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# Repurchase Options in the Market for Lemons

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**Abstract**

We study repurchase options (repo contracts) in a competitive asset market with asymmetric information. Gains from trade emerge from a liquidity need, but private information about asset quality prevents the full realization of trade. We obtain a unique equilibrium, which features a pooling repo contract and full participation among borrowers. The equilibrium repo contract resolves adverse selection: the embedded repurchase option prevents the market unraveling that occurs in asset-sale markets. However, the contract is inefficient due to cream skimming. Competition to attract high-quality borrowers through the terms of the repurchase option inefficiently lowers liquidity. The equilibrium contract has a closed form and is portable to many applications.

**Keywords:** Repurchase Agreement, Collateralized Debt, Private Information, Optimal Contracts

**JEL: D82, G23, G32**

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# 1 Introduction

Many financial contracts such as collateralized debt, bridge loans, factoring, and loan discounts are *de facto* asset-sale contracts that embed a repurchase option. Strip away legal differences and all of these contracts have a common structure: they all specify a collateral security, a loan amount, and a loan principal.[1] Because these contracts are non-recourse or rarely settled in court, borrowers have the option to default. The option to default is, *de facto,* an option to repurchase the asset. We refer to contracts embedding repurchase options broadly as repos. Why do so many financial contracts take the form of repos? Why don't borrowers simply sell securities and buy them back later?

This paper argues that, in any asset market where asymmetric information is prevalent, repos emerge as a natural market response to adverse selection. We consider a market where borrowers need funds to carry out investment projects. Borrowers raise funds by trading pre-existing assets. The issue is that only borrowers know the assets' payoffs. If borrowers can only sell assets, as in the classic lemons market, the ones with good assets desist from selling because the price is too low. Adverse selection ensues (Akerlof, 1970). Lenders can resolve this adverse selection by offering a repurchase option. Indeed, *any* borrower who is unwilling to sell an asset at a low price will change her mind if she is also offered the option to repurchase the asset at a price below its worth, even if this repurchase price has a slight premium (an implicit interest charge). Of course, the borrowers with bad assets will take advantage by selling their lemons and never returning for them, implicitly defaulting. If rightly set, the premia can compensate lenders for default losses and induce good borrowers to participate, thereby resolving adverse selection.

By virtue of resolving the adverse selection problem, repos can possibly increase the funding of investment projects. However, lender competition brings about another problem, cream skimming. The issue with repos is that their bidimensional price nature enables lenders to poach good borrowers, by lowering the repurchase premium of their contracts. Ultimately, cream skimming is detrimental to liquidity to the point that it can even offset the advantages of repos.

To understand the contribution of this paper, consider the example in Table 1. A set of borrowers have investment opportunities that yield a 20% return.[2] Among the borrowers, half own assets worth \$40, and the other half own assets worth \$80. If only asset sales are possible, owners of the \$80 asset would not sell at the average price of \$60: A 20% return on \$60 yields \$72 whereas the asset is worth \$80. Thus, selling the \$80 asset leads to a loss of \$8 and the

[1] These contracts are not a modern financial innovation. In fact, an early form of collateralized debt contract, the *Pignus,* was already stipulated in Roman Law. Holmstrom (2014) considers pawning dating back to Tang-dynasty China, *circa* 650 AD, as the precursor of these contracts.

[2] In the original Akerlof (1970) setup, buyers (hereafter lenders) have higher valuations for assets than sellers (hereafter borrowers), but we can equivalently think of gains from trade arising from borrowers having a higher need for cash than lenders, albeit both having the same valuation for the asset. What is critical for the impediment to trading is that borrowers have private information about the asset.

Table 1: Illustrative Example

| Collateral | Value | | | |
|---|---|---|---|---|
| Low Quality | \$40 | | | |
| High Quality | \$80 | | | |
| | Sales Price | Repurchase Price | Average Funds Lent | Added Value |
| Sale | \$40 | $\infty$ | \$20 | \$4 |
| Repo | \$50 | \$60 | \$50 | \$10 |

market therefore unravels. Things change if a lender offers a repo where the borrowers are paid \$50 for the asset, but can repurchase it for \$60. A 20% return on \$50 yields \$60 so the good borrower breaks even. The example illustrates how repos attract all borrowers, lenders break even,[3] and the total funding liquidity *more than doubles*.

Although the example illustrates the advantages of repos, it does not describe an actual market outcome. In the example, a new entrant lender could offer a repo for \$45 with a repurchase price of \$50. Because the implicit rate is much lower, the offer would attract the \$80 borrower.[4] This cream skimming would in turn lead to less funding. The example is therefore silent about which contracts we should expect to emerge. The rest of the paper is devoted to the study of market competition for repos in this lemons market. The paper characterizes the set of equilibrium contracts, distills the operating forces, and characterizes conditions under which repos increase investment funding.

To conduct the analysis, the paper adopts a modern approach to competitive markets with private information. Asset qualities can be drawn from any arbitrary continuous distribution. We consider the timing in Netzer and Scheuer (2014), in which lenders can post contracts but can withdraw them after observing the contracts posted by their competitors. This is a foundation for the well-known Miyazaki-Wilson-Spence equilibrium (henceforth MWS). The positive analysis of the environment reveals that the equilibrium is *unique* and features a single *pooling* contract that induces *full participation*.

A key result is that any possible equilibrium features a default threshold quality—borrowers with collateral worse than the threshold quality default. This property narrows the set of traded contracts in any possible equilibrium to two traded contracts at most: a highest sales-price contract that attracts the default borrowers and a highest nondefault value contract that attracts the

[3] We know that low-quality borrowers will not exercise the repurchase option so each low-quality borrower costs the lender \$10: the lender pays \$50 and ends with a \$40 asset. However, lenders make \$10 on high-quality borrowers offsetting this loss.

[4] Low-quality borrowers would prefer the original $\{\$50, \$60\}$ contract; they will default regardless of the contract. The new contract would attract high-quality borrowers, who obtain an investment payoff of \$54 dollars. The deviation is profitable to the lender and would cannibalize the original contract.

nondefaulters. The repurchase option in the highest nondefault value contract allows borrowers to invest without losing their assets, as in the example. This guarantees full participation.

Pooling occurs when the highest sales-price contract and highest nondefault value contract coincide. If the two contracts differ, there exists another profitable contract that offers a higher nondefault value. This means that an entrant lender can offer a different contract to attract the best borrowers of the incumbent contracts, thus prompting the withdrawal of its competitors. The equilibrium must therefore pool all borrowers into one contract. Among the set of possible pooling outcomes, a unique equilibrium contract emerges, a contract that solves an auxiliary problem. The auxiliary problem maximizes the nondefault value subject to a zero-profit pooling condition. The solution to that problem has a unique analytic solution for any asset-quality distribution.

Turning to the normative analysis, the market equilibrium is suboptimal relative to a constrained-planner problem that maximizes total gains from trade. The planner's problem is also characterized through an auxiliary problem. In this case, rather than maximizing the nondefault value, the planner maximizes the sales price subject to the zero-profit pooling condition. The parallel between the planner and the market auxiliary problems illustrates that the source of inefficiency is exclusively cream skimming.[5] Intuitively, the planner's optimal contract provides more liquidity than the market contract because its sales price is higher. A higher sales price induces a higher default rate and a higher implicit interest rate, which hurts the best borrowers, taking them to be indifferent between participating or not. In the market equilibrium, as illustrated by the simple example, lenders poach nondefault borrowers. They can do so by lowering the implicit interest rate, which is only possible by also lowering the sales price. Hence, the resulting market equilibrium maximizes the nondefault value by trading off a lower interest rate for a lower sales price. Since cream skimming induces lower sales prices, overall investment is inefficient. Relative to an equilibrium restricted to asset sales, repos improve efficiency when the benefits of increasing participation exceed the detrimental effects of cream skimming, a property we characterize via a sufficient statistic.[6]

The idea that richer contracts can resolve adverse selection is also present in security design (see Gorton and Pennacchi, 1990; Demarzo and Duffie, 1999; Biais and Mariotti, 2005).[7] Security design rests on the assumption that borrowers commit to the terms of a contract, prior to knowing their assets' quality.[8] This commitment is essential to a borrower who wants to guarantee a high

[5]In the example above, the $\{\$50, \$60\}$ contract, in fact, corresponds to the solution of the constrained planner problem, but as shown above, that contract is not robust to competition.

[6]For example, if we modify the example setting the high quality asset to \$50 such that the asset quality dispersion is mild, the result reverts and asset sales dominate repos.

[7]This literature uses the terminology of collateralized debt, but we can view this security as akin to repos. They also use the terminology of equity issuances, which is essentially asset sales.

[8]In Demarzo and Duffie (1999) the precise assumption is as follows: "after designing the security, but prior to the sale of the security to outside investors, the issuer or underwriter handling the sale receives information relevant to the payoff of the security."

sales price, even when she ends up with a bad asset. In fact, the optimal security design solution in Biais and Mariotti (2005) coincides with the planner's solution here, which guarantees a high sales price for all borrowers. To guarantee such high prices, borrowers must commit to repos where the interest rate equals the return on investment, thus losing all their surplus when their assets turn out good. By contrast, the paper studies a market equilibrium where lenders tempt borrowers to break that commitment. The testable implications are very different. With commitment, the extent of asymmetric information manifests in the sales price and the default rates, but not in the implied interest, which always equals the return on investment. By contrast, in our market equilibrium, the default rate is constant regardless of the asset quality distribution. In turn, the extent of asymmetric information manifests in a wedge between the implied interest and the return on investment. This feature of the market solution is in line with empirical observations that are seen as a puzzle.[9]

We conclude the theoretical analysis by studying variations to the model that showcase the roles of different modeling assumptions. To study the role of the equilibrium concept, we recast the analysis in the competitive search environment of Guerrieri et al. (2010). An implicit assumption in their model is that lenders serve contracts to at most one borrower. This capacity constraint affects the nature of competition because, when deviating, lenders do not consider all the borrowers that could be attracted by their contracts, but form the belief that only those that are most attracted will show up. Under competitive search, the outcome is also a unique pooling equilibrium, but the one that *minimizes* the sales price. Nonetheless, under competitive search, repos always improve liquidity in relation to outright sales. This showcases that the advantages of repos hold across different equilibrium concepts. To study the role of lender commitment, we also study a variation where lenders obtain the option to default and retain the assets with a certain probability. In this case, the equilibrium is also pooling, but may no longer feature full participation, something that reintroduces adverse selection and reduces the advantages of repos.

The organization of the manuscript is as follows. We lay out the environment in Section 2. Section 3 characterizes the equilibrium. Section 4 discusses the efficiency results. Section 4.1 presents the constrained planner (security design solution), and Section 4.2 compares the repo equilibrium to an equilibrium where only outright sales are allowed. Section 5 studies the variations to the environment. Section 6 presents an illustration of the theory in connection with some empirical facts established by the literature. Before we proceed, we establish some connections with the literature.

[9]Loan amounts in repo markets fluctuate substantially relative to their notional values (Gorton and Metrick, 2012) whereas default rates are stable. This is seen as a puzzle because, without asymmetric information, lenders and borrowers would treat repos as a call option. Arbitrage pricing would maximize loan amounts. As Dang et al. (2013) put it, "the existence of repo haircuts is a puzzle, as standard finance theory would suggest that risk simply be priced."

**Related Literature.** Originally motivated by trade in used goods markets, the lemons model of Akerlof (1970) is also foundational for finance: asymmetric information is a prevalent friction that explains asset market liquidity (Eisfeldt, 2004). The study of asymmetric information in macroeconomics and finance has evolved along two branches. One branch enriches the underlying environment in Akerlof (1970), whereas the other enriches the contract space. This paper contributes directly to the latter branch, by studying competition in a richer contract space, but has implications for the first.

The literature that enriches the contract space has primarily focused on security design, following Demarzo and Duffie (1999) and Biais and Mariotti (2005). A frequent result is that collateralized debt is ideal.[10] This paper departs from security design because the market trades *after* the asset quality is known. The distinction is material: First, commitment may be a good assumption only in some contexts. Second, the default rate in our equilibrium is invariant to information, a different testable implication. Third, in security design, contracts are efficient by construction, something that leaves no room for policy design.[11]

The second branch that studies the richer lemons markets is vast. This literature incorporates features such as dynamic trade and learning (Daley and Green, 2010; Fuchs and Skrzypacz, 2019; Guerrieri and Shimer, 2014), competitive and random search (Guerrieri et al., 2010; Lester et al., 2019), information acquisition (Gorton and Ordoñez, 2014), multidimensional screening (Chang, 2017; Guerrieri and Shimer, 2018), layered trades (Dang et al., 2013), nonexclusive contracts (see Attar et al., 2011; Kurlat, 2016), equilibrium with signals (Kurlat and Scheuer, 2020), etc. With few exceptions, these studies are restricted to asset sales. One exception is Madison (2017), who studies an optimal security design problem with random search. Other exceptions are Dang et al. (2013) and Yang (Forthcoming) who study security design problems, taking into account collateral reuse (as in Brunnermeier and Pedersen, 2009). We contend that, due to cream skimming, allowing for repos in such richer environments is a natural extension and can dramatically change outcomes.[12]

The focus on asymmetric information found here complements previous work that explains the use of repos in response to transaction costs.[13] In Duffie (1996), repos are the only way to hold long positions in securities. In Parlatore (2019) and Monnet and Narajabad (2017),

[10]The security design solution relates to the durable goods monopoly model of Hendel and Lizzeri (2002), where a monopolist offers leasing contracts. Because the terms of the lease are fixed, this acts as a form of commitment akin to that of the security design literature.

[11]The main normative observation is that default is inefficiently low in the market equilibrium, which relates to other studies of collateral with default (e.g. Geanakoplos and Zame, 2014).

[12]When studying outright sales, Guerrieri et al. (2010) obtain full separation with rationing. Here, we obtain a pooling equilibrium without rationing that Pareto dominates the separating equilibrium.

[13]The standard argument based of transaction costs is that it is more convenient to pledge collateral rather than to sell collateral, if there is a social value from keeping the asset with borrowers. This explanation is only partial. For one thing, repurchase options are often used in markets where the collateral are financial claims that have no special user value. And for another, observed haircut rates and over-collateralization rates are surprisingly high. Implied transaction costs are simply too large to account for high over-collateralization rates.

repos minimize endogenous transaction costs. In Gottardi et al. (2019), the repurchase price is state-contingent and lenders reuse collateral which allows the transfer of wealth to states where collateral is more valuable. Another point of contact is with the literature on collateral use. In this paper, assets are used as collateral because investment projects cannot be self financed (as in Kiyotaki and Moore, 2008). There is also a vast literature that studies different uses for collateral to relax asymmetric information problems regarding, for example, project risks (Stiglitz and Weiss, 1981; Besanko and Thakor, 1987), costly state verification problems (Townsend, 1979), or moral-hazard problems (Holmstrom and Tirole, 1997). If we introduce these agency frictions into our model, the exogenous return on investment here, is replaced by an endogenous return to relaxing those agency frictions with internal funds. This shadow value of relaxing supports trade under private information (Bigio, 2015).

# 2 The Environment

Consider a two-period economy, $t = 1, 2$. There is a measure-one continuum of borrowers and a large number of ex-ante homogeneous lenders, indexed by $j \in \mathcal{J} = \{1, 2, ..., J\}$. All agents are risk neutral and maximize their $t = 2$ payoff. At $t = 1$, each borrower is endowed with an indivisible asset and has access to an investment project. At $t = 1$, assets pay dividends and the investment projects yield a return. Assets are heterogeneous in quality: they payout a different dividend, denoted by $\lambda \in \Lambda = [\underline{\lambda}, \bar{\lambda}]$. By contrast, projects are identical. Each project generates a gross return $(1 + r)x$ with certainty at $t = 2$ where $x$ is the amount of cash invested in the project at $t = 1$. The net investment return is fixed at $r > 0$.[14] In order to invest, the borrowers need external finance because project returns are nonpledgeable, as in Hart and Moore (1994). To obtain external funding, the only option for the borrowers is to perform a transaction with their existing assets. Lenders provide liquid funds and engage in those transactions competitively.

