Heterogeneous Credit Constraints and Optimal Monetary Policy

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Abstract

The optimal response to adverse external shocks in an economy involves the choice of a exchange rate policy. While the traditional Mundell-Flemming inspired theories support a floating exchange rate, evidence shows that central banks intervene in foreign exchange markets regularly. One of the reasons for these interventions relies on the consequences of large depreciations triggering negative balance sheet effects in economies with dollarized liabilities as shown by Benigno et al. (2013) and Devereux and Poon (2011). This paper extends this literature by introducing heterogeneity in credit constraints across sectors. Our findings support that “leaning against the wind” policy responses are optimal even when only a sector of the economy is affected by the credit constraints. Thus, relative price distortions provide an additional justification for these policies. We show that the vulnerability of the economy to large negative external shocks depends not only on the overall unhedged foreign debt, but also on its distribution across sectors.

Key words: Exchange rate dynamics, Exchange rate intervention, Monetary policy.

JEL Classification: E5, F3, G15.

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1 Introduction

Monetary policy in developing countries presents many challenges. Besides the complications stemming from underdeveloped financial systems (i.a.: credit constraints, dollarization of liabilities, lack of credibility), policy makers have to respond to critiques based on the literature built to explain the macroeconomic dynamics in developed economies. This is the case for the debate over the optimal exchange rate policy; one of the most controversial topics in the international macroeconomics literature. While the workhorse sticky prices macroeconomic models support some level of exchange rate stabilization and terms of trade manipulation, the conventional view backs the Mundell-Flemming tradition of a flexible exchange rate as the optimal response to external shocks.\(^1\) Regardless, central banks in several small open economies have maintained a firm position backing foreign exchange rate interventions and exchange rate smoothing, as documented by Calvo and Reinhart (2002) and Mihaljek (2005).

The reason why developing countries depart from the traditional view and aim for some exchange rate smoothing might be related to dollarization, which presents a series of challenges for developing countries. For instance, a higher amount of foreign reserves or pre-established credit lines is required by the monetary authority to perform its role of lender of last resort in a credible fashion (Calvo et al. (2013), Ito and McCauley (2019)). Other findings in the literature are an increased instability of money demand (more prevalent under currency substitution) and currency mismatches, which increase the vulnerability of the financial system to a sudden-stops.\(^2\) Levi-Yeyati (2006) provides an empirical counterpart to these findings, showing a positive correlation between the degree of dollarization and the sensitivity of prices to money creation, the propensity to systemic banking crisis and the volatility of output growth.

Out of these challenges, one that has been particularly important due to its role in episodes of financial crises is the presence of currency mismatches in liabilities. In a nutshell, a sufficiently high variation in the exchange rate could trigger debt service difficulties to loans contracted in foreign currency by firms and agents whose assets and income are expressed in domestic currency. The rupture of the chain of payments could end in the appearance of credit constraints, with consequences that have been widely studied in the literature. Under the presence of balance sheet effects, depreciations can be contractionary, as opposed to the conventional mechanism of stabilization. Hausmann et al. (2001) has stressed this channel as the main reason why central banks exhibit fear of floating. In the literature, Céspedes et al. (2004) and Gertler et al. (2007) have studied the role of credit constraints and optimal exchange rate policy, finding that flexible rates are preferred to other regimes. Nonetheless, they agree that the presence of liability dollarization reduces welfare and increases the volatility produced by external shocks. Devereux et al. (2006) build a model with financial constraints à la Gertler et al. (2007), differentiating

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\(^1\)See Faia and Monacelli (2008), Sutherland (2005), De Paoli (2009a) and De Paoli (2009b)

\(^2\)For a theoretical explanation of the effects of currency substitution in the volatility of money demand see Miles (1978), McKinnon (1982) and Borensztein and Berg (2000).
between a non-traded and exportable sector.

Aghion et al. (2009) presents a model in which financial crisis are triggered by private sector balance sheet constraints, relying on not fully enforceable credit contracts as in Kiyotaki and Moore (1997). This type of contract makes optimal for lenders to keep the size of the loans under a threshold expressed in foreign currency. A large depreciation can push firms into a region in which default becomes optimal, triggering a banking crisis. Chang (2018) presents a dynamic general equilibrium model using an occasionally binding constraints but in the balance sheet of banks. In this setup, the central bank will find a region in the state-space in which “leaning against the wind” (LAW, for short) foreign exchange interventions relax the borrowing constraints faced by private sector banks. Similarly, Benigno et al. (2012) use a simple setup in which the presence of financial frictions makes optimal monetary responses less counter-cyclical, with the aim of affecting the real exchange rate and relaxing the borrowing constraints.

Although these results find novel channels in which foreign exchange rate policy interacts with financial frictions, they remain silent regarding the heterogeneity observed in credit composition across different firms. As Caballero and Krishnamurthy (2005) point out, after a crisis, firms producing tradable goods can use their production to guarantee loans (trade credit) while the non-tradable sector remains constrained, since their product is of not value for international lenders.³ Tornell and Westermann (2002) provide evidence on the asymmetric financing opportunities between the tradable and non-tradable sectors. Using level data from a panel of over 3800 firms the author construct a probit model to test for asymmetries in severity of credit constraints finding that non-exporters face more obstacles to fulfill their financing needs an that, among exporters, a higher share of exports in firms output is correlated with easier access to credit. Finally, they study the role of collateral as an obstacle to obtain financing, determining that firms in the non-tradable sector report more severe constraints of this sort.

The present paper aim is to contribute in the design of an optimal monetary and foreign exchange rate policy design by explicitly accounting for the heterogeneous financial constraints across sectors. We restrict ourselves to a simple setup that illustrates this idea by following Devereux and Poon (2011). In this model, we explicitly account for occasionally binding constraints which push firms away from their optimal level of output. Our results suggest that the unhedged debt in foreign currency affects the optimal monetary policy design as in Devereux and Poon (2011). Moreover, the composition of the dollarization matters: for a given level of overall dollarization, the more dollarized the non-tradable sector is, the more vulnerable the economy is to external shocks and the stronger the response needed by the central bank to stabilize the economy. Finally, we find a risk sharing motive for LAW policies as the central bank becomes less expansionary outside the constrained region to reduce the severity of its response during the crisis. This element contributes to the discussion regarding ex-ante and

³Ganesh-Kumar et al. (2001) argument in the lines of Ghironi and Melitz (2005) that exporting provides a signal regarding competitiveness and efficiency, granting easier access to financial sources.
ex-post policies prevalent in the literature.\footnote{For a discussion see Korinek and Jeanne (2013), Mendoza and Bianchi (2010), Chen et al. (2013), Benigno et al. (2013).}

The present document is organized as follows. Section 2 presents the model. Section 3 gives a diagrammatic analysis. Section 4 presents the results of the numerical exercises and discusses the optimal monetary policy. Last section concludes.

2 The Model

The model follows Devereux and Poon (2011) and Obstfeld and Rogoff (2000), modified to include a non-tradable sector which faces an occasionally binding collateral constraint that limits its capacity to import inputs for production.

