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Paula Lourdes Hernández Verme

Mónica Karina Rosales Pérez

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Paula Lourdes Hernández Verme and Mónica Karina Rosales Pérez*
paulaherver@gmail.com karina_1388@hotmail.com
Departamento de Economía y Finanzas
Universidad de Guanajuato

Abstract

There was nothing in the fundamentals of the Mexican economy that would suggest at the moment the beginning of a crisis of such magnitude. The 1994 crisis was extremely unexpected for households and domestic and foreign firms, because there were good economic indicators so far, together with the financial stability of the previous years.

The Mexico of 1994 had a *de jure* fixed exchange rate regime, but in practice, it was an intermediate peg, not a serious hard peg. Our goal is to try to find out whether things would have been different if there would have been instead either a floating exchange rate regime or a hard peg in Mexico at the time of the crisis of 1994. In our aim at trying to answer this question, we set up a Dynamic Stochastic General Equilibrium Model (DSGE) that shared the main stylized characteristics of the Mexican economy of that time. We considered a pure exchange, monetary, small open economy with a DSGE framework in discrete time that obtains from micro-foundations.

Resumen

No había nada en los fundamentos de la economía mexicana de ese momento que pudiera sugerir el inicio de una crisis de tal magnitud. La crisis de 1994 fue extremadamente inesperada tanto para los hogares como para empresas nacionales y extranjeras, porque había buenos indicadores económicos hasta ese momento y debido a la estabilidad financiera de años previos.

El México de 1994 tenía un régimen *de jure* de tipo de cambio fijo, pero, en la práctica, se trataba de un régimen cambiario intermedio, no de un régimen cambiario fijo serio.

Nuestro objetivo es tratar de averiguar si las cosas hubieran sido diferentes si hubiera habido un régimen de tipo cambiario flotante o un régimen cambiario fijo serio en México en el momento de la crisis de 1994. En nuestro objetivo a tratar de responder a esta pregunta, hemos creado un modelo de equilibrio general dinámico y estocástico (DSGE) que comparte las principales características estilizadas de la economía mexicana de la época. Se consideró una economía pequeña y abierta, monetaria y de intercambio puro, con un marco DSGE en tiempo discreto que se obtiene de micro-fundamentos.

Keywords: *exchange rate regimes, sudden stops of international capital, bank panics, dynamic stochastic general equilibrium, monetary policy, small open economy*
JEL codes: E13, E52, E58, F33, G21

* Contact author and presenter.

1. Introduction

Mexico has experienced different economic crises along its history, but the particular characteristics of the local and international environment made the 1994 Mexican crisis the worst crisis occurred in Mexico since 1930. During this crisis, known as "The December Mistake" or internationally called the "Tequila Effect", different economic, social and political problems emerged, leading the government to making some choices that were clearly not the best it could make. During the six years' administration of Carlos Salinas de Gortari (1988-1994), the Federal Government implemented a series of policies that were intended to help the Mexican economy to achieve a quick transition to Capitalism, giving substantial weight to foreign direct investment and to the (speculative) inflow of international capital. This administration also privatized the national banking industry and other government-owned firms, utilizing the proceeds from these transactions to finance mostly investment in infrastructure, with the aim of taking advantage of the benefits from the North American Free Trade Agreement (NAFTA)[1]. All these events combined produced very good economic indicators for the first part of Salinas de Gortari's administration, such as high rates of GDP growth, high stability and low inflation. According to Aguirre (2002), in the period of 1989-1993, the foreign reserves increased from USD 6,379 billion to USD 24,538 billion while the government debt decreased from USD 81,003 billion to USD 78,747 billion and the devaluation reached 35.39% since the exchange rate increased from $2.295 on 30/12/1988 to $3.1071 on 30/12/1993. In addition, the country's exports more than doubled in the same period, increasing from USD 20,545 billion to USD 51,886 billion (a 152.5% increase). Another important indicator for the period was the accumulated inflation in those five years, which was 123.37%, while by the end of 1993 the annual inflation reached the historic low level of 8.01%. All of these indicators for 1993 predicted even better outcomes to be expected for 1994. In addition, 1994 was an important year for the Mexican citizens since it was an election year.

In summary, there was nothing in the fundamentals of the Mexican economy that would suggest at this point the beginning of a crisis of such magnitude. The 1994 crisis was extremely unexpected for both domestic and foreign firms and households, because of all the good economic indicators so far together with the financial stability of the previous years.

The standard chronology of this crisis points to a series of unfortunate events that occurred during 1994 as the origin of this crisis. In particular, the social unrest by the end of 1993 led to the armed rebellion of the EZLN (Ejército Zapatista de Liberación Nacional) in Chiapas in early January of 1994 and to the assassination of presidential candidate Luis Donaldo Colosio in late March of that year. These two events are seen as the main cause of the substantial reduction in the country's foreign reserves; according to Aguirre (2002), the first hit of the crisis took the reserves from USD 26,273 billion to USD 24,649 billion. However, the reserves continued their way downward and later that year (in June), the reserves reached a historical low of USD 15,998 billion, and they continued to fall. This trend was intensified after the assassination of Jose Francisco Ruiz Massieu -who was the Secretary General of the PRI[2] on September 28th 1994. By December of 1994, when Ernesto Zedillo took charge as the Mexican President, the crisis was imminent and the Mexican debt was significantly above the country's foreign reserves.

[1] NAFTA is the trade agreement signed by the governments of Canada, Mexico and the United States, creating a trilateral trade block in North America. The agreement came into force on January 1st, 1994. The goal of the NAFTA was to eliminate barriers to trade and investment between the three countries.

[2] The acronym PRI stands for "Partido Revolucionario Institucional." It is the political party that held power in Mexico for more than 70 years, and it was also the party in power in 1994.

The debt issued by the Mexican government took mainly two forms: The Tesobonos and the CETES. Regarding the first debt instrument, the name of Tesobonos was the acronym for "Bonos de la Tesorería de la Federación" (Federal Treasury Bonds); it was a form of debt issued by the Mexican government that was denominated in foreign currency but payable in the national legal tender. With respect to the second debt instrument, CETES was the acronym for "Certificados de la Tesorería de la Federación" (Federal Treasury Certificates) and it was a debt instrument denominated in domestic currency. Aguirre (2002) commented that the debt in the form of Tesobonos reached the level of USD 18,384 billion of dollars, while the foreign reserves reached rock bottom at USD 12,470 billion. By December 19th of that year, President Zedillo decided to devaluate the Mexican peso by 15%, from $3.5 to $4.03 per dollar. According to the INEGI[3], at the end of 1994 Mexico owed as well 39,701 million pesos in CETES, while the foreign reserves continued to fall until they reached only USD 6,148.2 billion.

This uncertain environment influenced adversely the expectations that both domestic and foreign investors had regarding the performance of the Mexican economy; among other things, the nominal exchange rate increased in 80% between December 20th 1994 and February 13th 1995. This explained in part why the foreign investors were fearful of investing good money in such an unstable country, all of which led to a sudden stop of international capital. The Mexican government tried to roll over some of its debts, but the foreign investors were unwilling to purchase new debt from Mexico, but especially from the Mexican government, given all the unfavorable events. This sudden stop of capital, plus the devaluation of the Mexican peso were the origin of the terrible crisis that affected millions of Mexican households and firms.

From 1994 to 1995, the Mexican economy experienced a decrease in its GDP; according to the CEFP[4], this reduction was of the order of 6.2%. This contraction of the economic activity represented, in accordance with INEGI, an increase in the unemployment rate from 3.2% to 6.6%, raise that was equivalent to 2,310 thousands of newly unemployed persons. This increase in unemployment was accompanied by an increase in the indicators of poverty: the number of poor people increased to 44 million of persons, almost half of the Mexican population in 1995. Moreover, the Bank of Mexico's Annual Report for 1995 established that during that year the aggregate consumption fell in 11.7%, while the total spending -measured by the aggregate demand- fell in 10.2%. A good approximate indicator of the magnitude of the sudden stop experienced by the Mexican economy was the significant reduction in the share of the foreign direct investment on the GDP: it was reduced from 7.8% of the GDP in 1994 to 0.3% in 1995. Moreover, the balance of payments had a deficit of the order of USD 654 million, thus increasing Mexico's debt with the rest of the world. Those among the Mexican people who had issued debt denominated in USD or had acquired loans from the banking sector were adversely affected by the devaluation of the Mexican peso: according to the statistics of the CEFP, the interest rates increased abruptly to 89.48% in March of 1995 against an annualized inflation rate of 20.4%.

The impact of this crisis is told in part by the amount of international help needed by the Mexican economy. According to Banda and Chacon (2005), Mexico received USD 17,800 billion from the International Monetary Fund, USD 2,000 billion from the World Bank, USD 500 billion from the United States and USD 83 billion from Canada, among others.

[3] The INEGI is an institution that is independent from the Mexican government. It is in charge of elaborating the official statistics in Mexico, and of making surveys to the population every ten years. These surveys provide information about the GDP, unemployment, investment, consumption, etc.

[4] CEFP is the acronym for "Centro de Estudios de las Finanzas Públicas" (Center for the Study of Public Finance), that is managed by the Chamber of Deputies of Mexico.

Another characteristic of the crisis of 1994 is that it also spread from Mexico to other Latin American countries, thus hurting the expectations that foreign investors had about them as well. An example was the case of Chile; Marhur, Gleason, Dibooglu and Singh (2002) found that the Mexican crisis affected the Chilean stock market, since Chile and Mexico had a bilateral trade agreement from 1991 to 1994.

We focused our study on the events surrounding this crisis in Mexico because of all the negative effects and repercussions it brought not only to Mexico but to other countries as well; as we mentioned earlier, there were high levels of unemployment which were coupled with reductions in consumption and in savings that contributed to deteriorate the quality of life of the Mexican people.

The Mexico of 1994 had a de jure fixed exchange rate regime, but in practice, it was an intermediate peg, not a serious hard peg. Our goal is to try to find out whether things would have been different if there would have been a floating exchange rate regime in place in Mexico at the time of the crisis of 1994 (instead of the fixed exchange rate regime that was actually in place). In our aim at trying to answer this question, we set up a Dynamic Stochastic General Equilibrium Model (DSGE) that shared the main stylized characteristics of the Mexican economy of that time. We followed most of the features of the model by Diamond and Dybvig (1983), since they model banks that can arise endogenously as transformers of maturities and liquidity providers; but we departed from their model significantly, since we introduced two important and nontrivial features: we introduced exchange rate regimes into the environment together with the fact that the domestic country was a small open economy. We chose to model a small open economy because of two reasons: Firstly, Mexico was classified (and still is) as a small open economy and an emerging country; secondly, there was empirical evidence showing that it was the emerging countries the ones that suffer the most in the event of a sudden stop of capital, since their counterparties in more developed countries can always borrow from their respective counterparties. Given the chosen focus on an open economy, we decided to take from the work by Chang and Velasco (2001) as well. However, a main distinction between our work and theirs is that we do take seriously the fiat component of both the domestic and foreign currency, and we do not introduce fiat money in the utility function as these authors did; we introduced instead nontrivial demands for a domestic and a foreign fiat currency that had no intrinsic value; both currencies may have circulated together and compete with each other. In addition, we modified accordingly the design of the deposit contracts offered by domestic banks to households: our main goal in this respect was to obtain more flexible contracts that would allow us to analyze the potentially negative effects that a sudden stop would have had on this economy.

We must also highlight that, from a technical standpoint, we have introduced some improvements with respect to the work by Diamond and Dybvig (1983) and Chang and Velasco (2001) in at least two ways. In the first place, we embedded the main features of the framework worked by these authors into a truly dynamic framework, in an economy that has an infinite horizon but where agents have finite lives. The second improvement relates with the setting up of a re-optimization algorithm, which is the way in which domestic banks adjust their contingent plans in the presence of unanticipated shocks.

We started our analysis of the problem at hand by focusing on a pure exchange economy based upon the micro-foundations of what would become the general equilibrium model of this economy: we modeled the behavior of the individual domestic households and banks in a very particular environment, where banks could be thought of as coalitions of households that may arise endogenously when it is in the best interest of the domestic households to do so: that is, when their expected utility is maximized. Next, we introduced alternative exchange rate regimes that in each case were coupled with a simple monetary policy rule; we considered two very distinct regimes: floating versus fixed exchange rates. On the one hand, we modeled an economy with floating exchange rates; this policy was coupled with an independent monetary policy that consisted in a constant rate of growth of the money supply. Thus, in the case where the nominal exchange rate was market-determined, the money supply was chosen exogenously by the monetary authority and there was a well-defined monetary rule in place. On the other hand, we also investigated the effects of a policy of fixed exchange rates, where the monetary authority set exogenously the nominal exchange rate; it is common knowledge that a policy of fixing the nominal exchange rate comes also with a cost: the central bank must give up an independent monetary policy, since it has to stand ready to buy or sell foreign exchange in exchange for domestic currency at the fixed price. As a consequence, the money supply is determined endogenously and the monetary authority gives up a valuable policy tool.

The overlapping generations of households in the domestic economy live for only two periods, but the economy itself has an infinite horizon. The latter allowed us to solve the model in the structural form by means of obtaining the equilibrium of this open economy in the form of a nontrivial dynamic system. We analyzed both the stationary and the transitional dynamic equilibria represented by the reduced-form dynamic equations that solved the general equilibrium for this model economy. We must draw attention to the fact that we worked on each exchange rate regime independently from one another, with the goal of comparing the contingent consumption bundles that arose in equilibrium under each of the regimes considered. Our goal was to compare the relative merits of these two different exchange rate regimes and decide whether a policy of floating exchange rates would have worked better for a model economy that replicated the main features that the Mexican economy had by the end of 1994. There were five stylized facts that we believe the Mexican economy of 1994 shared with the East-Asian countries of 1997-98. We list these characteristics below:

1) Increased risky-lending behavior by domestic banks.
2) The lack of sound financial structure worsened with the ill-oriented process of financial and capital liberalization.
3) The banks' financial assets constituted the majority of their total assets –instead, for instance, of financing in capital markets.
4) Borrowing from foreign banks was a significant portion of the domestic debt of the private sector.
5) The majority of these countries had intermediate pegs in place.

We have confidence that our model captures all five stylized facts. As we mentioned earlier, we considered a pure exchange, monetary, small open economy with a DSGE framework in discrete time that obtains from micro-foundations. *Ex-ante* identical domestic agents face an uncertainty whose realization is private information. There is a standard problem in coordination that may lead to strategic complementarities and crises of a self-fulfilling type, with a potential for multiple equilibria that may or may not be realized. There is free international mobility of financial capital without any legal restriction on the holdings of foreign currency other than the reserve requirements mentioned earlier, ensuring that the domestic country inherited and took as given the prices and interest rates that were determined in the appropriate world markets. The foreign debt instruments considered may display different maturities and maturity-dates at fixed world interest rates. Moreover, in this economy the private sector is a net debtor of the rest of the world and there is an exogenous upper limit to the amount of long-term debt lending from foreign banks at each point in time. The latter permitted us to obtain equilibrium allocations that were not only determinate but also globally unique.

With the set-up we just described we were able to analyze separately the effects that sudden stops and bank runs might have on the aggregate economy, as well as how these problems relate to each other. As in Hernández-Verme and Wang (2009), domestic investment is subject to multiple reserve requirements that take the form of currency reserves; however, in our model the monetary authority chooses exogenously to pay an interest on these currency reserves. Moreover, the monetary authority chooses to back a fraction of the domestic money supply by holding interest-bearing foreign-reserve assets. We model two such economies that are similar in every respect, except for their choice of exchange rate regimes.

We must mention that the dynamic system that resulted from a regime of floating exchange rates displayed a very convoluted dynamic behavior. In particular, the equilibrium system that results from this regime was a decoupled dynamic system where both the short-term foreign debt position of domestic banks and the contingent consumption bundles inherited their dynamics from what we call a core dynamical sub-system. On the one hand, the core sub-system consisted of five first order difference equations that determined five state variables. A striking characteristic of these state variables was that they were determined independently of the foreign interest rates, thus adding to their determinacy. On the other hand, the caused dynamic sub-system of foreign debt and contingent consumptions consisted each of a nonlinear and second order difference equation, with a potential for very complicated transitional dynamics that may display endogenously-arising volatility.

Now we turn to the small open economy that operates with fixed exchange rates. The resulting dynamic system was also decoupled, with the same structure of causality than the one arising from floating exchange rates. However, since this economy inherited the world inflation rate (which we assumed was exogenous and constant,) the dynamic behavior displayed by the main economic variables of this economy was significantly simpler: all the equations of both dynamic sub-systems were linear and of first order. Moreover, the equilibrium dynamical paths did not reveal any endogenously-arising volatility, and they approached the steady-state equilibria monotonically.

Our analysis proceeded in two stages. First, we solved for the full-information benchmark equilibrium, in the absence of early liquidation, sudden stops or bank panics. Secondly, because we wanted to replicate the main cause of the 1994 Mexican crisis, we introduced the possibility of a sudden stop of capital and we analyzed the consequences of this unexpected event under the two alternative exchange rates regimes mentioned above. In the environment we modeled, we used a very simple definition of a sudden stop: a sudden stop is a cause of intrinsic uncertainty that originates in an unanticipated shock that reduces significantly the inflows of international capital with respect to what would be promised or expected in the standard contingent contracts. The big idea behind all this is that a sudden stop could affect, and possibly even reverse the structure and direction of the international flow of international capital that are commonly associated with this model economy. And the latter could have severe consequences for the economy as a whole. Furthermore, we establish that bank runs in our economy can occur if and only if a bank becomes illiquid and insolvent.

Thus, in the second stage of our inquiry, we analyzed the consequences of having several alternative combinations of the fundamental and policy parameters for these two economies. We worked with domestic banks that formulated contingent plans regarding their choice variables. In particular, a key element of the banks' contingent plan was their need to borrow the quantity of short-term debt that maximized the utility of their depositors under full information. This choice is what domestic banks expected from foreign banks in the absence of a crisis. We find that the amount of short-term debt that is part of banks' contingent contracts is larger for the economy that operates under fixed exchange rates than for floating exchange rate in the event of a sudden stop of capital.

