure 13: Model Replication: Market equity contains more cash-flow relevant information than book equity)
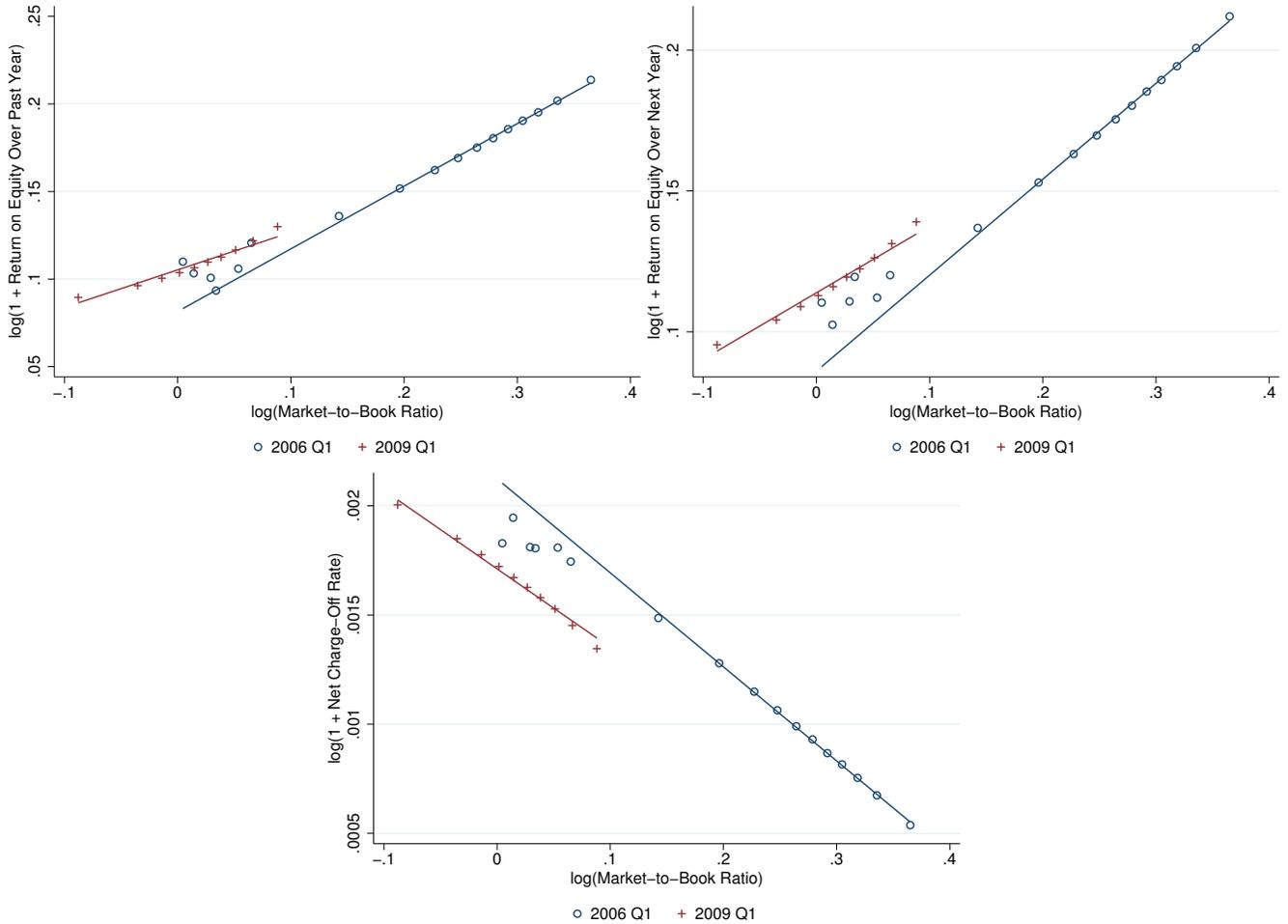
Notes: These figures show cross-sectional binned scatter plots of log outcomes on the log market-to-book equity ratio, in 2006 Q1 and 2009 Q1. All plots control for log book equity by residualizing the variables on log book equity, and then adding back the mean of each variable to maintain centering. Data are generated from 10,000 simulated banks using the model at estimated parameters. ROE over the past year is defined as book net income over the last four quarters divided by book equity four quarters ago; ROE over the next year is defined the one lead of this variable (i.e. profits over the next four quarters divided by current book equity). The net charge-off rate is loan charge-offs over the next quarter minus loan recoveries over the next quarter, divided by total loans this quarter.

D   Model Appendix: Numerical Solution

We solve the model using the finite-differences method with an upwind scheme for the choice of forward or backward differences. Specifically, we compute the numerical derivatives of the value function $v(\lambda, q)$ using finite differences and use the first order conditions to solve for policies (c, ι), and iterate on the HJB equation. A detailed description of this algorithm for a general class of models known as mean-field games can be found in Achdou et al. (2020).

Our model, which belongs to this class, is simpler to solve because we keep prices constant, but presents an added complication in that the size of the jump depends on the endogenous state variables. In particular, starting from a point (λ, q), upon receiving a Poisson shock the bank jumps to $\left(\lambda + J^\lambda, q + J^q\right) = \left(\lambda + \frac{\epsilon(\lambda+1)}{1-\epsilon(\lambda+1)}, q - \epsilon\frac{(1-\tau q)}{(1-\tau\epsilon q)}q\right)$. To avoid having to interpolate the value function, we take advantage of the fact that the size of the default shock ε is constant when constructing the grid.

We build a non-uniform grid iteratively: starting from an initial grid point (λ_0, q_0), with $\lambda_0 \approx 0$ and $q_0 = 1$, we pick the following points in the grid using the recursion $(\lambda_n, q_n) = \left(\lambda_{n-1} + J^{\lambda_{n-1}}, q_{n-1} + J^{q_{n-1}}\right)$. This way, upon receiving the default shock the bank always jumps to a point that belongs to the grid. We depart from this grid construction scheme in two regions:

1. when λ is large, the size of the jump is also large, so the grid may become too coarse. We add points to the grid wherever we have $\lambda_n - \lambda_{n-1}$ above certain threshold, by setting $\lambda_0 = (\lambda_n + \lambda_{n-1})/2$ and adding points to the grid following the previous recursion;

2. when λ is close to but below $\xi/(1 - \xi)$, or when $q < \xi\left(1 + \lambda^h\right)/\lambda^h$ for some high value λ^h, the stationary distribution features close to zero mass, and the value function features less curvature. In these two regions of the state space we use a uniform grid and interpolate $\left(\lambda + J^\lambda, q + J^q\right)$ using the closest point.

To compute the stationary distribution, we simulate the model for enough periods such that the mean and standard deviation of λ and q are approximately constant. Finally, to aggregate variables to a quarterly frequency, we set time steps $dt = 1/90$ and for every 90 time steps we use the last value for stocks and the mean for flows.