2.1 Firms

Firms produce using labor and intermediate inputs. We allow for differences in technology, given by:

\[
Y^T_t = A^T F(L^T_t, I^T_t) \\
Y^N_t = A^N G(L^N_t, I^N_t)
\]

where $Y$ stands for output, $A$ is the overall productivity, $L$ represents labour and $I$, investment. The supra-indexes $T$ and $N$ represent tradable and non-tradable sectors. Households are heterogeneous in the labour services they provide and enjoy market power on the provision of these services:

\[
L_t = \left[ \int_0^1 (L_t(i))^{\frac{\rho-1}{\rho}} \, di \right]^{\frac{\rho}{\rho-1}}
\]

where we have indexed workers by $i$ along the unit interval and $\rho > 1$, represents the elasticity of demand for households $i$ services. Profits to firms in tradable and non-tradable sectors are given by:

\[
\Pi^T_t = P^T_t Y^T_t - W_t L^T_t - S_t Q^*_t I^T_t \\
\Pi^N_t = P^N_t Y^N_t - W_t L^N_t - S_t Q^*_t I^N_t
\]

where $S$ stands for the nominal exchange rate and $Q^*_t$ represents the price of imported inputs, expressed in foreign currency. It follows that both sectors face the same price over inputs. Non-tradable sector firms are forced to use their net worth to finance the purchase of imported inputs:

\[
S_t Q^*_t I^N_t \leq \bar{N}_t - D^p_t - S_t D^*_t
\]
where \( \bar{N}_t \) is the value of assets of the non-tradable firms expressed in local currency while \( D_t^p \) and \( D_t^* \) represents the preexistent debt in domestic and foreign currency held by non-tradable sector firms, \( D_t^* \). For the rest of paper, we will assume that \( N_t = \bar{N}_t - D_t^p \) assets less liabilities in domestic currency. Non-tradable firms will be exposed to exchange rate fluctuations given their short position in dollars. A depreciation can make the borrowing constraint in (3) binding, limiting their capacity for using intermediate inputs in production. The preference of non-tradable firms for holding debt on a different currency is a prevalent feature in middle-income countries. Honohan and Shi (2001) provide evidence that in order to maintain their profitability and satisfy the pent-up demand for loans, banks end up on lending a large share of their dollar deposits domestically, effectively transerring the currency risk to their unhedged clients. For simplicity we take the ratio of dollar-denominated debt as exogenous, given that dollarization measures are highly persistent.\(^5\) We proceed to describe the model under both scenarios: when borrowing constraints are binding and when they are not.

2.1.1 Non-binding borrowing constraints

For simplicity, we will assume homogeneous factor intensity across sectors to highlight the technology parameters as the main factors determining the relative price. Under this assumption, production in the tradable and non-tradable sectors follow:

\[
Y_t^T = A_t^T \left( L_t^T \right)^\omega \left( I_t^T \right)^{1-\omega} \]
\[
Y_t^N = A_t^N \left( L_t^N \right)^\omega \left( I_t^N \right)^{1-\omega} \]

We assume full labour mobility across sectors, equating the marginal product to factor payments:

\[
W_t = \omega \frac{P_t^N Y_t^N}{L_t^N} = \omega \frac{P_t^T Y_t^T}{L_t^T} \]
\[
S_t Q_t^* = \left( 1 - \omega \right) \frac{P_t^N Y_t^N}{I_t^N} = \left( 1 - \omega \right) \frac{P_t^T Y_t^T}{I_t^T} \]

Prices of non-tradable and tradable goods are given by:

\[
P_t^N = \kappa \frac{W_t^\omega (S_t Q_t^*)^{1-\omega}}{A_t^N} \tag{4} \]
\[
P_t^T = \kappa \frac{W_t^\omega (S_t Q_t^*)^{1-\omega}}{A_t^T} \tag{5} \]

where:

\[
kappa = \left( \frac{1}{1-\omega} \right)^{1-\omega} \left( \frac{1}{\omega} \right)^\omega \]

yielding the main result from this section: under non binding credit constraints, the relative price between tradable and non-tradable goods is given by the relative productivity:

\[
\frac{P_t^N}{P_t^T} = \frac{A_t^T}{A_t^N} \tag{6} \]

2.1.2 Binding borrowing constraints

When borrowing constraints bind, the non-tradable sector equilibrium changes. Assuming equation (3) binds:

\[ S_t Q_t^* I_t^N = N_t - S_t D_t^* \]

Firms will choose employment to maximize profits, yielding the implicit demand function:

\[ W(i) = \frac{\omega_A T L \omega T I_1}{1 - \omega T L T (L(i) \beta)} \]

Where the assumption of free labor mobility across sectors has been used. In the symmetric equilibrium, where \( L(i) = L \) and \( W(i) = W \) for all \( i \):

\[ W = \frac{P_N Y_N}{L_N} = \omega \frac{P_T Y_T}{L_T} \]

Thus, as in the case where constraints are not binding, the relative prices of both goods is determined by:

\[ \frac{P_T}{P_N} = \frac{Y_T L_N}{Y_N L_T} \]

When the restriction over intermediates binds, it is possible to express the equilibrium output as:

\[ Y_T = A_T L_T^\omega I_T^{1-\omega} \]  
\[ Y_N = A_N L_N^\omega \left( \frac{N - S D^*}{S Q^*} \right) \]  

2.2 Households

There is a continuum of households, indexed by \( j \), which derive utility from consumption, leisure and real money balances:

\[ U_t(j) = \log C_t(j) + \chi \log \left( \frac{M_t(j)}{P_t} \right) - \eta \frac{L(j)^{1+\psi}}{1+\psi} \]  

\( C_t \) represents total consumption and \( \frac{M_t}{P_t} \) the holdings of real money balances. Overall and home goods consumption composites are given by:

\[ C_t = \left( C_t^H \right)^\alpha \left( C_t^F \right)^{1-\alpha} \]
\[ C_t^H = \left( C_t^N \right)^\theta \left( C_t^T \right)^{1-\theta} \]

which corresponding price indexes:

\[ P_t = \left( \frac{P_t^H}{\alpha} \right)^\alpha \left( \frac{S_t P_t^{F,*}}{1 - \alpha} \right)^{(1-\alpha)} \]
\[ P_t^H = \left( \frac{P_t^N}{\theta} \right)^\theta \left( \frac{P_t^T}{1 - \theta} \right)^{(1-\theta)} \]
where $P^F_t$ represents the price of foreign goods, and the parameters $\alpha \in [0, 1]$ and $\theta \in [0, 1]$ represent the preference of home goods relative to foreign goods, and the relative preference of non-tradables over tradables, respectively. Households maximize utility subject to the following budget constraint:

$$P_tC_t(j) + M_t(j) = W_t(j)L_t(j) + M_{t-1}(j) + T_t(j) + \Pi_t(j)$$  \hspace{1cm} (11)

where $M_{t-1}$ represents the initial holdings of money balances, $T$ stands for transfers and $\Pi$ is the sum of profits from firms and the labour union. Maximization of (10) subject to the budget constraint in (11) yields the demand for tradable and non-tradable home goods, the foreign goods demand and the demand for money balances:

$$C^F_t(j) = (1 - \alpha)\frac{P_tC_t(j)}{P^F_t}$$
$$C^N_t(j) = \frac{\theta \alpha P_tC_t(j)}{P^N_t}$$
$$C^T_t(j) = \alpha (1 - \theta)\frac{P_tC_t(j)}{P^T_t}$$
$$M_t(j) = \chi P_tC_t(j)$$

Nominal wages, set and fixed at the beginning of each period, cannot adjust to shocks within the period. This assumption will create a role for monetary, adjusting to smooth consumption across different states. Considering this constraint, agents will set the wage to maximized the expected utility across states:

$$W_t = \eta \frac{P}{1 - \rho} E_t \left( L_t^{1 + \psi} \right)$$  \hspace{1cm} (12)

where the wage includes a mark-up given by the elasticity of demand of labour of $\frac{\rho}{1 - \rho}$.

