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Effects of U.S. Quantitative Easing on Latin American Economies

César Carrera Fernando Pérez Forero Nelson Ramírez-Rondán

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César Carrera

Fernando Pérez Forero

Nelson Ramírez-Rondán

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Abstract

Emerging economies have been largely affected by the Fed's quantitative easing (QE) policies. This paper assesses the impact of these measures in terms of key macroeconomic variables for four small open economies in Latin America: Chile, Colombia, Mexico and Peru. We identify a QE policy shock in a structural VAR with block exogeneity (\dot{a} la Zha, 1999), and we impose a mixture of zero and sign restrictions (\dot{a} la Arias et al., 2014). Overall, we find that this QE policy shock has significant effects on financial variables such as aggregate credit and the exchange rate. These effects are larger than the ones produced on output and prices.

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César Carrera (E-mail: cesar.carrera@bcrp.gob.pe) is a researcher in the Macroeconomic Modeling Department; Fernando Pérez Forero (E-mail: fernando.perez@bcrp.gob.pe) is a researcher in the Macroeconomic Modeling Department; and Nelson Ramírez-Rondán (E-mail: nelson.ramirez@bcrp.gob.pe) is a researcher in the Research Division at the Central Reserve Bank of Peru, Jr. Miró Quesada 441, Lima, Peru.

1 Introduction

There has been widespread concern among policymakers in emerging economies about the effects of quantitative easing (QE) policies implemented in developed economies. This comes from the fact that these measures have triggered large surges in capital inflows to emerging countries, leading to exchange rate appreciation, high credit growth, and asset price booms. However, it is unclear whether these mentioned effects were transmitted to economic activity and inflation, and, if so, we do not know the propagation mechanism of these shocks. The latter is related to the fact that most central banks in these economies have implemented macroprudential policies with the purpose of mitigating any potential systematic risk.

Unconventional monetary policy measures were implemented in developed countries with the purpose of stimulating economic activity, since standard monetary policies became ineffective (the short-term interest rate reached its zero lower-bound (ZLB)). Walsh (2010) highlights that central banks do not directly control the money supply, inflation, or long-term interest rates (likely to be most relevant for aggregate spending); however, they can have a close control over narrow reserve aggregates such as the monetary base or a short-term interest rate. In short, operating procedures (the relationship between central bank instruments and operating targets) were crucial for the implementation of a QE policy.

A central bank that implements QE buys a specific amount of financial assets from financial institutions, thus increasing the monetary base and lowering the yield of those assets. Furthermore, QE may be used by monetary authorities to stimulate the economy by purchasing assets of longer maturity and thereby lowering longer-term interest rates further out on the yield curve (see Jones and Kulish, 2013).

Regarding the U.S. and the Fed, QE policy measures increased the private-sector liquidity, mainly through the purchase of long-term securities. That is, the QE episode in the U.S was characterized by a sharp increase in the size of the balance sheet of the Fed, together with an increase in money aggregates (e.g., M1), a decrease in the long versus short interest rates spread, and a short-term interest rate unchanged and very close to zero. Figure 1 depicts the policy rate close to zero starting in 2009 and, at the same time, how the spread between long- and short-term interest rates decreases at the same date. Figure 2 depicts the evolution of the Fed's balance sheet components. In particular, we can observe the switch toward securities, especially of mortgage-backed securities (MBS) and long-term Treasury bonds in early November 2008.

According to Baumeister and Benati (2012), unconventional policy interventions in the Treasury market narrowed the spread between long- and short-term government bonds. The latter triggered an increase in economic activity and a decline in inflation by removing duration risk from portfolios and by reducing the borrowing costs for the private sector. Moreover, according to Bernanke (2006), if the aggregate demand depends on long-term interest rates, then special factors that lower the spread between short- and long-term rates will stimulate the economy. Even more, Bernanke (2006) argues that if the term premium declines, then a higher short-term rate is required to obtain consistent financial conditions with maximum sustainable employment and stable prices.²

¹The central bank reduces the yields of long-term assets through the Large Scaled Asset Purchase (LSAP) program. As a result, the spread between long- and short-term rates decreases, since the short-term interest rate remains unchanged.

² Rudebusch et al. (2007) provide empirical evidence for a negative relationship between the term

7.0 -10-Year Treasury Constant Maturity Rate - - Effective Federal Funds Rate 6.0 5.0 4.0 3.0 2.0 QE2 1.0 0.0 2007 Q1 2008 Q1 2009 Q1 2010 Q1 2011 Q1 2012 Q1 2013 Q1

Figure 1. Long- and short-term interest rates

Source: Federal Reserve Economic Database (FRED).

Starting in 2009, central banks in the U.S., the U.K., Canada, Japan, and the Euro area reduced their policy rates close to the ZLB. At the same time, these institutions used alternative policy instruments and adopted macroprudential measures focused on close monitoring and supervision of financial institutions. Financial stability became one of the main policy targets. The expansion of the central bank's balance sheet through purchases of financial securities and announcements about future policy (influencing expectations) was the usual policy.³

Jones and Kulish (2013), Hamilton and Wu (2012), Gagnon et al. (2011), and Taylor (2011) analyze the effects of the QE policy on the global economy. However, most of these researchers focus their attention on financial variables such as long-term interest rates and aggregate credit. There are some other authors who analyze the QE policy effects on other key macroeconomic variables within the same economy: Glick and Leduc (2012) study the case of the U.S.; Lenza

premium and economic activity. The authors show that a decline in the term premium of 10-year Treasury yields tends to boost GDP growth.

³ Unconventional monetary policy measures are other forms of monetary policy that are used when interest rates are very close the ZLB. These measures include QE policy, credit easing, and signaling. Regarding credit easing, a central bank purchases private-sector assets in order to improve liquidity and credit access. The signaling policy is referred to central bank communication, i.e., the use of statements with the purpose of lowering market expectations of future interest rates. For example, during the credit crisis of 2008, the U.S. Fed indicated rates would be low for an "extended period" and the Bank of Canada made a "conditional commitment" to keep rates at the lower bound of 25 basis points until the end of the second quarter of 2010.