**Information.** The information regarding asset quality $\lambda$ is asymmetric: it is only known to the borrower who owns the asset. The asset quality follows a continuous distribution with c.d.f. $F(\cdot)$ and support $\Lambda \equiv [\underline{\lambda}, \bar{\lambda}]$. The distribution is common knowledge.

**Contract.** Borrowers and lenders can sign repo contracts. A *repo contract* is a pair of prices $\boldsymbol{p} = \{p_s, p_r\} \in \Lambda \times \Lambda$.[15] The first entry, $p_s$, is the sales price. It specifies the first leg of the

[14] The environment is equivalent to one where trade occurs because borrowers discount consumption by a factor $\beta$, as in Guerrieri and Shimer (2014). Consider the same two-period economy except that, instead of the liquidity need to finance investment projects, borrowers have a liquidity need because they discount time between $t = 1$ and $t = 2$ by a factor $\beta$ whereas lenders do not discount at all. To equate the two settings, one can simply set the borrower's discount factor to the inverse of the gross investment return, $\beta = \frac{1}{1+r} < 1$.

[15] The restriction of prices to the domain box $\Lambda \times \Lambda$ is without loss of generality.

contract: at $t = 1$, the lender transfers $p_s$ in cash to the borrower and the asset is transferred to the lender. The second entry, $p_r$, is the repurchase price, that is, the price to be repaid at $t = 2$ by the borrower. We assume limited commitment on the side of borrowers. Failure to repurchase is considered a default and, in that case, the asset is seized by the lender who becomes entitled to the dividend, without further recourse. If the borrower repays $p_r$, honoring the contract, the asset is returned. Although borrowers cannot commit, the legal environment can enforce the turnover of the asset if the borrower repays.[16] Note that an *asset-sale contract* is a special repo contract: the contract subspace, $\boldsymbol{p} \in \Lambda \times \left\{\bar{\lambda}\right\}$ features a repurchase price such that both parties know that no borrower would repurchase the asset. Finally, given that assets are indivisible, screening with quantities is impossible. Hence, prices are not contingent on the quantity.

**Equivalence between Repo and Collateralized Debt.** An alternative interpretation is collateralized debt: under this interpretation, $p_s$ is the loan size, $p_r$ the face value, and $p_r/p_s - 1$ the implied interest rate. Default occurs when the face value is not repaid. For the rest of the paper, we bear this equivalence in mind. In practice, there are legal distinctions between repo and collateralized debt, but we do not delve into them here. We use the repo representation for exposition purposes.

**Timing.** At $t = 1$, the repo markets open. We adopt the timing of the three-stage game by Netzer and Scheuer (2014), which is a foundation for the MWS equilibrium. The game goes as follows:

- Stage 1: Each lender offers a contract. The contract offered by lender $j$ is denoted by $\boldsymbol{p^j} \in \Lambda^2$. The set of offered contracts, $\mathbb{P}_0 = \{\boldsymbol{p}^j : \forall j \in \mathcal{J}\}$, is observed by all lenders.
- Stage 2: After observing $\mathbb{P}_0$, lenders can withdraw the contract they offered in the previous stage. Lender $j$'s withdrawal is denoted by $I^j \in \{0, 1\}$—with 0 denoting withdrawal. The set of contracts after withdrawals is

$$\mathbb{P} = \left\{\boldsymbol{p}^j \in \mathbb{P}_0 : I^j = 1, \forall j \in \mathcal{J}\right\}.$$

- Stage 3: Borrowers choose a contract in $\mathbb{P}$ or opt to keep their assets.

At $t = 2$, borrowers decide whether to default on repurchasing the asset. For the time being, and to simplify the exhibition, we assume that lenders offer at most one repo contract. Technically speaking, this is different from the MWS equilibrium, in which lenders can offer multiple contracts. We later show that this restriction is without loss of generality.

[16] The lack of commitment on the side of borrowers implies that a repo contract is equivalent to an asset-sale contract with an embedded repurchase option.

**Borrower's Problem.** Consider stage 3. A borrower decides whether to participate in the repo markets according to:

$$\max \{0, v(\lambda)\}, \tag{1}$$

where the maximum value obtained from participating in a repo transaction and choosing the optimal contract is:

$$v(\lambda) = \max_{\boldsymbol{p} \in \mathbb{P}} \{(1+r) p_s - \min \{\lambda, p_r\}\}. \tag{2}$$

The value function in (1) encodes the borrower's participation constraint: the borrower only brings the asset to the repo markets if $v(\lambda)$ is positive. Otherwise, she opts out. In the value function (2), the borrower chooses the contract that maximizes her wealth, conditional on participation. The choice follows a cost-benefit analysis: the benefit is the investment payoff, $(1+r) p_s$, and the cost is the repayment, $\min \{\lambda, p_r\}$. If the borrower defaults, the cost is losing the asset. If the asset is repurchased, the cost is the repurchase price. Because there are no default costs, the asset is repurchased if and only if its quality is above the repurchase price. The contract chosen by a borrower with an asset of quality $\lambda$ is denoted by the function $P : \Lambda \to \mathbb{P}$, where $P(\lambda) = \{P_s(\lambda), P_r(\lambda)\}$ is a two-valued set function. $\Gamma(\lambda | \boldsymbol{p}, \mathbb{P})$ represents the cumulative density of assets that choose contract $\boldsymbol{p} \in \mathbb{P}$, in a given equilibrium.

**Lender's Problem.** In stage 2, each lender decides whether to withdraw her offered contract. This decision depends on whether the profit from keeping the contract is positive, taking as given all other lenders' withdrawal decisions. If lender $j$ decides to withdraw, the set of remaining contracts would be

$$\mathbb{P}^{-j} = \left\{\boldsymbol{p}^k \in \mathbb{P}_0 : I^k = 1, \forall k \in \mathcal{J}/j\right\}.$$

The resulting profit for this lender is

$$\Pi^j \left(\boldsymbol{p}^j; \mathbb{P}^{-j}, \mathbb{P}_0^{-j}\right) = \max \left\{\int \min \left\{\lambda, p_r^j\right\} d\Gamma \left(\lambda | \boldsymbol{p}^j, \mathbb{P}^{-j} \cup \boldsymbol{p}^j\right) - p_s^j, 0\right\}. \tag{3}$$

The profit in equation (3) is given by the following: the lender knows that not withdrawing would result in paying $p_s^j$ in any scenario. The lender anticipates being repaid $\min \{\lambda, p_r^j\}$, from a transaction with a borrower with asset quality $\lambda$. Hence, the lender builds an expectation with respect to the probability distribution of borrowers that sign contract $\boldsymbol{p}^j$ given that the set of offered contracts is $\mathbb{P}^{-j} \cup \boldsymbol{p}^j$. That distribution has a cumulative density function $\Gamma(\lambda | \boldsymbol{p}^j, \mathbb{P}^{-j} \cup \boldsymbol{p}^j)$. Hence, $\int \min \{\lambda, p_r^j\} \Gamma(d\lambda | \boldsymbol{p}^j, \mathbb{P}^{-j} \cup \boldsymbol{p}^j)$ is the borrower's expected repayment. Naturally, the lender withdraws if the expected profit is negative.

In stage 1, each lender offers a contract that maximizes profit, taking as given all other offers

$\mathbb{P}_0^{-j} = \left\{ \boldsymbol{p}^k : \forall k \in \mathcal{J}/j \right\}$. For lender $j$, the optimal contract offer is

$$\underset{\boldsymbol{p}^j \in \Lambda^2}{\operatorname{argmax}} \, \Pi^j \left( \boldsymbol{p}^j ; \mathbb{P}^{-j}, \mathbb{P}_0^{-j} \right).$$

**Equilibrium Concept.** We cast an equilibrium as a finite extensive form game. Naturally, the equilibrium notion is subgame perfect Nash equilibrium. Since many lenders compete to attract borrowers, taking $\mathbb{P}^{-j}$ as given, the solution to the game constitutes a market equilibrium, as in Netzer and Scheuer (2014). A contract $\boldsymbol{p} \in \mathbb{P}$ is active if at least some borrowers choose $\boldsymbol{p}$. Otherwise, there is no trading volume and the contract is inactive. When we refer to a unique equilibrium, we refer to uniqueness among the set of active contracts.

# 3 Characterization

Following sequential form analysis, we first characterize the borrowers' contract choice and their subsequent default decision. We then proceed by backward induction to study the set of equilibrium contracts. We begin with two observations: opting out of any contract is always an option for borrowers. Second, the value function $v(\lambda)$ is weakly decreasing. We emphasize the "weakly" decreasing property due to the repurchase option in the contract. This distinguishes repos from outright asset sales, in which case the value function from the selling the assets would be strictly decreasing in $\lambda$. These observations lead to the following lemma:

**Lemma 1** (Full Participation and Partial Default)**.** *Consider an equilibrium with offered contracts $\mathbb{P}$. The borrowers' participation and default strategy satisfies:*

*(i) [Full participation] All borrowers sign a repo contract;*

*(ii) [Default threshold] There exists a unique default threshold $\lambda_d$ such that all qualities $\lambda \leq \lambda_d$ default.*

The first item of Lemma 1 states that all borrowers participate in the repo market. The result contrasts with the outcome in the standard lemon market equilibrium where, due to private information, high-quality assets do not participate, and adverse selection ensues as a result. Full participation here implies that adverse selection is fully resolved: there are no borrowers that self-select out of the collateral pool. The second item says contracts where the collateral quality is below the threshold $\lambda_d$ feature defaults.

The repurchase option embedded in repo contracts is critical to ensure full participation. In particular, if the repurchase price $p_r \leq (1+r)\, p_s$, the repurchase option allows high quality borrowers to invest and obtain a positive payoff if they repurchase their assets. Since the payoff of repurchasing the asset is independent of the asset quality, even the best borrowers are willing to participate. In equilibrium, lenders compete to guarantee that repurchasing the asset is indeed attractive for borrowers that do not default. Competition guarantees that the payoff of repurchasing the asset is indeed positive. If this were not the case, every borrower would default, but in that case a single lender could deviate and offer a lower repurchase price to induce the marginal borrowers to exercise the repurchase option and make profits himself. Since the option to repurchase always brings a positive value to the nondefaulters, there is always full participation.

The outcome of the default threshold is also natural. Take a borrower who decides to sign contract $\boldsymbol{p}$ and intends to default. Consider another borrower with a worse-quality asset who goes for contract $\tilde{\boldsymbol{p}}$ and intends not to default. The borrower with the worse asset had the option to sign contract $\boldsymbol{p}$, but signed contract $\tilde{\boldsymbol{p}}$ because it yields a higher value. This means that the borrower with the better asset could have gone to contract $\tilde{\boldsymbol{p}}$ and obtained a higher value by not defaulting. Hence, it must be that the borrower with worse-quality collateral that goes to$\tilde{\boldsymbol{p}}$must also default. This implies the existence of a threshold default asset. Next, we show that to characterize the borrowers' optimal choice, it is sufficient to consider only two contracts in $\mathbb{P}$. For any equilibrium, consider the contract that offers the highest sales price and the contract that offers the highest nondefault value. A highest sales-price contract is

$$\boldsymbol{p}^d \equiv \underset{\boldsymbol{p}\in\mathbb{P}}{\operatorname{argmax}}\ p_s,$$

and a highest nondefault value contract is

$$\boldsymbol{p}^n \equiv \underset{\boldsymbol{p}\in\mathbb{P}}{\operatorname{argmax}} \left\{(1+r)\, p_s - p_r\right\},$$

which is associated with a nondefault value $\bar{v}$. The terms of each contract correspondingly are $\boldsymbol{p}^d = \left\{p_s^d, p_r^d\right\}$ and $\boldsymbol{p}^n = \left\{p_s^n, p_r^n\right\}$.

We obtain the following characterization.

**Lemma 2** (Borrower Contract Choice)**.** *Borrowers who hold assets of quality below the default threshold $\lambda_d$ choose a contract with the highest sales price:*

$$P(\lambda) = \boldsymbol{p}^d \ \textit{and}\ v(\lambda) > \bar{v},\ \forall \lambda \in [\underline{\lambda}, \lambda_d)\,.$$

*Borrowers who hold assets of quality above $\lambda_d$ choose a contract with the highest nondefault value:*

$$P(\lambda) = \boldsymbol{p}^n \ \textit{and}\ v(\lambda) = \bar{v},\ \forall \lambda \in \left[\lambda_d, \bar{\lambda}\right].$$

The lemma states that all borrowers who default must select the highest sales-price contract. All borrowers who do not default must select the contract that offers the highest nondefault value. These choices are intuitive: borrowers who know they will default only care about the sales price. Therefore, they must select a contract that offers the highest sales price. Borrowers who intend to repurchase assets face a trade-off between a higher loan size and a lower implied interest rate. Those borrowers will choose a contract that maximizes the nondefault value. All the results so far tell us that we can boil down the equilibrium set of contracts into two contracts, which simplifies the analysis. In equilibrium, the two contracts could actually be the same contract. The next proposition establishes that this is indeed the case.

**Proposition 1** (Pooling Equilibrium)**.** *If an equilibrium exists, it features a unique pooling contract such that:*

*(i) [Repurchase price] The repurchase price equals the default threshold:*

$$p_r = \lambda_d; \tag{4}$$

*(ii) [ZPC] The contract satisfies:*

$$p_s = \mathbb{E}\left[\min\left\{\lambda, p_r\right\}\right]. \tag{5}$$

Proposition 1 is a stepping stone toward the main result. The proposition states that the only possible equilibrium must feature a single pooling contract that attracts all assets. Item 1 says that the repurchase price equals the default threshold. Item 2 says that the contract must satisfy a zero-profit condition (ZPC) associated with full participation, pooling, and the optimal default decision. This is the set of contracts that satisfy equation (5). To interpret it, notice that the left-hand side is the outflow of funds to the lender and the right-hand side is the expected recovery values from defaulting assets and repurchase payments, where the expectation is calculated with respect to $F$. For convenience, we define the function $Z : \Lambda \to \Lambda$ as the sales price consistent with the ZPC, that is

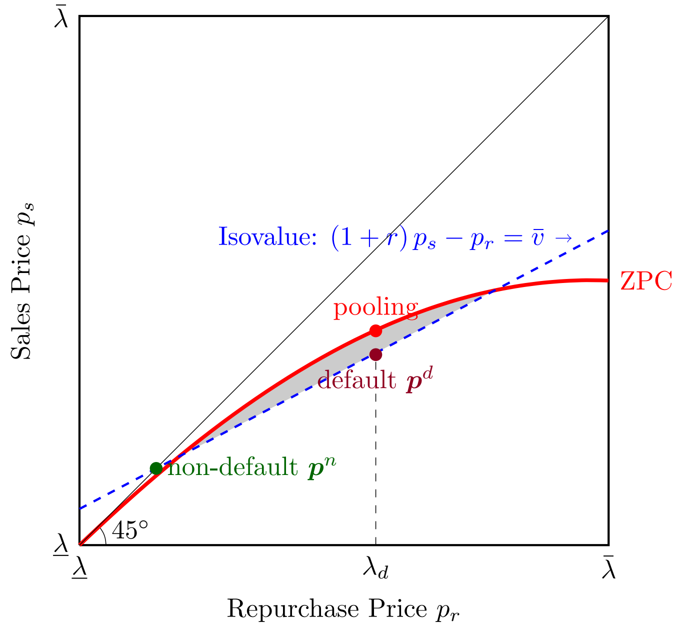
$$Z\left(p_r\right) \equiv \mathbb{E}\left[\min\left\{\lambda, p_r\right\}\right].$$

An important property is that $Z$ is strictly concave.[17]

We employ a graphical tool to explain why any separating equilibrium would break down. Figure 1 is constructed in a $\left[\underline{\lambda}, \bar{\lambda}\right] \times \left[\underline{\lambda}, \bar{\lambda}\right]$ box, the space of possible contracts. Sales prices are depicted on the y-axis and repurchase prices on the x-axis. The solid curve that crosses the figure plots $Z$ and thus represents the set of contracts that satisfy the ZPC. To show that a separating equilibrium breaks down, assume the false hypothesis that a separating equilibrium exists. By Lemma 2, we know that we could have at most two groups of contracts: one where

[17]Note that $Z'\left(p_r\right) = 1 - F\left(p_r\right) \geq 0$ and $Z''\left(p_r\right) = -f\left(p_r\right) < 0$.