Our results suggest that the fractions of deposits that the domestic banks must hold in form of currency reserves played a key role in most scenarios, regardless of the exchange rate policy in place. Conventional wisdom suggested that the Mexican crisis could be attributed to the decrease in foreign reserves experienced in 1994. We took this premise to be true and we run the thought experiment suggested by it, so that we could compare the consequences for each exchange rate regime. On the one hand, we found that under floating exchange rates, some combinations of parameters would cause a bank run in the case a sudden stop hit the domestic country; and, as one might expect, the domestic banks subsequently would suffer from illiquidity and insolvency. Regarding the economy that operated with fixed exchange rates - which was the case of the Mexican economy in 1994-, our analysis suggested as well that, for some combinations of deep and policy parameters, the sudden stop of capital would cause a banking crisis, with the associated suspension of convertibility. It would appear that it is more likely for a sudden stop to have caused a bank run when a fixed exchange rate regime is in place[5].

We found in the existing literature that crises arising in emerging countries, like Mexico, have been attributed to different causes. For example, authors like Aysun and Honig (2011) have found a relationship between the quality of institutions and the possibility of a crisis; they pointed out that a country with a weak institutional framework is more prone to crisis. More recently, Benmelech and Dvir (2011) tried to find out if there was a clear-cut relationship between the short-term debt held by the private sector and how vulnerable the economy as a whole was to financial crises. They explored two channels that could make it so: either the short-term debt increased a country's vulnerability to financial crises or the amount of short-term debt is an effect -rather than a cause- of a forthcoming crisis in emerging countries. They concluded that the short-term debt did not cause crises; they found instead that the short-term debt was more likely a reflection of the weak financial system of the country.

There is also a group of studies that have focused on the potential consequences of that a crisis could have on the economy as a whole. Among them, we found the work by Cooper and Ross (1998) and Ennis and Keister (2006). They studied how banks could adjust the deposit contracts they offered to households in the case a bank run occurred in equilibrium, together with the implied consequences of this change in their decisions. They concluded that it was evident that the probability of a run would affect the level of investment made by banks and that banks would formulate their deposit contracts based upon this level. Thus, they concluded that deposit contracts played a key role, since contracts that were not properly designed could leave banks vulnerable to runs. Following in this line of study we have the work by Furceri and Zdzienicka (2011), who studied the effects that debt crises could have on the Gross Domestic Product. They concluded that, in the case of a sudden stop (which they also call a debt crisis), the reduction that it would cause on the rate of economic growth would be larger in countries where the incidence of public debt is higher. Moreover, they draw attention to the fact that the negative impact that a debt crisis might have on growth would be larger than the effect caused by an increase in the debt of the public sector. Closer to our approach, Edwards (2007) studied the influence that capital inflows could have on external crises; he concluded that there was not sufficient evidence to suggest that high levels of capital mobility could definitely increase the probability of a crisis.

[5] This is an inference, not necessarily a result of the model.

Regarding some of the previous work that has focused on the study of the Mexican crisis, we found that Cole and Kehoe (1996) attributed the Mexican crisis to a self-fulfilling-debt crisis; in particular, they concluded that the main factor causing a crisis was the short maturity of Mexico's domestic debt. Concurrently, the work by Sachs, Tornell and Velasco (1996) tried to explain that the Mexican crisis was not the consequence of an overvalued exchange rate or the deficit in the balance of payments experienced by Mexico at the time; instead, they attributed the Mexican crisis to a self-fulfilling panic that was exacerbated by the fixed exchange rate regime in place. However, they also pointed out the caveats of their analysis: as any theoretical model built to improve our understanding of a particular event, they argued that the model they used to evaluate the impact of a debt crisis appears to have some limitations, and that their results should not be taken lightly.

We found a long list of events that could have caused the crises that were experienced by emerging countries; among them we have the following: the short maturity of their debt, the poor quality of their institutions, their degree of international capital mobility, the deficits in their balance of payments, a bad combination between exchange rate regimes and monetary policies, a pessimistic shift in households' expectations, etcetera.

We consider that our contribution to the existing literature does not lie in explaining what caused the crisis of 1994; instead, we explored the consequences that a sudden stop of capital could cause. We found that the underlying exchange rate regime and the monetary policy in place were central to understanding this phenomenon. However, the direction of the effects did not have opposite effects depending on the regime: instead, the exchange rate regimes explain the different incidence of a crisis, not its direction. We have restricted our attention to the line of thought that established that crises in emerging countries arose due to their short-term debt, either because of their short maturity or because of the size of their debt position. In particular, we have focused as well on how a change in the amount of their debt could have increased the likelihood of a run.

The remainder of this thesis proceeds as follows: in Section 2, we focus on what constitutes a "sudden stop." Next, in Section 3, we describe the main features of the small open economy that operates under floating exchange rates. In Section, we present our economy but under fixed exchange rates. Subsequently, in Section 5, we examine the particular consequences that a sudden stop could have on our model economy, making a distinction that was based upon the policy in place. Finally, in Section 6 we present the main conclusions of this work.

2. What is a "sudden stop"

According to the standard chronology, the term "sudden stop" was used first by Dornbusch and Werner (1994) in the context that followed the 1994 Mexican crises, but it was Dornbusch, Goldfajn and Valdes (1995) who cited the following bankers' adage: "It is not the speed that kills, it is the sudden stop." (p. 219). The term was popularized later on by Guillermo Calvo and his associates. Edwards (2007) (p. 86) provides us with a nice definition of what constitutes a sudden stop in practice:

> … a "sudden stop" episode as an abrupt and major reduction in capital inflows to a country that up to that time had been receiving large volumes of foreign capital. More specifically, I imposed the following requirements for an episode to qualify as a sudden stop: (a) the country in question must have received an inflow of capital (relative to gross domestic product [GDP]) larger than its region's third quartile during the two years prior to the sudden stop; (b) net capital inflows must have declined by at least 5 percent of GDP in one year.

3. The economy under floating exchange rates

In this section, we build a theoretical model of an economy that shares certain stylized structural characteristics that the Mexican economy displayed at the time of the so-called "Tequila crisis" of 1994. The goal of this section is a counterfactual exercise: we try to learn if things would have worked out differently if there had been a floating exchange rate regime in place in Mexico at the time of the crisis of 1994, instead of the intermediate peg that was actually in place. We would like to highlight the seminal work by Fischer (2001) and Bubula and Otker-Robe (2003) in this respect, who support what has been called the "bipolar" view of exchange rates and, in particular, Bubula and Otker-Robe (2003) (p. 3) summed up the adverse consequences of having an intermediate peg in place, especially for economies that are open to international capital mobility:

> Not all rigid exchange rate regimes are equally susceptible to crises, however; some rigid rates have lasted for decades or even centuries (Schuler, 1999). However, intermediate regimes between hard pegs and floating rates—that is, soft peg regimes and tightly-managed floats— has been at center stage in most major crises in recent years. In this light, there has been growing support for the view that such regimes will not be viable for any lengthy period of time, particularly for countries highly integrated with international capital markets—the so called bipolar view of exchange rate regimes. The viability of soft peg regimes in particular has been questioned: in many instances, soft pegs broke down as the authorities directed monetary policy toward domestic goals in an environment with high capital mobility. The proponents of the bipolar view hence argued that pegs could not be maintained under high capital mobility unless the country makes an irrevocable commitment to the peg (as in hard peg regimes) and is prepared to support it with necessary policies and institutions.

Our approach is very simple but complete. In particular, we abstract from the government's debt of any kind and from any explicit fiscal policy, while we focus on the consequences of a policy of floating exchange rates that is accompanied by a very simple monetary rule: the domestic money supply is exogenous and it grows at the constant rate $\sigma \geq 0$ every period. Something very important is that the private banking sector in this economy is a net debtor with respect to the rest of the world.

In this economy, households will live for only two periods, but the economy will go on after their death: we assume that the economy as a whole has an infinite horizon. There will be two sources of uncertainty. In the first place, and this is inherent to any model economy based on the spirit of Diamond and Dybvig (1983), there is a shock that will be realized at the same time every period, before the end of the households' youth: this shock is supposed to alter each of the households' preferences in terms of when they wish to consume; when the domestic households are born, they are aware that such a shock will hit them, one way or another: they know its stationary distribution and that it is i.i.d. across households and time, but its actual realization is private to each household and it cannot be observed directly, even when incurring in a monitoring cost. Regarding the second source of uncertainty, it is what we call intrinsic uncertainty since it will modify the fundamentals of this economy such as they are perceived by the public (as opposed to extrinsic uncertainty, which alters the beliefs of the public but not the fundamentals); however, this shock cannot be anticipated either by domestic or foreign entities, implying that households and banks will not be able to protect themselves from it. It is worth to highlight this difference: it is not only that they cannot know what will be its particular realization beforehand; what happens, instead, is that they do not know of its distribution nor can they even fathom the possibility that such an event could even affect the economy, which is why it is appropriate to use the term "sudden" stop of capital when referring to it.

The monetary policy in place seeks to regulate and guide the activities of the depositary institutions in this economy, which take the form of private financial intermediaries, in charge of canalizing loanable funds. The loanable funds in this economy can come from two possible sources: in the first place, there are the deposits made by domestic households in the domestic banking sector, while the second source contemplates the foreign borrowing from banks in the rest of the world, which we assume can be done only through financial institutions.

It is well known that models of monetary economies where a fiat currency circulates as the legal tender tend to be irregular economies, where there is typically a continuum of equilibria that are not locally unique. The indeterminacy of equilibria poses a problem for making predictions and forecasts for these economies, since for a given set of fundamentals there can be an infinite number of equilibria that are consistent with different beliefs of the public. This problem can grow to be even more complicated when the economy under study trades with the rest of the world, either in goods or in currencies, or both. The indeterminacy of the nominal exchange rate is a popular result for economies where no legal restrictions on transactions involving foreign exchange do exist. This fact has been studied almost exhaustively by the seminal work by Neil Wallace (1979) by John Kareken and Neil Wallace (1981), and more recently by Roger Farmer (2002) (p. 65), among others. Farmer (2002) explains very nicely what is behind the concept of indeterminacy:

> … I will argue that the concept of indeterminacy is important because the existence of indeterminate equilibria is associated with the possibility that beliefs can influence outcomes. To understand how agents behaved in a world of indeterminate equilibria, we must pin down their beliefs. If we are prepared to grant, to beliefs, the same methodological status that one usually reserves for preferences and endowments, there is a simple solution to the problem of indeterminacy. Beliefs pick the equilibrium, and each possible belief is associated with a different possible realization of the observable variables of the model.

There is much of an agreement that to avoid indeterminacy one has to introduce some type of institutional restrictions which should regulate the trading of currencies and will help to pin down the equilibria for this economy. In particular, according to Wallace (1979):

> … Without binding legal restrictions on asset holdings that prevent one currency from being substituted for another either directly or indirectly via international borrowing and lending, demands for different currencies are determined not in part but speculation, but entirely by speculation. One consequence is an indeterminacy proposition: *Without government intervention in foreign exchange markets and without binding restrictions on currency holdings, exchange rates, price levels, and in general all prices are indeterminate*. (p. 1, author's italics)

Following in this spirit, we introduced such a legal restriction into our model economy, and it took the form of binding reserve requirements: a minimum fraction of the deposits in the banking system must be kept in the form of domestic currency with the Central Bank, while another fixed fraction of the deposits must be kept in the form of foreign currency. In the case where both reserve requirements bind[6] one can obtain demands for domestic and foreign real money balances that are determinate, thus simplifying matters enormously. The particular form

[6] The reserve requirements bind when both currencies are dominated by deposits in real rate of return.

of this restriction will be explained in detail later on, together with the monetary and exchange rate policy, but we wanted to give a complete overview to the reader.

An interested reader may argue that the characteristics we describe for this economy are shared as well by other small open economies, such as the East-Asian countries during the 1997 crisis. This is so, and it only reinforces the point that we are trying to make, which is that such economies are financially fragile and mostly depending on expected inflows of capital.

3.1 The environment

Consider an open, small economy. There is a single endowment good that can be consumed in each period, but goods cannot be produced. The good is homogenous across time and countries. The population is constant and conformed of infinite overlapping generations; the households in each generation live for two periods. Time is discrete and indexed by $t = 0,1,2\ldots$

There are two groups of domestic participants: households and domestic banks. There is a continuum of households with unit mass, who are also the only depositors in the domestic country. Domestic banks may arise endogenously as coalitions of young households. There is also a central monetary authority in the domestic economy (the equivalent of a Central Bank) that behaves as a centralized clearinghouse and is also in charge of the domestic monetary and exchange rate policy. Finally, there is a group of foreign banks whose behavior is taken as given by the domestic agents: that is, we do not model their strategic behavior, we only take their decisions as given; they lend resources in fixed and exogenous amounts directly to domestic banks at different points in time and they charge the world gross real interest rates for each corresponding date and maturity. As a result, the private sector in this economy will be a net debtor to the rest of the world, and the domestic banks will the only ones holding these foreign debt instruments and responsible to make the appropriate payments when due.

3.2 The monetary policy under floating exchange rates

There are two fiat currencies that may circulate simultaneously in this economy: the domestic currency and a foreign currency. The variable $\hat{\vartheta}_t$ represents the value that the variable ϑ takes at t, when the exchange rate is left to float. On the one hand, the domestic legal tender is called the "peso"; $\hat{M}_t$ is the aggregate nominal stock of pesos issued by the domestic monetary authority at t, and $\hat{p}_t$ represents the domestic price level, measured as pesos per unit of the good at t. The domestic currency yields the gross real return $\left(\hat{p}_t/\hat{p}_{t+1}\right) > 0$ when held between t and $t+1$.

On the other hand, the foreign currency is an international means of payment that we will call the "dollar". The variable $\hat{Q}_t$ denotes the aggregate nominal stock of foreign currency circulating in the domestic country at t, while the exogenous world price level p_t^* is measured as dollars per unit of the good at t. Meanwhile, the real return of holding dollars between t and $t+1$ is given by $\left(p_t^*/p_{t+1}^*\right) = \left(1+\sigma^*\right)^{-1} > 0$, where $\sigma^* > 0$ is the constant and exogenous net inflation rate in the rest of the world.

We assume that there are no legal restrictions to the holdings of foreign currency in the domestic country other than a reserve requirement. Moreover, there are no legal restrictions either on the international trade of goods nor on the international flow of financial capital; this, together with the fact that the goods are homogeneous across countries, will imply that the Law of One Price will be satisfied at all times. Let the variable $\hat{e}_t$ denote the nominal exchange rate between pesos and dollars at t; under a policy of floating exchange rates, this price clears the market for foreign currency exchanged for pesos, ensuring that the net excess demand for foreign currency is zero. Regarding its units, $\hat{e}_t$ is measured as the amount of pesos that one must give up in order to obtain one dollar at $t \geq 0$. We can write down the Law of One Price as

$$\hat{e}_t \cdot p_t^* = \hat{p}_t, \tag{1}$$

which is satisfied at all times.

There is a monetary rule in place whereby the evolution of the nominal supply of pesos is given by:

$$\hat{M}_{t+1} = (1+\sigma) \cdot \hat{M}_t, \tag{2}$$

where $\sigma > 0$ is the rate of domestic money growth, which is chosen exogenously by the domestic monetary authority. Also, the monetary authority must hold $\hat{B}_t^*$ units of interest-bearing, dollar-denominated assets at t, which are destined to back the dollar-value of the domestic money supply in case unanticipated events arise; we assume that the gross real interest rate associated with these assets is constant over time, and it is denoted by $\varepsilon > 1$, which is determined in the appropriate world markets and taken as given by all domestic agents. The Central Bank's reserves in dollar-denominated assets must satisfy the following condition:

$$\hat{B}_t^* = \theta \cdot \left(\frac{\hat{M}_t}{\hat{e}_t} \right), \tag{3}$$

where $\theta \in [0,1]$ is an exogenous policy tool that represents the fraction of the dollar-value of the supply of pesos that is backed by the monetary authority.

There is a legal restriction in the financial system that requires of all commercial banks that they hold a minimum fraction of their deposits in the form of currency reserves in the vaults of the Central Bank. In particular, the domestic banks need to set apart at least a fraction $\phi_d \in (0,1)$ of their deposits and keep them with the Central Bank in the form of domestic currency reserves in pesos. Banks must do the same in terms of the foreign currency: they must keep at least the fraction $\phi_f \in (0,1)$ in the form of dollar reserves that are also kept with the Central Bank. We assume that all currencies are dominated in rate of return by domestic deposits, which implies that all money holdings in the economy are intermediated and that the real demands for the different currencies are determinate. In particular, $(\hat{m}_t/\hat{p}_t) \geq \phi_d \cdot (w + \hat{\tau}_t)$ represents the real holdings of domestic currency held by banks, while $\hat{q}_t \geq \phi_f \cdot (w + \hat{\tau}_t)$ are their real holdings of foreign currency; both expressions hold with equality when each of the reserve requirements bind. These reserves are held with the monetary authority in its vaults, and they cannot be accessed until the end of $t+1$. The return on these reserves is the same as the real return for the fiat currencies, i.e., $(\hat{p}_t/\hat{p}_{t+1})$ for the reserves held in pesos and (p_t^*/p_{t+1}^*) for the reserves denominated in dollars. Each of these reserve requirements will bind whenever the currency reserves are dominated in rate of return by the long-term investment. This obtains whenever $(\hat{p}_t/\hat{p}_{t+1}) < R$ and $(p_t^*/p_{t+1}^*) < R$ arise in equilibrium, where $R > 1$ denotes the long-term real return that the domestic banks obtain when they invest in their illiquid investment technology; the latter is the condition that is needed for the real demands for currencies to be determinate. We will assume that this is the case throughout the paper.

We must highlight that the domestic central bank pays an additional interest on these reserves, so that the effective return on them is $\rho \cdot \left(\hat{p}_t / \hat{p}_{t+1} \right)$ in the case of the domestic currency reserves and $\rho \cdot \left(p_t^* / p_{t+1}^* \right)$, respectively, for the foreign currency reserves. Moreover, we assume that $\rho \geq 1$ is an exogenous policy parameter chosen by the domestic monetary authority, representing the premium paid over the currency reserves by the central bank.