4000 - Securities Held Outright

Support for Specific Institutions

All Liquidity Facilities

2500 - 1500 - 1000 -

Figure 2. Fed's balance sheet

Source: Federal Reserve Economic Database (FRED).

et al. (2010) and Peersman (2011) study the Euro area; and Schenkelberg and Watzka (2013) cover the case of Japan. Gambacorta et al. (2012) perform a similar analysis for eight advanced countries. Belke and Klose (2013) and Fratzscher, Marcel and Lo Duca, Marco and Straub, Roland (2013) study the spillover effects between the U.S. and the Euro area. Baumeister and Benati (2012) quantify the QE policy effects in the U.S. and in the U.K. Finally, Curdia and Woodford (2011) work on a theoretical approach to the central bank balance sheet.

On the other hand, central banks from developing countries anticipated most of the negative effects from QE policies and adopted their own macroprudential policies. The purpose of these policies was to affect financial variables such as exchange rates, capital flows, credit markets, and asset prices.⁴

In this regard, a branch of the literature has analyzed the effectiveness of unconventional monetary policy measures taken by central banks in both advanced and emerging economies. In particular, policymakers are interested in assessing the impact of QE policies on output and inflation. However, little research has been conducted in regard to the spillover effects of these policy measures on emerging market economies.

⁴ The effects on the exchange rate are discussed in Eichengreen (2013). See also Cronin (2014) for the interaction between money and asset markets and its effect on emerging economies. See Aizenman et al. (2014) for the effect of tapering on financial variables in developing economies. Moreover, the case of Peru is documented in Quispe and Rossini (2011).

This paper focuses its attention on the QE policy measures implemented by the fed and their macroeconomic effects on four Latin American economies: Chile, Colombia, Mexico, and Peru. This group of small open economies shares some characteristics such as the fact that these economies apply the inflation targeting scheme, they have credit in both domestic and foreign currency, and they are recipients of important capital inflows. These characteristics allow us to have more confidence on the identification process as to what we denominate a QE policy shock.

In order to identify the QE policy shock, we estimate a structural vector autoregressive (SVAR) model with block exogeneity in the spirit of Zha (1999). Moreover, we identify QE policy shocks through a mixture of zero and sign restrictions in line with Arias et al. (2014). Given the identified shock, we assess how it is transmitted to each small open economy (SOE); i.e., we have a domestic block for each Latin American country. The advantage of our approach is that, by construction, U.S. shocks can affect each SOE but not the other way around.

Regarding the methodology, the list of previous papers that study other types of QE policies using SVAR models includes Schenkelberg and Watzka (2013), where they analyze the real effects of QE measures on the Japanese economy using zero and sign restrictions. They find that a QE policy shock generates a 7 percent drop in long-term interest rates and a 0.4 percent increase in industrial production. Baumeister and Benati (2012) estimate a time-varying SVAR identified through sign restrictions. They find that compressions in the long-term yield spread exert a powerful effect on both output growth and inflation in the U.S. and in the U.K.

The structure of the paper is as follows: section 2 presents the SVAR model with block exogeneity, section 3 discusses the identification of the QE shock, section 4 presents the estimation results, and section 5 concludes.

2 A SVAR model with block exogeneity

In this section, we closely follow Cushman and Zha (1997) and Zha (1999). They argue that block exogeneity in a SVAR is a natural extension for small open economies, since it rules out any unrealistic effects that could arise in a standard SVAR model, e.g., a significant effect in the big economy derived from a shock in the small one. Furthermore, the assumption of block exogeneity reduces tremendously the number of parameters to be estimated. Finally, we estimate the model using Bayesian techniques.

2.1 The setup

Consider a two-block SVAR model. We take this specification in order to be in line with a small open economy setup. In this context, the big economy is represented for t = 1, ..., T by

$$\mathbf{y}_t^{*\prime} \mathbf{A}_0^* = \sum_{i=1}^p \mathbf{y}_{t-i}^{*\prime} \mathbf{A}_i^* + \mathbf{w}_t^{\prime} \mathbf{D}^* + \varepsilon_t^{*\prime}, \tag{1}$$

where y_t^* is $n^* \times 1$ vectors of endogenous variables for the big economy; ε_t^* is $n^* \times 1$ vectors of structural shocks for the big economy $(\varepsilon_t^* \sim N(0, I_{n^*}))$; $\widetilde{\mathbf{A}}_i^*$ and \mathbf{A}_i^* are $n^* \times n^*$ matrices of structural parameters for $i = 0, \ldots, p$; \mathbf{w}_t is a $r \times 1$ vector of exogenous variables; \mathbf{D}^* is $r \times n$ matrix of structural parameters; p is the lag length; and, T is the sample size.

The small open economy is represented by

$$\mathbf{y}_{t}'\mathbf{A}_{0} = \sum_{i=1}^{p} \mathbf{y}_{t-i}'\mathbf{A}_{i} + \sum_{i=0}^{p} \mathbf{y}_{t-i}^{*}\widetilde{\mathbf{A}}_{i}^{*} + \mathbf{w}_{t}'\mathbf{D} + \varepsilon_{t}',$$
(2)

where y_t is $n \times 1$ vector of endogenous variables for the small economy; ε_t is $n \times 1$ vector of structural shocks for the domestic economy ($\varepsilon_t \sim N(0, I_n)$) and structural shocks are independent across blocks i.e. $E(\varepsilon_t \varepsilon_t^{*'}) = \mathbf{0}_{\mathbf{n} \times \mathbf{n}^*}$); \mathbf{A}_i are $n \times n$ matrices of structural parameters for $i = 0, \ldots, p$; and, \mathbf{D} is $r \times n$ matrix of structural parameters.

The latter model can be expressed in a more compact form, so that

$$\begin{bmatrix} \mathbf{y}_t' & \mathbf{y}_t^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_0 & \mathbf{0} \\ -\widetilde{\mathbf{A}}_0^* & \mathbf{A}_0^* \end{bmatrix} = \sum_{i=1}^p \begin{bmatrix} \mathbf{y}_{t-i}' & \mathbf{y}_{t-i}^{*\prime} \end{bmatrix} \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \widetilde{\mathbf{A}}_i^* & \mathbf{A}_i^* \end{bmatrix} + \mathbf{w}_t' \begin{bmatrix} \mathbf{D} \\ \mathbf{D}^* \end{bmatrix} + \begin{bmatrix} \varepsilon_t' & \varepsilon_t^{*\prime} \end{bmatrix} \begin{bmatrix} I_n & \mathbf{0} \\ \mathbf{0} & I_{n^*} \end{bmatrix},$$

or simply

$$\overrightarrow{\mathbf{y}}_{t}'\overrightarrow{\mathbf{A}}_{0} = \sum_{i=1}^{p} \overrightarrow{\mathbf{y}}_{t-i}'\overrightarrow{\mathbf{A}}_{i} + \mathbf{w}_{t}'\overrightarrow{\mathbf{D}} + \overrightarrow{\varepsilon}_{t}', \tag{3}$$