### 2.3 Equilibrium

Foreign demand from home goods is assumed unit elastic:

$$X^d_t = \tilde{X}_t \frac{S_t}{P^F_t}$$

where $\tilde{X}$ will be used to account for stochastic shocks to foreign demand. Also, prices are normalized to the price of foreign goods, assuming $P^F_t = 1$. As symmetric imperfectly competitive equilibrium is defined by the set of allocations $\Theta = \{C^T, C^N, C^F, L^T, L^N, M, Y^T, Y^N, Y, I^N, I^T\}$ and the set of prices, $\varphi = \{W, S, P^T, P^N, P^H\}$ for a given $Q^*$ and $\tilde{X}$ such that:

1. Firms in both sectors maximize profits;
2. Wage maximizes expected utility;
3. Households maximize utility over consumption and real money balances subject to ex-post budget constraints;

4. Money market clearing condition is satisfied:

\[ M_t = M_{t-1} + T_t \]

5. Home good markets clearing conditions hold:

\[ P_t^T Y_t^T + P_t^N Y_t^N = \alpha P_t C_t + \tilde{X} S_t \]

6. Non-tradable market goods clear:

\[ Y_t^N = C_t^N \]

Now we proceed to characterize the equilibrium under binding and non-binding credit constraints to the non-tradable sector.

2.3.1 Equilibrium with non-binding constraints:

From the the profit maximization condition, under non-binding constraints, payments to labour imply:

\[ P_t C_t = W_t L_t = W_t (L_t^T + L_t^N) = \omega \left( P_t^T Y_t^T + P_t^N Y_t^N \right) \]

substituting in the money market clearing condition:

\[ M_t = \chi P_t C_t = \chi \omega \left( P_t^T Y_t^T + P_t^N Y_t^N \right) \]

Now replacing (13) in the goods market clearing condition:

\[ P_t^T Y_t^T + P_t^N Y_t^N = \frac{1}{1 - \alpha \omega} \tilde{X} S_t \]

The system determining the imperfectly competitive equilibrium is composed by equations (14)-(16), jointly with optimal pricing from equations (4) and (5), which can be solved for \( \{ P^T, P^N, S, Y^H, Y^T \} \), given the realizations of \( \tilde{X} \) and the value for \( W \) set at the beginning of the period. Wage is determined by equation (12), obtained from the values of employment, prices and consumption associated with the realizations of \( \tilde{X} \) and \( M \).

2.3.2 Equilibrium with binding collateral constraints

When collateral constraints bind, the budget constraint faced by households is given by:

\[ P_t C_t = P_t^N Y_t^N - N_t + S_t D_t^* + \omega P_t^N Y_t^N \]
which yields the following money market and goods clearing conditions:

\[ M_t = \chi P_t C_t = \chi (P_t^{N} Y_t^{N} - (N_t - S_t D_t^*) + \omega P_t^T Y_t^T) \]

\[ P_t^T Y_t^T = \frac{1 - \alpha \theta}{\gamma} \bar{X} S_t - \frac{\alpha (1 - \theta)}{\gamma} [N_t - S_t D_t^*] \]

\[ P_t^N Y_t^N = \frac{\omega \alpha \theta}{\gamma} \bar{X} S_t - \frac{\alpha \theta}{\gamma} [N_t - S_t D_t^*] \]

where \( \gamma \equiv 1 - \alpha + \alpha (1 - \theta)(1 - \omega) \). These results together with (2) and (9) constitute the system of equations determining the equilibrium, which can be solved for \( \{ \frac{P_t^T}{P_t^H}, L_t^T, L_t^N, S_t, Y_t^H, Y_t^T \} \) given the realization of \( \bar{X} \) and \( W \), with the latter determined by equation (12).

### 2.4 The collateral constraint binding region

The only source of stochasticity in this economy comes from \( \bar{X} \). Given its realization and the monetary policy response, the economy will either be in a region where the collateral constraint is binding or not binding. In each region, equilibrium values are determined in a different manner. For this reason, we must define these two areas of the state space, which are delimited by a cutoff exchange rate.

In the unconstrained region:

\[ SQ^* I = SQ^* (I_t^T + I_t^N) = (1 - \omega) (P_t^T Y_t^T + P_t^N Y_t^N) = \frac{1 - \omega M}{\omega} \]

while in the constrained region, the overall expenditure in intermediate goods is determined by:

\[ \frac{1 - \omega M}{\omega} \bar{X} S_t - (1 - \omega) P_t^T Y_t^T \leq N - S D^* \]

From the last equation it is possible to determined cutoff exchange rate, determined by the net value of assets of the non-tradable sector firms, the size of the tradable sector and the domestic money supply:

\[ \bar{S} = \frac{1}{D^*} \left[ N + (1 - \omega) P_t^T Y_t^T - \frac{1 - \omega M}{\omega} \right] \]

Therefore, when the nominal exchange rate is below \( \bar{S} \), the constraint will not bind. Notice that the cutoff rate has a negative relationship with the money supply, thus, a contractionary monetary policy could pull the non-tradable firms out of the constrained region. Further substitution yields:

\[ \bar{S} = \frac{N \left( \frac{1 - \alpha}{\gamma} - \frac{1 - \omega M}{\omega} \bar{X} \right)}{D^* \left( \frac{1 - \alpha}{\gamma} + (1 - \omega) \frac{1 - \theta}{\gamma} \bar{X} \right)} \] (17)

Equation (17) shows that a less expansionary - or contractionary - monetary policy helps relaxing the credit constraint by generating a positive balance sheet effect, which increases the net worth of indebted firms. This is the LAW motive shared with Devereux and Poon (2011) and Benigno et al. (2016). In the next section we explore how the presence of tradable and non-tradable sectors changes the optimal monetary policy in the model.
3 Diagrammatic Analysis

Now we briefly deviate from the quantitative analysis, to show the diagrammatic version of the model. This will help to show, by contrasting our results to the ones obtained by Devereux and Poon (2011), how the presence of non-tradable goods changes the optimal policy.