Since both reserve requirements bind, we can pin down the demands for real money balances. We denote the per capita real balances of pesos by

$$\left(\frac{\hat{m}_t}{\hat{p}_t} \right) = \phi_d \cdot \left(w + \hat{\tau}_t \right). \tag{4.1}$$

Meanwhile, the per capita real holdings of dollar-denominated assets are defined as follows

$$\hat{b}_t^* \equiv \left(\frac{\hat{B}_t^*}{p_t^*} \right). \tag{4.2}$$

The real demand for foreign currency in equilibrium will be represented by

$$\hat{q}_t \equiv \frac{\hat{Q}_t}{p_t^*}. \tag{4.3}$$

The budget constraint in per capita terms that is faced by the monetary authority at the end of t is given by:

$$\hat{\tau}_t = \frac{\hat{M}_t - \hat{M}_{t-1}}{\hat{p}_t} - \frac{\hat{B}_t^* - \hat{B}_{t-1}^* \cdot \varepsilon \cdot \left(\frac{p_t^*}{p_{t+1}^*} \right)}{p_t^*}. \tag{5}$$

The right hand side of equation (5) has two terms, and each one represents a different activity by the Central Bank. The first term represents the real value of any changes in the nominal supply of domestic currency, as a result of new money creation (seigniorage) by the Central Bank, while the second term indicates the effects on the government finances of any real changes in the Central Bank's reserves position of dollar-denominated assets. Next, we define $\hat{z}_t \equiv \left(\hat{M}_t / \hat{p}_t \right) \geq 0$ as the real per capita supply of the domestic fiat currency. By combining (1), (2), (3) and (5), we can obtain the per capita budget of the monetary authority in terms of the real money balances:

$$\hat{\tau}_t = \hat{z}_t \cdot \left(\frac{\sigma}{1+\sigma} - \theta \right) + \hat{z}_{t-1} \cdot \varepsilon \cdot \theta. \tag{6}$$

As we can see, the expression in (6) is a first order linear difference equation in $\hat{z}_t$ representing the evolution of the per capita monetary transfer over time; later, we will analyze this equation in more detail.

3.3 The households' behavior under floating

A typical household is born at the beginning of period t, and we say that she is young during her first period of life. During her second and last period of life, at $t+1$, she is called old. All households, regardless of the generation they belong to, are ex-ante identical; however, a series of i.i.d. shocks with a stationary probability distribution may alter each household's preferences so that she may become impatient –with probability λ- or patient –with probability $(1-\lambda)$- at the end of her youth. The parameter $0<\lambda<1$ is exogenous and part of the public information available to households. Since there is a continuum of agents, the Law of Large Numbers applies and λ also represents the fraction of households that become impatient in a given generation, while the remainder fraction $(1-\lambda)$ of households becomes patient. As we have mentioned earlier, the probability distribution of the shock is public information, but a particular realization will be private to each young agent by the end of her youth. Agents can learn information only from observing market transactions; there is no monitoring they can do that will avail them the information.

For the analysis that follows it will be very helpful to split each period into two: morning and afternoon. In the morning of t, the young households receive (exogenously) $w>0$ units of the endowment good and a real lump-sum monetary transfer equivalent to $\hat{\tau}_t$ units of the single good from the monetary authority[7]. Next, they may choose to go to a bank and deposit the total of their $(w+\hat{\tau}_t)$ income inelastically, or to remain in autarky, depending on what they perceive as more profitable for them. When the shock alters their preferences, if they become impatient, households wish to consume $\hat{c}_{1,t}>0$ units of the good only when young and nothing when old, so they may be in need of liquidating early whatever assets they have accumulated so far. However, if they become patient, they derive utility only from consuming $\hat{c}_{2,+1}>0$ units of the good when old at $t+1$. Then, we can represent the households' preferences by the following expected utility function

$$E\left[u\left(\hat{c}_{1,t},\hat{c}_{2,t+1}\right)\right]=\lambda\cdot\ln\left(\hat{c}_{1,t}\right)+(1-\lambda)\cdot\ln\left(\hat{c}_{2,t+1}\right). \quad (7)$$

Households and domestic banks can access, indistinctly, a long-term illiquid storage technology that yields the exogenous gross real return $R>1$ at the end of $t+1$ for each unit of the good stored at t, if they can wait (i.e. if they become patient). However, it could also be the case that households became impatient instead, and they would be in need of consuming early, for which they may need to liquidate this investment early. However, early liquidation is costly: they would obtain only the exogenous gross real return $0<x<1<R$ at the end of t per each unit of good stored.

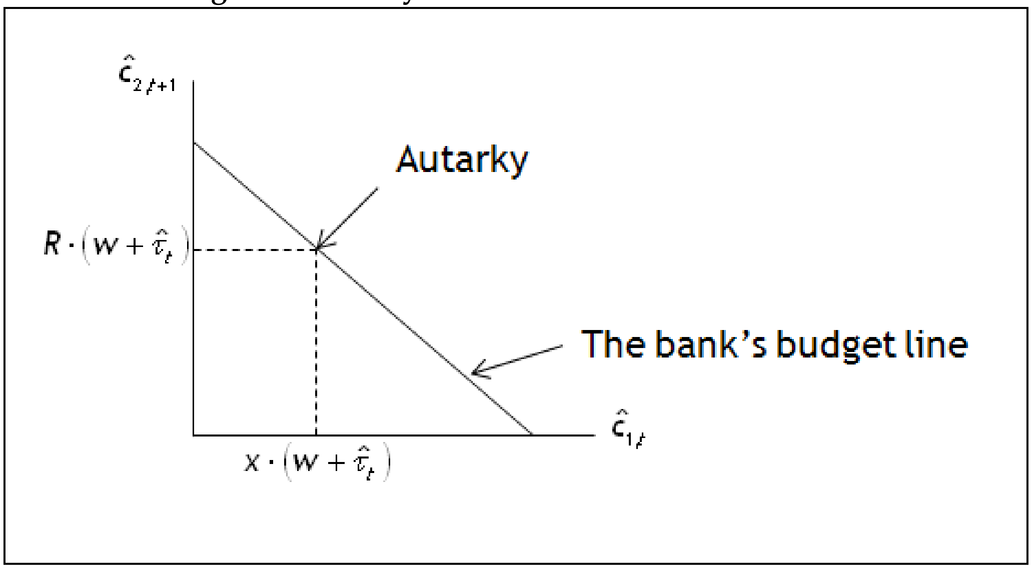
3.3.1 The importance of financial intermediation

Domestic banks can pool together the total of the economy's resources, given by $(w+\hat{\tau}_t)$ goods in the morning of t, thus allowing households to diversify risk and get insurance at the same time. Regarding the insurance provision, banks can perform this function because they have the ability of pulling closer together the returns associated with each of the two different states of the world. As we can see in Figure 1, their menu of choices now –with intermediation- has more

[7] This is the means used by the domestic central bank for injecting new fiat currency into the economy, and $\hat{\tau}_t$ represents the injection per young agent.

alternatives available for the households' contingent consumption goods: they can attain any of the $\left(\hat{c}_{1,t}, \hat{c}_{2,t+1}\right) \gg 0$ bundles in the continuum of feasible allocations along the bank's budget line, which must be compared against the unique bundle that they can access in autarky[8], where they can only consume $\hat{c}_{1,t} = x \cdot \left(w + \hat{\tau}_t\right)$ goods by the end of t if she is impatient or $\hat{c}_{2,t+1} = R \cdot \left(w + \hat{\tau}_t\right)$ goods by the end of $t+1$ if she is patient instead. Thus, in autarky the households lack the ability of diversifying risk.

Figure 1. Autarky versus financial intermediation



a) The full-information benchmark

The full information benchmark arises when the flow of information available to everybody is perfect (i.e., in the full information benchmark.) In this case, it is possible for everybody to observe the type of each of the households and, thus, households would always behave according to their true type.

In particular, in case households turned out to be patient, they would always wait until the end of $t+1$ to consume $\hat{c}_{2,+1}$ goods: since they are recognized as patient, banks will know that they can wait, and they will give priority to the impatient households, who must absolutely consume $\hat{c}_{1,t}$ goods at t or face a prohibitive punishment[9].

b) The case with imperfect flow of information

When the flow of information is incomplete, it is impossible for neither households nor banks to learn the other households' type, even if they chose to perform prohibitively monitoring functions through which they cannot learn anything new. The households' types can only be learned from market transactions.

It is important to highlight that only the patient households could credibly misrepresent their type[10]: they could withdraw $\hat{c}_{1,t}$ goods, like any impatient household would, and nobody

[8] We refer to autarky as the allocation of resources in the absence of banks.
[9] Since the utility function satisfies the Inada conditions as well as strict concavity, the price for consuming cero is infinite.
[10] The impatient households cannot credibly pretend to be of the patient type, since they must consume at t or face dire consequences: they cannot wait to consume, as patient households can.

would be the wiser. But, since they derive utility only from consuming the good when old, they would have to find a way of transferring this purchasing power to the future. Patient households could do this by saving the proceeds from their withdrawals and investing them in the short-term storage technology, that yields the one-period lower real return x by the end of t. This storage technology yields the same short-term return as liquidating the long-term illiquid investment (deposits) early. As a result, a patient household who pretended to be of the impatient type would withdraw $\hat{c}_{1,t}$ units of the good from the bank by the end of t; next, she would invest this amount into the storage technology and leave it there until the end of $t+1$, when they would obtain $x \cdot \hat{c}_{1,t}$ units of the good and consume them. However, if patient households were to choose this line of action, they would withdraw from the economy's pool of resources and they would receive a lower return. Thus, except in the case of extreme circumstances, it is in the best interest of the economy as a whole to try to avoid this situation and to force households to reveal their true type. But this is only possible when the expected return of acting as a patient household (their true type) exceeds the return of storing the good. This logic is what is behind the idea of the incentive compatibility or truth–telling constraint that we write below:

$$x \cdot \hat{c}_{1,t} < \hat{c}_{2,+1}. \tag{8}$$

Thus, only when (8) obtains will a patient agent behave as her true type instead of pretending to be impatient. We will see later on that we would need any deposit contract the households sign to promote truth-telling, so the economy may aim at replicating the full information benchmark and increase social welfare accordingly.

3.4 Domestic financial intermediation

As we brought up earlier, the domestic banks are coalitions of households/depositors that may arise endogenously when it is profitable for them to do so. The goal of the banks is to intermediate the domestic investment in the long-term technology and to provide insurance to domestic households. Banks have two sources of funds that are available to them at the beginning of t: there are the deposits that they keep for the households plus whatever they can borrow long-term from foreign banks, at the interest rate r_2. The domestic banks act as financial intermediaries in this respect and they offer a single type of demand deposits contract to the young households. We must point out that only the banks can access the world credit markets by trading with their counterparts who reside in the rest of the world (i.e. the economy's foreign borrowing is always intermediated.) In addition, there is a regulation that requires of domestic banks that they keep two reserves accounts with the domestic monetary authority: in the first account they will deposit the pesos required of them, while they will keep the dollars in the second account.[11]

Table 1. Instruments of foreign debt available to domestic banks

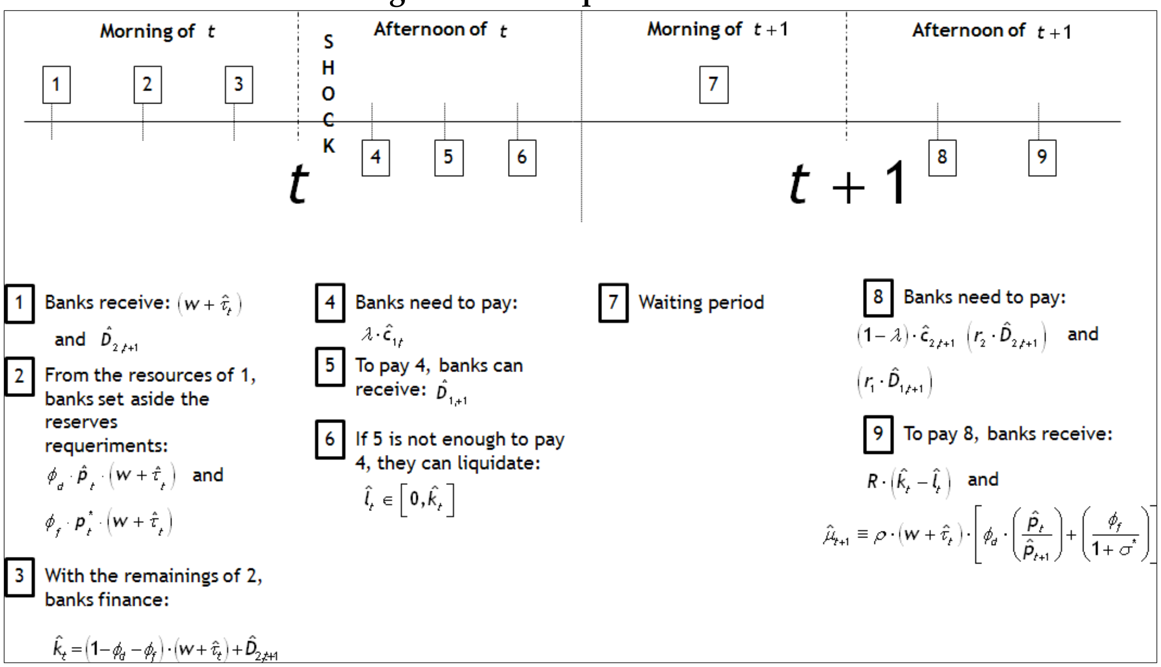
Instrument of debt	Issue date	Maturity date	World interest rate
Long-term debt $\hat{D}_{2,+1}$	Morning of t	Afternoon of $t+1$	$r_2 > 1$
Short-term debt $\hat{D}_{1,+1}$	Afternoon of t	Afternoon of $t+1$	$r_1 > 1$

[11] We are implicitly assuming, for simplicity, that the domestic banks could keep a deposit account denominated in dollars with the central bank. But this is not essential to the model and it could easily be modified.

The domestic banks can access two different debt instruments in the appropriate world markets. Because we have assumed that the domestic country is a small open economy, it follows that the banks in the home country will take as given the foreign interest rates that clear the world markets for the debt instruments that they can access. As we can see in Table 1, $\hat{D}_{1,+1} > 0$ represents the amount of short-term debt that banks will issue[12] in the afternoon of t and that will mature by the end of $t+1$, charging the real gross interest rate r_1. On the other hand, $\hat{D}_{2,+1} > 0$ is the amount of the long-term debt that banks will issue in the morning of t and they will have to repay at the end of $t+1$, when they will be charged the gross real interest rate $r_2 > 1$. We further assume that the world interest rates r_1 and r_2 are deterministic and constant over time, and that $r_2 > r_1 > 1$ holds since $\hat{D}_{2,+1}$ has a longer term than $\hat{D}_{1,+1}$ has.

3.4.1 The sequence of events

Figure 2. The sequence of events



In the morning of t, the domestic banks receive $(w+\hat{\tau}_t)$ goods from each young depositor/household; at this time, they can also access the long-term loan $\hat{D}_{2,+1}$ from the foreign banks. With these resources, they must first set aside the currency reserves required of them: they deposit the fraction ϕ_d of deposits in the form of pesos, while the fraction ϕ_f must take the form of dollars. This means that each commercial bank will deposit $\phi_d \cdot \hat{p}_t \cdot (w+\hat{\tau}_t)$ pesos and

[12] Even though this decision is not made effective by the banks until the end of t, they still have to plan for it in the morning of t, which is the time at which they must formulate their contingent plans based upon maximizing their expected utility. The same is true about the contingent consumption bundle $(\hat{c}_{1,t}, \hat{c}_{2,t+1})$. As a consequence, at the beginning of t, the domestic banks must elaborate a plan that contemplates the amount $\hat{D}_{1,t+1}$ of short-term debt that they expect to borrow at the end of t, as well as how much they would plan on consuming if they turned to be either impatient or patient.

$\phi_f \cdot p_t^* \cdot \left(w + \hat{\tau}_t\right)$ dollars, respectively, in the corresponding accounts that she must keep with the central bank. In addition, banks will combine whatever is left of their deposits and the long-term debt they issued, and they will use these funds to finance their investment into the economy's illiquid technology in the amount of $k_t > 0$ goods. This translates into the following budget constraint, which arises in the morning of t:

$$\hat{k}_t = \left(1 - \phi_d - \phi_f\right) \cdot \left(w + \hat{\tau}_t\right) + \hat{D}_{2,t+1}. \qquad (9)$$

In the afternoon of t, the banks need to pay the withdrawals of the impatient households[13] in the amount of $\lambda \cdot \hat{c}_{1,t}$. At this time, they can also borrow $\hat{D}_{1,+1}$ goods from the foreign banks to complement their income, which they will have to repay in the afternoon of $t+1$. In case they needed more funds to face their obligations to the impatient households, they could count on liquidating early a part of the long-term investment in the amount of $\hat{l}_t \in \left[0, \hat{k}_t\right]$ goods; however, but they will try to avoid this option because early liquidation is costly, since it offers a lower return. These activities lead to the following budget constraint at the end of the afternoon of t:

$$\lambda \cdot \hat{c}_{1,t} \leq x \cdot \hat{l}_t + \hat{D}_{1,+1}. \qquad (10)$$

The morning of $t+1$ can be considered as a waiting period, since the young impatient households have already consumed and the banks are pooling their resources together to repay their foreign debt, both long-term and short-term, in the afternoon of period $t+1$.

Next, in the afternoon of $t+1$, the banks need to pay the withdrawals of the patient agents that did not misrepresent their type in the amount $\left(1-\lambda\right) \cdot \hat{c}_{2,t+1}$; they also need to repay the total amount due of their foreign debt: the long-term debt $\left(r_2 \cdot \hat{D}_{2,t+1}\right)$ and the short-term debt $\left(r_1 \cdot \hat{D}_{1,t+1}\right)$. To pay this, the banks will use the return of the remaining long-term investment in the amount $R \cdot \left(\hat{k}_t - \hat{l}_t\right)$ together with the return yielded by their currency reserves. These operations give rise to the following budget constraint for the afternoon of $t+1$

$$\left(1-\lambda\right) \cdot \hat{c}_{2,t+1} + r_1 \cdot \hat{D}_{1,t+1} + r_2 \cdot \hat{D}_{2,t+1} \leq R \cdot \left(\hat{k}_t - \hat{l}_t\right) + \hat{\mu}_{t+1}, \qquad (11)$$

where $\hat{\mu}_{t+1} \equiv \rho \cdot \left(w + \hat{\tau}_t\right) \cdot \left[\phi_d \cdot \left(\frac{\hat{p}_t}{\hat{p}_{t+1}}\right) + \left(\frac{\phi_f}{1+\sigma^*}\right)\right]$ represents the return on the banks' legal reserves. We now turn to examining the banks' problem in the next section.

3.4.2 The domestic banks' problem in the absence of panics

Banks offer deposit contracts to the young households in the morning of t. As we mentioned earlier, one can think of domestic banks as coalitions of depositors that arise whenever it is convenient to them, so that they can maximize the expected utility of its members. As a result, these contracts promise the state-contingent pair $\left(\hat{c}_{1,t}, \hat{c}_{2,t+1}\right)$ that maximizes the expected utility

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[13] We are assuming, for now, that in equilibrium no patient household chooses to withdraw early, thus misrepresenting her type.

described in (7) subject to the restrictions in (8) - (11). The reader should notice that $\hat{l}_t \in \left[0, \hat{k}_t\right]$ represents the amount of early liquidation of the long-term investment that is available to banks.