Figure 1: Pooling of default and nondefault assets




no borrower defaults and another where all borrowers default. For the sake of argument, assume that each group has one contract.

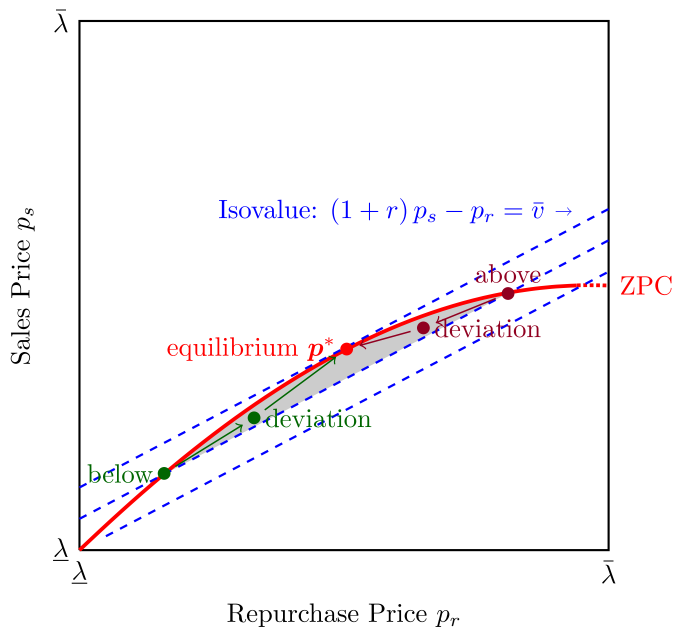
Figure 1 depicts the hypothetical candidate separating equilibrium together with the actual pooling equilibrium. To understand the unraveling of a separating equilibrium, we must first be assured that, in any separating equilibrium, the default contract must be located at some point below the $Z$ curve. In the appendix proof, we show that if this is not the case, then the sum of profits from both separating contracts is negative and at least one must be withdrawn. By continuity, we also know that an isovalue line intersects both contracts of the separating equilibrium because the threshold asset must be indifferent between the default and the nondefault contract. In the figure, this isovalue line is represented by the blue-dashed line.

Once we understand the location $\boldsymbol{p}^d$ in any possible separating equilibrium, it suffices to show that there's a profitable deviation. Since there exists other isovalue lines under the ZPC (the gray area in the figure) there is another contract that can be offered, that is profitable and attracts either all borrowers or nondefault borrowers. This is true for any candidate separating equilibrium, which means that no separating equilibrium exists.

Since the equilibrium is pooling, the only possibility is that the contract set is a singleton, $\mathbb{P} = \{\boldsymbol{p}\}$, if it exists at all. Because the equilibrium features full participation, the asset quality distribution underlying the pooling contract is

$$\Gamma\left(\lambda;\boldsymbol{p},\mathbb{P}\right) = F\left(\lambda\right),\ \forall\lambda\in\Lambda.$$

Figure 2: Equilibrium refinement




Given that any equilibrium must feature a single pooling contract which attracts the participation of all types, we define the following subset of the contract space:

$$\mathbb{Z} \equiv \left\{ \boldsymbol{p} \in \Lambda^2 \middle| \, p_s \leq Z\left(p_r\right) \text{ and } \left(1+r\right) p_s - p_r \geq 0 \right\}.$$

This set includes all pooling contracts that induce full participation and yield (weak) positive profits to the lender. We also define the upper boundary of the set, which we denote by $\bar{\mathbb{Z}}$ as the members of $\mathbb{Z}$ such that $p_s = Z\left(p_r\right)$.

Proposition 1 shows that the equilibrium contract is given by equations (4) and (5). Equation (4) results from the optimal default decision and (5) results from competition. Of course, there are many possible contracts that satisfy those equations. Proposition 1 gives us only two equations for three unknowns $\{p_s, p_r, \lambda_d\}$. The next proposition establishes which among these possible solutions is the actual equilibrium.

**Proposition 2** (Uniqueness)**.** *There exists a unique equilibrium featuring a* ***single*** *zero-profit pooling contract* $\boldsymbol{p}^*$*. The contract is*

$$\boldsymbol{p}^* = \underset{\boldsymbol{p} \in \bar{\mathbb{Z}}}{\operatorname{argmax}} \left\{ \left(1+r\right) p_s - p_r \right\}, \tag{6}$$

*which provides the highest nondefault value* $\bar{v}^* = \left(1+r\right) p_s^* - p_r^*$ *among all zero-profit pooling contracts.*

Proposition 2 states that the equilibrium contract $\boldsymbol{p}^*$ is the one for which the isovalue of nondefault is tangent to $Z$, i.e., the contract that maximizes the value of nondefault assets, subject to the ZPC. We argue that $Z$ is concave and the isovalue curves are affine. Hence, the solution exists and is unique and interior.

To explain why the solution boils down to the optimization problem in (6), we rely on Figure 2. Consider a pooling contract located on the ZPC, but with a sales price above or below $p_s^*$, as depicted in the figure. If the equilibrium contract does not maximize the nondefault value, there exists a deviating contract that delivers a higher value to nondefault assets. In fact, we can pick any contract in the shaded area of the figure. That deviating contract will attract all nondefault assets because the nondefault value is higher. Of course, if the deviating contract is offered in stage 1, the incumbent contract will be withdrawn due to the loss of nondefault borrowers who were cross-subsidizing the losses from default. Thus, the deviating contract remains alone and all assets would end up with the deviating contract. The deviating contract is strictly profitable because it is under the ZPC curve. The maximization representation follows because the only contract that can survive these deviations is $\boldsymbol{p^*}$, the contract that maximizes the nondefault value. Any attempt to attract the nondefaulters or the defaulters away from $\boldsymbol{p^*}$ would lead to losses, which guarantees that $\mathbb{P}_0 = \{\boldsymbol{p^*}\}$ is indeed an equilibrium.

The characterization of the equilibrium showcases that the economic force driving the equilibrium toward $\boldsymbol{p}^*$ is cream skimming. Cream skimming results from a trade-off. On one hand, the nondefaulters do not want to cross-subsidize the defaulters. If defaults can be reduced, the implied interest rate can be made smaller, and this increases the nondefault value. To induce fewer defaults, the repurchase price (loan principal) must fall to reduce the default threshold. When the repurchase price falls, competition drives down the sales price (loan size). A trade-off emerges because, on the other hand, borrowers also want to scale up their investments as much as possible. Thus, a higher sales price allows them to do so. In equilibrium, the contract that maximizes the nondefault value equates on the margin the benefit of a lower borrowing cost (less cross-subsidization) against the cost of reducing the loan size (less investment). Because lenders make money from the nondefaulters, the competition takes the equilibrium toward $\boldsymbol{p^*}$.

The trade-off between cross-subsidization and investment shows up as an interest rate wedge. Since $\bar{v} > 0$, the implied rate of $\boldsymbol{p}^*$ is lower than the return on investment, $p_r^*/p_s^* - 1 < r$. Proposition 2 resonates a similar result in Wilson (1977) where, in the context of insurance markets and two risk types, the equilibrium contract maximizes the value of low-risk types, subject to a zero profit condition. Below we discuss that cream skimming is a source of market inefficiency that a planner with commitment to the terms of a contract would resolve. Before that, we present the solution to the maximization problem in (6).

**Analytic Solution.** The repo equilibrium characterized in Proposition 2 has a simple analytic solution. We obtain it by solving the maximization problem in equation (6).

**Corollary 1** (Analytic solution)**.** *The unique equilibrium contract that solves (6) has prices*

$$p_r^* = F^{-1}\left(\frac{r}{1+r}\right) \tag{7}$$

$$p_s^* = \mathbb{E}\left[\min\left\{\lambda, F^{-1}\left(\frac{r}{1+r}\right)\right\}\right], \tag{8}$$

*and a default rate*

$$d = \frac{r}{1+r}. \tag{9}$$

Because of full participation, the aggregate liquidity that funds investment is $p_s^*$. A testable implication of this solution is that, given $r$, the default rate $d$ is independent of the asset distribution, $F$. The intuition relies on how the highest nondefault value $\bar{v}$ is obtained. The highest nondefault value is obtained when an isovalue line, which has a slope, $1/(1+r)$, is tangent to $Z$. This tangency implies that, if a lender were to increase the sales price by 1 dollar, the repurchase price should increase by $1+r$ dollars. Given that lenders break even, it must be that the $1/(1+r)$ fraction of borrowers do not default and the remaining $r/(1+r)$ fraction default. This property must hold for any $F$. A testable implication of this result is that the margin of adjustment to greater information asymmetries is the loan size, but not the default risk. This outcome is a clear example that a contraction in lending resulting from aggravated asymmetric information can manifest in the amount lent without leading to higher observed default rates.

**Discussion: Single-Contract Posting.** So far we have assumed for simplicity that lenders can post at most one contract. This assumption precludes the possibility of lenders offering a set of contracts and also precludes cross-subsidization across different contracts. The next proposition says that allowing for multiple contracts would not alter the equilibrium.

**Proposition 3** (Multiple Contracts)**.** *Consider a modification of the environment where lenders can individually offer multiple contracts in stage 1. Then, the same pooling equilibrium characterized in Corollary 1 is obtained.*

The proofs are immediate because none of the proofs for the earlier results rely on the profitability of individual contracts. Importantly, the result differs from other results in insurance models, where restrictions to single contracts are not without loss of generality (Mimra and Wambach, 2019). A common result in those models is that allowing for multi-contract offerings can sustain separating equilibria that would otherwise not be possible.

# 4 Efficiency

In this section, we analyze efficiency and contrast repos against sales contracts. We investigate the effects on efficiency of three forces, (i) participation, (ii) the pooling outcome, and (iii) cream skimming. To set the stage, we need an appropriate efficiency notion. We adhere to Holmstrom and Myerson (1983) and consider a most natural notion: constrained ex ante Pareto efficiency.[18] To that end, we contemplate that at a prior date to $t = 1$, at $t = 0$, borrowers are ex ante identical and endowed with homogeneous assets. Then, at $t = 0$ borrowers draw $\lambda$ from $F(\cdot)$. Because all agents are ex ante identical, the appropriate efficiency notion is to maximize the expected $t = 2$ borrower wealth.

Given the information friction, if borrowers could agree to sell all their assets at $t = 0$ at the unconditional expected value, they would. An adequate efficiency notion must take into account that, due to the borrower's lack of commitment to contracts, there is an ex post participation at $t = 1$, and that the decision to participate is made once the borrower knows $\lambda$.

## 4.1 Optimal Repo Contract Design

To study efficiency, we contemplate a planner who maximizes the ex ante borrower payoff, is subject to the same information constraint as lenders, and takes into account the ex post participation. Let the planner offer a menu of repo contracts $P : \Lambda \to \Lambda^2$ that satisfy a balanced budget. Because borrowers cannot commit to an ex-ante contract, it does not matter if the planner offers the contract at $t = 0$ or at $t = 1$.

Formally, the planner chooses the menu of contracts and the participation threshold to maximize:

$$\max_{\{P(\cdot),\lambda^p\}} \int_{\underline{\lambda}}^{\lambda^p} \left((1+r) P_s(\lambda) - \min\{\lambda, P_r(\lambda)\}\right) dF(\lambda) \tag{$\mathcal{M}$}$$

subject to the incentive-compatibility constraint, a participation constraint, and a budget constraint:

$$(1+r) P_s(\lambda) - \min\{\lambda, P_r(\lambda)\} \geq (1+r) P_s\left(\tilde{\lambda}\right) - \min\left\{\lambda, P_r\left(\tilde{\lambda}\right)\right\}, \forall \lambda, \tilde{\lambda} \in [\underline{\lambda}, \lambda^p] \tag{IC}$$

$$(1+r) P_s(\lambda^p) - \min\{\lambda^p, P_r(\lambda^p)\} \geq 0 \tag{PC}$$

$$\int_{\underline{\lambda}}^{\lambda^p} \left(\min\{\lambda, P_r(\lambda)\} - P_s(\lambda)\right) dF(\lambda) \geq 0. \tag{BC}$$

The planner's problem ($\mathcal{M}$) differs from the competitive outcome along two dimensions. The first difference is that the planner does not maximize profits and, therefore, does not cream skim. The second difference is that the planner can choose a separating contract. In the market equilibrium, competition prevents this.

We consider the cases where there is enough dispersion in asset quality for adverse selection under asset sales: situations in which the highest quality asset is unwilling to sell at a price equal

[18] The market outcome is ex post efficient: the planner's solution cannot improve liquidity without hurting the value of nondefault borrowers.

to the unconditional average quality. Otherwise, the planner could simply offer that price, all qualities would participate, and the planner would recover the first-best allocation. Thus, we impose:

**Condition 1** (Heterogeneity.)**.** $(1+r)\,\mathbb{E}\left[\lambda\right] < \bar{\lambda}$.

The next proposition shows that, although the planner mechanism is also pooling, it offers a different pooling repo contract than the market equilibrium.

**Proposition 4** (Constrained Efficiency)**.** *Under Condition 1, the planner's optimal contract is a zero-profit full-participation pooling contract* $\boldsymbol{p}^p$ *such that*

$$\boldsymbol{p}^p = \underset{\boldsymbol{p}\in\bar{\mathbb{Z}}}{\text{argmax}}\ p_s. \tag{10}$$

*In the solution, the participation constraint is binding,*

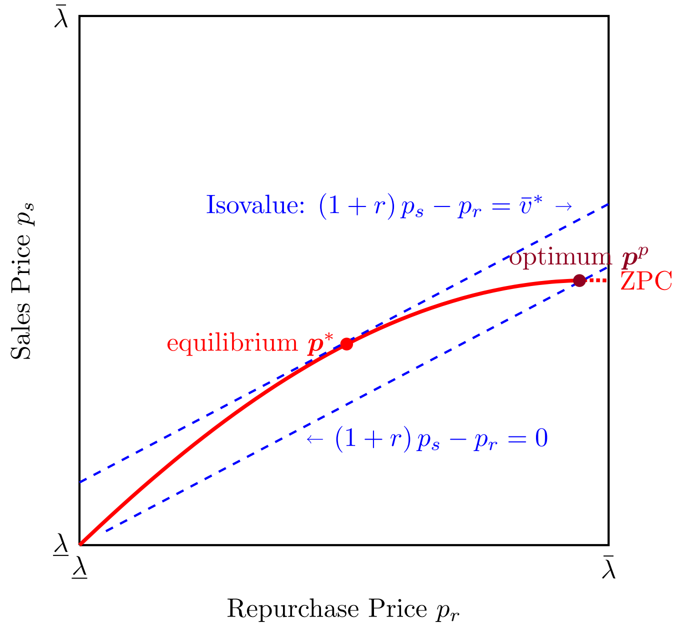
$$\bar{v}^p = (1+r)\,p_s^p - p_r^p = 0.$$

The planner's solution is similar to the market outcome in that the solution is also a zero-profit pooling contract. The difference is that, rather than maximizing the nondefault value, the planner's solution maximizes total liquidity subject to the participation constraint of nondefault assets, $\bar{v} \geq 0$. Since the planner's solution differs from the market equilibrium, the market equilibrium is inefficient.