3.1 Unconstrained Regime

The economy for a given nominal wage can be represented in a simple IS-LM fashion using the money and good markets clearing conditions. In the unconstrained case, money market equilibrium is given by:

\[ M_t = \chi \omega \kappa W_t^\omega (S_t Q_t^*)^{1-\omega} \left( \frac{Y_t^T}{A_t^T + \frac{Y_t^N}{A_t^N}} \right) \]  

while the goods markets are given by:

\[ Y_t^T = A_t^T \frac{1}{\kappa} \left( \frac{1 - \alpha \theta \omega}{1 - \alpha \omega} \bar{X} \frac{S_t^T}{W_t^\omega (Q_t^*)^{1-\omega}} \right) \]  
\[ Y_t^N = A_t^N \frac{1}{\kappa} \left( \frac{\alpha \theta \omega}{1 - \alpha \omega} \bar{X} \frac{S_t^N}{W_t^\omega (Q_t^*)^{1-\omega}} \right) \]

yielding the following expression for overall output:

\[ Y_t = \frac{P_t^T}{P_t^H} Y_t^T + \frac{P_t^N}{P_t^H} Y_t^N \]

by normalizing this ratio to 1 in the unconstrained regime, we get, as expected, the Mundell-Fleming representation of the economy, where, as usual, a negative demand shock given by a fall of \( \bar{X} \) shifts the IS curve to the left, while a positive shock to money supply pushes the LM curve to the right.

3.2 Constrained Regime

When the collateral constraint binds, the IS and LM schedules in (18) and (21) must be amended. The money market clearing condition becomes:

\[ M_t = \chi \omega \kappa W_t^\omega (S_t Q_t^*)^{1-\omega} \left( \frac{Y_t^T}{A_t^T + \frac{Y_t^N}{A_t^N}} \right) \]

\[ (P_t^N P_t^H) Y_t^T + P_t^N Y_t^N \]

by normalizing this ratio to 1 in the unconstrained regime, we get, as expected, the Mundell-Fleming representation of the economy, where, as usual, a negative demand shock given by a fall of \( \bar{X} \) shifts the IS curve to the left, while a positive shock to money supply pushes the LM curve to the right.
The LM schedule still depicts a negative relationship in the \( S, Y \) space. Despite the non-linear relationship between output and money balances, the mechanism prevails. An exchange rate depreciation reduces purchases of intermediate inputs, despite the increased demand from the tradable sector. Nominal prices in both sectors increase, due to a cost push in one of the production factors, increasing the demand for money. To reach the equilibrium, output must fall to allow the money market to clear. However, the effect will be asymmetric. Notice that the relative price in the constrained region is given by:

\[
\frac{P^N}{P^T} = \frac{A^T}{A^N} \left( 1 - \omega \right) \frac{\tilde{X}_t S_t - \alpha \theta}{N_t - S_t D^*_t} \right)^{1-\omega} \tag{22}
\]

Differentiating this expression with respect to the exchange rate yields:

\[
\frac{\partial \left( \frac{P^N}{P^T} \right)}{\partial S_t} = \frac{A^T}{A^N} \left( 1 - \omega \right)^{2-\omega} \frac{\omega \alpha \theta}{\gamma} \left( \frac{\tilde{X}_t S_t - \alpha \theta}{N_t - S_t D^*_t} \right)^{1-\omega} \tag{23}
\]

given that \( N_t - S_t D^*_t > 0 \), the above expression is always positive, for positive prices. Thus, output will fall in an asymmetric way across sectors to reach the equilibrium. In the case of the IS, the tradable and non-tradable sectors will exhibit a different behaviour. The goods market equilibria are given by:

\[
Y^T_t = \frac{A^T}{\kappa W^T_t (S_t Q^*_t)^{1-\omega}} \left[ \frac{1 - \alpha \theta}{\gamma} \tilde{X}_t S_t - \frac{\alpha (1 - \theta)}{\gamma} (N_t - S_t D^*_t) \right] \tag{24}
\]

and:

\[
Y^N_t = \frac{A^N \omega}{W^N_t (S_t Q^*_t)^{1-\omega}} \left[ \frac{\omega \alpha \theta}{\gamma} \tilde{X}_t S_t - \frac{\alpha (\theta)}{\gamma} (N_t - S_t D^*_t) \right]^{\omega} \left[ \frac{N_t - S_t D^*_t}{N - S_t D^*_t} \right]^{1-\omega} \tag{25}
\]

where we have introduced below each term the reaction to a change in the nominal exchange rate.

In the case of the tradable sector, a depreciation has two positive effects in the demand: it increases the foreign demand for home tradable goods and lowers the demand for imported inputs. By contrast, the exchange rate depreciation increases the price of the intermediate inputs in domestic currency, which acts as a cost-push shock, increasing the final price of tradable goods, having a negative effect in its demand. The overall result will depend on the value of the parameters. Regarding the non-tradable sector, we observe a similar positive effect on the demand, however, the increase in exports has a different impact for the demand of non-tradable (second term in equations 24 and 25). Another difference is that the depreciation will directly affect the production of non-tradables by tightening the collateral constraint. Thus,
we should observe a more vertical IS curve in the case of the non-tradable sector, or even a negative slope in the $S,Y$ mapping. While under reasonable parameter values, we would expect a positive relationship between the tradable sector output and the exchange rate.

Here the model exhibits a key difference with respect to the one sector benchmark of Devereux and Poon (2011). In that setup, the less expansionary monetary policy is justified by the presence of a negative-sloped IS curve when the borrowing constraint binds. The negative relationship between output and the exchange rate is the result of credit constraints in the only productive sector. In this situation, a less expansionary policy could stabilize output. In the model with a tradable and non-tradable sector, even under a positively sloped IS curve, a “leaning-against-the-wind” policy is still justified. When the economy is pushed into the constrained region, the Central Bank will face an incentive to intervene to correct relative price distortions stemming from the suboptimal use of intermediate inputs in the non-tradable sector. Thus, the central bank will have an allocative efficiency motive, beyond expanding output.

The model exhibits strong non-linearities as the size of the shock matters for monetary policy reaction. It follows that the optimal reaction to a negative foreign demand shock that does not trigger the credit constraint is to let the exchange rate to fully adjust, minimizing the fall in output. However, if the shock is big enough to push the economy to the constrained region, the response will change drastically. When the non-tradable sector constraint binds, relative price distortions will emerge and, in the case of strong balance sheet effects, the supply of non-tradables might even decrease with the exchange rate depreciation. Here the Central Bank might find optimal to “lean against the wind” and reduce money supply, dampening the exchange rate burst. In turn, this will relax the credit constraints and reduce the relative price distortions.

4 Optimal Monetary Policy

Given the explicit non-linear nature of the model presented, the optimal response to external shocks will depend on the state-space region in which the economy starts. The Central Bank faces a complex problem as it has to worry about the relative prices and efficiency in production, as well as in the overall effects of negative shocks to foreign demand for tradable goods. In the case where credit constraints are not symmetric, the Central Bank will have to worry about the effects of a depreciation in the access to credit and its consequences in relative prices.

As discussed in the previous section, in the unconstrained region optimal monetary policy should allow for the exchange rate to cushion external shocks. Following Obstfeld (1998) and Devereux and Poon (2011), we calculate the one period utility function. It will result instrumental to express utility in terms of the ratio $\frac{S}{W}$. From (15), we know that the utility from money balances are a linear transformation of consumption. Utility from consumption is given
Figure 1: IS-LM model under non binding constraints

Note: Depreciations increase tradable and non-tradable output along the IS curve, while a depreciation reduces the total output of the economy on the LM curve. The equilibrium point is given by E.