In this section, we first solve the domestic banks' problem in the absence of panics: we assume that there are no unforeseen events -other than the shock to the households' preferences that could force the banks to liquidate their long-term investment early. That is, we restrict our attention to allocations where $\hat{l}_t = 0$ obtains[14], since we want to replicate the allocations under the full information benchmark.

We can combine equation (7) with the budget constraints (8) - (11) and rewrite the banks' problem as follows, in terms of the state variables:

$$E\left[u\left(\hat{D}_{1,t+1}, \hat{D}_{2,t+1}\right)\right] = \lambda \cdot \ln\left(\frac{\hat{D}_{1,t+1}}{\lambda}\right) + (1-\lambda) \cdot \ln\left(\frac{R \cdot \hat{k}_t + \hat{\mu}_{t+1} - r_1 \cdot \hat{D}_{1,t+1} - r_2 \cdot \hat{D}_{2,t+1}}{1-\lambda}\right), \qquad (12.1)$$

To solve the indeterminacy of equilibria, we must further assume that the amount of long-term debt is rationed by foreign banks, which leads to the following credit-rationing equation

$$\hat{D}_{2,t+1} \leq f_2, \qquad (12.2)$$

where $f_2 > 0$ represents the exogenous upper limit to long-term borrowing that is imposed by the banks in the rest of the world. In particular, we assume that banks will always borrow long-term to the maximum that they are allowed to, leading to

$$\hat{D}_{2,t+1} = f_2. \qquad (12.3)$$

This reduces the number of unknowns in the problem that banks must solve, leaving to banks only the choice of the state variable $\hat{D}_{1,t+1}$. Combining (12.1) with (12.2) and (12.3) we obtain the following reduced-form objective function for the domestic banks:

$$E\left[u\left(D_{1,t+1}\right)\right] = \lambda \cdot \ln\left(\frac{\hat{D}_{1,t+1}}{\lambda}\right) + (1-\lambda) \cdot \ln\left(\frac{R \cdot \hat{k}_t + \hat{\mu}_{t+1} - r_1 \cdot \hat{D}_{1,t+1} - r_2 \cdot f_2}{1-\lambda}\right). \qquad (12.4)$$

At this point, we must obtain the necessary first order condition with respect to the only state variable $\hat{D}_{1,t+1} \geq 0$ that we are left with, focusing on an interior solution. The following expression defines implicitly the domestic banks' demand for short-term foreign debt:

$$\hat{D}_{1,t+1} = \frac{\lambda \cdot \left(R \cdot \hat{k}_t + \hat{\mu}_{t+1} - r_2 \cdot f_2\right)}{r_1}. \qquad (13.1)$$

Now, we proceed to calculate the state-contingent consumption by the impatient and the patient households, respectively. We can combine (10) with (13.1) in order to obtain a reduced-form expression for the consumption by impatient households, $\hat{c}_{1,t}$:

$$\hat{c}_{1,t} = \frac{R \cdot \hat{k}_t + \hat{\mu}_{t+1} - r_2 \cdot f_2}{r_1} = \frac{\hat{D}_{1,t+1}}{\lambda}. \qquad (13.2)$$

[14] The purpose is to represent the full information benchmark.

Next, we can combine (11) with (13.1) in order to find a reduced-form expression for the consumption by patient households, $\hat{c}_{2,t+1}$:

$$\hat{c}_{2,t+1} = R \cdot \hat{k}_t + \hat{\mu}_{t+1} - r_2 \cdot f_2 = \frac{r_1 \cdot \hat{D}_{1,t+1}}{\lambda}. \tag{13.3}$$

The expressions in (13.1) to (13.3) describe the reaction functions chosen by domestic banks in the case where the full information benchmark is binding. In the next section, we turn to describe the equations that clear the different markets that we considered, with the aim at finding the conditions that must be satisfied in a general equilibrium with floating exchange rates.

3.5 General equilibrium with floating exchange rates and in the absence of panics

In the general equilibrium, several conditions must be satisfied simultaneously. We must combine the banks' reaction functions that we obtained in the previous section with the different conditions that are needed for the markets to clear. As a result, we will obtain the reduced-form dynamic system that arises in equilibrium when the nominal exchange rate is market-determined.

3.5.1 Market clearing

The first condition that must be satisfied is the Law of One Price, implying that the nominal floating exchange rate $\hat{e}_t$ must adjust to ensure that (1) holds at all times.

A connection between the interests rates of the instruments of foreign debt and that of the investment technologies must be established, which is represented by the following arbitrage condition:

$$R = r_2 > r_1. \tag{14}$$

The strict inequality in (14) arises because $\hat{D}_{2,t+1}$ has a longer term than $\hat{D}_{1,t+1}$, in spite of the fact that both mature simultaneously in the afternoon of $t+1$.

To ensure that the reserves requirements are binding, we must impose (15.1) and (15.2) below, which imply that the return of each of the two currencies that we consider in the model is lower than that of the deposits. The latter means that only banks will hold currency reserves and that they will choose to hold as little of them as possible, as implied by the following two inequalities

$$R > \left(\frac{\hat{p}_t}{\hat{p}_{t+1}} \right), \tag{15.1}$$

$$R > \frac{1}{\left(1+\sigma^*\right)}. \tag{15.2}$$

As we mentioned in the previous section, we needed to simplify the set of endogenous state variables as much as possible, with the goal of building a very parsimonious model of a regular economy. With this in mind, we assumed that the long-term foreign debt was rationed and equal to the exogenous limit of $\hat{f}_2$ units of the single good early in the morning of t; one could think of different reasons that justify this assumption. What we assume is that the long-term credit is rationed: foreign banks would like to borrow arbitrarily large amounts of long-term debt, but the banks in the rest of the world will not allow it: i.e., we choose to assume that this constraint has been imposed exogenously by the banks in the rest of the world, which we think it a reasonable constraint for emerging countries. Then, the following equality must hold in the general equilibrium of this economy

$$\hat{D}_{2,t+1} = \hat{f}_2 . \qquad (16)$$

(16) will always holds in this economy, regardless of the exchange rate regime that is in place, as we will see in section 4.

In equilibrium, since the domestic currency reserve requirement binds, the domestic money real balances will be given by the fraction ϕ_d of total deposits, $\left(w+\hat{\tau}_t\right)$, and the domestic price $\hat{p}_t$ clears the market for domestic currency exchanged for goods, yielding:

$$\hat{z}_t = \frac{\hat{m}_t}{\hat{p}_t} = \phi_d \cdot \left(w+\hat{\tau}_t\right). \qquad (17)$$

Regarding the domestic holdings of the foreign currency, a condition similar to (17) must be satisfied as well, which implies that the following equation is satisfied in equilibrium, where the reserve requirement on foreign currency binds:

$$\hat{q}_t = \phi_f \cdot \left(w+\hat{\tau}_t\right). \qquad (18)$$

The central bank must back the dollar-value of the domestic money supply with reserves of dollar-denominated assets, and thus (3) and (4.2) must both hold. To complete our equilibrium conditions, we must find the equilibrium real return on domestic real money balances. We use the definition of $\hat{z}_t$ to find that $\hat{p}_t = \dfrac{\hat{M}_t}{N \cdot \hat{z}_t}$ and obtain the following reduced-form expression for the real return on domestic currency in equilibrium

$$\left(\frac{\hat{p}_t}{\hat{p}_{t+1}}\right) = \frac{\hat{z}_{t+1}}{\hat{z}_t \cdot \left(1+\sigma\right)}. \qquad (19)$$

3.5.2 The reduced-form dynamic system and its structure

The reduced-form dynamic system is the solution to our model in equilibrium. After combining (6) with (17), we obtained the equilibrium law of motion for $\hat{z}_t$ under floating exchange rates, which is given by

$$\hat{z}_t = a_1 + a_2 \cdot \hat{z}_{t-1}. \qquad (20.1)$$

Next, we combined equations (6) and (20.1) to obtain the equilibrium law of motion for $\hat{\tau}_t$:

$$\hat{\tau}_t = g_1 + g_2 \cdot \hat{z}_{t-1}. \qquad (20.2)$$

Also, we rearranged equations (4.2), (4.3) and (9) in order to obtain the equilibrium laws of motion for the per capita central bank's dollar-denominated reserves, the real foreign currency balances and the investment in the long-term technology, respectively. In order to obtain the law of motion in equilibrium for (4.2), we combined this equation with (20.1) and obtained the following equilibrium law of motion for b_t:

$$\hat{b}_t = j_1 + j_2 \cdot \hat{z}_{t-1}; \qquad (20.3)$$

next, the law of motion for the real foreign currency balances in equilibrium arises from the combination of (4.3) with (20.2):

$$\hat{q}_t = i_1 + i_2 \cdot \hat{z}_{t-1} . \tag{20.4}$$

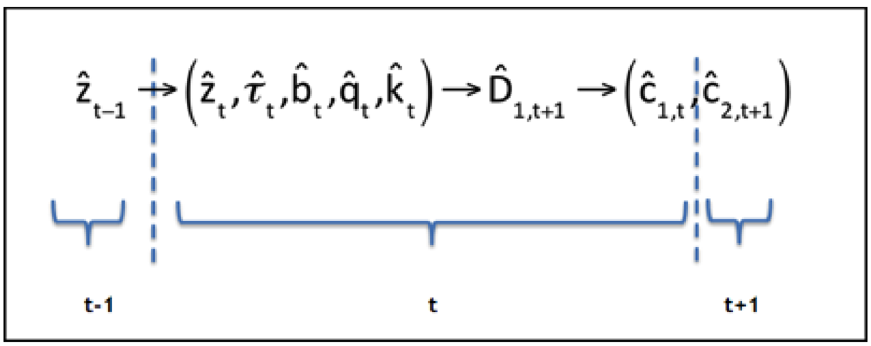
Finally, the combination of (9) with (20.2) will give us the law of motion for the long-term investment in equilibrium

$$\hat{k}_t = h_1 + h_2 \cdot \hat{z}_{t-1} . \tag{20.5}$$

Equations (20.1) to (20.5) together constitute a linear first order dynamic sub-system of difference equations that we have chosen to call the core dynamic sub-system, where all five variables $\left(\hat{z}_t, \hat{\tau}_t, \hat{b}_t, \hat{q}_t, \hat{k}_t\right) >> 0$ are independent of the world interests rates and inherit their dynamics directly from $\hat{z}_{t-1}$, given the monetary policy rule in place.

As we will explain in brief, the reduced-form dynamic overall system has a decoupled structure, which is a very interesting technical property of the equilibrium dynamic system. Moreover, the overall equilibrium dynamic system is composed of three dynamic sub-systems that feed recursively on each other, expressing a very clear order of causality, as we can see from Figure 1. The independence of the monetary policy rule under floating implies that the domestic real money balances govern the causality in this dynamic system.

Figure 3. Order of causality in the dynamic general equilibria with floating exchange rates



Once $\hat{z}_{t-1}$ is chosen at $t-1$ by the previous generation, it becomes an exogenous state variable that bequeaths its dynamic properties to the core dynamic sub-system; the latter, together with the world interests rates, will give rise to the second dynamic sub-system, composed of the foreign debt instruments: since we assumed that $\hat{D}_{2,t+1}$ is rationed and equal to f_2, it only rests for $\hat{D}_{1,t+1}$ to be determined at this stage in the second subsystem. Its equilibrium law of motion is given by the following second order nonlinear difference equation that we obtained from the combination of (13.2) with (20.2) and (20.5)

$$\hat{D}_{1,t+1} = m_1 + m_2 \cdot \hat{z}_{t-1} + m_3 \cdot \left(\frac{\hat{z}_{t-1}}{\hat{z}_t}\right) . \tag{21}$$

To complete the overall dynamic system, we needed to obtain the reduced-form equations for the state-contingent consumptions by impatient and patient households. It is apparent from (13.3) and (13.4) that both state contingent consumptions depend directly from $\hat{D}_{1,t+1}$. In particular, after we combine (13.3) and (13.4) with (21), we obtain the equilibrium laws of motion for the consumption by impatient and patient households, respectively, as we see in the two following expressions

$$\hat{c}_{1,t} = \frac{m_1}{\lambda} + \frac{m_2}{\lambda} \cdot \hat{z}_{t-1} + \frac{m_3}{\lambda} \cdot \left(\frac{\hat{z}_{t-1}}{\hat{z}_t} \right), \tag{22.1}$$

$$\hat{c}_{2,t+1} = \frac{r_1 \cdot m_1}{\lambda} + \frac{r_1 \cdot m_2}{\lambda} \cdot \hat{z}_{t-1} + \frac{r_1 \cdot m_3}{\lambda} \cdot \left(\frac{\hat{z}_{t-1}}{\hat{z}_t} \right). \tag{22.2}$$

Equations (22.1) and (22.2) set up our third and last dynamic sub-system. Each of these two equations is a second order nonlinear difference equation that inherits its dynamics from (21).

3.6 The steady-state equilibria with floating exchange rates

We now turn to evaluating the dynamic system we developed in the previous section, in stationary equilibrium allocations. Next, we turn to analyze whether the stationary equilibrium is unique or not. And, finally, we develop some comparative-statics analysis.

3.6.1 Finding the steady-state equilibria under floating

We start our analysis with the core sub-system. After imposing that the variables in (20.1) to (20.5) are constant over time, we obtain the following reduced-form expressions for steady-state domestic inflation and the stationary-equilibrium values of the five variables that set up the first sub-system

$$\left. \frac{\hat{p}_{t+1}}{\hat{p}_t} \right|_{ss} = \left(1 + s\right) \tag{23.1}$$

$$\hat{z} = \frac{\phi_d \cdot w \cdot \left(1+\sigma\right)}{1+\sigma-\phi_d \cdot \left[\sigma-\theta\cdot\left(1+\sigma\right)\right]-\phi_d\cdot\varepsilon\cdot\theta\cdot\left(1+\sigma\right)}, \tag{23.2}$$

$$\hat{\tau} = g_1 + g_2 \cdot \hat{z}\ , \tag{23.3}$$

$$\hat{b} = j_1 + j_2 \cdot \hat{z}\ , \tag{23.4}$$

$$\hat{q} = i_1 + i_2 \cdot \hat{z}\ , \tag{23.5}$$

$$\hat{k} = h_1 + h_2 \cdot \hat{z}\ . \tag{23.6}$$

Next, we turn to the foreign debt dynamic sub-system. After imposing stationarity on (21), we obtain the following expression for the short-term foreign debt in the steady-state equilibrium, as part of the second dynamic sub-system:

$$\hat{D}_1 = m_1 + m_3 + m_2 \cdot \hat{z}\ , \tag{24}$$

Finally, we turn to the third sub-system of state contingent commodities. We impose stationarity on (22.1) and (22.2) and obtain the values of both state-contingent consumptions, which yields:

$$\hat{c}_1 = \frac{m_1}{\lambda} + \frac{m_3}{\lambda} + \frac{m_2 \cdot \hat{z}}{\lambda}, \tag{25.1}$$

$$\hat{c}_2 = \frac{r_1 \cdot m_1}{\lambda} + \frac{r_1 \cdot m_3}{\lambda} + \frac{r_1 \cdot m_2 \cdot \hat{z}}{\lambda}. \tag{25.2}$$

It is important to remember that there is no early liquidation in this section, since we want to replicate the equilibria that would arise in the full-information benchmark. Thus, no bank panics arise in equilibrium and the conditions (14) to (16) are met.

3.6.2 Uniqueness of the steady-state general equilibrium with floating

Once that we obtain the steady state equilibria we need to analyze its properties of local/global uniqueness and determinacy, focusing on the mapping from allocations to prices of equilibria. According with the General Equilibrium theory, the equilibria are "regular" when the number of equilibria is finite and where there is a one-to-one mapping between the vectors of relative prices and the excess demand function near to the equilibrium allocation. We can say that our equilibria are regular since they accomplish the two characteristics described above; moreover, we can say that the steady-state equilibrium is globally unique and determinate.

The core dynamic sub-system in the steady state equilibrium, that is, $\left(\hat{z}, \hat{\tau}, \hat{b}, \hat{q}, \hat{k}\right)$ is always determinate and unique since it is not connected with the vector of foreign interest rates. Moreover, for a given value of the short-term debt $\left(\hat{D}_1\right)$ there is a vector of gross real interest rates $\left(r_1, r_2 = R\right)$ that satisfies the equilibrium conditions. There is a one-to-one mapping that leads us to have locally unique and determinate equilibria.

3.6.3 Comparative statics in the steady-state general equilibrium with floating

To complete the study of the steady-state equilibria, we analyze what will happen to our key variables when we change each of the policy parameters, one at a time. We obtain the comparative statics for the instrument of short-term debt and for the contingent consumptions.

<u>3.6.3.1 Comparative statics for the short-term debt</u>

a) Domestic inflation and short-term debt

$\frac{\partial \hat{D}_1}{\partial \sigma} < 0$; as one can see from equation (13.2), when the domestic rate of inflation increases (decreases), the return on the currency reserves denominated in pesos will decrease (increase) causing a decrease (increase) in the amount of the short-term debt.

b) Foreign interest rate on short-term debt versus short-term debt

$\frac{\partial \hat{D}_1}{\partial r_1} < 0$; the interest rate r_1 represents the price charged to domestic banks when they borrow short-term from the rest of the world. The intuition behind this result is consistent with the law of demand, which states that when the price of a good goes up (goes down), this typically causes the quantity demanded for that good to fall (increase). As a consequence, the quantity of short-term debt that domestic banks would wish to borrow will decrease (increase) since in general, consumers prefer low prices for the goods/assets that they demand.

c) Inflation in the rest of the world versus short-term debt

$\frac{\partial \hat{D}_1}{\partial \sigma^*} < 0$; this inequality responds to an explanation along the same lines as the one for the domestic inflation rate: an increase in the inflation in the rest of the world reduces the return on foreign currency; we can see from equation (13.2) that when the inflation rate in the rest of the world increases (decreases), the return of the currency reserves denominated in foreign currency will decrease (increase) causing a decrease (increase) in the quantity of the short-term debt demanded by the domestic banks.

d) Reserve requirements versus short-term debt

$\frac{\partial \hat{D}_1}{\partial \phi_d} > 0$ and $\frac{\partial \hat{D}_1}{\partial \phi_f} > 0$; if banks need to hold a higher fraction of their deposits, either in domestic or foreign currency, in the central bank's vault, they probably will suffer from illiquidity in t and thus they will need an alternative source of liquidity to face all the withdrawals in this period of time.

e) The central bank's remuneration on currency reserves versus short-term debt

$\frac{\partial \hat{D}_1}{\partial \rho} > 0$; when the central bank increases in the exogenous policy parameter $r\ ^{3}\ 1$, it signifies that the complementary remuneration on currency reserves paid by the domestic central bank (on top of the implicit return of each currency) is now higher. This higher return on currency reserves in $t+1$ in turn will causes the amount of short-term debt will increase.

f) The return on the long-term illiquid technology versus short-term debt

$\frac{\partial \hat{D}_1}{\partial R} > 0$; as we can see from equations (11) and (13.2), the amount that domestic banks borrow short-term will increase (decrease) with an augment (diminution) in the gross real return on the long-term illiquid storage technology.