Under Condition 1, with outright sales, higher quality borrowers opt out of the market due to adverse selection. By utilizing the repurchase option and adjusting the repurchase price, the planner can offer a repo contract that induces full participation, as long as the nondefault value is positive. Full participation improves the outcome. Also, the solution satisfies the ZPC because otherwise the planner can more efficiently (effectively) redistribute its revenues by raising the sales price of any contract offered. A final salient feature of the efficient solution is that, despite the opportunity to separate borrowers into different contracts, the planner solution also pools all borrowers into one contract, like the market outcome. The rationale behind pooling is that separation induces a waste of resources. To induce separation, the planner must offer a lower sales price to the nondefaulters. However, the planner can always do better by raising a nondefault sales price by $\varepsilon$ and the nondefault repurchase price by $\varepsilon(1+r)$, thus moving along the same isovalue line. This new contract induces the same default threshold because it falls in the same isovalue line, but it clearly raises planner's revenues by $r\varepsilon$. Since the planner solution satisfies the ZPC, this means that the planner solution cannot be separating.

The discussion clarifies the nature of the inefficiency of the competitive market. Clearly, (i) full participation plays no role because both the planner and market solutions involve full

Figure 3: Optimal repo contract




participation. Also, (ii) the pooling nature of the market equilibrium plays no role because the planner's solution does not exploit the ability to separate across contracts. It is only (iii) cream skimming, what creates the market inefficiency. Cream skimming, which results from the desire to attract better nondefaulters, ends up lowering the sales and repurchase prices to reduce the default and interest rate. The planner is never tempted to attract better borrowers. On the contrary, the planner cares only about total investment and thus goes all the way to raising the sales price to the point that the implicit interest rate equals $r$.

The planner's solution can be interpreted as an optimal security design problem. In fact, the planner solution here coincides with the optimal contract in Biais and Mariotti (2005). Security design has an embedded commitment assumption to a contract at $t = 0$ and are designed prior to learning information. For that reason, cream skimming is not a possibility, because lenders do not compete for the nondefaulters once the terms of the contract are set. The market solution does not possess that commitment and, thus, allows for inefficient cream skimming. Despite its inefficiency, the equilibrium repo contract can bring substantial efficiency gains over equilibria that are restricted to asset sales only, as we explain below.

**Discussion: A Tax on Repo?** According to Gorton et al. (2020), adverse selection in financial markets was an important motivation behind the policy interventions in the repo and asset-backed security markets during 2008-2009. Motivated by those arguments, Philippon and Skreta (2011) and Tirole (2012) study optimal policy interventions in markets but use sales contracts without considering repo contracts explicitly. These interventions are not justified if contracts are designed efficiently, as in the security design approach. However, as we showed earlier, in competitive repo markets, the inefficiency may warrant intervention.

One possible intervention is to set taxes that distort the market equilibrium in order to reduce cream skimming. We now show that the planner's solution cannot be decentralized with self-financed taxes. To that end, consider a policy intervention where the planner subsidizes the sales price by $\tau_s$ and finances the subsidy with a tax on the repurchase price $\tau_r$. We assume that, if borrowers default, they do not pay the tax. In this case, the zero-profit condition for lenders is:

$$p_s = \int_{\underline{\lambda}}^{p_r+\tau_r} \lambda dF(\lambda) + p_r (1 - F(p_r + \tau_r)),$$

whereas the budget balance for the planner requires:

$$\tau_s = \tau_r (1 - F(p_r + \tau_r)).$$

We can add the two conditions above and obtain a consolidated zero-profit condition:

$$p_s + \tau_s = \mathbb{E}[\min\{\lambda, p_r + \tau_r\}].$$

Using the same arguments to obtain the market equilibrium in Proposition 2, we know that the market solution will fall in

$$\boldsymbol{p}^* \in \underset{p_s+\tau_s=\mathbb{E}[\min\{\lambda,p_r+\tau_r\}]}{\operatorname{argmax}} (1+r)(p_s+\tau_s) - (p_r+\tau_r).$$

The solution to the repo contract with subsidies and taxes is:

**Corollary 2** (Taxed Contract)**.** *Given $\tau_s$ and $\tau_r$, the unique equilibrium contract has a repurchase price of:*

$$p_r = F^{-1}\left(\frac{r}{1+r}\right) - \tau_r.$$

*and a sales price of*

$$p_s = \mathbb{E}\left[\min\left\{\lambda, F^{-1}\left(\frac{r}{1+r}\right)\right\}\right] - \tau_s.$$

*Furthermore, the nondefault value, the default threshold, and the default rate remain unchanged.*

The proof is immediate by change of variables, so we omit it. The intuition is simple: whatever the tax rates are, if they apply to all contracts uniformly, they are undone by market competition! As simple as it is, the corollary is meaningful. It implies that the second-best allocation cannot be implemented with uniform taxes. Instead, the planner must either directly dictate the terms of trade of the contract or contribute resources. Because the intervention cannot be self financed, an optimal intervention must be explicit about the cost of the intervention.

## 4.2 Repo vs. Outright Sales: Efficiency Comparison

We now characterize conditions under which repo contracts increase market liquidity in relation to outright sales. Recall that repo contracts encompass outright sales—setting the repurchase price above the highest asset quality. Restricting our subsample to sales contracts only and using the market timing formulation of Netzer and Scheuer (2014), we obtain a unique equilibrium with a single pooling contract. This equilibrium coincides with the Pareto dominant equilibrium in Akerlof (1970). We first characterize this equilibrium with an alternative representation. We define the following statistics:

$$Z_a(\lambda) \equiv \mathbb{E}\left[\tilde{\lambda}\,\middle|\,\tilde{\lambda} \leq \lambda\right] \text{ and } L_a(\lambda) \equiv \mathbb{E}\left[\tilde{\lambda}\,\middle|\,\tilde{\lambda} \leq \lambda\right] F(\lambda), \ \forall \lambda \in \Lambda.$$

The term $Z_a(\lambda)$ is the pooling price (in an asset-sale contract) consistent with zero profits for a given an arbitrary quality threshold $\lambda$. The term $L_a(\lambda)$ denotes the corresponding level of aggregate liquidity obtained for that price and threshold; it represents the total value of assets up to that threshold. The solution to the lemons problem is:

**Lemma 3** (Asset-Sale Equilibrium)**.** *The equilibrium with outright sales is given by a threshold participation*

$$\lambda_a = \underset{\lambda \in \Lambda}{\operatorname{argmax}} \left\{\lambda \,|\, (1+r) Z_a(\lambda) - \lambda \geq 0\right\}. \tag{11}$$

*The equilibrium price is given by $p_a = Z_a(\lambda_a)$ and the liquidity provided is $x_a = L_a(\lambda_a)$ .*

Equation (11) characterizes the highest possible quality willing to participate in the pooling contract and thus the unique equilibrium of the sales contracts under the Netzer and Scheuer (2014) formulation.[19] This characterization allows us to present a sufficient statistic to guarantee that repos dominate outright sales. We specialize the distributions along one dimension:

**Assumption 1.** *Let $F(\cdot)$ be such that $(1+r)(\lambda - Z_a(\lambda)) < \frac{F(\lambda)}{f(\lambda)}$ for all $\lambda \in \Lambda$.*

This assumption ensures that we consider the class of distributions for which the Akerlof (1970) equilibrium is unique without further refinements.[20]

**Proposition 5** (Repo vs. Outright Sales)**.** *Let Assumption 1 hold. When Condition 1 holds, there is partial participation in the asset-sale equilibrium. The repo equilibrium generates more*

[19] The solution exists and is unique since $(1+r) Z_a(\underline{\lambda}) - \underline{\lambda} = r\underline{\lambda} \geq 0$. Under Condition 1, $(1+r) Z_a(\bar{\lambda}) - \bar{\lambda} < 0$, which implies partial participation, $\lambda_a < \bar{\lambda}$. However, it is possible that $Z_a(\lambda)$ intersects $\lambda$ multiple times. The solution under Netzer and Scheuer (2014) corresponds to the largest root of the equation. See Azevedo and Gottlieb (2017) for a thorough analysis of the regulating conditions such that the price correspondence is continuous with respect to the primitives.

[20] The condition guarantees that $(1+r) Z_a(\lambda) - \lambda$ is strictly decreasing. When Condition 1 holds, since $(1+r)Z_a(\underline{\lambda}) - \underline{\lambda} \geq 0$ and $(1+r) Z_a(\bar{\lambda}) - \bar{\lambda} \leq 0$, then $(1+r) Z_a(\lambda) - \lambda = 0$ has exactly one root in $\Lambda$. A unique root is necessary to obtain the sufficient statistic below.

*aggregate liquidity than the asset-sale equilibrium if and only if*

$$(1+r)\, Z_a \left(L_a^{-1}\left(p_s^*\right)\right) < L_a^{-1}\left(p_s^*\right). \tag{12}$$

*When Condition 1 does not hold, there is full participation and the asset-sale equilibrium generates more aggregate liquidity than the repo equilibrium.*

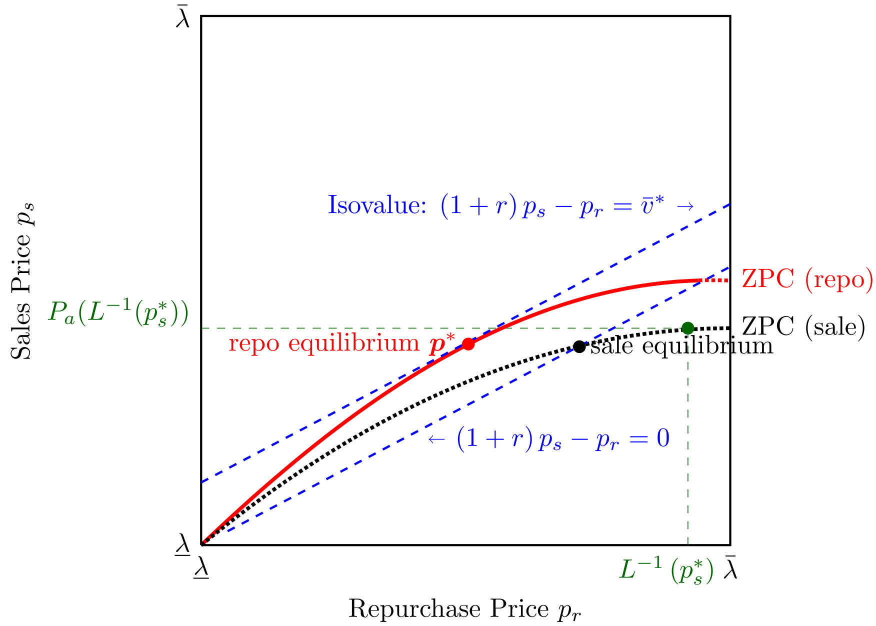
We start by examining situations where Condition 1 holds. When inequality (12) holds, the repo equilibrium provides more aggregate liquidity than the asset-sale equilibrium, and thus, repos bring about efficiency gains. To understand inequality (12), fix $\boldsymbol{p}^*$. In the repo market equilibrium, $p_s^*$ also represents total liquidity (because of full participation). In turn, $L_a^{-1}$ projects a given level of liquidity into a participation threshold in the asset-sale equilibrium. Therefore, Condition 1 checks if the participation constraint of the sales equilibrium is satisfied for the threshold, $L_a^{-1}\left(p_s^*\right)$. If it is not satisfied, then the threshold of the sales equilibrium must be lower than $L_a^{-1}\left(p_s^*\right)$ and, by Assumption 1, the equilibrium under sales must deliver less liquidity than the repo equilibrium. Figure 4 provides a graphical illustration. From the value of $p_s^*$, we obtain a sales equilibrium threshold $L_a^{-1}\left(p_s^*\right)$ on the x-axis and $P_a\left(L_a^{-1}\left(p_s^*\right)\right)$ the corresponding sales price depicted on the y-axis. In the figure, $\left\{L_a^{-1}\left(p_s^*\right), P_a\left(L_a^{-1}\left(p_s^*\right)\right)\right\}$ does not satisfy the participation constraint. The sales equilibrium features a threshold quality to the left of $L_a^{-1}\left(p_s^*\right)$, implying lower efficiency under outright sales. Conversely, when the informational asymmetries very mild the efficiency loss from cream skimming is greater than the advantage of reduced adverse selection brought about by repos.

When Condition 1 doesn't hold, it is because adverse selection under the sales-only equilibrium is mild. In such instances, repos bring about little efficiency gains by increasing participation because adverse selection, the problem to solve, is mild to begin with. However, in that scenario, repos allow for cream skimming that ultimately impairs cross-subsidization and thus reduces aggregate liquidity.

To further illustrate the trade-off behind the introduction of repos, we conduct a comparative static exercise with respect to the asset quality distribution $F$. For ease of exposition, we consider the example of a uniform quality distribution, as in Hendel and Lizzeri (2002). This case has closed-form solutions for both the repo and outright sales. Details are included in Appendix A.9.

**Example** (Uniform Distribution)**.** Let the asset quality follow a uniform distribution $\lambda \sim U(1-\sigma, 1+\sigma)$, where the bound of distribution $\sigma \in [0,1]$. The average asset quality $\mathbb{E}[\lambda] = 1$. Absent any information frictions, aggregate liquidity equals 1. As we increase $\sigma$ from 0 to 1, we are inducing a mean-preserving spread in the distribution of qualities. In the case of a uniform distribution, condition (1) translates into $\sigma > r$. When there is little dispersion, i.e., $\sigma \leq r$, the asset-sale equilibrium features full participation and obtains aggregate liquidity of 1. The

Figure 4: Comparison of repo contract vs. asset sales




expressions for the equilibrium repo contract is:

$$p_s^* = 1 - \frac{\sigma}{(1+r)^2} \quad \text{and} \quad p_r^* = 1 - \frac{1-r}{1+r}\sigma.$$

When there is sufficient dispersion, $\sigma > r$, the asset-sale equilibrium has a participation cutoff, sales price, and aggregate liquidity given by:[21]

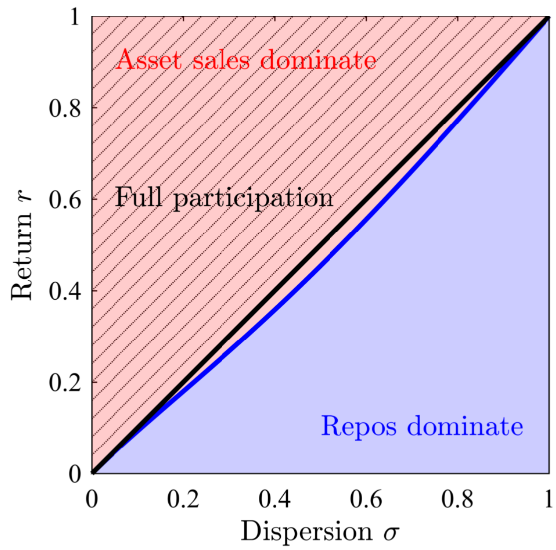
$$\lambda_a = \frac{1+r}{1-r}(1-\sigma), \; p_a = \frac{1}{1-r}(1-\sigma), \text{ and } x^a = \frac{r}{(1-r)^2}\frac{(1-\sigma)^2}{\sigma}.$$

When $\sigma = r$, there is full participation in the asset-sale equilibrium and, thus, the asset-sale equilibrium Pareto dominates the repo equilibrium. As the dispersion increases beyond that point, the aggregate liquidity in sales equilibrium declines due to a lower participation and a lower sales price. The details for the calculations are included in Appendix A.9. In Figure 5, we plot the threshold level of dispersion $\sigma$, above which the repo contract dominates outright asset sales, given the investment return $r$.[22] Of course, with a higher investment return, even the highest quality assets are more eager to sell. As a result, the repo dominance threshold is higher. When investment return is reasonably low, say 5%, the repo dominance threshold is just

[21] When the asset quality follows uniform distribution, the Akerlof asset-sale equilibrium has a participation threshold $\lambda_a$ characterized by $(1+r)\mathbb{E}[\lambda\,|\lambda \le \lambda_a] - \lambda_a = \frac{1}{2}[(1+r)\underline{\lambda} + (1-r)\lambda_a] = 0$. Under Condition (1), there is a unique interior root , $\lambda_a = \frac{1+r}{1-r}\underline{\lambda}$.