Figure 2: IS-LM model under binding constraint in the non tradable sector

Note: Apreciations increase tradable but reduces non-tradable output along the respective IS curves, while a depreciation reduces the total output of the economy on the LM curve. The equilibrium point is given by E.
by:

\[ C_t = \frac{\omega}{1 - \alpha \omega} \frac{X_t S_t P_t}{W_t^{\alpha \omega}} = \Lambda \frac{X_t S_t^{\alpha \omega}}{W_t^{\alpha \omega}} = \left( \frac{\omega}{1 - \alpha \omega} \right) \frac{X_t}{\Lambda} (S_t^{\alpha \omega} W_t) \]

where \( \Lambda \) is a function of parameters. Using the previous result, we can express households’ expected utility of consumption as:

\[ EU = \Lambda' + \alpha \omega E \tilde{X} \ln \left( \frac{S}{W} \right) \]  

(26)

where \( \Lambda' \) is a function of parameters, and the only stochastic component comes from the foreign demand shock \( \tilde{X} \), which allows us to drop the time subscript.

We assume monetary policy is a function of the state of the world \( \tilde{X} = \exp(X) \), where \( X \sim N(0, 0.4)^6 \), denoted by \( M(\tilde{X}) \). To compute expectations, we will use Gaussian quadrature with three abscissas: \( \tilde{X}_1, \tilde{X}_2 \) and \( \tilde{X}_3 \). The first point will center the distribution \( \tilde{X} = 1 \), while the second and third will be above average and below average, respectively. The optimal monetary rule is given by the vector of state contingent responses:

\[ M_i = M(\tilde{X}_i), \quad i = \{1, 2, 3\} \]

that maximizes the expected utility of the households.

Here we depart from Devereux and Poon (2011) as they assume a discrete distribution for \( \tilde{X} \in \{X(1), \ldots, X(Z)\} \), with given probabilities. Now we use (14) and (15) to rewrite (??):

\[ EU = \Lambda'' + \alpha \omega E \tilde{X} \ln \left( \frac{M}{\left( E \tilde{X} M^{1+\psi} \right)^{\frac{1}{1+\psi}}} \right) \]

(27)

where \( '' \) collects parameters. The optimal policy is obtained using the first order condition of the expected utility maximization problem:

\[ \frac{\pi_i}{M_i} = \frac{\pi_i}{M^\psi} \left( \frac{M^\psi}{\left( E \tilde{X} M^{1+\psi} \right)^{\frac{1}{1+\psi}}} \right) \]

(28)

The solution to (28) yields \( M_i = M = 1, \forall i \). Therefore, the optimal monetary policy involves a fixed level of money balances: \( M = \bar{M} \). This result is the one that replicates the flexible wage equilibrium, which is the first best under commitment.\(^7\)

4.1 Calibration and Results

We will calibrate values for the case of a small open economy. We set the share of intermediate goods in output at 20 percent, yielding a value of 0.8 for \( \omega \). The share of foreign goods in consumption is \( 1 - \alpha \) is calibrated around 0.4 percent (\( \alpha = 0.6 \)). This value reflects the low share

\(^6\)Under this assumption, \( \tilde{X} \) exhibits an exponential distribution.

\(^7\)Devereux and Poon (2011) prove this result. Under commitment, monetary policy is unable to consistently exploit the market power that the economy has over home goods.
of final goods imports in the consumption basket in small open economies. Regarding the share of consumption of non-tradables we follow Mendoza (2005) who calibrated a similar parameter for the case of Mexico. We assume that non-tradable consumption accounts for half of overall consumption, yielding $\theta = 0.5$. We will test results for different levels of dollarization. See the appendix B for more details regarding how optimal monetary policy response is calibrated.

Tables 1 and 2 present the results of the main simulations. The first column of each table presents the distribution of variable for each shock, in addition to the mean and its standard deviation. The second and third columns of each table show the values of each variable in each state, with different types of monetary policy. The first element of each vector 1x3 of a variable, is the baseline state (when $\bar{X} = 1$, the mean), while the second element presents a state with a positive shock (when $\tilde{X} > 1$, greater than the mean) while the last element is a state with a negative demand shock (when $\tilde{X} < 1$, less than the mean). Since the negative shock may cause the economy to enter a region where the credit constraints bind, results emphasize this state. Thus, a negative foreign demand shock will generate a fall in net exports (fall in $\bar{X}$) that will depreciate the exchange rate. Whether or not the economy ends in the constrained region will depend on the demand for imported inputs being larger than the companies’ debt capacity. As we can see, in Table 1, the negative shock that does not bring the economy into a constraint regime, and the optimal monetary policy is a fixed monetary policy. Table 2 presents the case in which firms higher indebtedness. In this case, the negative realization of the shock ($\tilde{X}$) pushes the economy into the constrained region.

Table 2 shows that a pro-cyclical monetary policy might be advisable in this case. The LAW police affects the relative price between the tradable and non-tradable sectors, pushing it closer the productivity parameters ratio (the optimal relative price). This LAW policy reduces the demand for intermediate inputs by the tradable sector and relaxes the restrictions on non-tradable production. By contrasting the results under a fixed monetary policy and optimal policy, we can observe how imports of intermediate goods by the non-tradable sector are higher under the latter. By helping the non-tradable sector access imported inputs, the central bank improves the factor allocation efficiency in the production of these goods, reducing its price and expanding its production, vis-a-vis the fixed monetary policy benchmark. Therefore, a LAW policy measure reduces the distortions caused by credit constraints which translates in more efficient production of non-tradable goods, and a higher overall productivity. From this point of view of production, credit constraints lead to sub-optimal allocation of labour and intermediate inputs across sectors. Reduced money supply reduces the value of wages and allows some substitution of intermediate inputs by labour. This helps to correct mismatches, while the lower provision of labour services increases the utility of the household.

It is important to discuss monetary policy role after a positive shock. As Table 2 shows, monetary policy is also relatively contractionary when the economy is hit by a positive foreign demand shock. The reason for this relies on the fixed wage assumption. Agents will fix their
Table 1: Distribution of variables under low dollarization scheme