<u>3.6.3.2 Comparative statics of the state-contingent consumptions</u>

a) Domestic inflation versus state-contingent consumptions

$\frac{\partial \hat{c}_1}{\partial \sigma} < 0, \frac{\partial \hat{c}_2}{\partial \sigma} < 0$; from (13.3) and (13.4), we can see that when the domestic rate of inflation increases (decreases), this reduces the term $\hat{\mu}_{t+1}$, which represents the return on the domestic currency reserves, thus reducing both state-contingent consumptions $\hat{c}_1$ and $\hat{c}_2$, since

b) The inflation in the rest of the world versus state-contingent consumption

$\frac{\partial \hat{c}_1}{\partial \sigma^*} < 0, \frac{\partial \hat{c}_2}{\partial \sigma^*} < 0$; when the rate of inflation in the rest of the world increases (decreases), it is easy to see from equation (13.3) and (13.4), that the quantity of $\hat{c}_1$ and $\hat{c}_2$ will decrease (increase) due to the diminution in the return of the currency reserves in dollars.

c) Reserve requirements versus the state-contingent consumption by the impatient

$\frac{\partial \hat{c}_1}{\partial \phi_d} > 0, \frac{\partial \hat{c}_1}{\partial \phi_f} > 0$; when banks are forced to hold a higher fraction of their deposits in the central bank's vaults, either in domestic or foreign currency, the consumption by impatient households in t will increase since the short-term debt increases too and there is a direct relationship between these two variables.

d) Reserve requirements versus the state-contingent consumption by the patient

$\frac{\partial \hat{c}_2}{\partial \phi_d} > 0, \frac{\partial \hat{c}_2}{\partial \phi_f} > 0$; $\hat{c}_2$ will increase when the central bank augments the fraction of the deposits that domestic banks must hold in their vaults, either in dollar or pesos. This is because the return that banks will receive in $t+1$ for their reserves, will increase since they deposited a high amount in the Central Bank; so they will have more goods to give to the patient agents, augmenting their consumption.

e) The interest rate on short-term debt versus the state-contingent consumption by the impatient

$\frac{\partial \hat{c}_1}{\partial r_1} < 0$; if the interest rate that domestic banks need to pay for the short-term debt increases (decreases), the quantity of short-term debt that domestic banks desire will decrease (increase); then banks will have less resources to cover the withdrawals of impatient households causing a diminution (augmentation) in $\hat{c}_1$.

f) The interest rate on short-term debt versus the state-contingent consumption by the patient

$\frac{\partial \hat{c}_2}{\partial r_1} = 0$; the consumption of the patient households does not depend on the interest rate on short-term debt, as we can see from equation (13.4).

g) The central bank's remuneration on currency reserves versus the state-contingent consumption by the impatient

$\frac{\partial \hat{c}_1}{\partial \rho} > 0$; if there is a high rate of return on the currency reserves, the domestic banks will receive a high return in $t+1$; this causes an increment in that amount of the short-term debt, augmenting as well the consumption of the impatient households.

h) The central bank's remuneration on currency reserves versus the state-contingent consumption by the patient

$\frac{\partial \hat{c}_2}{\partial \rho} > 0$; if there is a high rate of return on the currency reserves, the domestic banks will receive a high return in $t+1$; with this "extra" money, banks increment the quantity of consumption for the patient agents.

i) The return on the illiquid long-term technology versus the state-contingent consumption by the impatient

$\frac{\partial \hat{c}_1}{\partial R} > 0$; as we can see from equations (11) and (13.2), the quantity of short-term debt will increase (decrease) whit an augment (diminution) in the gross real return on the long-term illiquid storage technology; causing an increment in the consumption of impatient households as well.

j) The return on the illiquid long-term technology versus the state-contingent consumption by the patient

$\frac{\partial \hat{c}_2}{\partial R} > 0$; if the gross real return on the long-term illiquid storage increases, the banks' available resources in $t+1$ will increase as well; increasing the consumption en $t+1$.

3.7 The dynamic general equilibria with floating exchange rates

As we mentioned earlier, the dynamic system consists of the core sub-system, the foreign debt sub-system and the contingent consumptions sub-system. We will analyze them in that order due to the structure of the causality relationships that we have explained in the Figure 3.

3.7.1 The core dynamic sub-system under floating

As we can recall, this sub-system is set up by the following five state variables in $\left(\hat{z}_t, \hat{\tau}_t, \hat{b}_t, \hat{q}_t, \hat{k}_t\right)$. This sub-system includes equations (20.1) to (20.5) and we can easily see that they display linear dynamics and all of them inherit their dynamics from $\hat{z}_{t-1}$. These variables display monotonic dynamics along the equilibrium paths around the stationary core. As these variables display linear dynamics, they will have a unique characteristic root that will be given by the coefficient associated with $\hat{z}_{t-1}$. All of these associated coefficients are positive, implying that all the characteristic roots in this sub-system are unique and positive.

3.7.2 The dynamic sub-system for the short-term foreign debt instrument under floating exchange rates

Now, we study the sub-system that governs the dynamics of short-term debt. This dynamic sub-system displays nonlinear dynamics, as we can see from equation (21); this equation is a second order, nonlinear difference equation in $\hat{z}_t$.To solve for the dynamic path of $\hat{D}_{1,t+1}$, we first augment the state-space by using the following definition that creates the new state variable $\hat{y}_t$

$$\hat{y}_{t+1} = \hat{z}_t ; \qquad (26.1)$$

and we use it in (21). As a result, the order of the system is reduced from two to one, and our first order nonlinear dynamic system in two unknowns consists of (26.1) together with the following new expression for the short-term debt

$$\hat{D}_{1,t+1} = m_1 + m_2 \cdot \hat{y}_t + m_3 \cdot \left(\frac{\hat{y}_t}{\hat{z}_t}\right). \qquad (26.2)$$

Now the dynamic system for $\hat{D}_{1,t+1}$ is given by (26.1) and (26.2). Next, we linearize this system in a neighborhood of the stationary equilibrium by using a First Order Taylor expansion.

Where the trace and determinant of J^1 in the steady-state are given by:

$$T(J^1) = \left.\frac{\partial \hat{D}_{1,t+1}}{\partial \hat{z}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\frac{m_3}{\hat{z}} < 0 . \qquad (27.1)$$

$$D(J^1) = -\left.\frac{\partial \hat{D}_{1,t+1}}{\partial \hat{y}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\left(m_2 + \frac{m_3}{\hat{z}}\right) < 0 . \qquad (27.2)$$

Both expressions are continuous and monotonic functions of the parameters. It is apparent that both trace and determinant are negative numbers; thus, the discriminant $\Delta^1 \equiv \left[T\left(J^1\right)\right]^2 - 4 \cdot D\left(J^1\right)$ is always positive. This ensures us that the pair of eigenvalues is real and distinct. This pair of distinct and real characteristic roots is given by $\left(\Omega_1^1, \Omega_2^1\right)$, where Ω_1^1 is the largest eigenvalue from the quadratic conjugate. As one can expect, the eigenvalues have opposite signs; moreover, the relationship $\Omega_1^1 < \left|\Omega_2^1\right|$ obtains. Later, we will analyze the dynamic behavior of these roots under different scenarios.

3.7.3 The dynamic sub-system of the state-contingent consumptions with floating exchange rates

Finally, we analyze the last sub-system of our dynamic system, the sub-system that includes the consumption of impatient and patient households. From equations (13.3) and (13.4) we can see that the contingent consumptions will inherit their dynamics from $\hat{D}_{1,t+1}$ and it is more evident in (22.1) and (22.2). Then, the dynamic equations of the contingent consumptions display nonlinear behavior as the short-term debt does; they are a second order, nonlinear, difference equation in $\hat{z}_t$. Moreover, in order to solve for the dynamic path of the contingent consumptions, we will use the same method that we used for obtaining $\hat{D}_{1,t+1}$, since it is a second order difference equation as well. First, we augment the state-space using (26.1) and we rearrange equations (22.1) and (22.2):

$$\hat{c}_{1,t} = \frac{m_1}{\lambda} + \frac{m_2 \cdot \hat{y}_t}{\lambda} + \frac{m_3 \cdot}{\lambda}\left(\frac{\hat{y}_t}{\hat{z}_t}\right), \tag{28.1}$$

$$\hat{c}_{2,t+1} = \frac{r_1 \cdot m_1}{\lambda} + \frac{r_1 \cdot m_2 \cdot \hat{y}_t}{\lambda} + \frac{r_1 \cdot m_3 \cdot}{\lambda}\left(\frac{\hat{y}_t}{\hat{z}_t}\right). \tag{28.2}$$

As a result, we obtain the first order nonlinear system for $\left(\hat{c}_{1,t}, \hat{c}_{2,t+1}\right)$ in $\left(\hat{z}_t, \hat{y}_t\right)$ given by (26.1), (28.1) and (28.2). Then, we linearize the system in a neighborhood of the steady state using a First Order Taylor approximation.

Now, we obtain the trace and determinant, in the steady state, of the Jacobian for the impatient consumption that is of J^2

$$T(J^2) = \left.\frac{\partial \hat{c}_{1,t}}{\partial \hat{z}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\frac{m_3}{\lambda \cdot \hat{z}} < 0, \tag{30.1}$$

$$D(J^2) = -\left.\frac{\partial \hat{c}_{1,t}}{\partial \hat{y}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\left(\frac{m_2}{\lambda} + \frac{m_3}{\lambda \cdot \hat{z}}\right) < 0. \tag{30.2}$$

As in the case of the short-term foreign debt, both, trace and determinant are negative. Also, both expressions are continuous and monotonic functions of the parameters.

We now follow the same methodology for the state-contingent consumption by the patient. The trace and determinant of J^3 evaluated in the steady state, are set by:

$$T(J^3) = \left.\frac{\partial \hat{c}_{2,t+1}}{\partial \hat{z}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\frac{r_1 \cdot m_3}{\lambda \cdot \hat{z}} < 0, \tag{31.1}$$

$$D(J^3) = -\left.\frac{\partial \hat{c}_{2,t+1}}{\partial \hat{y}_t}\right|_{(\hat{z},\hat{y})} \Rightarrow -\left(\frac{r_1 \cdot m_2}{\lambda} + \frac{r_1 \cdot m_3}{\lambda \cdot \hat{z}}\right) < 0. \tag{31.2}$$

Since $\hat{c}_{2,t+1}$ inherits its dynamic behavior from $\hat{D}_{1,t+1}$ and $\hat{c}_{1,t}$, it is apparent that both the trace and determinant, are negative as well.

The discriminant for J^2 is given by $\Delta^2 \equiv \left[T\left(J^2\right)\right]^2 - 4 \cdot D\left(J^2\right)$ and it is evident that is always positive; since, as we said before, trace and determinant are both negative. From this arises that there are two real and distinct characteristic roots denoted by $\left(\Omega_1^2, \Omega_2^2\right)$, one negative and one positive.

$\Delta^3 \equiv \left[T\left(J^3\right)\right]^2 - 4 \cdot D\left(J^3\right)$ is the discriminant for J^3 which is always positive, as in the two previous cases. Then the characteristic roots $\left(\Omega_1^3, \Omega_2^3\right)$ have opposite signs, are real and distinct too. Since $\hat{c}_{2,t+1}$ inherits its dynamic behavior from the short-term debt and the consumption of impatient households, we will see that its dynamic behavior is less stable and more volatile than the dynamics orbits displayed by $\hat{D}_{1,t+1}$ and $\hat{c}_{1,t}$. Now, we proceed to the analysis of the dynamic behavior of these eigenvalues that we just obtained.

3.8 Analysis of the equilibrium dynamics of the short-term foreign debt and the contingent consumptions

We now describe how the dynamic properties of the short-term debt and the contingent consumptions change as we consider scenarios with different combinations of parameters.

3.8.1 Dynamic behavior under a baseline scenario

We first proceed to define a baseline scenario, which is given by the following values of the parameters $\lambda = 0.2$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $R = 1.05$, $r_1 = 1.02$, $\sigma^* = 0.05$, $\varepsilon = 1.02$ and $\rho = 1.1$. In this scenario, the steady-state equilibrium value of the short-term foreign debt $\left(\hat{D}_{1,t+1}\right)$ is a sink with eigenvalues that have opposite signs, which leads to fluctuations and endogenously arising volatility. Both characteristic roots lye inside the unit circle and the negative eigenvalue determines the size of the fluctuations around the steady-state equilibrium. Higher domestic steady-state inflation (increases in σ) increases the magnitude of the positive eigenvalue (thus increasing the scope for instability) while the norm of the negative eigenvalue becomes smaller, thus reducing the size of the oscillations.

Consumption inherits its dynamic behavior from short-term foreign debt. In particular, the equilibrium values of state-contingent consumption $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$, they each have eigenvalues with opposite signs, and they are each a saddle with paths that approach monotonically the steady-state. In particular, the positive eigenvalue is the stable root (since it lies inside the unit circle) and it determines the properties of dynamic equilibria along the stable manifold. Higher steady-state inflation increases the scope for stability by reducing the size of the positive eigenvalue.

3.8.2 Dynamic behavior in different scenarios

Many combinations between the values of the parameters can be done; however, we just analyze some of them. We find that the parameters that make a huge change in the dynamic behavior of $\hat{D}_{1,t+1}$, $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$ are the proportion of impatient households (λ) and the backing of the domestic money supply (θ).

Table 2: Summary of Scenarios

	$\theta = 0.1$	$\theta = 1$
$\lambda = 0.8$	Scenario 1	Scenario 2
$\lambda = 0.05$	Scenario 3	Scenario 4

As we can see in Table 2, we set up four scenarios based upon distinct values that λ and θ may take; we combine a small and large proportion of impatient households ($\lambda = 0.05$ and $\lambda = 0.8$, respectively) with a low and high backing of the domestic money supply ($\theta = 0.1$ and $\theta = 1$). Each scenario presents different dynamics in a neighborhood of the steady-state equilibrium of the short-term debt and the contingent consumptions. We resume these results as follows:

Scenario 1. Regarding the short-term foreign debt $\left(\hat{D}_{1,t+1}\right)$, the dynamic orbits approaching the steady-state equilibrium display a scope for a saddle with monotonic convergence, regardless of the different possible combinations of the policy parameters, together with the foreign interest rates. In particular, as σ increases, the scope for stability of this debt instrument increases. With respect to the households' consumption $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$, the scope for instability is reduced as σ increases, and a saddle with monotonic convergence along the stable manifold arises in most cases, thus reducing the possibility of unstable equilibria.

Scenario 2. In this scenario, dynamic equilibria approach the long-run value of $\hat{D}_{1,t+1}$, either as a sink with fluctuations or as a saddle with a positive stable eigenvalue. This dynamic behavior is not only inherited but also magnified in the case of consumption: the magnification effect is larger for $\hat{c}_{2,t+1}$, increasing the scope for instability. Thus, the consumption of the impatient agents and that of the patient agents has a well-defined region where they are a source and display oscillatory divergence. As in the previous scenario, a larger growth rate of the domestic money supply tends to increase the scope for stability of the system.

Scenario 3. In the third scenario, the steady-state value of $\hat{D}_{1,t+1}$ is a sink with fluctuations, since it has two roots with opposite signs, but both stable. In the case of the households' consumption $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$, they exhibit a scope for a saddle with monotonic convergence in most of the cases. Also, increases in σ also increase the scope for stability of this dynamic sub-system.

Scenario 4. In this case, the stationary value of the short-term foreign debt $\hat{D}_{1,t+1}$ is always a sink that displays fluctuations. Meanwhile, dynamic equilibria around the steady-state values of $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$ can be either a saddle with monotonic convergence or a source with oscillatory divergence, depending upon the values of R and ε together with the rate of the domestic money supply. High values of the interest rate $\left(1.15 < R \leq 1.2\right)$ together with high values of σ gives rise to a region with oscillatory divergence. When $\varepsilon \geq 1.14$, the steady-state value of $\hat{c}_{1,t}$ and $\hat{c}_{2,t+1}$ become a source with fluctuations, and increases in σ augment the stability of the system by increasing as well the determinacy of dynamic equilibria.

We now sum up the results that we obtained about the orbits along the equilibrium dynamic paths. The consumptions' dynamic behavior is not only inherited from $\hat{D}_{1,t+1}$ but it is also magnified: the marginal effect is larger for $\hat{c}_{1,t}$, but it is even larger for $\hat{c}_{2,t+1}$; however, there are less stable and more volatile than the short-term debt. Also, a larger growth rate of the domestic money supply tends to increase the stability of the system by increasing, as well, the determinacy of dynamic equilibria. Given our assumptions and our scenarios, changes in the backing of the domestic money supply do not alter significantly the properties of the dynamic equilibria of the short-term debt; thus, we cannot make any policy recommendations in this regard. However, the contingent consumptions do suffer little changes. This result indicates us that a high fraction of the reserves held in foreign currency can destabilize the system; as well as high levels in the interest rates.

4. The economy under fixed exchange rates

In this section we try to reproduce the scenario around which the 1994 Mexican crisis took place; thus, we use a small, open economy and a fixed exchange rate. However, we warn the readers that the exchange rate regime in place was an intermediate peg, rather than a hard peg. In particular, we continue with the basic economy that we developed in Section 3, but with a big difference: there is a fixed exchange rate regime in place. Recalling, there is a single endowment good in our economy that can be consumed in each period, but goods cannot be produced. Also, in the domestic country, there is a continuum of households and domestic banks (that may arise endogenously as coalitions of young households). Moreover, there is a domestic monetary authority and a group of foreign banks. The main difference is that now, the nominal exchange rate will be set exogenously once and for all by the monetary authority, which will be associated with a policy of endogenous money supply.