where
$$\overrightarrow{\mathbf{y}}_t' \equiv \begin{bmatrix} \mathbf{y}_t' & \mathbf{y}_t^{*'} \end{bmatrix}$$
, $\overrightarrow{\mathbf{A}}_i \equiv \begin{bmatrix} \mathbf{A}_i & \mathbf{0} \\ \widetilde{\mathbf{A}}_i^{*} & \mathbf{A}_i^{*} \end{bmatrix}$ for $i = 1, \dots, p$, $\overrightarrow{\mathbf{D}} \equiv \begin{bmatrix} \mathbf{D} \\ \mathbf{D}^{*} \end{bmatrix}$ and $\overrightarrow{\varepsilon}_t' \equiv \begin{bmatrix} \varepsilon_t' & \varepsilon_t^{*'} \end{bmatrix}$.

System (2) represents the small open economy in which its dynamics are influenced by the big economy block (1) through the parameters $\tilde{\mathbf{A}}_i^*$, \mathbf{A}_i^* and \mathbf{D}^* . On the other hand, the big economy evolves independently, i.e. the small open economy cannot influence the dynamics of the big economy.

Even though block (1) has effects over block (2), we assume that the block (1) is independent of block (2). This type of *Block Exogeneity* has been applied in the context of SVARs by Cushman and Zha (1997), Zha (1999) and Canova (2005), among others. Moreover, it turns out that this is a plausible strategy for representing small open economies such as the Latin American ones, since they are influenced by external shocks like the mentioned Unconventional Monetary Policy (UMP) measures implemented in the U.S. economy.

2.2 Reduced form estimation

The system (3) is estimated by blocks. We first present a foreign, then a domestic block, and finally introduce a compact form system i.e. stack both blocks into a one system.

2.2.1 Big economy block

The independent SVAR (1) can be written as

$$\mathbf{y}_{t}^{*\prime}\mathbf{A}_{0}^{*} = \mathbf{x}_{t}^{*\prime}\mathbf{A}_{\perp}^{*} + \varepsilon_{t}^{*\prime}$$
 for $t = 1, \dots, T$;

where

$$\mathbf{A}_{+}^{*\prime} \equiv \begin{bmatrix} \mathbf{A}_{1}^{*\prime} & \cdots & \mathbf{A}_{p}^{*\prime} & \mathbf{D}^{*\prime} \end{bmatrix}, \ \mathbf{x}_{t}^{*\prime} \equiv \begin{bmatrix} \mathbf{y}_{t-1}^{*\prime} & \cdots & \mathbf{y}_{t-p}^{*\prime} & \mathbf{w}_{t}^{\prime} \end{bmatrix},$$

so that its reduced form representation is

$$\mathbf{y}_t^{*\prime} = \mathbf{x}_t^{*\prime} \mathbf{B}^* + \mathbf{u}_t^{*\prime} \quad \text{for } t = 1, \dots, T;$$

where $\mathbf{B}^* \equiv \mathbf{A}_+^* (\mathbf{A}_0^*)^{-1}$, $\mathbf{u}_t^{*\prime} \equiv \varepsilon_t^{*\prime} (\mathbf{A}_0^*)^{-1}$, and $E[\mathbf{u}_t^* \mathbf{u}_t^{*\prime}] = \mathbf{\Sigma}^* = (\mathbf{A}_0^* \mathbf{A}_0^{*\prime})^{-1}$. Then the coefficients \mathbf{B}^* are estimated from (4) by OLS, so that

$$\widehat{\mathbf{B}^*} = \left[\sum_{t=1}^T \mathbf{y}_t^{*\prime} \mathbf{x}_t^*\right] \left[\sum_{t=1}^T \mathbf{x}_t^{*\prime} \mathbf{x}_t^*\right]^{-1},$$

and Σ^* is recovered through the estimated residuals $\widehat{\mathbf{u}_t^{*\prime}} = \mathbf{y}_t^{*\prime} - \mathbf{x}_t^{*\prime} \widehat{\mathbf{B}^*}$.

2.2.2 Small open economy block

The SVAR (2) is written as

$$\mathbf{y}_t'\mathbf{A}_0 = \mathbf{x}_t'\mathbf{A}_+ + \varepsilon_t'$$
 for $t = 1, \dots, T$;

where

$$\mathbf{A}'_{+} \equiv \begin{bmatrix} \mathbf{A}'_{1} & \cdots & \mathbf{A}'_{p} & \widetilde{\mathbf{A}}^{*}_{0} & \widetilde{\mathbf{A}}^{*}_{1} & \cdots & \widetilde{\mathbf{A}}^{*}_{p} & \mathbf{D}' \end{bmatrix}$$

$$\mathbf{x}'_{t} \equiv \begin{bmatrix} \mathbf{y}'_{t-1} & \cdots & \mathbf{y}'_{t-p} & \mathbf{y}^{*\prime}_{t} & \mathbf{y}^{*\prime}_{t-1} & \cdots & \mathbf{y}^{*\prime}_{t-p} & \mathbf{w}'_{t} \end{bmatrix}.$$

The reduced form is now

$$\mathbf{y}_t' = \mathbf{x}_t' \mathbf{B} + \mathbf{u}_t' \text{ for } t = 1, \dots, T;$$
 (5)

where $\mathbf{B} \equiv \mathbf{A}_{+} \mathbf{A}_{0}^{-1}$, $\mathbf{u}_{t}' \equiv \varepsilon_{t}' \mathbf{A}_{0}^{-1}$, and $E\left[\mathbf{u}_{t} \mathbf{u}_{t}'\right] = \mathbf{\Sigma} = (\mathbf{A}_{0} \mathbf{A}_{0}')^{-1}$. As we can see, foreign variables are treated as predetermined in this block, i.e. it can be considered as a VARX model (Ocampo and Rodriguez, 2011). In this case, coefficients \mathbf{B} are estimated from (5) by OLS, and $\mathbf{\Sigma}$ is recovered through the estimated residuals $\hat{\mathbf{u}}_{t} = \mathbf{y}_{t}' - \mathbf{x}_{t}' \hat{\mathbf{B}}$.