[22] The threshold condition is $\sigma > \left((r+1)^2(r^2+1) - (r^2-1)\sqrt{r^2(r^2+4r+6)+1}\right)/(2(1+r(r(r+3)-1)))$.

Figure 5: The welfare improvements of repo contracts over asset sales



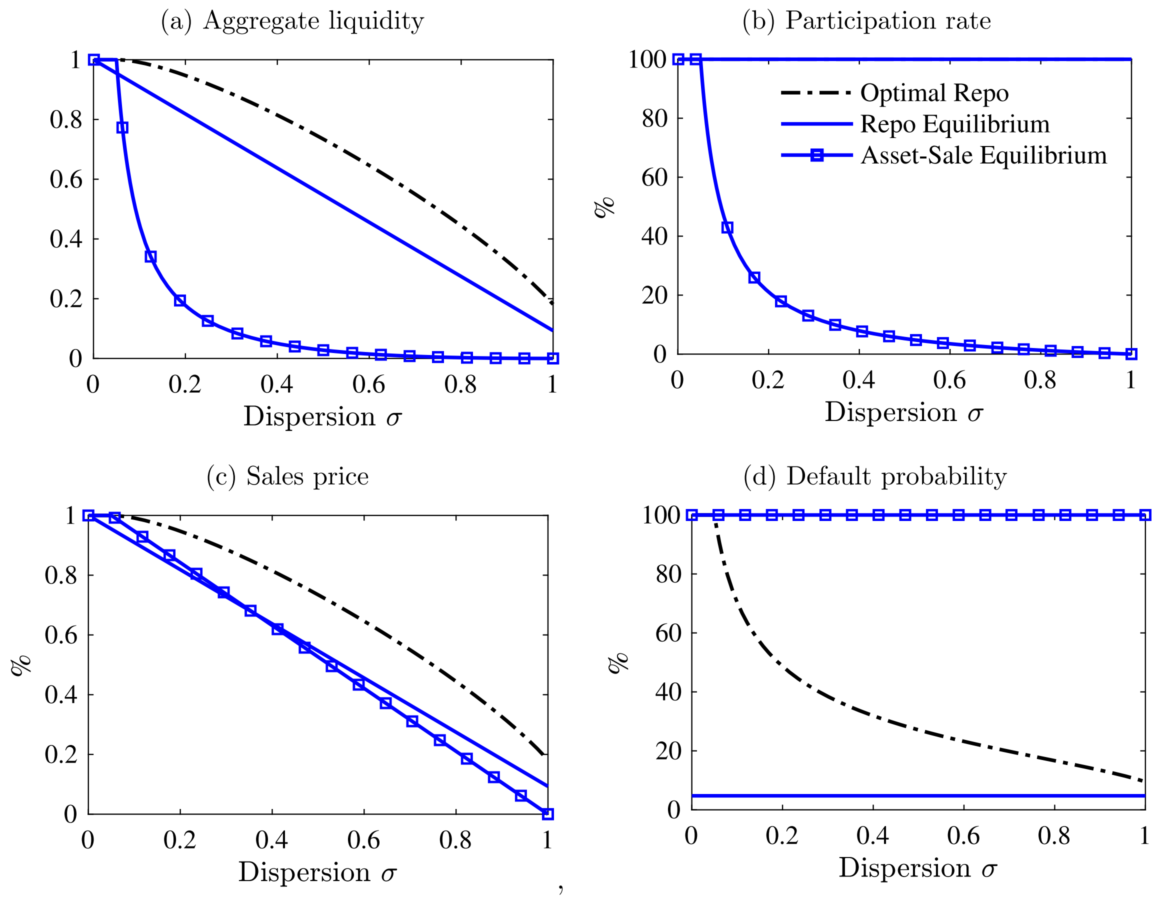

slightly higher.

For further illustration, we set the investment return $r$ to be 5% and plot the resulting contracts in Figure 6, as we vary $\sigma$. Panel (a) shows the aggregate liquidity in the repo and sales equilibria. With a small dispersion in asset quality and hence mild adverse selection, the repo equilibrium and the asset-sale equilibrium both obtain a level of aggregate liquidity close to the full information benchmark. In that case, selling assets dominates repo contracts. As asset quality becomes more dispersed and adverse selection aggravates, aggregate liquidity plummets in the sales equilibrium. Eventually, for a sufficiently high dispersion, repos dominate outright asset sales and ameliorate the lemons problem.

Panel (b) reveals that when adverse selection is severe, the gains from repos are due to higher participation. Repo contracts bring all assets to the market thanks to the repurchase option. Furthermore, the cross-subsidization from a better pool of high quality assets allows for a higher sales price. With asset sales, high quality assets opt out of the market because the sales price is unappealing. When $\sigma$ increases from 0.05 to 0.2, the fraction of sold assets decreases from 100% to around 20% under the sales equilibrium.

Panel (c) shows an additional channel by which repos improve over asset sales, once information asymmetry is severe: the sales price is higher in the repo equilibrium contract than in outright sales. For the repo equilibrium, recall that the default probability is fixed, as shown in panel (d). The decline in the aggregate liquidity occurs as lenders anticipate the average quality decreasing and hence offer a lower sales price. The repo equilibrium experiences mild decline, while in the asset sales case, the equilibrium aggregate liquidity declines sharply. Although simple, this example showcases that repos are a natural outcome in markets with severe adverse

Figure 6: The welfare improvements of repo contracts over asset sales




selection.

We can conclude the section by stating that repos are a valuable financial innovation, and are only detrimental when the asymmetric information is not a big problem to begin with.

# 5 Variations

**Competitive Search for Repos.** In this section, we investigate the effects of introducing repos into an alternative market environment. As a particular example, we introduce repos into the competitive search environment with asymmetric information developed by Guerrieri et al. (2010). There are two meaningful departures relative to the equilibrium we have studied so far. First, under competitive search, lenders self select into contracts that differ not only in prices but also in trading probability. Second, there is a capacity constraint where each lender can at most service one contract. The latter departure is important as it affects the off-equilibrium beliefs and in turn the equilibrium outcome.

Let's begin by discussing outright sales equilibrium in the competitive search framework.

With outright sales, the solution is diametrically different to the one discussed above. Assuming away contract posting costs, Guerrieri et al. (2010) show that the equilibrium is fully separating and all assets are sold at their true quality $\lambda$. Separation is possible because the trading probabilities differ across contracts. With a continuum of qualities, the trading probability of a given quality $\lambda \in \Lambda$ is $\Theta(\lambda) = (\underline{\lambda}/\lambda)^{\frac{1+r}{r}}$. In turn, the aggregate liquidity is $\mathbb{E}\left[(\underline{\lambda}/\lambda)^{\frac{1+r}{r}}\lambda\right]$. Compared to the repo or sales equilibrium shown above, the aggregate liquidity is lower. Details are included in the Online Appendix.

We extend the competitive search framework to allow for repos. When repo contracts are offered, the competitive search solution is no longer separating but becomes pooling once again.[23] The reason is that competition forces inhibit the use of trading probabilities as a screening device, when a repurchase price can be used.[24] Despite featuring a unique pooling contract, the nature of the problem changes relative to the outcome under the Netzer and Scheuer (2014) setting. With repos, the unique pooling contract is given by:

$$\boldsymbol{p}^{GSW} = \{\underline{\lambda}, \underline{\lambda}\} .$$

This contract induces full participation and zero default. However, it is the worst contract among the contracts that satisfy the ZPC. Thus, the repo market equilibrium under the Netzer and Scheuer (2014) formulation dominates the repo equilibrium under competitive search. Still, under competitive search, repos improve liquidity relative to outright sales only. We can see this from $\underline{\lambda} \geq \mathbb{E}\left[(\underline{\lambda}/\lambda)^{\frac{1+r}{r}}\lambda\right]$.[25]

**Lender's Lack of Commitment.** We now relax the assumption that the lenders can commit to returning the asset to borrowers. In particular, we grant lenders the option to retain the

[23]Under competitive search, we can show that the equilibrium is characterized following identical steps to those that lead to Proposition 1, adding an intermediate step showing that all contracts are serviced with probability of one, i.e., no rationing. That is, any equilibrium must produce full participation, must be pooling, and must fall on the ZPC. Hence, the differences in the equilibrium outcome are generated by how the off-equilibrium beliefs in competitive search affects potential deviations.. Because of the capacity constraint, the off-equilibrium belief is that deviations attract assets that have the most to gain and hence can bear the lowest trading probability. Deviations toward higher prices will attract only low-quality default assets. This creates a force to never raise prices. Deviations downward have an embedded off-equilibrium belief that the deviation will attract high-quality nondefault assets. This creates an force toward offering the zero-profit contract with the lowest possible price.

[24]The pooling result here relates to other pooling outcomes in competitive search models. In Chang (2017) and Guerrieri and Shimer (2018), sellers possess multidimensional private information, which breaks the single-crossing property that allows separation in Guerrieri et al. (2010). Here, multidimensional contracting breaks the single-crossing property and hence the possibility of separating equilibria.

[25]The competitive search solution has an unappealing feature: with an arbitrary small amount of assets with zero quality, $\underline{\lambda} = 0$, and we reach a "no-trade" equilibrium. Hence the amount of aggregate liquidity vanishes to zero. In that case, repos and outright sales both produce zero liquidity, something that does not occur under the baseline formulation.

asset with probability, $\eta$.[26] In equilibrium, whenever they have the option to keep the asset, the lender will keep the asset. If the contract features a default from the borrower, the lender will keep the asset. If the borrower does not intend to default, the lender knows that it is because the asset is worth more than $p_r$, so it is convenient to keep the asset. Hence, the asset ends with the lender with probability $\eta$.

With this modification, the borrower's value function (2) becomes

$$v(\lambda) = (1+r)\,p_s - (1-\eta)\min\{\lambda, p_r\} - \eta\lambda.$$

The modified value function considers that, with probability $(1-\eta)$, the borrower's cost is $\min\{\lambda, p_r\}$, as before and that, with probability $\eta$, the cost is $\lambda$ because either the borrower or the lender will default. Thus, without full commitment on the side of the lender, the contract becomes a blend between an asset sale and a repo. The zero-profit condition in (5) is modified to:

$$p_s = \mathbb{E}\left[(1-\eta)\min\{\lambda, p_r\} + \eta\lambda \,|\lambda < \lambda^*\right], \tag{13}$$

where the condition now includes a threshold participant quality denoted by $\lambda^*$. The ZPC is modified to capture that, with probability $1-\eta$, the lender obtains the same expected revenue as before and, with probability $\eta$, the lender will keep the asset. A key distinction is that now the participation threshold $\lambda^*$ might be interior. The participation threshold solves:

$$\lambda^* = \underset{\lambda\in\Lambda}{\operatorname{argmax}}\,\{\lambda\,|v(\lambda) \geq 0\}\,. \tag{14}$$

Relative to the equilibrium with full commitment, the system of equations includes an extra equation—the participation constraint. To solve for the repo contract and obtain $\{p_s, p_r, \lambda^*\}$, we follow the same principle as before. We find that the equilibrium contract maximizes the nondefault value

$$\max\{(1+r)\,p_s - (1-\eta)\,p_r\}$$

but is now subject to the zero-profit condition (13) and the participation constraint (14).

Some observations emerge. We consider situations where Condition 1 holds. When $\eta$ is very small, all qualities still participate. The insight that repo resolves adverse selection extends to more general settings when allowing for a small degree of limited commitment on the side of lenders. In the other extreme, as $\eta \to 1$, the solution collapses to the Akerlof equilibrium, because no asset would be returned. For intermediate values, we have partial participation. The reason is that the value for borrowers is no longer constant among the nondefaulters but decreasing in the value of the asset. By inducing partial participation, the lack of commitment

[26] One can endogenize $\eta$ by letting lenders receive signals about the underlying asset quality. But for now, we keep the analysis simple.

by lenders reintroduces adverse selection as a force that, together with cream skimming, shapes the repo market equilibrium.

# 6 Interpretation and Evidence

We think of our theory as applying to any asset market that features repo-like structures (collateralized debt, bridge loans, factoring, and loan discounts) including the repo market itself, which is an important source of funding in securities markets. In this section we show that the market equilibrium uncovered in this paper is consistent with observed patterns in repo markets.[27] To that end, we recast the model to capture the additional institutional details of actual repo markets, which allows us to map the model to data.

**An Extended Model.** We embed our model in an environment in which repo markets are a funding source in securities trading. Borrowers and lenders are both risk-neutral, as before. The timing is as follows. At $t = 0$ borrowers agree to buy a security at a known market price. We assume that borrowers buy at most one security and have no funds to purchase the security themselves. At $t = 1$, borrowers are obliged to come up with funds to settle the transaction. The borrower can settle the transaction by drawing on a credit line that has an implicit interest cost of $r$. The credit line requires no collateral by the borrower and is full recourse. To avoid using the credit line, the borrower can access a repo market that works exactly as in Section 2. As before, lenders in the repo market have no cost of capital so repo markets are intrinsically a cheaper source of funding. However, the information problem of this repo market is that, at $t = 0$, the borrower receives a perfect signal regarding the future price of the security at $t = 2$ while lenders don't.

To let the model fit a cross-section of asset classes, we allow for asset heterogeneity and multiple asset classes. In particular, there is a continuum of assets indexed by $i \in [0, 1]$. Each asset belongs to one of a discrete number of asset classes, indexed by $n \in \{1, 2, ..., N\}$. The function $\alpha(i) : [0, 1] \to \{1, 2, ..., N\}$ maps each asset to its class. Securities in the same class have common attributes. Denote the $t = 0$ price of security $i$ by $p_i^0$. The price evolves to $p_i^2$ at $t = 2$, following a stochastic process,

$$\frac{p_i^2}{p_i^0} \equiv (1 + r) \cdot \exp\left(\sigma_{\alpha(i)} X_i - \frac{1}{2}\sigma_{\alpha(i)}^2\right).$$

Here, $\sigma_{\alpha(i)}$ is the volatility of the corresponding asset class to which $i$ belongs. The term $X_i$ is a standard normal distribution random variable which is *i.i.d.* We subtract $\frac{1}{2}\sigma_{\alpha(i)}^2$ to normalize the

[27] In practice, repo contracts are subject to lender recourse. However, failures in repo markets are rarely taken to courts. For that reason, we think that repo markets are a perfect environment to align our theory with the facts.

mean growth to zero. The growth in the price $(1+r)$ implies that it is profitable on average to buy the asset, even in absence of the repo market. Naturally, it's cheaper to fund the purchase in the repo market.

We think of borrowers and lenders as differing in their ability to process information that is relevant for the price formation process. Namely, borrowers are specialists in identifying securities and learn $X_i$ at $t=0$. Hence, they perfectly identify the growth in the price of the security—we can leave some unpredictable component, but this is not necessary to make the point. Lenders, on the contrary, cannot observe the signal. Lenders can observe the class where the security belongs but they cannot distinguish a security $i$ from security $j$: in mathematical terms, lenders can observe $\alpha(i)$ but not $i$. Finally, $p_i^0$ and $\sigma_{\alpha(i)}$ are known to everyone. All in all, the asymmetry of information only regards the future price, and the risk for the lender is the same within the asset class.