4.1 The monetary policy under fixed exchange rates

Under this regime, the domestic money supply, denoted by $\bar{M}_t$, is endogenous and it will change as needed to meet the condition imposed by the monetary authority that the nominal exchange rate, represented by $\bar{e}$ pesos per each dollar, must be constant over time, in the spirit of a very hard peg. This implies that the monetary policy is subordinated to the exchange rate policy. Also, under this regime the purchasing power parity still holds, but it must be modified to account for the new exchange rate regime:

$$\bar{e} \cdot p_t^* = \bar{p}_t \ , \qquad (32)$$

The expression in (32) reflects reflect the new policy. As with floating exchange rates, the injection of domestic currency is fulfilled through the monetary lump-sum transfers in the amount of $\bar{\tau}_t$ goods to the young households.

As before, there are two fiat currencies (pesos and dollars) that may circulate simultaneously in this economy. As we said before, the pesos' supply is denoted by $\bar{M}_t$ and it yields the gross real return $\left(\bar{p}_t / \bar{p}_{t+1}\right) > 0$ when held between t and $t+1$. $\bar{Q}_t$ denotes the aggregate nominal stock of foreign currency circulating in the domestic country at t with the new regime, where the real return of holding dollars between t and $t+1$, is given by $\left(\dfrac{p_t^*}{p_{t+1}^*}\right) = \left(\dfrac{1}{1+\sigma^*}\right) > 0$; where $\sigma^* > 0$ is the constant and exogenous net inflation rate in the rest of the world.

We assume that there are no legal restrictions in the use of foreign currency in the domestic country other than a reserve requirement. Also, the monetary authority chooses to hold reserves in the form of foreign reserves assets to contribute to the sustainability of the fixed exchange rates in place. According to this, we must modify equation (3) and we obtain

$$\bar{B}_t = \theta \cdot \left(\frac{\bar{M}_t}{\bar{e}} \right). \qquad (33)$$

In accordance with all the assumptions that we have made so far, we obtain the following per capita representation of the budget constraint faced by the monetary authority at the end of t:

$$\bar{\tau}_t = \bar{z}_t \cdot (1-\theta) + \bar{z}_{t-1} \cdot \left[(\varepsilon \cdot \theta) - \frac{1}{1+\sigma^*} \right]. \qquad (34)$$

4.2 Households' behavior under fixed exchange rates

We follow the same households' behavior as in the case under floating exchange rates. A typical household is born at the beginning of period t, when we say that she is young; during her second and last period of life, at $t+1$, she is called old. They suffer a shock that may alter their preferences; with probability λ they may become impatient or impatient with probability $(1-\lambda)$ at the end of t.

In the morning of t, the young households receive their endowment good and the real lump-sum monetary transfer. Next, they may choose to go to a bank and deposit the total of their $(w+\bar{\tau}_t)$ income inelastically. When the shock alters their preferences, if they become impatient, they will wish to consume $\bar{c}_{1,t}$ units of the endowment good only when young and nothing when old. However, if they become patient, they will derive utility only from consuming $\bar{c}_{2,+1}$ units of the good when old at $t+1$. As under floating exchange rates, the households' preferences can be represented by the following expected utility function:

$$E\left[u\left(\bar{c}_{1,t}, \bar{c}_{2,t+1}\right)\right] = \lambda \cdot \ln\left(\bar{c}_{1,t}\right) + (1-\lambda) \cdot \ln\left(\bar{c}_{2,t+1}\right). \qquad (35)$$

As under floating exchange rates, domestic banks and households can access to the same long-term illiquid investment. Also, banks can borrow from the two instruments of foreign debt that we described in Table 1.

The domestic banks under a hard peg will face a problem very similar to the one they had to solver under floating exchange rates. The structure of the problem is similar, but the specifics are different, as we can see in (36) - (39), where the difference is more than just using different notation. Under this regime, the domestic banks want to maximize (35) subject to:

$$x \cdot \bar{c}_{1,t} < \bar{c}_{2,+1}, \qquad (36)$$

$$\bar{k}_t = \left(1-\phi_d-\phi_f\right) \cdot \left(w+\bar{\tau}_t\right) + \bar{D}_{2,t+1}, \qquad (37)$$

$$\lambda \cdot \bar{c}_{1,t} \leq x \cdot \bar{l}_t + \bar{D}_{1,+1}, \qquad (38)$$

$$(1-\lambda) \cdot \bar{c}_{2,t+1} + r_1 \cdot \bar{D}_{1,t+1} + r_2 \cdot \bar{D}_{2,t+1} \leq R \cdot \left(\bar{k}_t - \bar{l}_t\right) + \bar{\mu}_{t+1}. \qquad (39)$$

As we can recall, equation (36) denotes the incentive compatibility that ensures us that the households will never misrepresent their type. (37) is the domestic banks' budget constraint that arises in the morning of t. The obligations that banks must cover in the afternoon of t are given by (38). Finally, (39) summarizes the resources available and payments that banks will do in the afternoon of $t+1$, where $\bar{\mu}_{t+1} = \left(w+\bar{\tau}_t\right) \cdot \rho \cdot \left[\phi_d \cdot \left(\frac{\bar{p}_t}{\bar{p}_{t+1}} \right) + \phi_f \cdot \left(\frac{1}{1+\sigma^*} \right) \right]$. This problem is solved in the same way that we solved the banks' problem under floating exchange rates; so we need to keep in mind equations (13.2) -(13.4).

4.3 General equilibrium with fixed exchange rates

As in the general equilibrium with floating exchange rates, we impose several conditions that must be satisfied. The equations related to the restrictions on the international transactions, i.e. equations (14) and (32), must hold. We need to ensure that the reserves requirements are still binding, so equation (15.2) holds and we rewrite equation (15.1) to:

$$R > \left(\frac{\bar{p}_t}{\bar{p}_{t+1}} \right). \tag{40}$$

Also, we assume that the amount of the long-term borrowed by banks from the rest of the world is given exogenously by f_2 units of goods: we keep the same spirit of credit rationing that we started in Section 3. As a consequence, the following relationship summarizes the demand for long-term foreign debt:

$$\bar{D}_{2,t+1} = f_2, \tag{41}$$

The next equation arises for the new price level in equilibrium, which must clear the market for the domestic real money balances

$$\bar{z}_t = \frac{\bar{m}_t}{\bar{p}_t} = \phi_d \cdot \left(w + \bar{\tau}_t \right). \tag{42.1}$$

Also, the market for foreign currency clears when:

$$\bar{q}_t \equiv \frac{\bar{Q}_t}{p_t^*} = \phi_f \cdot \left(w + \bar{\tau}_t \right). \tag{42.2}$$

One of the main differences between the floating exchange rates and the fixed exchange rates is the rate of return on real domestic balances and foreign balances; as we can recall from the previous section, there was no clear link between the domestic and foreign inflation rates with a policy of floating exchange rates. But now, the fixed exchange rates buy us a closer relationship between both inflation rates, yielding:

$$\left(\frac{\bar{p}_t}{\bar{p}_{t+1}} \right) = \left(\frac{p_t^*}{p_{t+1}^*} \right) = \left(\frac{1}{1+\sigma^*} \right). \tag{43}$$

This equation implies that the domestic country inherits the world inflation rate.

4.3.1 The reduced-form dynamic system in equilibrium and its structure

In order to continue with the analysis of the general equilibrium, now we need to obtain the equations that will set up our dynamic system under fixed exchange rates. As in the previous section, our dynamic system has a decoupled structure and is composed by three dynamic sub-systems; the core dynamic sub-system, dynamic sub-system for the foreign debt instruments and, finally, the dynamic sub-system for the contingent consumptions dynamic sub-system. As we can see from Figure 4, there is a very clear order of causality; moreover, we can infer from Figure 4 that each sub-system depend recursively on each other[15].

First we obtain the equilibrium law of motion for $\bar{\tau}_t$; since, the dynamic system inherits their dynamics directly from $\bar{\tau}_{t-1}$. We combine (34) with (42.1) and we obtain:

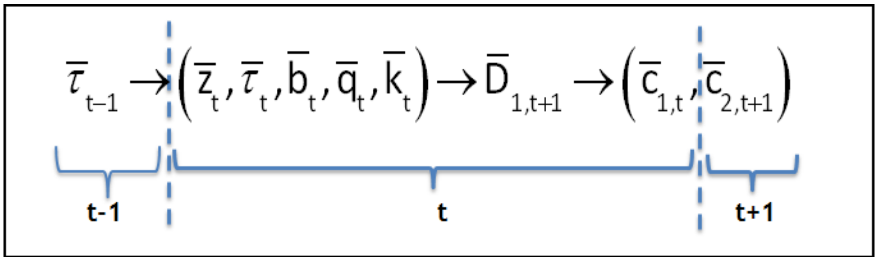
[15] Through this section we will use variables with an upper bar to difference from the equations of the floating exchange rates.

$$\bar{\tau}_t = \beta_1 + \beta_2 \cdot \bar{\tau}_{t-1}. \qquad (43.1)$$

Next, the combination of equations (42.1) and (43.1) yields

$$\bar{z}_t = \alpha_1 + \alpha_2 \cdot \bar{\tau}_{t-1}. \qquad (43.2)$$

Figure 4. Order of causality in dynamic general equilibria with fixed exchange rates



The equilibrium law of motion for the per capita central bank's dollar-denominated reserves is given by:

$$\bar{b}_t = \varphi_1 + \varphi_2 \cdot \bar{\tau}_{t-1}. \qquad (43.3)$$

Moreover, we can rearrange the real foreign currency balances held by banks on their account with the central bank, and we obtain the following reduced-form equation:

$$\bar{q}_t = \psi_1 + \psi_2 \cdot \bar{\tau}_{t-1}. \qquad (43.4)$$

The combination of (37) with (43.1) gives us the equilibrium law of motion for the long-term investment in this regime and it is:

$$\bar{k}_t = \eta_1 + \eta_2 \cdot \bar{\tau}_{t-1}. \qquad (43.5)$$

All the previous equations, that is (43.1) - (43.5), set up the core dynamic sub-system and together constitute a linear first order system of difference equation.

Then, we obtain the foreign debt instruments sub-system; this sub-system is only integrated by $\bar{D}_{1,t+1}$ since the assumption that we did in equation (41). From equation (13.2), (43.1) and (43.5) we can obtain the equilibrium law of motion for the short-term debt under fixed exchange rates

$$D_{1,t+1} = \xi_1 + \xi_2 \cdot \bar{\tau}_{t-1}. \qquad (44)$$

This sub-system is also a linear first order system of difference equations. Finally, we must obtain the dynamic sub-system for the state-contingent consumptions. The combination of (13.3) and (44) give us the law of motion in equilibrium of the impatient consumption

$$\bar{c}_{1,t} = \frac{\xi_1}{\lambda} + \frac{\xi_2}{\lambda} \cdot \bar{\tau}_{t-1}, \qquad (45.1)$$

also, the law of motion for the consumption of patient agents is given by:

$$\bar{c}_{2,t+1} = \frac{r_1 \cdot \xi_1}{\lambda} + \frac{r_1 \cdot \xi_2}{\lambda} \cdot \bar{\tau}_{t-1}. \qquad (45.2)$$

Like the previous sub-systems, this last sub-system is also a linear first order system of difference equations that inherits its dynamic from (44) and, evidently, from $\bar{\tau}_{t-1}$.

4.4 The steady-state equilibria with fixed exchange rates

Once we obtained all the equations that belong to our dynamic system, we will proceed to evaluate our system in stationary equilibrium allocations. Then, we turn to analyze whether the stationary equilibrium is unique or not. And, finally, we develop some comparative-statics analysis.

4.4.1 Finding the steady-state equilibria under fixed exchange rates

First, we start this analysis with the core sub-system that explains the dynamic behavior of $\left(\overline{\tau}_t, \overline{z}_t, \overline{b}_t, \overline{q}_t, \overline{k}_t,\right)$. We imposed that the variables in (43.1) to (43.5) are constant over time and we obtain the following reduced-form expressions for the stationary equilibrium values:

$$\overline{\tau} = \frac{\phi_d \cdot w \cdot \left[(1-\theta)\left(1+\sigma^*\right) + \varepsilon \cdot \theta \cdot \left(1+\sigma^*\right) - 1\right]}{\left(1+\sigma^*\right)\left[1-\phi_d \cdot (1+\theta)\right] - \phi_d \cdot \left[\varepsilon \cdot \theta \cdot \left(1+\sigma^*\right) - 1\right]}, \quad (46.1)$$

$$\overline{z} = \alpha_1 + \alpha_2 \cdot \overline{\tau}, \quad (46.2)$$

$$\overline{b} = \varphi_1 + \varphi_2 \cdot \overline{\tau}, \quad (46.3)$$

$$\overline{q} = \psi_1 + \psi_2 \cdot \overline{\tau}, \quad (46.4)$$

$$\overline{k} = \eta_1 + \eta_2 \cdot \overline{\tau}. \quad (46.5)$$

Next, we obtain the reduced-form expression for the short-term foreign debt in a steady state equilibrium, which is described by:

$$\overline{D}_1 = \xi_1 + \xi_2 \cdot \overline{\tau}, \quad (47)$$

Finally, we impose stationarity on (45.1) and (45.2) in order to obtain the reduced formulas in steady state equilibria of the third sub-system composed by the contingent consumptions:

$$\overline{c}_1 = \frac{\xi_1}{\lambda} + \frac{\xi_2}{\lambda} \cdot \overline{\tau}, \quad (48.1)$$

$$\overline{c}_2 = \frac{r_1 \cdot \xi_1}{\lambda} + \frac{r_1 \cdot \xi_2}{\lambda} \cdot \overline{\tau}. \quad (48.2)$$

4.4.2 Uniqueness of the steady-state equilibria with fixed exchange rates

Now, we must analyze the properties of local uniqueness and determinacy, focusing in the mapping from allocations to prices of equilibria. As in the case with floating exchange rates, our equilibria are regular according with the two conditions that we defined in the previous section. Thus, our equilibria are locally unique and determinate.

The core dynamic sub-system in the steady state, that is, $\left(\overline{\tau}, \overline{z}, \overline{b}, \overline{q}, \overline{k}\right)$ is always determinate and unique since is not associated with the vector of foreign interest rates. Moreover, for a given vector $\left(\overline{D}_1\right)$ there is a vector of interest rates $\left(r_1, r_2 = R\right)$ that together satisfy the equilibrium conditions. There is a one-to-one mapping that gives us locally unique and determinate equilibria.

4.4.3 Comparative statics in the steady-state equilibrium with a fixed exchange rates

Now, we proceed to analyze what will happen with our key variables when we change a policy parameter in the steady state equilibria. Through the comparative statics we study how the changes in the policy parameters affect the instrument of short-term debt and the contingent consumptions.

4.4.3.1 Comparative statics of the short-term debt

a) Inflation in the rest of the world versus short-term debt

$\frac{\partial \bar{D}_1}{\partial \sigma^*} < 0$; an increase in the inflation in the rest of the world reduces the return on foreign and domestic currency; we can see from equation (39) that when the inflation rate in the rest of the world increases (decreases), the return of the currency reserves, either in domestic or foreign currency, will decrease (increase) causing a decrease (increase) in the quantity of short-term debt demanded by the domestic banks.

b) Foreign interest rate on short-term debt versus short-term debt

$\frac{\partial \bar{D}_1}{\partial r_1} < 0$; the interest rate r_1 represents the price charged to domestic banks when they borrow short-term from the rest of the world. The intuition behind this result is consistent with the law of demand, which states that when the price of a good goes up (goes down), this typically causes the quantity demanded for that good to fall (increase). As a consequence, the quantity of short-term debt that domestic banks would wish to borrow will decrease (increase) since in general, consumers prefer low prices for the goods/assets that they demand.

c) Reserve requirements in domestic currency versus short-term debt

$\frac{\partial \bar{D}_1}{\partial \phi_d} < 0$; when banks are forced to hold a high fraction of their deposits in the Central Bank's vaults in the form of domestic currency, the quantity of short-term debt demanded by domestic banks will decrease.

d) Reserve requirements in foreign currency versus short-term debt

$\frac{\partial \bar{D}_1}{\partial \phi_f} > 0$; as we can see from equation (39), when there is an increment in the fraction of the deposits in foreign currency that domestic banks must hold at the Central bank's vaults, they will increase, as well, their desired quantity of short-term debt in order to meet their obligations in t.

e) The central bank's remuneration on currency reserves versus short-term debt

$\frac{\partial \bar{D}_1}{\partial \rho} > 0$; increases in the exogenous policy parameter $\rho \geq 1$ signify that the complementary remuneration on currency reserves paid by the domestic central bank is now higher. This higher return on currency reserves in $t+1$ in turn will causes the amount of short-term debt will increase.

f) The return on the long-term illiquid technology versus short-term debt

$\frac{\partial \bar{D}_1}{\partial R} > 0$; as we can see from equation (39) an increment (diminution) in the gross real return on the long-term illiquid investment will augment (reduce), too, the quantity of short-term.

4.4.3.2 Comparative statics of the state-contingent consumptions

a) The inflation in the rest of the world versus state-contingent consumption

$\frac{\partial \bar{c}_1}{\partial \sigma^*} < 0, \frac{\partial \bar{c}_2}{\partial \sigma^*} < 0$; when the rate of inflation in the rest of the world increases (decreases), it is easy to see from equations (38) and (39), that the quantity of $\bar{c}_1$ and $\bar{c}_2$ will decrease (increase) due the diminution in the return of the currency reserves in form of dollars.

b) Reserve requirements in domestic currency versus the state-contingent consumption

$\frac{\partial \bar{c}_1}{\partial \phi_d} < 0, \frac{\partial \bar{c}_2}{\partial \phi_d} < 0$; when banks are forced to hold a high fraction of their deposits in the Central Bank's vaults in the form of domestic currency, both consumptions will decrease since there will be a decrease in the short-term debt demanded by domestic banks.

c) Reserve requirements in foreign currency versus the state-contingent consumption

$\frac{\partial \bar{c}_1}{\partial \phi_f} > 0, \frac{\partial \bar{c}_2}{\partial \phi_f} > 0$; when banks hold a high fraction of their deposits in the Central Bank's vaults in the form of foreign currency, the consumption in t will increase since the short-term debt increases too. $\bar{c}_2$ will increase because the return that banks will receive in $t+1$ for their reserves is higher; causing an augmentation in $\bar{c}_2$.

d) The interest rate on short-term debt versus the state-contingent consumption by the impatient

$\frac{\partial \bar{c}_1}{\partial r_1} < 0$, the quantity of short-term debt that domestic banks desire will decrease (increase) if the interest rate that domestic banks need to pay for that debt increases (decreases); then banks will have less resources to cover the withdrawals of impatient households causing a diminution (augmentation) in $\bar{c}_1$.

e) The interest rate on short-term debt versus the state-contingent consumption by the patient

$\frac{\partial \bar{c}_2}{\partial r_1} = 0$, the consumption by patient households does not depend on the interest rate on short-term debt.

f) The central bank's remuneration on currency reserves versus the state-contingent consumption by the impatient

$\frac{\partial \bar{c}_1}{\partial \rho} > 0$; the short-term debt will augment if there is a high rate of return on the currency reserves; the domestic banks will receive a high return in $t+1$, augmenting as well the consumption of the impatient households.

g) The central bank's remuneration on currency reserves versus the state-contingent consumption by the patient

$\frac{\partial \bar{c}_2}{\partial \rho} > 0$; when the return on the reserves that banks will receive in $t+1$ increases, banks will have more resources in this period leading an augmentation in the consumption of patient households.

h) The return on the illiquid long-term technology versus the state-contingent consumption by the impatient

$\frac{\partial \bar{c}_1}{\partial R} > 0$; the quantity of short-term debt will increase (decrease), as we can see from equation (39), whit an augment (diminution) in the gross real return on the long-term illiquid storage technology; causing an increment in the consumption of impatient households as well.

i) The return on the illiquid long-term technology versus the state-contingent consumption by the patient

$\frac{\partial \bar{c}_2}{\partial R} > 0$; if the return on the long-term illiquid investment augments, banks will augment too the consumptions of the patient households since they have more available resources.