2.2.3 Compact form

It is worth to mention that the two reduced forms can be stacked into a single model, so that the SVAR model (3) can be estimated by usual methods. The model can be written as

$$\overrightarrow{\mathbf{y}}_{t}'\overrightarrow{\mathbf{A}}_{0} = \overrightarrow{\mathbf{x}}_{t}'\overrightarrow{\mathbf{A}}_{+} + \overrightarrow{\varepsilon}_{t}' \text{ for } t = 1, \dots, T;$$

where

The reduced form is now

$$\overrightarrow{\mathbf{y}}_{t}' = \overrightarrow{\mathbf{x}}_{t}' \overrightarrow{\mathbf{B}} + \overrightarrow{\mathbf{u}}_{t}' \quad \text{for } t = 1, \dots, T;$$
 (6)

where $\overrightarrow{\mathbf{B}} \equiv \overrightarrow{\mathbf{A}}_+ \left(\overrightarrow{\mathbf{A}}_0\right)^{-1}$, $\overrightarrow{\mathbf{u}}_t' \equiv \overrightarrow{\varepsilon}_t' \left(\overrightarrow{\mathbf{A}}_0\right)^{-1}$, and $E\left[\overrightarrow{\mathbf{u}}_t \overrightarrow{\mathbf{u}}_t'\right] = \overrightarrow{\Sigma} = \left(\overrightarrow{\mathbf{A}}_0 \overrightarrow{\mathbf{A}}_0'\right)^{-1}$. In this case, if we estimate $\overrightarrow{\mathbf{B}}$ by OLS, this must be performed taking into account the block structure of the system imposed in matrices $\overrightarrow{\mathbf{A}}_i$, i.e. it becomes a restricted OLS estimation. Clearly, it is easier and more transparent to implement the two step procedure described above and, ultimately, since the blocks are independent by assumption, there are no gains from this joint estimation procedure (Zha, 1999). Last but not least, the lag length p is the same for both blocks and it is determined as the maximum obtained from the two blocks using the Akaike information criterion (AIC).

2.2.4 **Priors**

We adopt Natural conjugate priors for reduced form coefficients. The latter implies that the prior, likelihood and posterior come from the same family of distributions (Koop and Korobilis, 2010). The introduction of priors is desirable, since the number of parameters to be estimated is very high and the number of observations is limited. Therefore, this a plausible strategy for reducing the amount of posterior uncertainty and, at the same time, it is useful for disciplining the data. In this regard, it is important to remark that we introduce priors for the reduced form coefficients, but this does not mean that we impose any prior information about the structural form. The latter is out of the scope of this paper. Nevertheless, more details can be found in Canova and Pérez-Forero (2014) and Baumeister and Hamilton (2013).

We assume that the prior distribution of the object $(\mathbf{B}, \mathbf{\Sigma}^{-1})$ is Normal-Wishart, so that

$$\beta \mid Y, \Sigma \sim N(\beta, \Sigma \otimes \underline{V})$$

$$\Sigma^{-1} \mid Y \sim W(\underline{S}^{-1}, \underline{\nu}),$$

where $\underline{\beta} = vec(\underline{\mathbf{B}})$ and $(\underline{\mathbf{B}}, \underline{V}, \underline{S}^{-1}, \underline{\nu})$ are prior hyperparameters with $\underline{\nu} = K + 1$. In particular, we parametrize:

$$\mathbf{B} = \mathbf{0}, S = hI_n, V = 100 * I_K,$$

with $\underline{h} = 0.1$ being a hyperparameter and K the number of regressors. As a result, the posterior distribution is

$$\beta \mid Y, \Sigma \sim N(\overline{\beta}, \Sigma \otimes \overline{V})$$

$$\Sigma^{-1} \mid Y \sim W\left(\overline{S}^{-1}, \overline{\nu}\right),$$

where

$$\overline{V} = \left[\underline{V}^{-1} + \sum_{t=1}^{T} \mathbf{x}_{t} \mathbf{x}'_{t} \right]^{-1}$$

$$\overline{\mathbf{B}} = \left[\underline{\mathbf{B}} \underline{V}^{-1} + \widehat{\mathbf{B}} \left(\sum_{t=1}^{T} \mathbf{x}_{t} \mathbf{x}_{t}' \right) \right] \overline{V},$$

and

$$\overline{S} = \sum_{t=1}^{T} \widehat{\mathbf{u}}_{t} \widehat{\mathbf{u}}_{t}' + \underline{S} + \widehat{\mathbf{B}} \left[\sum_{t=1}^{T} \mathbf{x}_{t} \mathbf{x}_{t}' \right] \widehat{\mathbf{B}}' + \underline{\mathbf{B}} \underline{V}^{-1} \underline{\mathbf{B}}'$$

$$-\overline{\mathbf{B}} \left[\underline{V}^{-1} + \sum_{t=1}^{T} \mathbf{x}_{t} \mathbf{x}_{t}' \right] \overline{\mathbf{B}}'$$

$$\overline{\nu} = T + \underline{\nu}.$$

We apply the same procedure for the two estimated blocks in order to produce draws of $(\mathbf{B}, \mathbf{\Sigma})$ from its posterior distribution.

2.3 Identification of structural shocks

2.3.1 General task

Given the estimation of the reduced form, now we turn to the identification of structural shocks. In short, we need a matrix $\overrightarrow{\mathbf{A}}_0$ in (3) that satisfies a set of identification restrictions. To do so, here we adopt a partial identification strategy. That is, since the model size $(\overrightarrow{n} = \dim \overrightarrow{\mathbf{y}}_t)$ is potentially big, the task of writing down a full structural identification procedure is far from straightforward (Zha, 1999). In turn, we emphasize the idea of partial identification, since in general we are only interested in a portion of shocks $\underline{n} < \overrightarrow{n}$ in the SVAR model, e.g., domestic and foreign monetary policy shocks. In this regard, Arias et al. (2014) provide an efficient routine to achieve identification through zero and sign restrictions. We adapt their routine for the case of block exogeneity.