Now, we map this enriched model to the original model. Consider asset $i$. We define the intrinsic quality of the security at $t=1$ by

$$\lambda_i = \frac{p_i^2}{p_i^0}\frac{1}{1+r}.$$

The asset quality follows $\lambda_i \sim LN\left(-\frac{1}{2}\sigma^2_{\alpha(i)}, \sigma_{\alpha(i)}\right)$ with an average value of 1. We now apply the formulas uncovered earlier in Corollary 1, as the repo market will work the same way. The repurchase price is $p_{r,i} = \Phi^{-1}_{\alpha(i)}\left(r/\left(1+r\right)\right)$ and the sales price is $p_{s,i} = \mathbb{E}_{\alpha(i)}\left[\min\left\{\lambda_i, \Phi^{-1}_{\alpha(i)}\left(r/\left(1+r\right)\right)\right\}\right]$. Following the jargon of repo markets, the haircut of the asset class of security $i$:

$$h_i = 1 - \mathbb{E}_{\alpha(i)}\left[\min\left\{\lambda_i, \Phi^{-1}_{\alpha(i)}\left(\frac{r}{1+r}\right)\right\}\right], \tag{15}$$

is the difference between a notional value at $t=0$, $p_0^i$, and the sales price ,$p_s^i$, for the repo market, relative to the notional price. This expression uses the convention of denoting by $\Phi_{\alpha(i)}$ the c.d.f. of $\lambda_i$. The implied interest rate (also called the repo rate in the jargon of repo markets) is

$$r_i = \frac{\Phi^{-1}_{\alpha(i)}\left(\frac{r}{1+r}\right)}{\mathbb{E}_{\alpha(i)}\left[\min\left\{\lambda_i, \Phi^{-1}_{\alpha(i)}\left(\frac{r}{1+r}\right)\right\}\right]} - 1. \tag{16}$$

This term amounts to the ratio of the repurchase price to the sales price minus 1. Finally, note that the default rate, or in the jargon of repo markets the "failures," is $d = r/\left(1+r\right)$.

Before we discuss some empirical evidence on repo markets, we provide some context of the types of assets involved in repo transactions. An asset class can be a set of fixed-income securities of a specific maturity issued by the same issuer—e.g., the set of three-month Treasury securities. Another class can be a collection of unrelated securities—e.g. for example, a menu of asset-backed securities, or a collection of corporate bonds or stocks. The theory equally applies to *homogeneous* as to *heterogeneous* asset classes. We can think of a homogeneous asset class as, for example, the class of government bonds. Despite there being no individual asset characteristics that are payoff relevant, the price of fixed income securities can fluctuate. The notion of asymmetric information is that borrowers are better informed about the predictable component, the price of bonds. Hence, the notion of private information is not about the payoff of a given serial number, but rather about the greater ability to process macroeconomic information. The information story therefore applies to homogeneous assets too, for example the extensive trade of government securities in repo markets. For a heterogeneous class, asymmetric information stems from the ability to process specific information about a security. For example, in the case of non-investment grade securities, the serial number is relevant for the asset payoff. Thus, in our formulation the signal $X_i$ captures both idiosyncratic information as well as macroeconomic information. Hence, an alternative interpretation for the lemons problem in our theory is that borrowers are better at forecasting future security prices than lenders. We embrace this financial interpretation here.

**Evidence from Repo Markets.** We are now equipped with the right objects to discuss how the model relates to empirical findings in the literature. Despite the importance of repos as a major source of short-term liquidity funding for security dealers and hedge funds, data on repo markets is only slowly being collected. Most evidence on the behavior of repo markets is either anecdotal or obtained from specific investor segments. Nevertheless, the known evidence points toward adverse selection as a driver of the key features in repo markets.

The central prediction of our theory is that repo failure rates are unresponsive to the degree of asymmetric information and only respond to the underlying investment return $r$; see equation (9). Rather, the margin of adjustment to scenarios with more asymmetric information is the loan size (the haircut) and the corresponding repo rate, see equations (15-16). In contrast, the security design approach predicts that only the haircut would move, but the interest rate would remain fixed, as illustrated by Figure 6.

We start with some background of repo markets. As Copeland et al. (2014) explains, repo markets are divided into two categories: bilateral and tri-party repos. Bilateral repo markets operate just like in our model, where buyers and sellers agree on contracting terms. In tri-party repos, an intermediary is hired to manage the collateral, setting strict rules such as verifying collateral eligibility and centralizing settlements. According to Copeland et al. (2014), active participants in bilateral repo markets are hedge funds, asset managers, and security dealers, and these participants can be on either side of the trades. In the tri-party repo markets, typically only security dealers act as the borrowers in our model, and money market mutual funds and security dealers act as the lenders.

Julliard et al. (2019) and Auh and Landoni (2016) are among the few studies on bilateral repo markets. Using transaction-level data of six major banks that participate in the U.K. repo markets, Julliard et al. (2019) find that, consistent with our theory, maturity (which affects the investment return $r$ and the extent of informational asymmetries in our model) and collateral quality (which reflects the extent of asymmetric information) are the main determinants of haircuts. The authors also find that the identity of counterparties matters for haircuts. For example, they note that hedge funds as borrowers receive higher haircuts after controlling for counterparty risk. Our theory is also consistent with this finding if we conceive hedge funds as the best informed players in the market. Julliard et al. (2019) conclude their study by arguing, "We find evidence in favor of an adverse selection explanation of haircuts, but little evidence in support of lenders' liquidity position or default probabilities affecting haircuts." In the case of the U.S. repo markets, Auh and Landoni (2016) use a sample of repo transactions reported by hedge funds from 2004-2007. The authors find that, as loan risk increases (which we interpret as leaving more room for private information), lenders require both higher rates and haircuts.[28]

Our model can also help interpret different findings in the literature that are hard to reconcile without an information story. In the context of bilateral repos, an influential study by Gorton and Metrick (2012) show dramatic increases in haircuts during 2007-2008. For some asset qualities, haircuts reached levels beyond 40% without corresponding increases in failures. Yet, for similar asset classes, Krishnamurthy et al. (2014) find that haircuts for tri-party repos were stable around the same time. We interpret this striking difference as supporting evidence of our model. We attribute the evidence that bilateral repo markets have larger and more volatile haircuts than tri-party markets to the fact that the participating borrowers in bilateral repos are usually the best informed specialists in the finance industry (hedge funds and security dealers). By contrast, borrowers in tri-party repo markets are less sophisticated.

We now turn to a final discussion focused on observed haircut patterns across different asset classes. According to the evidence in these works, we expect that asymmetric information is more pronounced in the bilateral repo market. Unfortunately, only tri-party repo data, where information asymmetries are less severe, is readily available to us. Still, the cross-sectional patterns in tri-party repos are consistent with an asymmetric information story. Using data on tri-party repo haircuts, we investigate the necessary variation in informational advantage, the value of $\sigma_{\alpha(i)}$, needed to capture the size of observed repo haircuts across different asset classes. We factor in an annualized cost of capital of 1% and think of the repo terms as lasting one week. Figure 7 describes the result from the exercise. In the top panel, the y-axis corresponds to the

[28]The authors find that a 1 point higher rate is associated with 9 points of haircut. The authors claims that "there is no obvious theoretical reason why the lender should take more risk (as evidenced by the higher spread) when the collateral quality drops." We, however, interpret higher spreads not as lenders taking more risk, but as cream-skimming behavior by lenders.

haircut measured in percentage. The x-axis presents the value of the informational advantage needed to explain a given level of haircut. The dots in the figure correspond to the haircut data of different asset classes. We observe that haircuts are increasing in the riskiness of the collateral assets. For example, the lowest average haircut of 2% corresponds to the safest homogeneous asset classes, US Treasury and agency securities, followed by international agency debt, agency collateralized mortgage obligations (CMO), and money market shares. The haircuts for more risky heterogeneous asset classes, such as municipal debt, corporate debt, asset-backed securities (ABS), private-label CMOs, collateralized debt obligations (CDO), and equities, ascend from 4% to 9%, while wholesale loans have 10% haircuts. In addition, for the same asset type, the non-investment grade requires a larger haircut than the investment grade. The pattern is visible: asset categories where there is a greater scope for informational difference feature higher haircuts. The bottom panel presents the model implied repo rates associated with each category. Consistent with these empirical patterns and the evidence in Julliard et al. (2019) and Auh and Landoni (2016), the model produces a positive correlation between haircuts, as well as rates, and the extent of quality dispersion that is not present in the security design outcome. Finally, observe that the model does not require substantial information differentials to rationalize the haircuts. Take Treasury securities: to explain the 2% haircut in Treasuries, the model needs borrowers to be able to predict 0.5% of the weekly fluctuations in Treasury securities that borrowers can't. For wholesale loans, borrowers need to have a 3.5% advantage.

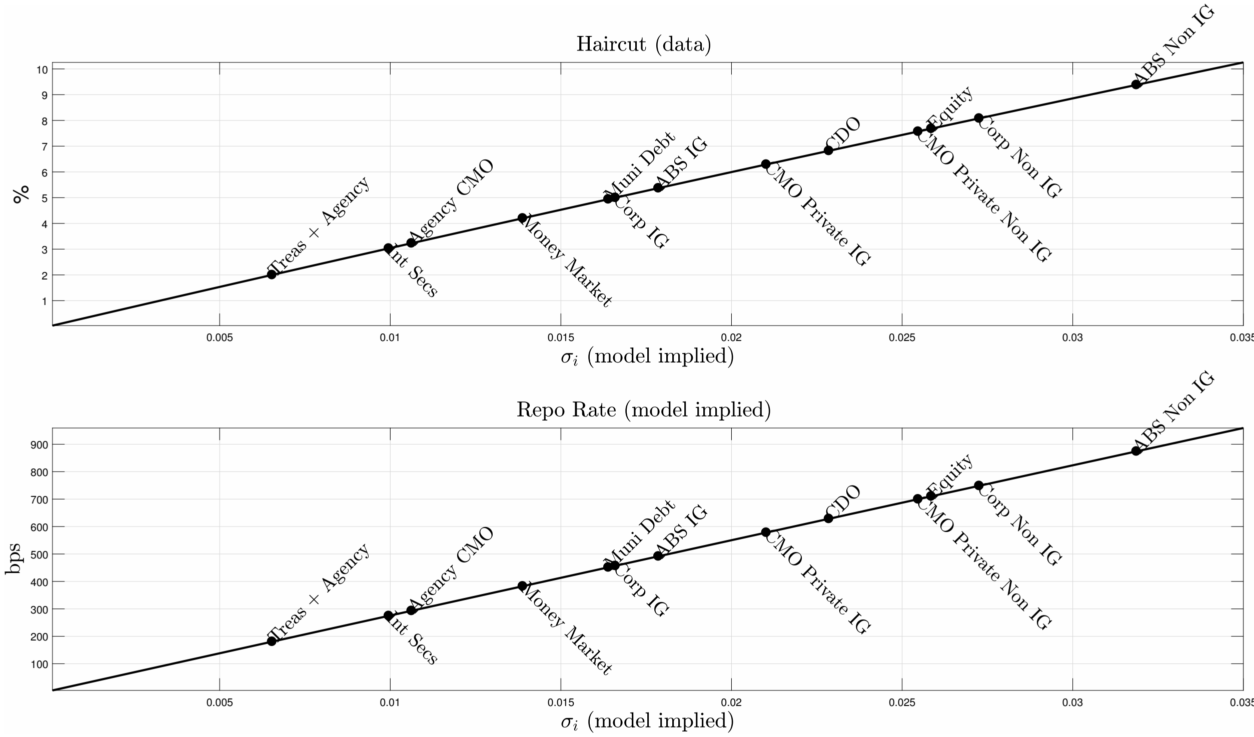
# 7 Conclusion

This paper exhibits a competitive market model of financial contracts that have repo like structures. The main insight is that, whenever asymmetric information is prevalent, lenders can offer repurchase options to attract borrowers that would otherwise not participate in these markets due to adverse selection. This improves the asset pool but comes at the expense of cream skimming of the high quality ones. The equilibrium contract is characterized in closed form.

Recent financial crises spurred an interest in models that incorporate private information into asset market models. These studies link changes in asset quality or information dispersion to firm funding and, through funding, into macroeconomic variables (Kurlat, 2013; Gorton and Ordoñez, 2014; Bigio, 2015). It is not difficult to see how the framework can be applied to environments with limited insurance, as in Alvarez and Jermann (2000), or to environments that feature collateral multipliers and endogenous prices (Brunnermeier and Pedersen, 2009; Simsek, 2013). A modeling advantage is that the contracts here render closed forms, making the theory readily portable to other settings, unlike other popular environments such as costly state verification by Townsend (1979).

Being generic, the theory only touches the surface of the questions postulated in the in-

Figure 7: Haircuts in the Data and Model Fit




*Notes*: In the first panel, the haircut data are measured by the average of the monthly median haircut for tri-party repo transactions in the U.S. from May 2010 to July 2019. The data are obtained from the Federal Reserve Bank of New York (https://www.newyorkfed.org/data-and-statistics/data-visualization/tri-party-repo/index.html#interactive/margins). The model-implied standard deviation is the one required by the model to match the observed average haircut in the data for each asset class, using a calibration of the model with $r = 5\%$, and the distribution $F$ is log-normal with mean 1. The second panel plots the model-implied interest rate.

troduction. The specific use of collateralized debt or repos typically depends on economic or institutional forces that are out of the scope of this paper. Further research could explore which market forces induce the same assets to be traded simultaneously in sales markets and repo markets and the coexistence of bilateral and tri-party repo markets.

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# A Proofs

## A.1 Proof of Lemma 1 and Lemma 2

To prove Lemma 1, we start by studying the borrower's contract choice strategy in stage 3 and subsequent default.

**Proof of Lemma 1: Default Threshold.** We first show that $v(\lambda)$ is weakly decreasing in $\lambda$. For any $\lambda_1 < \lambda_2$, let their optimal contract choices be $P(\lambda_1) = \boldsymbol{p}^1 = \{p_s^1, p_r^1\}$ and $P(\lambda_2) = \boldsymbol{p}^2 = \{p_s^2, p_r^2\}$. The values obtained satisfy the following:

$$\begin{aligned} v(\lambda_1) &\equiv (1+r)\, p_s^1 - \min\left\{\lambda_1, p_r^1\right\} \\ &\geq (1+r)\, p_s^2 - \min\left\{\lambda_1, p_r^2\right\} \\ &\geq (1+r)\, p_s^2 - \min\left\{\lambda_2, p_r^2\right\} = v(\lambda_2)\,. \end{aligned} \tag{17}$$

In equation (17), the first inequality follows from the optimal contract choice of $\lambda_1$: it obtains a higher value from contract $\boldsymbol{p}^1$ than from contract $\boldsymbol{p}^2$ . The second inequality is due to $\lambda_2 > \lambda_1$. The two inequalities may not be strict due to the repurchase option $\min\{\lambda, p_r\}$. That is, the value function is weakly decreasing in $\lambda$.

Next, we show that all nondefault assets obtain the same value. Continuing from the arguments above, if neither asset defaults, the repurchase prices must satisfy $\lambda_1 \geq p_r^1$ and $\lambda_2 \geq p_r^2$. Their values now also satisfy,

$$\begin{aligned} v(\lambda_2) &= (1+r)\, p_s^2 - p_r^2 \\ &\geq (1+r)\, p_s^1 - \min\left\{\lambda_2, p_r^1\right\} \\ &= (1+r)\, p_s^1 - p_r^1 = v(\lambda_1)\,. \end{aligned} \tag{18}$$

In obtaining equation (18), the first inequality comes from the optimal contract choice of $\lambda_2$: it obtains a higher value from contract $\boldsymbol{p}^2$ than from contract $\boldsymbol{p}^1$. The second equality is due to $\lambda_2 > p_r^1$: if quality $\lambda_2$ were to choose the contract for $\lambda_1$, it wouldn't default either because it is worth more that $\lambda_1$, and $\lambda_1$ does not default. Combining equations (17) and (18), we obtain that the two nondefault qualities obtain the same value, $v(\lambda_1) = v(\lambda_2)$.