4.5 The dynamic equilibria with fixed exchange rates

All the dynamic sub-systems inherit their dynamics from $\bar{\tau}_{t-1}$ since the exchange rate regime that is in place. We will star our analysis of the dynamic equilibria with the core sub-system; later, with the foreign debt sub-system and finally, the contingent consumptions sub-system.

4.5.1 The core dynamic sub-system under fixed

Equations (43.1) to (43.5) set up the core dynamic sub-system: all of these are difference equation of first order that display linear dynamics. As these variables display linear dynamics, they will have a unique root given by the coefficient associated to $\bar{\tau}_{t-1}$. We can see that all associated coefficients are positive; so, all the roots in this sub-system are unique and positives.

4.5.2 The dynamic sub-system for the foreign debt instrument under fixed exchange rates

Equation (44) is a first order difference equation and its dynamic is inherited from $\bar{\tau}_{t-1}$. Unlike the case of the core dynamic sub-system, there will be a unique root given by ξ_2, that is negative.

4.5.3 The dynamic sub-system of state-contingent consumption with fixed exchange rates

This dynamic sub-system is given by equations (45.1) and (45.2). As we said before, they inherit their dynamics from $\bar{D}_{1,t+1}$ and, clearly, from $\bar{\tau}_{t-1}$. So, as expected, they have a negative and unique root too; the root for the impatient consumptions is given by $\frac{\xi_2}{\lambda}$, and the one for the patient consumption is $\frac{r_1 \cdot \xi_2}{\lambda}$.

4.6 Analysis of equilibrium dynamics of the short-term debt and contingent consumptions

We now describe the dynamic properties of the short-term debt and the contingent consumptions. Also, we construct different scenarios that analyze different combinations of parameters in order to make our study more inclusive.

4.6.1 Dynamic behavior under a baseline scenario

We defined a baseline scenario with the following values of the parameters: $\lambda = 0.2$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $R = 1.05$, $r_1 = 1.02$, $\varepsilon = 1.02$ and $\rho = 1.1$. The dynamic properties of equilibria change significantly once the exchange rate is fixed: dynamic system is linear and of first order, determinacy is not a real issue here, but stability and volatility do matter. The characteristic root of $\bar{D}_{1,t+1}$ is typically negative, but it could lie inside or outside the unit circle. Thus, the steady-state equilibria display a scope for stable fluctuations for most values in the parameter space. This dynamic is inherited by the households' consumption $\bar{c}_{1,t}$ and $\bar{c}_{2,t+1}$. However, for some values of θ that are sufficiently high, the eigenvalue becomes positive and stable, thus reducing the scope for volatility, while increasing ϕ_d when it is close enough to 1 leads to unstable fluctuations.

4.6.2 Dynamic behavior in different scenarios

As in the case with floating exchange rates, we complete our analysis of this regime by studying what would happen to dynamic orbits when the equilibria lie in different partitions of the parameter space. With fixed exchange rates, however, we only had to evaluate different values for the backing of the domestic money supply (θ); we do not change the proportion of impatient households (λ) because this parameter does not alter the dynamics of the system under a hard peg. We will have three different scenarios, each for one of the values that θ may take, which we describe below.

Scenario 1. The first scenario is when the backing of the domestic money supply is fairly low, and we chose the particular value $\theta = 0.05$. Regarding the three state variables $\bar{D}_{1,t+1}$, $\bar{c}_{1,t}$ and $\bar{c}_{2,t+1}$, the dynamic orbits that approach the steady-state equilibrium display stable fluctuations in most of the cases. However, when ϕ_d is close enough to 1 and the world inflation rate (σ^*) is sufficiently low, the fluctuations become unstable. Moreover, as the foreign inflation rate increases, the stability of the system increases as well.

Scenario 2. The second scenario consists of a medium to high backing of the domestic money supply, which we set to $\theta = 0.8$. Independently of the values of the parameters, for high values of σ^*, the dynamic equilibria around the steady-state values of $\bar{D}_{1,t+1}$, $\bar{c}_{1,t}$ and $\bar{c}_{2,t+1}$ converge monotonically. However, values of σ^* that are low enough lead to stable fluctuations and endogenously arising volatility. In general, increasing σ^* reduces the volatility of the system as well.

Scenario 3. The third scenario contemplates a backing of the domestic money supply that is very close to 100% $(\theta = 0.98)$. As θ approaches a threshold value of 0.98, the stability of the system increases and the scope for volatility is eliminated. In particular, the dynamic orbits around the steady-state equilibrium values of $\bar{D}_{1,t+1}$, $\bar{c}_{1,t}$ and $\bar{c}_{2,t+1}$ become unstable as σ^* increases.

As in the case of floating exchange rates, the contingent consumptions inherited their dynamic behavior from the instrument of short-term debt; basically, they displayed dynamic paths with the same sign of the eigenvalues, but the effects were magnified. In this respect, we find that the backing of the domestic money supply plays the role of stabilizing the dynamic system: when the backing of the money supply (θ) increases, the stability of the system increases as well (i.e., the magnitude of the characteristic roots become smaller). Moreover, under our assumptions, a high rate of the world inflation tends to reduce the volatility of the system; but the combination of high values in the backing of the domestic money supply with high rates of world inflation increases the volatility of the system.

5. The consequences of a sudden stop of international capital on the domestic economy

In this section, we analyze in some detail what would be the effects on the aggregate economy in case a sudden stop arose. We star with some definitions and then to proceed to the analysis of each exchange rate regime.

5.1 A sudden stop in the economy

In this section, we introduce the possibility of a sudden stop under each of the exchange rate regimes considered: floating and a hard peg[16]. We define a sudden stop as an intrinsic and unanticipated shock that consists in a significant reduction of the inflows of international capital with respect to what would be promised or expected in the standard contingent contracts. We will analyze the consequences of this shock and we will determine under which circumstances a banking crisis would arise in equilibrium for both floating and fixed exchange rates.

5.1.1 Extrinsic versus intrinsic uncertainty

In economic theory, we deal most of the time with two types of uncertainty: extrinsic and intrinsic uncertainty, respectively.

Throughout this section, we will focus on the effects of a particular kind of intrinsic uncertainty, but first must define how these two types of uncertainty differ, especially in the context of our model.

The extrinsic uncertainty is the kind of uncertainty that does not come from a variation in the economic fundamentals; it is, instead, the uncertainty that could be represented by, for instance, sunspots. A clear-cut example of extrinsic uncertainty would be when all the households share a nasty nightmare about all the banks closing: there is no good reason for it, just like a sunspot, but the public has a firm believe that this would happen, and they will behave accordingly. This kind of uncertainty could be perpetuated by the presence of self-fulfilling prophecies, since, by definition, they could cause something to become true through the public's actions in the markets.

Meanwhile, the intrinsic uncertainty arises when there is a completely unanticipated change in one or more of the fundamentals of the economy as they are perceived by the public. Of course this change might itself trigger changes in the public's expectations as well. We will introduce this type of uncertainty in our model in the form of a sudden stop: this unanticipated change in the fundamentals of the economy will come up as a reduction on the quantity of the short-term funds that banks could borrow from the rest of the world in the afternoon of t. Banks originally planned to borrow $D_{1,t+1}$ goods at the moment when they formulated their contingent plans. But, without any anticipation possible they learn in the afternoon of t that they will get instead only $d_{1,t+1} < D_{1,t+1}$ as opposed to what their contingent plans were counting on: in this sense, a sudden stop could be thought as a rationing on the credit amount of $D_{1,t+1}$ that domestic banks can borrow.

5.1.2 Liquidity and solvency

When this reduction in the quantity of the amount available to borrow short-term at the end of t hits the economy, it reduces the resources available to banks, which are reduced to $d_{1,t+1}$

[16] We must mention that during the 1994 crisis, Mexico was under an intermediate peg. Thus, the scenario with the hard peg does not replicate what happened: the central bank was not fully committed to sustaining the peg.

goods, where $0 \le d_{1,t+1} < D_{1,t+1}$ must hold. Now the banks' contingent plan needs to change and, as a consequence, it could lead to a lack of liquidity, of solvency or both.

Banks are in an illiquid position when they do not have enough liquid assets to meet their payment obligations. In our model, at the end of t, we say that banks have an illiquid position when the following condition holds[17]:

$$c_{1,t} > x \cdot k_t . \qquad (49)$$

(49) means that banks will be illiquid if all the depositors decided to withdraw all their deposits and banks did not have enough resources to pay them all, even if they liquidated early all of their long-term investment, that is: when $l_t = k_t$ obtains. Note that in our model, equation (49) is a necessary but not sufficient condition for a bank run to occur in equilibrium.

However, the banks' illiquid position could just be the reflection of a temporary problem; even if equation (49) is satisfied, it could still be feasible to save a bank if one proved that the bank is illiquid but solvent. If on the contrary, the bank is illiquid and insolvent, maybe it is not a good plan to save it.

Banks are illiquid and solvent when they have enough resources left after pay the depositors' withdrawals, even when banks liquidate early all their long-term investment and they use the real value of the new short-term debt. A bank is illiquid and insolvent if the next equation holds:

$$c_{1,t} > x \cdot k_t + d_{1,t+1} . \qquad (50)$$

If (50) is satisfied, then (49) is also satisfied, and it will be very probable that the bank will have to close.

5.1.3 Early liquidation and bank-runs

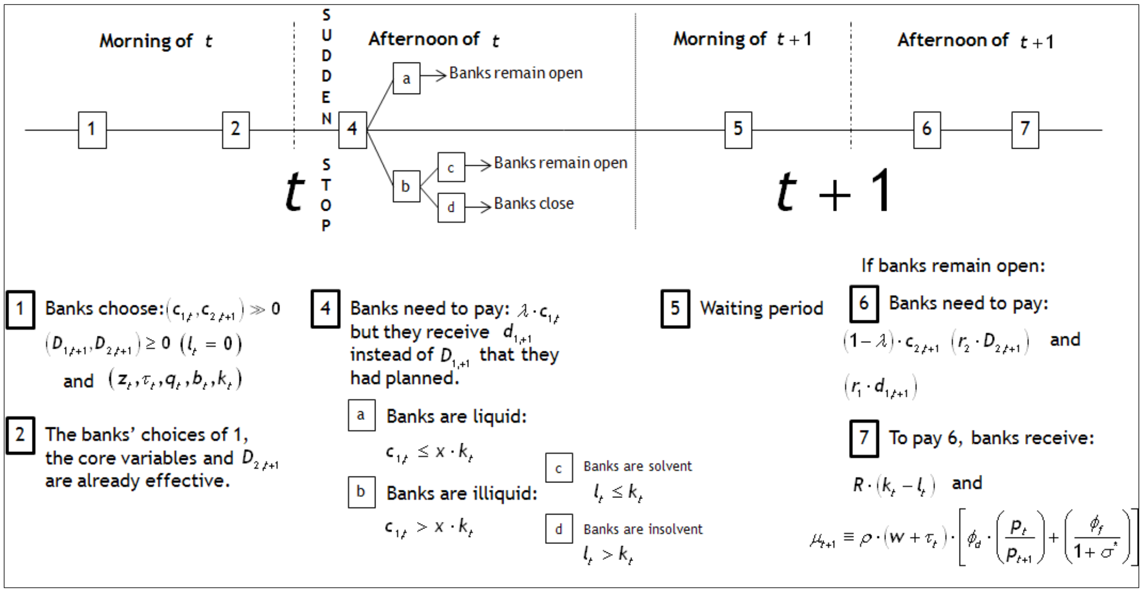
In the previous sections, we set up an environment where households would act according to their true type and banks would not need to liquidate their investments early. We established that at the beginning of t, banks formulated a plan for their pair of contingent consumptions $\left(c_{1,t}, c_{2,t+1}\right)$, and also, the plan that involved the state variables $\left(z_t, \tau_t, b_t, q_t, k_t\right)$, the debt vector $\left(D_{1,t+1}, D_{2,t+1}\right) > 0$ and no early liquidation $\left(l_t = 0\right)$. At the end of t, banks would pay $c_{1,t}$ goods to each of the $0 < \lambda < 1$ impatient households without the need of early liquidation, since this was according to their plan. Next, at the end of $t+1$, the banks would pay $c_{2,t+1}$ goods to each of the $0 < \left(1-\lambda\right) < 1$ patient households, together with the principal plus interest associated to the loans that they acquired from the foreign banks. But now, an unexpected shock hits the economy in the afternoon of t and this plan needs to be modified accordingly.

In the absence of this shock, banks would receive $D_{1,t+1}$ units of the good from the foreign banks in the afternoon of t, and this quantity was enough to pay the withdrawals of the impatient households as they had planned for it; but now they just learned that they will just receive $d_{1,t+1}$, which is a smaller quantity of goods and it is not sufficient to cover the withdrawals. Thus, banks would need to liquidate a fraction, or the full quantity, of their long-

[17] This equation and the next one are the same for both regimes, floating and fixed exchange rates.

term investment[18] in order to compensate for the reduction on the short-term debt available to them, and to be able to pay the withdrawals by the impatient households. This sequence of events is represented in Figure 5.

Figure 5: Sequence of events with a sudden stop of international capital



We need to find out how much of the long-term investment the domestic banks need to liquidate in order to determine the situation of banks in terms of liquidity and solvency. Using equation (10) together with the fact that $l_t > 0$, we can solve for the implied l_t; notice that, according to the banks' contingent plan

$$l_t = \frac{\lambda \cdot c_{1,t} - D_{1,t+1}}{x} = 0. \qquad (51.1)$$

But we need to modify this equation since banks now receive only $d_{1,t+1}$ instead of $D_{1,t+1}$ goods; so, equation (51.1) changes to:

$$l_t = \frac{\lambda \cdot c_{1,t} - d_{1,t+1}}{x} > 0 \qquad (51.2)$$

Then equation (51.2) represents the quantity of early liquidation l_t that banks would need to do in order to be able to afford the payment of the withdrawals that they had planned for the end of t.

5.1.4 Suspension of convertibility

Following the work of Diamond and Dybvig (1983) and Chang and Velasco (2001), the suspension of convertibility is a tool that can be used by banks in order to avoid bank runs. Banks use this tool when they decide to suspend payments to depositors[19] in the afternoon of t after they have paid $c_{1,t}$ goods only to a fraction λ of the agents. After this, the banks will stop

[18] We will find the conditions that make $l_t \leq k_t$ or $l_t > k_t$.

[19] This is the equivalent of period 1 in Diamond and Dybvig (1983) and Chang and Velasco (2001).

paying withdrawals (since they will assume that all the impatient agents have withdrawn) and they will open again in the afternoon of $t+1$.

We must remember that in the original problem, it was the imposition of the expression in (8) what ensured that no household would be willing to misrepresent her type. Assuming this, banks formulated their contingent plans that included the payment of $c_{1,t}$ goods to the λ impatient agents and $c_{2,t+1}$ goods to the $(1-\lambda)$ patient agents. In this scenario banks did not need the early liquidation in the afternoon of t since they had enough resources to do the payments they planned to the impatient agents. However, when such a shock hits the economy in the afternoon of t, things are not the same anymore: there are fewer resources available at the end of t and the patient agents can feel nervous thinking that they might not receive their payment if they choose to wait as planned.

The suspension of convertibility can relax these assumptions and, at the same time, decrease the possibility of bank runs in the economy. If a patient household knows that banks will only serve the first λ depositors by the end of t, then she knows with almost certainty that banks will have enough resources to pay at least $c_{2,t+1}$ goods each to the $(1-\lambda)$ patient agents by the end of $t+1$. Since $c_{2,t+1} > c_{1,t}$ holds, it is a dominant strategy for the patient households not to misrepresent their type. As we can see from equation (10), if banks establish that they will pay only to the first λ withdrawals, patient households will not fear that there will not be enough resources to pay them: the banks will not suffer from any troubles because that is the quantity that they planned in the first place without the need of liquidating early. With this tool, the likelihood that banks will have an illiquid position or solvency problems is fairly low, unless the reduction in $D_{1,t+1}$ is too severe.

5.2 Do banks close or not under floating exchange rates?

In this subsection we will find the conditions under which a bank closes or remains open with a policy of floating exchange regime in place. We establish, on the one hand, that if banks are illiquid but solvent, then $\hat{l}_t \leq \hat{k}_t$ obtains, and they do not have to close. On the other hand, if banks are illiquid and insolvent, $\hat{l}_t > \hat{k}_t$ would obtain instead and there would be a fairly high possibility that they will have to close.