2.3.2 The algorithm

The algorithm for the estimation is as follows⁵

- 1. Set first K = 2000 number of draws.
- 2. Draw $(\mathbf{B}^*, \mathbf{\Sigma}^*)$ from the posterior distribution (foreign block).
- 3. Denote \mathbf{T}^* such that $(\mathbf{A}_0^*, \mathbf{A}_+^*) = ((\mathbf{T}^*)^{-1}, \mathbf{B}^*(\mathbf{T}^*)^{-1})$ and draw an orthogonal matrix \mathbf{Q}^* such that $((\mathbf{T}^*)^{-1}\mathbf{Q}^*, \mathbf{B}^*(\mathbf{T}^*)^{-1}\mathbf{Q}^*)$ satisfy the zero restrictions and recover the draw $(\mathbf{A}_0^*)_k = (\mathbf{T}^*)^{-1}\mathbf{Q}^*$.
- 4. Draw (\mathbf{B}, Σ) from the posterior distribution (domestic block).
- 5. Denote **T** such that $(\mathbf{A}_0, \mathbf{A}_+) = (\mathbf{T}^{-1}, \mathbf{B}\mathbf{T}^{-1})$ and draw an orthogonal matrix **Q** such that $(\mathbf{T}^{-1}, \mathbf{B}\mathbf{T}^{-1})$ satisfy the zero restrictions and recover the draw $(\mathbf{A}_0)_k = \mathbf{T}^{-1}\mathbf{Q}$.
- 6. Take the draws $(\mathbf{A}_0)_k$ and $(\mathbf{A}_0^*)_k$, then recover the system (3) and compute the impulse responses.
- 7. If sign restrictions are satisfied, keep the draw and set k = k + 1. If not, discard the draw and go to Step 8.
- 8. If k < K, return to Step 2, otherwise stop.

In this regard, it is worth to remark two aspects related with this routine:

- In contrast with a Structural VAR estimated through Markov Chain Monte Carlo methods (Canova and Pérez-Forero, 2014), draws from the posterior are independent each other.
- Draws from the reduced form of the two blocks (\mathbf{B}, Σ) and (\mathbf{B}^*, Σ^*) are independent by construction.

⁵See details from Arias et al. (2014) in Appendix C.

3 Identifying a QE policy shock

In this section, we assess the transmission mechanism of a structural QE policy shock. We first consider the U.S. as the big economy and Chile, Colombia, Mexico, or Peru as the small one. Regarding the U.S., we include the variables: i) economic policy uncertainty index (EPU_{US}), ii) an indicator related with the spread between long- and short-term interest rates (Spread), iii) a money aggregate (M1_{US}), iv) the Federal Funds Rate (FFR), v) the consumer price index (CPI_{US}), and vi) the industrial production index (IP_{US}). Regarding the SOE we include the variables: i) the real exchange rate (RER), ii) the interbank interest rate in domestic currency (INT), iii-iv) the aggregate credit of the banking system in U.S. dollars (Cred_{FC}) and in domestic currency (Cred_{DC}), v) the consumer price index (CPI), and vi) an indicator of economic activity (GDP).

We identify the QE shock imposing minimal restrictions in the U.S. economy block. On the other hand, we do not impose any restriction in the SOE block, since at this point we are completely agnostic about the endogenous spillover effects that can be generated by this shock. In short, we identify a QE shock such that it increases the level of U.S. money aggregates and, at the same time, decreases the level of spreads in the yield curve spreads, keeping the federal funds rate unchanged because of the ZLB (see Table 1). In addition, since the price level and the economic activity are nonpolicy variables, we assume that they only react to the policy shock after one period (i.e., they are considered to be *slow* variables).

Table 1. Identification restrictions for a QE policy shock in the U.S.

Variable	QE shock $(h = 0)$	QE shock $(h = 1, 2)$
U.S. economic policy uncertainty index (EPU _{US})	?	?
Term spread indicator $(Spread)$	_	_
$M1 \text{ money stock } (M1_{US})$	+	+
Federal funds rate (FFR)	0	?
U.S. consumer price index (CPI_{US})	0	?
U.S. industrial production index (IP_{US})	0	?
Domestic block	?	?

Note: ? = left unconstrained.

Similar identification strategies for unconventional monetary policy shocks through sign and zero restrictions can be found in Peersman (2011), Gambacorta et al. (2012), Baumeister and Benati (2012), Schenkelberg and Watzka (2013). In line with this literature, the QE policy shock is identified using a mixture of zero and sign restrictions. Moreover, we impose that those sign restrictions must be satisfied for a three-month horizon. One aspect that deserves to be mentioned is the flexibility of our identification scheme. In this regard, since we do not know the true model that contains this shock, we capture the parameter region that satisfies our restrictions, which might include several different structural models. On top of that, our QE policy shock is such that the monetary policy instrument might be either the money aggregate or the term spread. For that reason, we do not normalize the shock to one particular variable, since there is no consensus in the literature about the key policy instrument.

 $^{^6}$ See Appendix A for details regarding the data transformation, etc.

4 Results

Results for the U.S. economy are depicted in Figure 3, in which the shaded areas show the sign restrictions. A QE policy shock increases the money stock $(M1_{US})$, reduces the level of the spread between the long- and short-term interest rates (Spread), and keeps the federal funds rate (FFR) close to zero. Strictly speaking, this is an expansionary unconventional policy shock, and, as a result, it produces a positive and significant effect in the industrial production (IP_{US}) and prices (CPI_{US}) in the medium-term.

These effects are significant in the short run and are in line with Peersman (2011), Gambacorta et al. (2012), Baumeister and Benati (2012), and Schenkelberg and Watzka (2013). Moreover, it can also be observed that the effect on spreads is not persistent and vanishes rapidly, in line with Wright (2012).

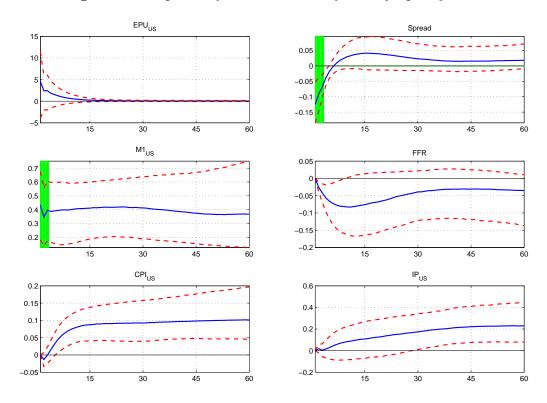


Figure 3. Response of U.S. variables after a QE policy shock

Note: median value (solid line) and 68% bands (dotted lines).

According to our empirical strategy, the identified QE policy shock hits the SOE as in (2). In this step, we consider how this shock affects four Latin American countries: Chile, Colombia, Mexico, and Peru. We present those results in Figure 4.