Denote the value obtained by nondefault assets by

$$\bar{v} \equiv \max_{\boldsymbol{p} \in \mathbb{P}} (1+r)\, p_s - p_r.$$

Consider the highest quality asset that ends up defaulting in equilibrium, i.e., $\lambda_d \equiv \max_{\lambda \leq P_r(\lambda)} \lambda$.

It must be that

$$v(\lambda_d) = (1+r) P_s(\lambda_d) - \lambda_d \geq \bar{v}.$$

For any quality $\lambda < \lambda_d$, we have

$$v(\lambda) \geq (1+r) P_s(\lambda_d) - \lambda > (1+r) P_s(\lambda_d) - \lambda_d \geq \bar{v}. \tag{19}$$

In equation (19), the first inequality states that the value obtained by quality $\lambda$ is at least above the level it can derive by deviating to the contract for $\lambda_d$ and subsequently defaulting. With this deviation, the value obtained has to be above the nondefault value $\bar{v}$ since $\lambda < \lambda_d$. Hence the second inequality is strict. It must be that asset quality $\lambda$ defaults in equilibrium to obtain a value above $\bar{v}$. To conclude, all assets with quality below $\lambda_d$ must also default in equilibrium. **QED.**

**Proof of Lemma 1: Full Participation.** We consider two hypothetical scenarios to show that there must be full participation. In the first scenario, suppose that there is at least one participating borrower that doesn't default. Then the nondefault value is $\bar{v} \geq 0$. Hence, it must be that all assets with higher quality can at least obtain the same nondefault value by choosing the same contract and not defaulting. Thus, there is full participation.

Suppose now that in all active contracts, all participating borrowers strictly prefer to default. Thus, it must be that for all participating borrowers, $\lambda \leq \lambda_d$, and that the repurchase price is above the quality $P_r(\lambda) > \lambda_d$. Then, in stage 1, a lender could deviate and offer an alternative contract $\{P_s(\lambda_d), \lambda_d\}$. With this contract, the entrant lender would attract all assets above the default threshold and obtain a value $v(\lambda_d) = (1+r) P_s(\lambda_d) - \lambda_d \geq 0$; therefore all qualities would participate. The proposed contract is a profitable since it would attract more high quality assets and obtain a greater amount of cross-subsidization within its asset pool. Hence, there must be full participation. **QED.**

**Proof of Lemma 2.** By the previous results, if an agent chooses not to default, it must select a contract that offers the highest nondefault value $\bar{v}$. If the agent chooses to default, it must select a contract with the highest sales price. We therefore restrict attention to two groups of contracts, those that offer the highest nondefault isovalue and those that offer the highest sales price. **QED.**

## A.2 Auxiliary Lemmata

In this section, we prove two auxiliary Lemmata. According to Lemma 2, we can restrict our attention to two sets of contracts: those that offer the highest sales price and those that offer the highest nondefault value. We restrict to one contract $\boldsymbol{p}^d$ among the highest sales-price set and one contract $\boldsymbol{p}^n$ among the highest nondefault value set. Later in the proof we deal with multiple contracts in each set. Thus, for the time being, we consider only two contracts. We show the following properties.

**Continuity of Value Function.** First, we show that the default contract $\boldsymbol{p}^d$ and the nondefault contract $\boldsymbol{p}^n$ must deliver the same nondefault value; i.e. they must be located along an isovalue line. In particular, the default threshold asset must be indifferent between defaulting or not. Formally,

**Lemma 4** (Continuity)**.** *The value function must be continuous at the default threshold, i.e., $v(\lambda_d) = \bar{v}$. That is, the default threshold asset is indifferent between the two contracts:*

$$\bar{v} = (1+r)\, p_s^n - p_r^n = (1+r)\, p_s^d - \lambda_d. \tag{20}$$

*Proof.* Suppose by contradiction that $v(\lambda_d) > \bar{v}$, then the nondefault types slightly above the default threshold can obtain a higher payoff by switching to the contract for $\lambda_d$. Formally, there exists a small $\varepsilon > 0$ such that, for $\lambda_+^d = \lambda_d + \varepsilon$, the deviation pays off, since $(1+r)\, p_s^d - \lambda_+^d > \bar{v}$. □

Next, we show that the isovalue line that crosses the two contracts must necessarily cross the ZPC, given by equation (5), in equilibrium.

**Isovalue line crosses the ZPC.** Consider a hypothetical separating equilibrium where only two contracts are offered, a highest sales-price contract $\boldsymbol{p}^d$ and highest nondefault value contract $\boldsymbol{p}^n$. Using the optimal default actions, the profits per contract are correspondingly:

$$\Pi^d\left(\boldsymbol{p}^d, \lambda_d\right) \equiv \mathbb{E}\left[\lambda \,|\, \lambda \le \lambda_d\right] - p_s^d \text{ and } \Pi^n\left(\boldsymbol{p}^n\right) \equiv p_r^n - p_s^n.$$

Recall the zero-profit equation (5) and the contract set $\mathbb{Z}$. The contract $\{Z(\lambda_d), \lambda_d\}$ is in the set of $\bar{\mathbb{Z}}$. Since $Z(p_r)$ is concave, $\{\boldsymbol{p} \in \Lambda^2 |\, p_s \le Z(p_r)\}$ is a convex set and so is $\{\boldsymbol{p} \in \Lambda^2 |\, (1+r)\, p_s - p_r \ge 0\}$. Being the intersection of two convex closed sets, $\mathbb{Z}$ is also convex and closed. We have the following Lemma.

**Lemma 5** ($\boldsymbol{p}^d$ must be inside $\mathbb{Z}$.)**.** *Consider a possible separating equilibrium with contracts $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$ that induces a default threshold $\lambda_d < \bar{\lambda}$. Then, the default contract has a sales price below $Z(\lambda_d)$ and, thus, $\boldsymbol{p}^d \in int\mathbb{Z}$.*

*Proof.* The proof is by contradiction. Suppose that $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$ are offered and that $p_s^d \ge Z(\lambda_d)$. We show that if that is the case, the sum of lender profits are negative and, as a result, this situation cannot occur in equilibrium. By Lemma 4, an isovalue line must cross both contracts and the default threshold asset must be indifferent between the two contracts. Rewriting equation (20), we have

$$\lambda_d = p_r^n + (1+r)\left(p_s^d - p_s^n\right).$$

Next, observe the following relationship:

$$\Pi^n\left(\boldsymbol{p}^n\right) = p_r^n - p_s^n < p_r^n - p_s^n + r\left(p_s^d - p_s^n\right) = \underbrace{p_r^n + (1+r)\left(p_s^d - p_s^n\right)}_{=\lambda_d} - p_s^d \equiv \Pi^n\left(\left\{p_s^d, \lambda_d\right\}\right). \quad (21)$$

The inequality follows from the fact that $r > 0$ and $p_s^d > p_s^n$ where the latter condition is necessary to have a separating equilibrium. Note then that:

$$\begin{aligned}\Pi^d\left(\boldsymbol{p}^d, \lambda_d\right) F\left(\lambda_d\right) + \Pi^n\left(\boldsymbol{p}^n\right)\left(1 - F\left(\lambda_d\right)\right) &< \Pi^d\left(\boldsymbol{p}^d, \lambda_d\right) F\left(\lambda_d\right) + \Pi^n\left(\left\{p_s^d, \lambda_d\right\}\right)\left(1 - F\left(\lambda_d\right)\right) \quad (22)\\ &\leq \mathbb{E}\left[\min\left\{\lambda, \lambda_d\right\}\right] - Z\left(\lambda_d\right) = 0.\end{aligned}$$

The first inequality follows from (21) and $F(\lambda_d) < 1$—which is also the case in a separating equilibrium. The second inequality follows from $p_s^d \geq Z(\lambda_d)$ and thus, when $\boldsymbol{p}^d \notin \mathrm{int}\mathbb{Z}$. Since $\Pi^d + \Pi^n < 0$, when $\boldsymbol{p}^d \notin \mathrm{int}\mathbb{Z}$, at least one contract must be withdrawn. Hence, a separating equilibrium cannot occur with $\boldsymbol{p}^d \notin \mathrm{int}\mathbb{Z}$. □

In the proof of Lemma 5 we use two contracts. We argued above that proving the result for only one default contract is without loss of generality. We can restrict to one contract $\boldsymbol{p}^d$ among the highest sales-price set, because in a separating equilibrium, the highest sales-price contracts feature default on all assets and, thus, $p_r^d$ is immaterial. Now consider a situation where more than one nondefault contract is offered and denote that set by $\mathbb{P}^n$. Inequality (21) holds for any $\boldsymbol{p}^n$ in the set of nondefault contract. Hence, if we had multiple nondefault contracts, (22) can be written as:

$$\begin{aligned}\Pi^d\left(\boldsymbol{p}^d, \lambda_d\right) F\left(\lambda_d\right) + \sum_{\boldsymbol{p}^n \in \mathbb{P}^n} \Pi^n\left(\boldsymbol{p}^n\right)\Gamma\left(\lambda; \boldsymbol{p}^n, \mathbb{P}\right) &< \Pi^d\left(\boldsymbol{p}^d, \lambda_d\right) F\left(\lambda_d\right) + \Pi^n\left(\left\{p_s^d, \lambda_d\right\}\right)\left(1 - F\left(\lambda_d\right)\right)\\ &\leq \mathbb{E}\left[\min\left\{\lambda, \lambda_d\right\}\right] - Z\left(\lambda_d\right) = 0,\end{aligned}$$

which implies that the contradiction in the proof of Lemma 5 is there, even if there are more than two contracts in the separating equilibrium. We make use of both Lemmata in the results that follow.

## A.3 Proof of Proposition 1

In this section, we prove Proposition 1 and show that a separating equilibrium is impossible. According to Lemma 2, we can restrict our attention to two set of contracts. We prove the result for two contracts, respectively $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$. When we show that a separating equilibrium breaks down, we show that it would break down for all nondefault contracts that yield the same value, making the restriction to one contract in each set without loss of generality. Also, without loss of generality, we focus on partial default, that is $\lambda_d \in [\underline{\lambda}, \bar{\lambda})$.[29] Hence, in the rest of the proof we consider a separating equilibrium with two contracts and an interior threshold. We proceed to show that any separating equilibrium would break down.

By Lemma 4, $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$ are in the same nondefault isovalue line with value $\bar{v}$. By Lemma 5, $p_s^d < Z\left(\lambda^d\right)$ and $\boldsymbol{p}^d \in \operatorname{int}\mathbb{Z}$. Since $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$ falls in the isovalue line, but $p_s^n < p_s^d$, the isovalue line is not tangent to $Z$ and thus $\bar{v} < \bar{v}^*$.

To rule out the separating equilibrium, we consider a deviation to a contract:

$$\tilde{\boldsymbol{p}} = \left\{Z\left(\lambda^d\right) - \varepsilon, \lambda^d\right\},$$

where $\varepsilon$ is a small positive value such that $Z\left(\lambda^d\right) - \varepsilon > p_s^d$. The nondefault value associated with $\tilde{p}$ is above the existing level, $\tilde{v} = (1+r)\left(Z\left(\lambda^d\right) - \varepsilon\right) - \lambda^d > \bar{v}$. Thus $\tilde{\boldsymbol{p}}$ attracts all nondefault assets. It also attracts all default assets because it offers a higher sales price than $\boldsymbol{p}^d$, $Z\left(\lambda^d\right) - \varepsilon > p_s^d > p_s^n$. Therefore $\tilde{\boldsymbol{p}}$ attracts all assets from their original contracts. Finally, since the deviating contract $\tilde{\boldsymbol{p}} \in \operatorname{int}\mathbb{Z}$, it is strictly profitable so a lender will indeed offer it. To conclude, for any postulated separating equilibrium, we can find a profitable deviation that breaks the candidate separating equilibrium. Note that the deviation works independently of the number of original contracts offered, given that all nondefault contracts must yield the same value and default contracts must feature the same sales price.

Now, we show the remainder of Proposition 1, that is, if the equilibrium is pooling, the contract must satisfy the ZPC. Assume that the equilibrium is pooling, then, by Lemma 1, it must feature full participation and have a unique threshold. Suppose the contract does not satisfy the ZPC. In that case, the incumbent contract belongs to $\operatorname{int}\mathbb{Z}$. Then, consider a deviation $\tilde{\boldsymbol{p}} = \boldsymbol{p} + \{\varepsilon, 0\}$. Since the sales price is higher, the deviation must attract all borrowers. Since for sufficiently small $\varepsilon$ the contract remains in $\operatorname{int}\mathbb{Z}$, the deviation is profitable. Then the contract must satisfy the ZPC. **QED.**

[29] If there is default in all contracts, again, contracts must all have the same sales price and the repurchase price is irrelevant. Thus, the equilibrium is pooling.

## A.4 Proof of Proposition 2

From Proposition 1 we know the set of candidate equilibria must fall in the ZPC. We now prove that the equilibrium contract is $\boldsymbol{p}^*$. To prove the result, we proceed in two steps: first we show that $\boldsymbol{p}^*$ would break any other pooling equilibrium in the ZPC and, then, that if $\boldsymbol{p}^*$ is indeed offered, there are no profitable deviations.

**Claim 1: $\boldsymbol{p}^*$ dominates other ZPC-contracts.** Recall that $\boldsymbol{p}^* \in \mathbb{Z}$ and falls on the tangent plane of $\mathbb{Z}$ and, because of strict concavity of $Z(\cdot)$, is therefore unique. Thus, assume $\boldsymbol{p}$ is a pooling equilibrium that falls on the ZPC, as required by Proposition, and differs from $\boldsymbol{p}^*$. Since it is not on the tangent plane, then it yields a value $\bar{v} < \bar{v}^*$. Consider a deviation to

$$\tilde{\boldsymbol{p}} = \boldsymbol{p}^* - \{\varepsilon, 0\},$$

for some small $\varepsilon$, with associated nondefault value $\tilde{v} = \bar{v}^* - (1+r)\,\varepsilon$. Since $\boldsymbol{p}$ is not in the tangent plane of $\mathbb{Z}$, it is possible to find $\varepsilon$ such that $\bar{v}^* - (1+r)\,\varepsilon > \bar{v}$. If this is the case, $\tilde{\boldsymbol{p}}$ attracts all nondefault types from $\boldsymbol{p}$. However, since $\boldsymbol{p}$ falls in the ZPC, but loses all its nondefault assets—which bring positive marginal revenues, $\boldsymbol{p}$ has to be withdrawn. Since $\tilde{\boldsymbol{p}}$ remains the sole contract, but belongs to int$\mathbb{Z}$, the deviation is profitable. **QED.**

To verify that the zero-profit pooling contract $\boldsymbol{p}^*$ is indeed an equilibrium, we now argue that the introduction of another contract would not be profitable.

**Claim 2: $\boldsymbol{p}^*$ cannot be broken.** Suppose $\boldsymbol{p}^*$ is offered and lender deviates and offers $\tilde{\boldsymbol{p}}$. If $\tilde{\boldsymbol{p}}$ offers a higher nondefault value than $\boldsymbol{p}^*$, it must be that $\tilde{\boldsymbol{p}} \notin \mathbb{Z}$ since $\boldsymbol{p}^*$ maximizes the nondefault value in $\mathbb{Z}$. In that case $\tilde{\boldsymbol{p}}$ attracts all nondefault assets away from $\boldsymbol{p}^*$. But since $\boldsymbol{p}^*$ is in the ZPC, then $\boldsymbol{p}^*$ must be withdrawn. Thus, $\tilde{\boldsymbol{p}}$ must be left as a sole contract. However, since $\tilde{\boldsymbol{p}} \notin \mathbb{Z}$, it makes losses if it remains as the contract offered. Hence, the deviation is not profitable.