<table>
<thead>
<tr>
<th>Variable</th>
<th>Low Dollarization</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed M</td>
</tr>
<tr>
<td>M</td>
<td>[1,1,1]</td>
</tr>
<tr>
<td>Y</td>
<td>[0.3572,0.4103,0.3110]</td>
</tr>
<tr>
<td>E(Y)</td>
<td>0.3584</td>
</tr>
<tr>
<td>σ_Y</td>
<td>(0.0287)</td>
</tr>
<tr>
<td>S</td>
<td>[0.5500,0.2751,1.0996]</td>
</tr>
<tr>
<td>E(S)</td>
<td>0.5958</td>
</tr>
<tr>
<td>σ_S</td>
<td>(0.2467)</td>
</tr>
<tr>
<td>C</td>
<td>[0.2703,0.3666,0.1993]</td>
</tr>
<tr>
<td>E(C)</td>
<td>0.2745</td>
</tr>
<tr>
<td>σ_C</td>
<td>(0.1190)</td>
</tr>
<tr>
<td>L</td>
<td>[0.8000,0.8000,0.8000]</td>
</tr>
<tr>
<td>E(L)</td>
<td>0.8000</td>
</tr>
<tr>
<td>σ_L</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Y_N</td>
<td>[0.2001,0.2298,0.1742]</td>
</tr>
<tr>
<td>E(Y_N)</td>
<td>0.2007</td>
</tr>
<tr>
<td>σ_Y_N</td>
<td>(0.0161)</td>
</tr>
<tr>
<td>L_N</td>
<td>[0.2240,0.2240,0.2240]</td>
</tr>
<tr>
<td>E(L_N)</td>
<td>0.2240</td>
</tr>
<tr>
<td>σ_L_N</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>Y_T</td>
<td>[0.5144,0.5909,0.4479]</td>
</tr>
<tr>
<td>E(Y_T)</td>
<td>0.5161</td>
</tr>
<tr>
<td>σ_Y_T</td>
<td>(0.0414)</td>
</tr>
<tr>
<td>L_T</td>
<td>[0.5760,0.5760,0.5760]</td>
</tr>
<tr>
<td>E(L_T)</td>
<td>0.5760</td>
</tr>
<tr>
<td>σ_L_T</td>
<td>(0.0000)</td>
</tr>
<tr>
<td>P_N</td>
<td>[1.0000,1.0000,1.0000]</td>
</tr>
<tr>
<td>E(P_N)</td>
<td>1.0000</td>
</tr>
<tr>
<td>σ_P_N</td>
<td>(0.0000)</td>
</tr>
</tbody>
</table>

Note: The variables are in a 1x3 vector, which represents each realization of a shock. First number of the vector represents the base line shock (X = 1), the second represents the state where there is a positive shock, while the last one, a negative shock. In each column we compute equilibrium two types monetary policy rules: a Fixed Rule and an Optimal rule.
Table 2: Distribution of variables under high dollarization scheme

<table>
<thead>
<tr>
<th></th>
<th>High Dollarization</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Fixed M</td>
<td>Optimal M</td>
<td></td>
</tr>
<tr>
<td></td>
<td>[1,1,1]</td>
<td>[1.00,0.99,0.97]</td>
<td></td>
</tr>
<tr>
<td>$Y$</td>
<td>[0.3572,0.4103,0.3038]</td>
<td>[0.3592,0.4092,0.3045]</td>
<td></td>
</tr>
<tr>
<td>$E(Y)$</td>
<td>0.3572</td>
<td>0.3584</td>
<td></td>
</tr>
<tr>
<td>$\sigma_Y$</td>
<td>(0.0308)</td>
<td>(0.0302)</td>
<td></td>
</tr>
<tr>
<td>$S$</td>
<td>[0.5500,0.2751,1.0716]</td>
<td>[0.5500,0.2723,1.0642]</td>
<td></td>
</tr>
<tr>
<td>$E(S)$</td>
<td>0.5911</td>
<td>0.5894</td>
<td></td>
</tr>
<tr>
<td>$\sigma_S$</td>
<td>(0.2372)</td>
<td>(0.2353)</td>
<td></td>
</tr>
<tr>
<td>$C$</td>
<td>[0.2703,0.3666,0.1991]</td>
<td>[0.2713,0.3659,0.1966]</td>
<td></td>
</tr>
<tr>
<td>$E(C)$</td>
<td>0.2745</td>
<td>0.2746</td>
<td></td>
</tr>
<tr>
<td>$\sigma_C$</td>
<td>(0.1191)</td>
<td>(0.0491)</td>
<td></td>
</tr>
<tr>
<td>$L$</td>
<td>[0.8000,0.8000,0.7910]</td>
<td>[0.8054,0.7973,0.7804]</td>
<td></td>
</tr>
<tr>
<td>$E(L)$</td>
<td>0.7985</td>
<td>0.7999</td>
<td></td>
</tr>
<tr>
<td>$\sigma_L$</td>
<td>(0.0033)</td>
<td>(0.0092)</td>
<td></td>
</tr>
<tr>
<td>$Y_{N}$</td>
<td>[0.2001,0.2298,0.1691]</td>
<td>[0.2011,0.2292,0.1705]</td>
<td></td>
</tr>
<tr>
<td>$E(Y_{N})$</td>
<td>0.1998</td>
<td>0.2007</td>
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<tr>
<td>$\sigma_{Y_{N}}$</td>
<td>(0.0175)</td>
<td>(0.0170)</td>
<td></td>
</tr>
<tr>
<td>$L_{N}$</td>
<td>[0.2240,0.2240,0.2240]</td>
<td>[0.2255,0.2232,0.2187]</td>
<td></td>
</tr>
<tr>
<td>$E(L_{N})$</td>
<td>0.2240</td>
<td>0.2240</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{L_{N}}$</td>
<td>(0.0000)</td>
<td>(0.0025)</td>
<td></td>
</tr>
<tr>
<td>$Y_{T}$</td>
<td>[0.5144,0.5909,0.4432]</td>
<td>[0.5172,0.5893,0.4390]</td>
<td></td>
</tr>
<tr>
<td>$E(Y_{T})$</td>
<td>0.5153</td>
<td>0.5162</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{Y_{T}}$</td>
<td>(0.0427)</td>
<td>(0.0434)</td>
<td></td>
</tr>
<tr>
<td>$L_{T}$</td>
<td>[0.5760,0.5760,0.5670]</td>
<td>[0.5799,0.5741,0.5617]</td>
<td></td>
</tr>
<tr>
<td>$E(L_{T})$</td>
<td>0.5745</td>
<td>0.5759</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{L_{T}}$</td>
<td>(0.0033)</td>
<td>(0.0067)</td>
<td></td>
</tr>
<tr>
<td>$P_{r_{N}}$</td>
<td>[1.0000,1.0000,1.0356]</td>
<td>[1.0000,1.0000,1.0029]</td>
<td></td>
</tr>
<tr>
<td>$E(P_{r_{N}})$</td>
<td>1.0059</td>
<td>1.0005</td>
<td></td>
</tr>
<tr>
<td>$\sigma_{P_{r_{N}}}$</td>
<td>(0.0133)</td>
<td>(0.0011)</td>
<td></td>
</tr>
</tbody>
</table>

Note: The variables are in a 1x3 vector, which represents each realization of a shock. First number of the vector represents the base line shock ($\tilde{X} = 1$), the second represents the state where there is a positive shock, while the last one, a negative shock. In each column we compute the equilibrium using two types monetary policy rules: a fixed monetary rule (Fixed M) and an optimal monetary policy rule (Optimal M).
Figure 3: Expected Utility under different dollarization

Note: The different lines represent different levels of dollarization. The more they are on the right, the more dollarized the economy is. A line represents different combinations between dollarization of the sectors that preserve the same level of dollarization of the economy. Thus a shift from left to right, along a curve, implies that the tradable sector is more dollarized than the non-tradable one.

wages across realizations of $\tilde{X}$ taking monetary policy under consideration. A contractionary monetary policy in the good state, reduces the need for a more contractionary policy in the bad state. In this sense, the central bank helps households to smooth consumption across states: the central bank can make monetary policy less pro-cyclical in the bad state by enforcing a more contractionary policy in the good state. In this sense, the optimal policy is a combination of ex-ante and ex-post policies.$^8$

4.2 Balance Sheet Effects in Tradable and Non-Tradable Sectors

In this section, we solve for optimal monetary policy and associated utility for different compositions of unhedged dollarization in the tradable and non-tradable sectors. To see the differences in dollarization between sectors$^9$, we conducted the following analysis: Fixing the level of dollarization to the whole economy, we modify the share of foreign currency debt in the non-tradable sector. For each level, we compute the optimal policy response and corresponding expected utility of the households. Figures 3 and 4 show the results.$^{10}$ Each line signifies the

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$^8$ This is more prevalent when the negative shock has a high probability of occurrence. Since Devereux and Poon (2011) assume a different distribution for the external shock, the authors do no emphasize this result in their conclusions.