As we initially suspect, this QE shock has a differentiated effect in each country; however, some regularities arise regarding the direction in the response of macro variables.⁷

⁷We did not include Brazil in our sample because we want to identify the rebalance in the credit market, so we need credit in both domestic and foreign currency. Even though not all results are statistically significant, we point out that the direction in the responses to the QE shock provides interesting

In general, we observe a real appreciation (RER) in line with the massive entrance of capital to the region. Countries such as Peru may react the most in the presence of this type of shock which is consistent with a more active exchange-rate policy in the Forex market.⁸

The QE shock produces a credit expansion in both currencies ($Cred_{FC}$ and $Cred_{DC}$), which in turn triggers a positive response of the domestic interest rate (INT) in the medium-term. Here we notice that the credit expansion is stronger in foreign currency than in the domestic one, which suggests that credit conditions became more attractive for domestic firms. This portfolio effect seems to be stronger in Peru and Colombia than in Chile and Mexico. It can be noticed in Figure 4 that the reaction of financial variables such as credit is faster and stronger than the reaction of output (GDP) and prices (CPI) after a QE shock.

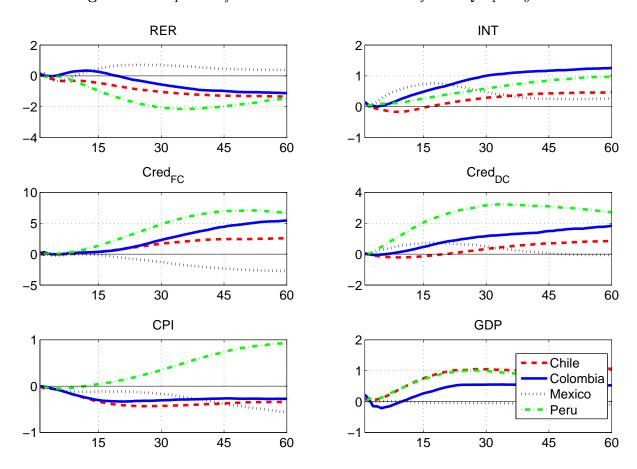


Figure 4. Response of Latin American variables after a QE policy shock

Note: median values.

feedback to this unusual type of shock. For details of confidence bands and statistical significance, see Appendix B.

⁸The higher degree of dollarization in Peru may explain this result, which is in line with a differentiated effect in inflation in contrast to other countries in the region.

5 Concluding Remarks

We identify a structural QE policy shock in an otherwise standard SVAR model. We quantify the effects derived from this shock (which has its origin in the U.S. economy) over four Latin American countries: Colombia, Chile, Mexico, and Peru. Regarding the transmission mechanism, we highlight the importance of financial markets.

Our results suggest larger effects derived from a QE policy shock on financial variables with respect to those over output and prices for these SOEs. The increase in international liquidity that follows after each QE round seems to transmit its effects on the macroeconomic variables of the SOEs through financial variables such as interest rates, aggregate credit, and exchange rates.

As recorded in the early literature, most central banks in developing countries anticipated those effects and adopted macroprudential policy measures. Those measures in general work toward credit growth (via reserve requirements) and exchange rate volatility (via foreign exchange interventions) and tended to mitigate the effects of a QE policy event.

The differentiated effect that exists between each QE round may bring up different results and a better identification strategy. Some researchers consider that QE1 was a rescue program, whereas QE2 and QE3 were programs focused on stabilizing and securing a steady growth path. Even inside of each round, it is possible to split the different components for each QE round. In our agenda, we have planned a more detailed identification of each QE based on the composition of each program.

The inclusion of variables that measure macroprudential policies in the SVAR model is also part of our research agenda. Even though we argue that those effects are already captured by the variables that are intended to be targets of those policies, we may reinforce our results by excluding all financial variables and plug in those variables that capture those macroprudential policies. For example, we can use reserve requirements rather than credit or use exchange rate interventions rather than the exchange rate itself.

Some exercises using different measures of capital flows are also in order, especially long-versus short-term flows. Even though there is agreement in regard to the massive capital inflows in the region, it is also true that central banks adopted macroprudential measures that diminish the full effect of those incoming capitals. Then, it is important to distinguish those capitals in order to robust our result to the measure of capital flow under investigation.

The evaluation of QE policy effects on the lending channel is also part of our research agenda. According to Carrera (2011), there is an initial deceleration in the lending process after 2007 as a result of a flight-to-quality process. Later on, credit growth expanded at a previous growth rate given the context of capital inflows in the region. The identified bank lending channel may play a role in understanding the mechanism of transmission of external shocks, taking into account their effects on the credit market.

⁹ For example, QE1 was announced on November 25, 2008, as a program to purchase agency debt and MBS for up to 600 billion U.S. dollars in order to provide greater support to mortgage lending and housing markets. This QE1 was expanded on March 18, 2009, and an additional 850 billion U.S. dollars of the same securities were approved in addition to 300 billion U.S. dollars in long-term Treasuries.

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A Data Description and Estimation Setup

We include raw monthly data for the period January 1996 - December 2014 with the exception of Colombia, where the sample stops at June 2014.

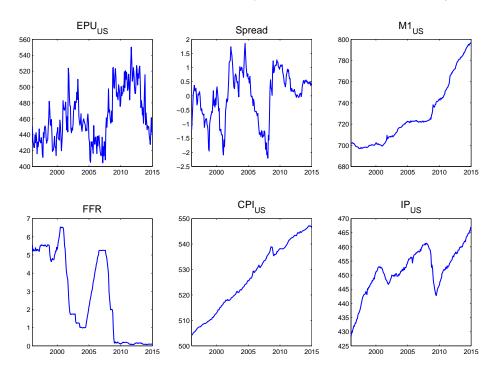
A.1 Big economy block variables y_t^*

We include the following variables from the U.S. economy:

- Economic policy uncertainty index from the U.S. (EPU $_{US}$).
- Spread indicator. 10
- M1 money stock, not seasonally adjusted.
- Federal funds rate (FFR).
- Consumer price index for all urban consumers: all items (1982-84=100), not seasonally adjusted.
- Industrial production index (2007=100), seasonally adjusted.

Data is in monthly frequency (1996:01-2014:12) and it was taken from the Federal Reserve Bank of Saint Louis website (FRED database). Interest rates were taken from the H.15 Statistical Release of the Board of Governors of the Federal Reserve System website.

Figure A.1. US time series (in 100*logs and percentages)



¹⁰This is calculated as the first principal component from all the spreads with respect to the federal funds rate: 3M, 6M, 1Y, 2Y, 3Y, 5Y, 10Y, 30Y from the Treasury. In addition we include AAA, BAA, State Bonds and Mortgages.