To find whether a bank will close or not, we first obtain the value of $\hat{l}_t$ implied by the sudden stop and then we proceed to compare it with the amount of long term investment. We also fix the value of $\hat{c}_{1,t}$ to what it was originally set, and we substitute it in (51.2), obtaining:

$$\hat{l}_t = \frac{\lambda}{r_1 \cdot x} \cdot \left[R \cdot \hat{k}_t + \rho \cdot (w + \hat{\tau}_t) \cdot \left(\frac{\phi_d}{1+\sigma} + \frac{\phi_f}{1+\sigma^*} \right) - r_2 \cdot f_2 \right] - \frac{\hat{d}_{1,t+1}}{x}. \quad (52)$$

Now that we have found the value of $\hat{l}_t$ we must compare it against $\hat{k}_t$; to facilitate this comparison we will first restructure (52) by defining some very helpful and necessary functions. In the first place, we define

$$\hat{n}_t \equiv \frac{\lambda \cdot R \cdot \hat{k}_t}{r_1 \cdot x}, \quad (53.1)$$

which is the first term of (52), while the second term of (52) is given by:

$$\hat{o}_t \equiv \frac{\lambda \cdot \rho \cdot \left(w + \hat{\tau}_t\right)}{r_1 \cdot x}\left(\frac{\phi_d}{1+\sigma} + \frac{\phi_f}{1+\sigma^*}\right). \tag{53.2}$$

Finally, we put together the third and fourth terms of (52) and obtain:

$$\hat{q}_t \equiv \frac{\lambda \cdot r_2 \cdot f_2}{r_1 \cdot x} + \frac{\hat{d}_{1,t+1}}{x}. \tag{53.3}$$

Now we must rewrite $\hat{l}_t$ from (52) using (53.1) -(53.3):

$$\hat{l}_t = \hat{n}_t + \hat{o}_t - \hat{q}_t, \tag{53.4}$$

Now we must compare this expression against $\hat{k}_t$.

We found it easier to first compare $\hat{n}_t$ versus $\hat{k}_t$, and then find the relationship between $\hat{o}_t$ and $\hat{q}_t$; finally, we put the results of this comparisons together, thus finding the relationship between $\hat{l}_t$ and $\hat{k}_t$. Depending of some assumptions, different scenarios arise in this comparison.

<u>Case 1</u>: $\frac{\lambda \cdot R}{r_1 \cdot x} > 1$ holds; then, it is easy to see that $\hat{n}_t > \hat{k}_t$ obtains. However, when we compared $\hat{o}_t$ versus $\hat{q}_t$, two distinct sub-cases arose:

a) When $\hat{o}_t \geq \hat{q}_t$ obtains, then $\left(\hat{n}_t + \hat{o}_t - \hat{q}_t\right) = \hat{l}_t > \hat{k}_t$ will arise, which implies that there will be panic in the economy and the banks will close.
b) When $\hat{o}_t < \hat{q}_t$ obtains, then we cannot determine whether $\hat{l}_t$ lies above or below $\hat{k}_t$ and we cannot ensure whether there will be a bank panic or not: the prediction is ambiguous.

<u>Case 2</u>: $\frac{\lambda \cdot R}{r_1 \cdot x} = 1$ obtains. In this case, $\hat{n}_t = \hat{k}_t$ and it is easy to see that $\hat{l}_t > \hat{k}_t$ obtains, which replicates the sub-case (a) of Case 1.

<u>Case 3</u>: $\frac{\lambda \cdot R}{r_1 \cdot x} < 1$ obtains. In this case, $\hat{n}_t < \hat{k}_t$ will always arise, but we must also specify two sub-cases that compare $\hat{o}_t$ and $\hat{q}_t$.

a) When $\hat{o}_t > \hat{q}_t$ obtains, we cannot determine unambiguously whether $\hat{l}_t > \hat{k}_t$ or $\hat{l}_t \leq \hat{k}_t$ holds. We would need additional information for solving the ambiguity and being able to predict whether banks will suffer runs from the public or not.

b) When $\hat{o}_t \leq \hat{q}_t$ obtains, then it will always be the case that $\hat{l}_t < \hat{k}_t$. Thus, no panics would be observed since banks would be illiquid but solvent, and banks will not have to close.

In what follows we present several numerical examples where the different cases may arise: we fix some of the parameter values and evaluate each of the conditions.

5.2.1 Alternative scenarios of sudden stops

In this subsection we will look for the combinations of parameters under which a bank run would obtain in equilibrium under floating.

Whenever $0.001 \le \hat{d}_{1,t+1} < 1.11$ obtains, this is enough to guarantee that $\hat{d}_{1,t+1} < \hat{D}_{1,t+1}$ will be observed in equilibrium under floating. Next, since (51.2) holds, we will be able to calculate the amount $\hat{l}_t$ of early liquidation that banks will need in order to compensate for the reduction in the short-term debt, as well as the long-term investment $\hat{k}_t$; as we said before, this relationship is important because it defines if banks will close or not.

The baseline scenario and Case 3. We define the baseline scenario, which is given by the following values of the parameters: $\lambda = 0.5$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $R = r_2 = 1.05$, $r_1 = 1.02$, $\sigma = 0.05$, $\sigma^* = 0.04$, $\varepsilon = 1.01$, $\rho = 1.3$, $w = 3$, $x = 0.9$ and $f_2 = 1$. Under this scenario, we found that $\frac{\lambda \cdot R}{r_1 \cdot x} < 1$ and the economy would be always in what we have called Case 3. Accordingly, with some combinations of parameters, the sub-cases c) and d) can be found. However, we eliminate the ambiguously of sub-case c) since we found that bank runs will never be equilibrium in this scenario. The exceptions are when $0.5 \le \phi_d \le 0.9$, $0.5 \le \phi_f \le 0.9$ or $0.7 \le \lambda \le 0.8$: in this case $\hat{l}_t > \hat{k}_t$ will obtain and banks will close. It makes sense that for high values of ϕ_d, ϕ_f and λ banks will close: in the first two cases, banks must hold a high proportion of their deposits in the central bank's vault, this will lead to lack of liquidity in the banks' means of payment causing their close. In the third case when a large proportion of households become impatient, there will be a high probability that banks will not have sufficient funds to pay them or if banks pay all the withdrawals to the impatient households it is very probable that banks will run out of sources in that period causing that banks close.

In order to complete our analysis, we set up different scenarios to study Cases 1 and 2.

Scenario 1. We take the same values as in the baseline scenario but we change the value of λ. The proportion of impatient households that we take for this scenario is $\lambda = 0.875$; through this value we ensure that the economy is always in what we have called Case 1 and 2. In this scenario the sub-cases a) and b) can arise, but now ambiguously appears in both sub-cases. Depending purely on the combination of some parameters, not of the sub-cases, banks will close or not. On the one hand, for example, values of $1.01 \le \varepsilon \le 1.15$ or $0 < \theta < 1$ coupled with $0.669 < \hat{d}_{1,t+1} < 1.11$, ensure us that banks will never close. But, on the other hand, we found that banks will close for sure when $0.3 \le \phi_d \le 0.9$ or $0.3 \le \phi_f \le 0.9$.

Scenario 2. In this scenario instead of change λ, we change the interest rate that banks will receive when they decide to liquidate early, we impose $x = 0.5$. As in the previous scenario, we find the sub-cases a) and b) with ambiguously in both. However, we find that for values of $1.01 \le \varepsilon \le 1.15$ or $0 < \theta < 1$ together with values of $0.409 \le \hat{d}_{1,t+1} < 1.11$ banks will remain open. But, when $0.5 \le \phi_d \le 0.9$ or $0.5 \le \phi_f \le 0.9$ banks will definitely close.

5.3 Do banks close or not under fixed exchange rates?

Following our purpose to replicate the 1994 Mexican crisis, now we introduce uncertainty in our model under fixed exchange rates. Like in the previous section, the uncertainty will come as an unanticipated change in the fundamentals of the economy, that is a reduction in $\bar{D}_{1,t+1}$. Now banks receive $\bar{d}_{1,t+1}$ goods from the foreign banks instead of $\bar{D}_{1,t+1}$.

Following the same explanation that under floating exchange rates, due this reduction in the short-term debt, banks can have an illiquid position and, in some cases, an insolvent position too. Remember that if (49) is satisfied, banks are illiquid but solvent and this condition is not sufficient for a bank run in our model. Also if (50) holds, banks are both, illiquid and insolvent, and the bank run is almost imminent.

As we said before, since the banks receive less debt from the foreign banks, they need to re-optimize their problem. Thus, they need to find the quantity of $\bar{l}_t$ that they will liquidate to compensate the reduction in $\bar{D}_{1,t+1}$.

Now we will find the conditions under which a bank closes or remains open. First we set up that if $\bar{l}_t \le \bar{k}_t$, banks do not close; otherwise, if $\bar{l}_t > \bar{k}_t$, the probability that they close is higher. As we did in the previous section, we find the value of $\bar{l}_t$ and then we compare it with $\bar{k}_t$. We rewrite equation (52) since now the fixed exchange regime is in place and we get:

$$\bar{l}_t = \frac{\lambda}{r_1 \cdot x} \cdot \left[R \cdot \bar{k}_t + \rho \cdot \left(w + \bar{\tau}_t \right) \cdot \left(\frac{\phi_d}{1+\sigma^*} + \frac{\phi_f}{1+\sigma^*} \right) - r_2 \cdot f_2 \right] - \frac{\bar{d}_{1,t+1}}{x}. \tag{54}$$

In this subsection, we also restructure (54) to facilitate the comparison with $\bar{k}_t$. The first term of (54) is now defined by

$$\bar{n}_t \equiv \frac{\lambda \cdot R \cdot \bar{k}_t}{r_1 \cdot x}, \tag{55.1}$$

the second term of (54) is now:

$$\bar{o}_t \equiv \frac{\lambda \cdot \rho \cdot \left(w + \bar{\tau}_t \right)}{r_1 \cdot x} \cdot \left(\frac{\phi_d}{1+\sigma^*} + \frac{\phi_f}{1+\sigma^*} \right), \tag{55.2}$$

and, to conclude, the third and fourth terms of (54):

$$\bar{q}_t \equiv \frac{\lambda \cdot r_2 \cdot f_2}{r_1 \cdot x} + \frac{\bar{d}_{1,t+1}}{x}. \tag{55.3}$$

A quick review to the previous section is sufficient to notice that equations (55.1) to (55.3) are very similar to the equations (53.1) to (53.3), they share a similar structure, since $\bar{l}_t = \bar{n}_t + \bar{o}_t - \bar{q}_t$ obtains. With this in mind, we will make the same assumptions that we did under floating exchange rates; Case 1 will arise when $\frac{\lambda \cdot R}{r_1 \cdot x} > 1$ holds, and the a) and b) sub-cases will take place too with their respective explanations. Moreover, Case 2 will be accomplished when $\frac{\lambda \cdot R}{r_1 \cdot x} = 1$, and we will follow the same explanation that under floating exchange rates. Finally, when $\frac{\lambda \cdot R}{r_1 \cdot x} < 1$ is satisfied, Case 3 will come up with its associated sub-cases c) and d).

5.3.1 Alternative scenarios of sudden stops

In this subsection we will find the relationship between $\bar{l}_t$ and $\bar{k}_t$ using different combinations of parameters, in order to determine if the banks will close or not.

When $0.001 \leq \bar{d}_{1,t+1} < 1.62$ obtains, this is sufficient to ensure that $\bar{d}_{1,t+1} < \bar{D}_{1,t+1}$ will be observed in equilibrium under fixed. Since (51.2) holds, we will be able to calculate the amount $\bar{l}_t$ of early liquidation that banks will need in order to compensate for the reduction in the short-term debt, as well as the long-term investment $\bar{k}_t$; because this relationship is important since it defines if banks will close or not.

The baseline scenario and Case 3. We define the baseline scenario, which is given by the following values of the parameters $\lambda = 0.5$, $\phi_d = \phi_f = 0.1$, $\theta = 0.2$, $R = r_2 = 1.05$, $r_1 = 1.02$, $\sigma^* = 0.04$, $\varepsilon = 1.01$, $\rho = 1.3$, $w = 3$, $x = 0.9$ and $f_2 = 1$. Under this scenario, we found that $\frac{\lambda \cdot R}{r_1 \cdot x} < 1$ and the economy would be always in what we have called Case 3. In accordance with some combinations of parameters, the sub-cases c) and d) can be found. Nonetheless, we eliminate the ambiguously of sub-case c) since we found that bank runs will never be equilibrium in this scenario. But, banks will close for sure only when $0.8 \leq \phi_d \leq 0.9$, $0.8 \leq \phi_f \leq 0.9$ or very low values of $\bar{d}_{1,t+1}$ coupled with $\lambda = 0.8$. This result makes sense because when the banks need to hold a high proportion of their deposits as a reserves in the central bank's vault, it is very probable that once that the households have decided to withdraw, the banks will face a problem of illiquidity and insolvency. Also, when a high fraction of households become impatient, banks will face illiquidity since they will not have enough resources to meet their payments. The insolvency will come too since banks will receive a very low quantity of short-term debt from the foreign banks. Due these events, banks will close for sure.

Now we define some scenarios to study Case 1 and 2 in our aim to complete our analysis.

Scenario 1. We take the same values as in the baseline scenario but we change the proportion of impatient households, we establish $\lambda = 0.875$; through this value we ensure that the economy is always in what we have called Case 1 and 2. In this scenario the sub-cases a) and b) can arise, but now ambiguously appears in both sub-cases. The event of bank run will depend purely on the combination of some parameters, not of the sub-cases. For instance, banks will never close when values of $1.01 \leq \varepsilon \leq 1.15$ or $0 < \theta < 1$ are together with $0.673 < \bar{d}_{1,t+1} < 1.62$. But, banks will close for sure when $0.4 \leq \phi_d \leq 0.9$ or $0.4 \leq \phi_f \leq 0.9$.

Scenario 2. Unlike the previous scenario, now we change x, we impose this parameter to be $x = 0.5$. As in the previous scenario, we find the sub-cases a) and b) with ambiguously in both.

Nevertheless, bank run will never be equilibrium when $1.01 \le \varepsilon \le 1.15$ or $0 < \theta < 1$ are coupled with values of $0.423 \le \bar{d}_{1,t+1} < 1.62$. But for values of $0.8 \le \phi_d \le 0.9$ or $0.8 \le \phi_f \le 0.9$ banks will close.

6. Conclusions

The goal of this thesis was to decide whether a policy of floating exchange rates would have worked better for a model economy that replicated the main features that the Mexican economy had by the end of 1994, instead of the fixed exchange rates that was in place. We decided to study this crisis since all the repercussions and consequences that brought; for instance, the high rates of inflation, the decreases in the main economic indicator as the consumption, the investment, etc. This crisis caused an increase in the Mexico's poverty and a decrease in the standard of living of the Mexican people.

To try to answer our question, we started our study from a pure exchange economy based upon the micro-foundations, for later find the general equilibrium of this economy. We considered a pure exchange, monetary, small open economy with a DSGE framework in discrete time. We introduced nontrivial demands for the domestic and foreign fiat currencies that had no intrinsic value and the multiple reserve requirements that take the form of currency reserves. Later, we first modeled an economy with floating exchange rates for the full-information benchmark equilibrium, and then with fixed exchange rates also in the absence of early liquidation, sudden stops or bank panics.

Under floating exchange rates, we found a locally unique and determinate equilibrium. The stability, volatility and determinacy of the system depend enormously on the proportion of impatient/patient households and on the fraction of the backing of the domestic money supply. Besides, increases in the fraction of the reserves that domestic banks need to set apart, in the form of pesos, in the Central bank's vaults reduce the scope for determinacy and increase the volatility of the system. Also, the fraction of impatient/patient households increases the scope for indeterminacy of the system. In summary, the reserve requirements play the role of destabilizing and the proportion of impatient households decreases the determinacy of the system.

Under fixed exchange rates, we found a locally unique and determinate equilibrium too, but with a less complex dynamic behavior. Under this regime, determinacy is not a real issue but stability and volatility do matter and they mainly depend on the backing of the domestic money supply. The backing of the domestic money supply plays the role of stabilizing the dynamics of the system. Since in this regime the domestic country inherits the inflation world, this variable is important in the dynamic behavior of the system too. We found that the world inflation rate coupled with medium to low backing of the domestic money supply tend to increase the stability of the system. While the combination of high values on the reserve requirements in the form of pesos and low levels in the world inflation rate, cause unstable fluctuations in the system. We concluded that the backing of the domestic money supply tends to stabilize the system.

Later, because we wanted to replicate the main cause of the 1994 Mexican crisis, we introduced the possibility of a sudden stop of capital and we analyzed the consequences of this unexpected event under the two alternative exchange rates regimes mentioned above. According to this, we were able to analyze separately the effects that sudden stops and bank runs might have on the aggregate economy, as well as how these problems relate to each other.

We found that the amount of short-term debt that is part of banks' contingent plans is larger for the economy that operates under fixed exchange rates than for floating exchange rate in the event of a sudden stop of capital. This leads us to think that maybe the domestic banks under fixed exchange rates depend more on the foreign banks since they need more money of them in order to fulfill their contracts. Then, a sudden stop of capital will affect more to the economy under fixed exchange rates than under floating exchange rates. Moreover, on the one hand, we found that under floating exchange rates, some combinations of parameters would cause a bank run in the case a sudden stop hit the domestic country; and, as one might expect, the domestic banks subsequently would suffer from illiquidity and insolvency. On the other hand, our analysis of the economy that operated with fixed exchange rates -which was the case of the Mexican economy in 1994-, suggested as well that for some combinations of deep and policy parameters, the sudden stop of capital would cause a banking crisis, with the associated suspension of convertibility. We also found that that no matter the exchange rate regime in place there will be a bank run when banks need to hold a high fraction of their currency reserves in the Central bank's vaults. Under both regimes, this policy parameter plays a huge role in the destabilization of our economy. Meanwhile, high values of the new short-term debt coupled with any fraction of the domestic money supply, or together with any level of the interest rate associated with the foreign-reserves assets in the form of dollars, will increase the stability of the economy, diminishing the probability of a crisis.

Our principal contribution was not to explain what caused the 1994 Mexican crisis; instead, we explored the consequences that a sudden stop of capital could have caused under floating exchange rates. We found that the underlying exchange rate regime and the monetary policy in place were key to understanding this phenomenon. However, we cannot tell under which regime the crisis would not have been so harmful for the Mexican economy, since we found that under both regimes the crisis was possible.

We are aware that our model has certain limitations; for instance, we did not introduce in our model variables to represent the social and political events that contributed to the event of a crisis, such as the assassination of the Presidential Candidate Colosio and the rebellion in the south of Mexico by the EZLN. However, any model is a simplified representation of the world, and we must pick the battles we want to fight and which ones we do not. The social and political events surrounding the 1994 crisis clearly belong to the second category, given the questions that we wanted to answer in this stage of the research.

Maybe later on, we could think about introducing a variable to represent the quality of the Mexican financial market in order to complete our model and to do a better reproduction of the 1994 Mexican economy. Another issue in our research agenda is to model the actual intermediate peg that operated in Mexico in 1994. We would like to venture that a possible result would be that bank runs are more probable under an intermediate peg than in a floating or fixed exchange rate regime, but we left this to further studies.

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