$^9$ For a given exchange rate, same foreign debt implies same dollarization

$^{10}$ An alternative figure is presented in Appendix C
Figure 4: Optimal Monetary Policy under different dollarization

![Graph showing Optimal Monetary Policy under different dollarization](image)

**Note:** The different lines represent different levels of dollarization. The more they are on the right, the more dollarized the economy is. A line represents different combinations between dollarization of the sectors that preserve the same level of dollarization of the economy. Thus a shift from left to right, along a curve, implies that the tradable sector is more dollarized than the non-tradable one.

same level of dollarization, ranging from lower (left) to higher (right). Furthermore, movements along a curve imply the same level of dollarization but with a different composition of sector dollarization.

Given a high level of dollarization (the lower curve), the more dollarized the tradable sector, the less dollarization of the non-tradable sector, the greater the expected utility. This is because the more dollarized the non-tradable sector is, the more likely it is that credit restrictions will bind, generating a negative change in expected utility. This analysis is similar for Figure 4 where a higher level of dollarization in the non-tradables implies a more leaning against the wind monetary policy, that is, less counter-cyclical monetary policy.

The results show that central banks should not only focus on overall dollarization, but concern as well with the sector holding the unhedged short positions in foreign currency. Thus, in order to improve the effectiveness of monetary policy in face of large negative shocks, central banks in countries with higher levels of financial dollarization should push for de-dollarization measures focused in the more vulnerable sector. Contreras et al. (2019) show how the Central Reserve Bank of Peru - a country with a pervasive financial dollarization of credit and an active foreign exchange intervention policy, has followed a de-dollarization programme in this line, focusing in sectors that do not exhibit a natural hedge, such as mortgages, car loans and credits.
to micro and small firms.

5 Conclusions

This paper presents a simple model with optimal monetary policy in a small open economy with two sectors, where the non-tradable sector faces binding collateral constraints. The model captures why monetary policy should be leaning against the wind even if only one sector, and not the whole economy is affected by credit constraints. This result complements the ones in Devereux and Poon (2011) and Benigno et al. (2013), by stating that the space for leaning against the wind policies is not limited to the overall financial access of the economy. In a nutshell, when the financial constraint hits one sector, distortions are transmitted to the rest of the economy through factor markets, affecting the overall productivity and welfare.

The results suggest that central banks should monitor more closely financial vulnerabilities present in the different sectors of the economy, focusing in the more leveraged sectors, instead of following broad debt and dollarization indicators. Additionally, relative price distortions can be an important source of inefficiency, granting targeted policies such as the ones followed by the Central Reserve Bank of Peru.

We leave for future work an extension of this framework to a dynamic general equilibrium model and incorporating the endogenous portfolio problem faced by real sector firms.
References


Appendix A. Solving the Model

5.1 Economy under non-binding constraints

Equations (1) - (5), (14)-(16) describe the economy under no constraints. Prices are given by (4) and (5).

\[ P_N = \kappa \frac{W(z \omega)}{A_N} \]  
(29)

\[ P_T = \kappa \frac{W(z \omega)}{A_T} \]  
(30)

Equations (19) and (20) define output for each sector:

\[ Y_T = A_T \frac{1}{\kappa} \left( \frac{1 - \alpha \theta \omega}{1 - \alpha \omega} \right) \tilde{X} \frac{S^\omega}{W^\omega(Q^*)^{1-\omega}} \]

\[ Y_N = A_N \frac{1}{\kappa} \left( \frac{\alpha \theta \omega}{1 - \alpha \omega} \right) \tilde{X} \frac{S^\omega}{W^\omega(Q^*)^{1-\omega}} \]

Consumer prices are obtained by combining (29) and (30) with the foreign price:

\[ P = \left[ \left( \frac{\kappa W^\omega(SQ^*)^{1-\omega}}{\alpha} \right) \left( \frac{1}{A_N^\theta} \right) \left( \frac{1}{A_T(1-\theta)} \right) \right] \alpha \left( \frac{SP^*_F}{1-\alpha} \right)^{1-\alpha} \]

\[ S^{1-\omega} W^\omega \left[ \left( \frac{\kappa (Q^*)^{1-\omega}}{\alpha} \right) \left( \frac{1}{A_N^\theta} \right) \left( \frac{1}{A_T(1-\theta)} \right) \right] \alpha \left( \frac{P^*_F}{1-\alpha} \right)^{1-\alpha} \]

Overall consumption is obtained combining (14) and (16)

\[ C = \frac{\omega}{1 - \alpha \omega} \tilde{X} \]

while sector consumption follows from the direct application of demand for non tradable and tradable goods.

\[ C_N = \theta \frac{\alpha PC}{P_N} = A_N \frac{1}{\kappa} \left( \frac{\alpha \theta \omega}{1 - \alpha \omega} \right) \tilde{X} \frac{S^\omega}{W^\omega(Q^*)^{1-\omega}} \]

\[ C_T = (1 - \theta) \frac{\alpha PC}{P_T} = A_T \frac{1}{\kappa} \left( \frac{(1-\theta) \alpha \omega}{1 - \alpha \omega} \right) \tilde{X} \frac{S^\omega}{W^\omega(SQ^*)^{1-\omega}} \]

Implying that exports are given by:

\[ Y_T - C_T = A_T \frac{1}{\kappa} \frac{\tilde{X} S^\omega}{W^\omega(Q^*)^{1-\omega}} = \tilde{X} \frac{S}{P_T} \]

Using determines the money market clearing condition with the money demand equation:

\[ M = \chi PC = \chi \frac{\omega}{1 - \alpha \omega} \tilde{X} \]

while wage is given by (12)

\[ W(i) = \eta \frac{\rho}{1 - \rho} \frac{E\left\{ L(i)^{1+\Psi} \right\}}{E\left\{ L(i) \right\} / PC(i)} \]
further replacing, gives us the following expression for the wage as a function of expected money supply:

\[ W = \left[ \eta \frac{\rho}{1-\rho} \right]^{\frac{1}{1+\psi}} \frac{1}{\chi} E \{ (M)^{1+\psi} \}^{\frac{1}{1+\psi}} \]