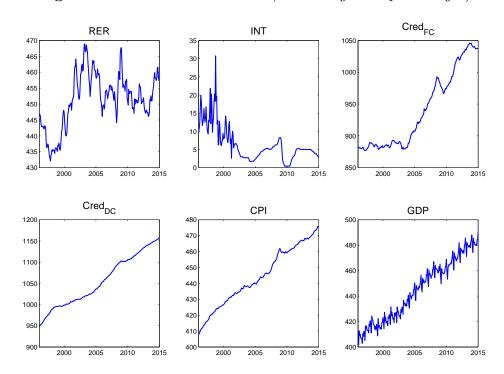
A.2 Chilean economy block variables y_t

We include the following variables from the Chilean economy:

- Real exchange rate.
- Interbank interest rate in Chilean pesos.
- Aggregated credit of the banking system in U.S. Dollars (foreign currency).
- Aggregated credit of the banking system in Chilean pesos (domestic currency).
- Consumer price index (2008=100).
- IMACEC Monthly indicator of economic activity (2008=100), not seasonally adjusted.

Data is in monthly frequency (1996:01-2014:12) and it was taken from the Central Bank of Chile website. All variables except interest rates are included as logs multiplied by 100. This transformation is the most suitable one, since impulse responses can now be directly interpreted as percentage changes.

Figure A.2. Chilean time series (in 100*logs and percentages)



A.3 Colombian economy block variables y_t

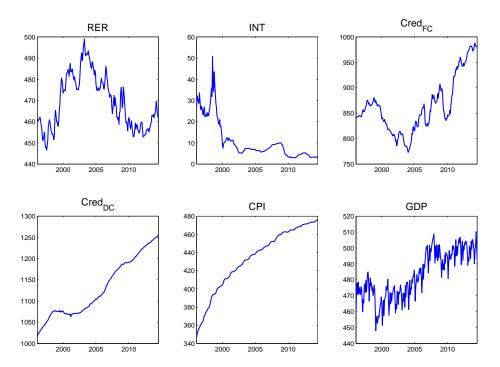
We include the following variables from the Colombian economy:

- Real exchange rate.
- Interbank interest rate in Colombian pesos.
- Aggregated credit of the banking system in U.S. Dollars (foreign currency).

- Aggregated credit of the banking system in Colombian pesos (domestic currency).
- Consumer price index (December 2008=100).
- Real industrial production index (1990=100), not seasonally adjusted.

Data is in monthly frequency (1996:01-2014:06) and it was taken from the Banco de la República website. All variables except interest rates are included as logs multiplied by 100. This transformation is the most suitable one, since impulse responses can now be directly interpreted as percentage changes.

Figure A.3. Colombian time series (in 100*logs and percentages)



A.4 Mexican economy block variables y_t

We include the following variables from the Mexican economy:

- Real exchange rate.
- Interbank interest rate (at 28 days) in Mexican pesos.
- Aggregated credit of the banking system commercial banks) in U.S. Dollars expressed in Mexican pesos (foreign currency).
- Aggregated credit of the banking system (commercial banks) in Mexican pesos (domestic currency).
- Consumer price index (December 2010=100).
- IGAE global economic activity index (2008=100), not seasonally adjusted.

Data is in monthly frequency (1996:01-2014:12) and it was taken from the Banco de México website. All variables except interest rates are included as logs multiplied by 100. This transformation is the most suitable one, since impulse responses can now be directly interpreted as percentage changes.

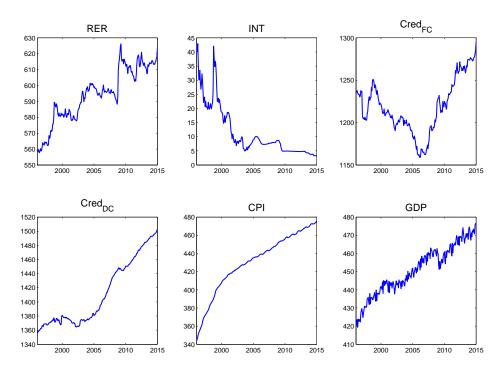


Figure A.4. Mexican time series (in 100*logs and percentages)

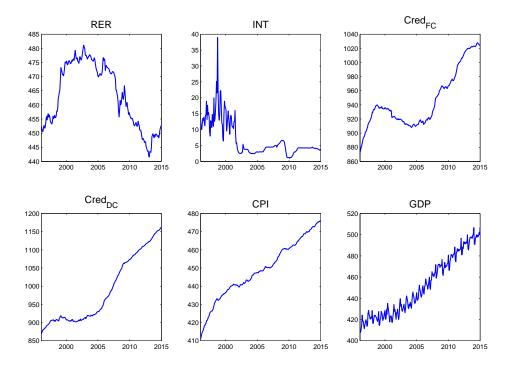
A.5 Peruvian economy block variables y_t

We include the following variables from the Peruvian economy:

- Real exchange rate index (2009=100).
- Interbank interest rate in Soles (in percentages).
- Aggregated credit of the banking system in U.S. Dollars (foreign currency).
- Aggregated credit of the banking system in nuevos Soles (domestic currency).
- Consumer price index for Lima (2009=100).
- Real gross domestic product index (2007=100), not seasonally adjusted.

Data is in monthly frequency (1996:01-2014:12) and it was taken from the Central Reserve Bank of Peru website. All variables except interest rates are included as logs multiplied by 100. This transformation is the most suitable one, since impulse responses can now be directly interpreted as percentage changes.

Figure A.5. Peruvian time series (in 100*logs and percentages)

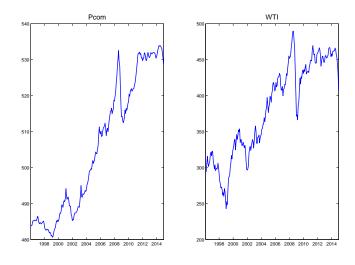


A.6 Exogenous variables \mathbf{w}_t

- Producer price index (all commodities).
- Crude oil prices: West Texas Intermediate (WTI) Cushing, Oklahoma.
- Eleven seasonal dummy variables.
- A constant term, a linear time trend (t) and quadratic time trend (t^2) .