Thus, it must be that $\tilde{\boldsymbol{p}}$ offers a (weakly) lower nondefault value than $\boldsymbol{p}^*$ does. If $\tilde{p}_s < p_s^*$, then the contract does not attract any borrowers from $\boldsymbol{p}^*$. Thus, the only case left to rule out is one where $\tilde{p}_s > p_s^*$ . In this case, $\tilde{\boldsymbol{p}}$ can attract default assets away from $\boldsymbol{p}^*$. Since $\boldsymbol{p}^*$ was originally earning zero profit and $\tilde{\boldsymbol{p}}$ attracts the worse quality among the assets in $\boldsymbol{p}^*$, $\boldsymbol{p}^*$ remains profitable. However, $\tilde{\boldsymbol{p}}$ is unprofitable. This concludes the proof of Proposition 2. **QED.**

## A.5 Proof of Proposition 3

Consider a separating equilibrium where a lender offers a nondefault contract and a default contract, $\boldsymbol{p}^n$ and $\boldsymbol{p}^d$, respectively. We allow one contract to cross-subsidize the other. First, we show that if we allow for cross-subsidization across contracts, at least one contract must make losses and there cannot be an equilibrium with cross-subsidized contracts.

**Observation 1: at least one contract must make losses.** Assume both contracts make profits. According to Lemma 4, we know that the threshold asset is indifferent between the two contracts, i.e., $\lambda_d$ satisfies equation (20). It cannot occur that both contracts make a profit. Otherwise, a competitor could offer two contracts that raise slightly $p_s^d$ and $p_s^n$ by the same amount, still make a profit while keeping the same default threshold. Hence, at least one contract must make a loss. **QED.**

**Observation 2: a pooling contract dominates the separating ones with cross-subsidies.** Now consider an equilibrium where at least one contract makes losses. Recall from Lemma 4 and Lemma 5 that the default contract would have to fall in $\mathbb{Z}$. The rest of the proof of Proposition 3 is a direct corollary of Proposition 1: In particular, the proof of Proposition 3 shows that if $\tilde{\boldsymbol{p}} = \left\{Z\left(\lambda^d\right) - \varepsilon, \lambda^d\right\}$ is offered, it attracts all borrowers, making $\boldsymbol{p}^d$ and $\boldsymbol{p}^n$ inactive, regardless of whether the two original contracts were offered by the same lender or two different lenders. Thus, a separating equilibrium with cross-subsidization across contracts would be broken by a lender that offers this pooling contract. **QED.**

## A.6 Proof of Proposition 4

The first distinction of the planner's problem from the lenders in the competitive equilibrium is that individual contracts don't need to satisfy zero-profits. Hence the planner has the possibility to cross-subsidize between two contracts that potentially separate default assets from nondefault assets. The second distinction is the objective. Next, we proof that it is optimal for the planner to pool all assets. We proceed with the following steps. First, we simplify the planner's problem, then we show that the planner will choose a pooling contract, and finalize the proof showing that the participation constraint of the nondefaulters must bind.

**Step 1: Simplification of the Planner's Problem.** In the first step, we simplify the mechanism design problem ($\mathcal{M}$) into a problem where the planner chooses only among two contracts. We simplify the problem by mimicking the arguments that lead to the threshold default asset in Lemma 1 and Lemma 2. Namely, the borrower's optimal strategy is to chose either the highest sales price or the highest nondefault value. Therefore, we study a planner's problem that chooses only two contracts: one for default assets $\boldsymbol{p}^d$ and the other for nondefault assets $\boldsymbol{p}^n$. The planner's problem is to maximize average ex-ante borrower payoff:

$$\max_{\boldsymbol{p}^d,\boldsymbol{p}^n,\lambda^p,\lambda_d\leq\lambda^p} \int_{\underline{\lambda}}^{\lambda_d} \left((1+r)\,p_s^d-\lambda\right)dF\left(\lambda\right)+\left((1+r)\,p_s^n-p_r^n\right)\left(F\left(\lambda^p\right)-F\left(\lambda_d\right)\right) \tag{$\mathcal{M}'$}$$

subject to

$$\begin{aligned}
p_s^d &\geq p_s^n &(23)\\
(1+r)\,p_s^n-p_r^n &\geq (1+r)\,p_s^d-p_r^d &(24)\\
(1+r)\,p_s^d-\lambda_d &\geq (1+r)\,p_s^n-p_r^n &(25)\\
\lambda_d &\leq p_r^d &(26)\\
\lambda_d &\geq p_r^n &(27)\\
(1+r)\,p_s^n-p_r^n &\geq 0 &(28)\\
\Pi\left(\boldsymbol{p}^d\right)F\left(\lambda_d\right)+\Pi\left(\boldsymbol{p}^n\right)\left(F\left(\lambda^p\right)-F\left(\lambda_d\right)\right) &= 0. &(29)
\end{aligned}$$

Condition (23) is an incentive compatibility constraint for default assets to choose $\boldsymbol{p}^d$. Condition (24) is an incentive compatibility constraint for nondefault assets to choose $\boldsymbol{p}^n$. Condition (25) states that the default threshold asset weakly prefers $\boldsymbol{p}^d$. Condition (26) states that if the agent chooses the default contract, it will indeed default. Condition (27), on the other hand, states that if the agent chooses the nondefault contract, she will indeed not default. Condition (28) is a participation constraint for nondefault assets. Condition (29) makes sure that the the menu of contracts does not use external funds. There are six choice variables, together with seven constraints. We solve the problem by eliminating constraints and variables.

**Step 2: Solving the Simplified Planner's Problem.** First, the objective function can be reformulated as a liquidity maximization problem. To see that, we re-arrange the objective setting it to:

$$\begin{aligned}
&\int_{\underline{\lambda}}^{\lambda_d} \left((1+r)\,p_s^d-\lambda\right)dF\left(\lambda\right)+\left((1+r)\,p_s^n-p_r^n\right)\left(F\left(\lambda^p\right)-F\left(\lambda_d\right)\right)\\
=&r\left[p_s^dF\left(\lambda_d\right)+p_s^n\left(F\left(\lambda^p\right)-F\left(\lambda_d\right)\right)\right]-\left[\Pi^d\left(\boldsymbol{p}^d,\lambda_d\right)F\left(\lambda_d\right)+\Pi^n\left(\boldsymbol{p}^n\right)\left(F\left(\lambda^p\right)-F\left(\lambda_d\right)\right)\right].
\end{aligned}$$

We proceed by making the following observations, which simplify the problem.

**Observation 1: in the solution to the planner's problem, the planner makes zero profits.** Suppose not. Then, the planner could simply raise both $p_s^d$ and $p_s^n$ by some small $\varepsilon$. Raising both sales prices satisfies all the constraints and $\lambda_d$ remains unchanged. However, the value of the objective would increase.

**Observation 2: $p_r^d \geq \bar{\lambda}$.** Observe that $p_r^d$ enters nowhere in the objective. Hence, conditions (24) and (26) can be trivially satisfied by setting $p_r^d = \bar{\lambda}$. Thus, we can drop $p_r^d$ and eliminate conditions (24) and (26) from the problem.

**Observation 3: $\lambda_d$ is interior.** If Condition (1) holds, it is not possible to offer a contract that pools all assets into a default contract and have the planner break even.[30] That is, the default threshold has to be interior, $\lambda_d < \bar{\lambda}$.

**Observation 4: $\lambda^p = \bar{\lambda}$.** The fourth observation is that the planner's problem must feature full participation. As in the proof of Lemma 1, if at least one nondefaulter participates, then all qualities weakly prefer to participate. We proceed by contradiction. Suppose that nondefaulters do not participate, without loss of generality, by trivially setting $\boldsymbol{p}^n = 0$ and hence, setting $\lambda^p = \lambda_d$. That is, the planner prefers an asset-sale contract rather than a repo contract. Recall observation 3 that, with asset sales, there must only be partial participation. Because the contract must make zero profits (observation 1), the solution would be

$$(1+r)\, p_s^d - \lambda_d = 0 \text{ and } p_s^d = \mathbb{E}\left[\lambda \middle|\, \lambda \leq \lambda_d\right].$$

In this case, the planner could offer the nondefault contract, $\tilde{\boldsymbol{p}}^n = \left\{p_s^d, \lambda_d\right\}$. Default assets clearly remain as default assets, but holders of any better-quality asset would participate. It is immediate to show that the planner's would generate revenue, while it would not reduce the value of its objective. Since by observation 1, the solution must earn zero revenues for the planner, partial participation is suboptimal.

**Observation 5: the default threshold asset is indifferent.** By the continuity result in Lemma 4, it must be that the default threshold asset is indifferent between the two contracts, i.e., condition (25) is binding. Hence,

$$\lambda_d = (1+r)\left(p_s^d - p_s^n\right) + p_r^n \geq p_r^n,$$

with equality if and only if $p_s^d = p_s^n$. This implies that equation (27) is redundant. We can substitute out $\lambda_d$ and eliminate condition (27).

Thus, with the observations we have collected so far, we have that the problem is:

$$\max_{p_s^d, p_s^n, p_r^n, \lambda_d} \left\{p_s^d F\left(\lambda_d\right) + p_s^n \left(1 - F\left(\lambda_d\right)\right)\right\} \tag{$\mathcal{M}''$}$$

[30] If Condition (1) does not hold, then the planner can buy all assets at their fair value after agents learn the quality of their assets. In that case, the planner can implement the first best allocation.

subject to

$$p_s^d \geq p_s^n \tag{30}$$

$$(1+r)\, p_s^d - \lambda_d = (1+r)\, p_s^n - p_r^n \tag{31}$$

$$(1+r)\, p_s^n - p_r^n \geq 0 \tag{32}$$

$$\Pi\left(\boldsymbol{p}^d\right) F\left(\lambda_d\right) + \Pi\left(\boldsymbol{p}^n\right)\left(1 - F\left(\lambda_d\right)\right) = 0. \tag{33}$$

**Observation 6: The planner must pool all assets together.** Suppose that the planner's solution is not pooling. Then, it is the case that $p_s^d > p_s^n$. However, consider a deviation along the direction $(1, 1+r)$, i.e., a contract such that $\tilde{\boldsymbol{p}}^n = \left(p_s^n + \varepsilon, p_r^n + (1+r)\,\varepsilon\right)$ for some small $\varepsilon$. Both (31) and (32) are satisfied and the default threshold remains unchanged. Furthermore, nondefault profits increase by $r\varepsilon$ and the objective also remains unchanged. However, since the deviation generates profits, by observation 1, we have that there is another solution with zero profits that yields greater value. Hence, it must be that $p_s^d = p_s^n = p_s$. This in turn implies that the default threshold satisfies $p_r^n = \lambda_d$.

**Step 3. Packaging the Solution.** Collecting the observations so far, the planner's problem becomes one of maximizing the sales price

$$\max_{p_s, \lambda_d} \; p_s \tag{$\mathcal{M}'''$}$$

subject to

$$(1+r)\, p_s - \lambda_d \geq 0 \tag{34}$$

$$p_s - \mathbb{E}\left[\min\left\{\lambda, \lambda_d\right\}\right] = 0 \tag{35}$$

We have shown that $p_s$ is increasing in $\lambda_d$ along the zero profit condition. Hence, the sales price is maximized when the participation constraint is binding at the point that maximizes liquidity.

## A.7 Proof for Corollary 1

Substituting the sales price as a function of the repurchase price according to the zero-profit condition, the problem of maximizing the nondefault value becomes

$$\max_{p_r \in \Lambda} (1+r)\left[\int_{\underline{\lambda}}^{p_r} \lambda dF\left(\lambda\right) + \left(1 - F\left(p_r\right)\right) p_r\right] - p_r.$$

The first-order condition with respect to $p_r$:

$$1 - F(p_r) = \frac{1}{1+r},$$

which implies that the nondefault rate is $\frac{1}{1+r}$ and the default rate is $\frac{r}{1+r}$. Hence we obtain the repurchase price in equation (7). Using the zero profit condition, we further obtain the sales price in equation (8).

## A.8 Proof for Proposition 5

When Condition 1 does not hold, the asset-sale equilibrium has full participation. It generates more liquidity than the repo equilibrium, since

$$\mathbb{E}[\lambda] > p_s^*.$$

In contrast, when Condition 1 holds, there is partial participation in the asset-sale equilibrium. In this scenario, we obtain equation (12), a condition that guarantees that the repo equilibrium dominates in providing higher aggregate liquidity. To obtain this condition, we consider a participation threshold in the asset sales market, which at the pooling price generates the same liquidity as the repo equilibrium $p_s^*$. This hypothetical participation threshold $L_a^{-1}(p_s^*)$ is on the right-hand side of equation (12). In Figure 4, it is depicted as the green dot in ZPC for selling assets. Notice that $L_a\left[\bar{\lambda}\right] = \mathbb{E}[\lambda] > p_s^*$ and $L_a\left[\underline{\lambda}\right] = 0 < p_s^*$ . Since $L_a(\cdot)$ is strictly increasing, it is ensured that $L_a^{-1}(p_s^*)$ is a unique interior point in the asset quality domain $\Lambda$. The left-hand side is the investment return from the pooling price $Z_a\left(L_a^{-1}(p_s^*)\right)$. The equation says that this threshold asset is unwilling to participate. When this occurs, the asset-sale equilibrium (depicted as the black dot in the figure) has a participation threshold below the hypothetical one $\lambda_a < L_a^{-1}(p_s^*)$. This implies that the asset-sale equilibrium must generate less liquidity than the repo equilibrium, $L_a(\lambda_a) < p_s^*$.

## A.9 Solutions under Uniform Distribution

**Akerlof Asset-Sale Equilibrium.** In the Akerlof asset-sale equilibrium, a pooling equilibrium is obtained. The pooling price $p^s$ and participation threshold $\lambda_a$ satisfy the following participation constraint and zero-profit condition:

$$(1+r)\,p_a - \lambda_a = 0$$

and

$$p_a = \mathbb{E}\left[\lambda \,|\lambda \leq \lambda_a\right].$$

The aggregate liquidity is $p_a F(\lambda_a)$. With a uniform distribution $\lambda \sim U(1-\sigma, 1+\sigma)$, we obtain a closed-form solution for the aggregate variables. If there is little dispersion $\sigma \leq r$,

$$\lambda_a = 1+\sigma, \;\; p_a = 1, \;\; \text{and } p_a F(\lambda_a) = 1.$$

When dispersion becomes larger $\sigma > r$, that is when Condition 1 holds,

$$\lambda_a = \frac{1+r}{1-r}(1-\sigma), \;\; p_a = \frac{1}{1-r}(1-\sigma), \;\; \text{and } p_a F(\lambda_a) = \frac{r}{(1-r)^2}\frac{(1-\sigma)^2}{\sigma}.$$

**Optimal Repo Contract.** In the optimal repo contract, when $\sigma > r$ is satisfied,

$$(1+r)\, p_s - \lambda_d = 0,$$

and

$$p_s^p = \mathbb{E}\left[\min\{\lambda, \lambda_d\}\right].$$

With a uniform distribution $\lambda \sim U(1-\sigma, 1+\sigma)$, the aggregate variables also have a closed-form solution:

$$\lambda_d^p = 1+\sigma - \frac{2\sigma}{1+r}\left(1 - \sqrt{\frac{r}{\sigma}(1+r) - r}\right)$$

and

$$p_s^p = \frac{1+\sigma}{1+r} - \frac{2\sigma}{(1+r)^2}\left(1 - \sqrt{\frac{r}{\sigma}(1+r) - r}\right).$$