Now, we solve for sectoral and aggregate labour:

\[ L_T = \omega \frac{P_T Y_T}{W} = \frac{\omega \left( \frac{1-\alpha \theta}{1-\alpha \omega} \right) \hat{X} S}{\left[ \eta \frac{\rho}{1-\rho} \right]^{\frac{1}{1+\psi}} \frac{1}{\chi} E \{ (M)^{1+\psi} \}^{\frac{1}{1+\psi}}} \]

\[ L_N = \omega \frac{P_N Y_N}{W} = \frac{\omega \left( \frac{\alpha \theta \omega}{1-\alpha \omega} \right) \hat{X} S}{\left[ \eta \frac{\rho}{1-\rho} \right]^{\frac{1}{1+\psi}} \frac{1}{\chi} E \{ (M)^{1+\psi} \}^{\frac{1}{1+\psi}}} \]

\[ L = \omega \frac{P_N Y_N + P_T Y_T}{W} = \frac{\omega \left( \frac{1}{1-\alpha \omega} \right) \hat{X} S}{\left[ \eta \frac{\rho}{1-\rho} \right]^{\frac{1}{1+\psi}} \frac{1}{\chi} E \{ (M)^{1+\psi} \}^{\frac{1}{1+\psi}}} \]

while investment is given by:

\[ I_T = (1-\omega) \frac{P_T Y_T}{SQ^*} = \frac{(1-\omega) \left( \frac{1-\alpha \theta \omega}{1-\alpha \omega} \right) \hat{X}}{Q^*} \]

\[ I_N = (1-\omega) \frac{P_N Y_N}{SQ^*} = \frac{(1-\omega) \left( \frac{\alpha \theta \omega}{1-\alpha \omega} \right) \hat{X}}{Q^*} \]

\[ I = \frac{(1-\omega) \left( \frac{1-\alpha \omega}{1-\omega} \right) \hat{X}}{Q^*} \]

Finally, the exchange rate is determined by the money market equilibrium:

\[ S = \frac{1}{\omega} \frac{1}{\chi} \hat{X} M \]

the model can be solved once \( W \) and \( \hat{X} \) are determined.

5.2 Economy under binding constraints

When constraints bind, the solution is harder to pin down. Prices are given by:

\[ P_T = \kappa \frac{W \omega (SQ^*)^{1-\omega}}{A_T} \]

\[ P_N = \frac{W}{\omega A_N} \frac{L^{1-\omega}}{(N^{1-\omega} - SD^{N^{1-\omega}})^{1-\omega}} \]

further substitution yields the price of non-tradables as a function of the wage of production:

\[ P_N = \left( \frac{1}{\omega} \right) W \left[ \frac{(SQ^*)^{1-\omega}}{A_N} \left( \frac{Y_N^{1-\omega}}{N - SD^*} \right) \right]^{\frac{1}{\omega}} \]
From (70) and (71):

\[ P_T Y_T = \frac{1 - \alpha \theta}{\gamma} \tilde{X} S - \frac{\alpha (1 - \theta)}{\gamma} [N - SD^*] \]

\[ P_N Y_N = \frac{\omega \alpha \theta}{\gamma} \tilde{X} S - \frac{\alpha \theta}{\gamma} [N - SD^*] \]

Overall expenditure pins down consumption as a function of prices:

\[ C = \frac{[SD^* - N]}{P_T} + \frac{\omega}{P_N} \tilde{X} S \]

while consumption for each type of good follows directly from the demand for tradable and non-tradable goods:

\[ C_N = \frac{\alpha \theta}{\gamma P_N} [SD^* - N] + \frac{\alpha \theta}{\gamma} \omega \tilde{X} S \]

\[ C_T = (1 - \theta) \frac{\alpha P_C}{P_T} = (1 - \theta) \frac{\alpha [SD^* - N]}{\gamma P_T} + (1 - \theta) \frac{\omega}{\gamma P_T} \tilde{X} S \]

Money market equilibrium is defined by:

\[ M = \chi P_C = \chi [\omega P_T Y_T + P_N Y_N - N + SD^*] \]

\[ = \chi \left[ \frac{[SD^* - N]}{\gamma} + \frac{\omega}{\gamma} \tilde{X} S \right] \]

Wage is given by (12):

\[ W(i) = \eta \frac{\rho}{1 - \rho} E \left\{ L(i)^{1+\Psi} \right\} \]

\[ W^{1+\Psi} = \eta \frac{\rho}{1 - \rho} \omega^{1+\Psi} \left[ \left( \frac{\alpha}{\gamma} [SD^* - N] + \frac{1 - \alpha \theta + \omega \alpha \theta}{\gamma} \tilde{X} S \right)^{1+\Psi} \right] \]

Profit maximization over labour yields:

\[ L_T = \frac{\omega}{\chi} \frac{P_T Y_T}{W} = \frac{\omega}{W} \left[ \frac{\alpha (1 - \theta)}{\gamma} [SD^* - N] + \frac{1 - \alpha \theta}{\gamma} \tilde{X} S \right] \] (35)

\[ L_N = \frac{\omega}{\chi} \frac{P_N Y_N}{W} = \frac{\omega}{W} \left[ \frac{\alpha \theta}{\gamma} [SD^* - N] + \frac{\omega \alpha \theta}{\gamma} \tilde{X} S \right] \] (36)

\[ L = \frac{\omega}{\chi} \left[ \frac{\alpha}{\gamma} [SD^* - N] + \frac{1 - \alpha \theta + \omega \alpha \theta}{\gamma} \tilde{X} S \right] \] (37)

Exchange rate is determined by the money market equilibrium:

\[ S = \frac{\gamma M}{(D^* + \omega \tilde{X}) \chi} + \frac{N}{D^* + \omega \tilde{X}} \]

Finally, investment in the tradable sector is given by:

\[ I_T = (1 - \omega) \frac{\alpha (1 - \theta)}{\gamma} [SD^* - N] + \frac{1 - \alpha \theta}{\gamma} \tilde{X} S \]

while \( I_N \) is obtained the credit constraint:

\[ I_N = \frac{N - SD^*}{SQ^*} \]
Appendix B: Finding Optimal Monetary Policy

Following Devereux and Poon (2011). We can compute optimal monetary policy with the next steps:
1. Set the initial state-contingent money response as:

\[ M^0 = [M^0_1; M^0_2; M^0_3] = [1; 1; 1] \]

2. Solve for the preset optimal fixed wage:

3. Use the wage to solve the remaining variables and compute expected utility as \( EU_0 \).

4. Define a vector \( \delta_{M^j} = [0; \delta_{M^j_2}; \delta_{M^j_3}] \), with \( j \in J \), where \( J \) is the policy space. This vector represents the exogenous money change of policy \( j \), carried out by the policymaker. Denote the new state-contingent money response as \( M_1 \), given by: \( M_1 = M^0 - \delta_{M^j} \)

5. Given \( M_1 \), solve \( W(M_1) \) and compute \( EU_1 \).

6. Repeat 4 and 5 \( n \) times. The money vector is:

\[ M = [1; 1 - \delta_{M^j_2}; 1 - \delta_{M^j_3}] \]

the combination of \( \delta_{M^j_2} \) and \( \delta_{M^j_3} \) that give the highest expected utility is the optimal monetary policy.

6 Appendix C: Additional tables and plots

![Figure 5: Distribution of variables under High dollarization scheme](image)
Figure 6: Distribution of under High dollarization scheme

Table 3: Parameters for calibration

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<th>Parameter</th>
<th>Value</th>
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<tr>
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<tr>
<td>$\rho$</td>
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<tr>
<td>$A_N = A_T$</td>
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