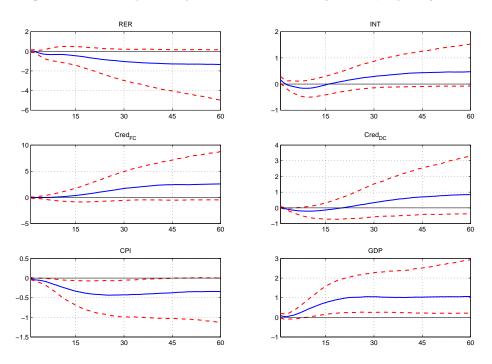
Data is in monthly frequency (1996:01-2014:12) and it was taken from the Federal Reserve Bank of Saint Louis website (FRED database).

Figure A.6. Time series included as exogenous variables (in 100*logs)



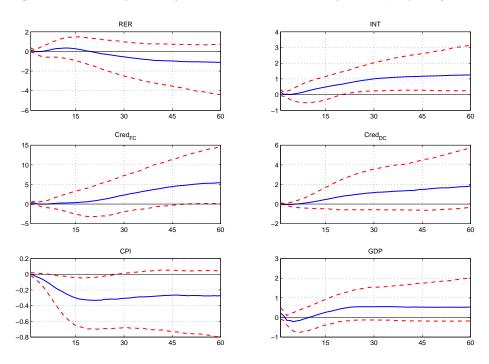
B Impulse responses in Latin America to a QE shock

Figure B.1. Response of Chilean variables after a QE policy shock



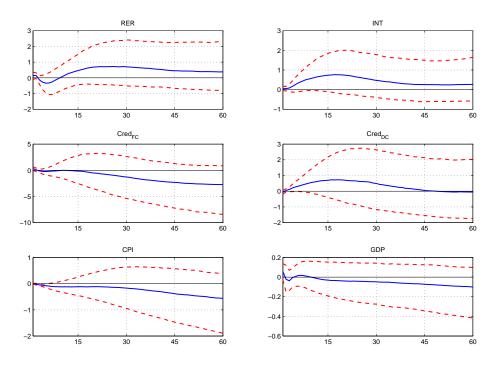
Note: median value (solid line) and 68% bands (dotted lines).

Figure B.2. Response of Colombian variables after a QE policy shock



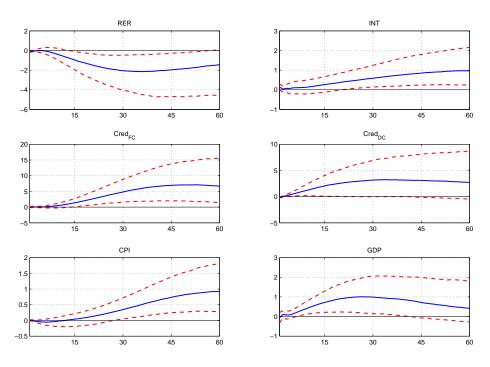
Note: median value (solid line) and 68% bands (dotted lines).

Figure B.3. Response of Mexican variables after a QE policy shock



Note: median value (solid line) and 68% bands (dotted lines).

Figure B.4. Response of Peruvian variables after a QE policy shock



Note: median value (solid line) and 68% bands (dotted lines).

C Estimation details

This section closely follows the work of Arias et al. (2014).

C.1 Implementing zero restrictions

For any orthogonal matrix **Q**, zero restrictions hold if

$$\mathbf{Z}_{j}f(\mathbf{A}_{0},\mathbf{A}_{+})\mathbf{Q}\mathbf{e}_{j}=\mathbf{Z}_{j}f(\mathbf{A}_{0},\mathbf{A}_{+})\mathbf{q}_{j}=0$$
; for $j=1,\ldots,n$.

In short, zero restrictions are linear restrictions in the columns of \mathbf{Q} . As a matter of fact, Theorem 2 of Arias et al. (2014) says that \mathbf{Q} satisfies the zero restrictions if and only if $\|\mathbf{q}_j\| = 1$ and

$$\mathbf{Z}_{j}\mathbf{R}_{j}\left(\mathbf{A}_{0},\mathbf{A}_{+}\right)\mathbf{q}_{j}=0; \text{ for } j=1,\ldots,n;$$

where

$$\mathbf{R}_{j}\left(\mathbf{A}_{0},\mathbf{A}_{+}
ight)\equiv\left[egin{array}{c} \mathbf{Z}_{j}f\left(\mathbf{A}_{0},\mathbf{A}_{+}
ight) \ \mathbf{Q}_{j-1} \end{array}
ight],$$

and
$$rank(\mathbf{Z}_j) \leq n - j$$
, $\mathbf{Q}_{j-1} = [\mathbf{q}_1 \quad \cdots \quad \mathbf{q}_{j-1}]$.

Moreover, Theorem 3 of Arias et al. (2014) shows how to obtain a **Q** that satisfies the zero restrictions given j = 1:

- 1. Compute \mathbf{N}_{j} , the basis for the null space $\mathbf{R}_{j}(\mathbf{A}_{0}, \mathbf{A}_{+})$.
- 2. Draw $\mathbf{x}_{j} \sim N(0, I_{n})$.
- 3. Compute $\mathbf{q}_j = \mathbf{N}_j \left(\mathbf{N}_j' \mathbf{x}_j \right) / \left\| \mathbf{N}_j' \mathbf{x}_j \right\|$.
- 4. If j = n stop, otherwise set j = j + 1 and go to step 1.

The random matrix $\mathbf{Q} = [\mathbf{q}_1 \cdots \mathbf{q}_n]$ has the uniform distribution with respect to the Haar measure on O(n), conditional on $(\mathbf{A}_0\mathbf{Q}, \mathbf{A}_+\mathbf{Q})$ satisfying the zero restrictions.

C.2 Implementing sign restrictions

It is standard in the literature (see e.g., Canova and De Nicoló (2002), Uhlig (2005) and Amir-Ahmadi and Uhlig (2009)) to implement sign restrictions as follows:

- 1. Draw (\mathbf{B}, Σ) from the posterior distribution.
- 2. Denote **T** such that $(\mathbf{A}_0, \mathbf{A}_+) = (\mathbf{T}^{-1}, \mathbf{B}\mathbf{T}^{-1})$.
- 3. Draw a $n \times n$ matrix $\mathbf{X} \sim MN_n$ (standard normal distribution).
- 4. Recover **Q** such that $\mathbf{X} = \mathbf{Q}\mathbf{R}$ is the QR decomposition. **Q** is therefore a random matrix with a uniform distribution with respect to the Haar measure on O(n).
- 5. Keep the draw if $\mathbf{S}_{j} f\left(\mathbf{T}^{-1}, \mathbf{B} \mathbf{T}^{-1}\right) \mathbf{Q} \mathbf{e}_{j} > 0$ is satisfied.