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Abstract

In most modern recessions there is a sharp increase in job destruction and a mild to moderate decline in job creation, resulting in unemployment. The Great Recession was marked by a significant decline in job creation particularly for young firms in addition to the typical increase in destruction. As a result job reallocation fell. In this paper, we explicitly propose a mechanism for financial shocks to disproportionately affect young (typically) smaller firms via credit contracts. We investigate the particular roles of credit frictions versus nominal rigidities in a New Keynesian model augmented by a banking sector characterized by search and matching frictions with endogenous credit destruction. In response to a financial shock, the model economy produces large and persistent increases in credit destruction, declines in credit creation, and an overall decline in reallocation of credit among banks and firms; total factor productivity declines, even though average firm productivity increases, inducing unemployment to increase and remain high for many quarters. Credit frictions not only amplify the effects of a financial shock by creating variation in the number of firms able to produce they also increase the persistence of the shock for output, employment, and credit spreads. When pricing frictions are removed, however, credit frictions lose some of their ability to amplify shocks, though they continue to induce persistence. These findings suggest that credit frictions combined with nominal rigidities are a plausible transmission mechanism for financial shocks to have strong and persistent effects on the labor market particularly for loan dependent firms. Moreover, they may play an important role in job reallocation across firms.

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1 Introduction

The Great Recession and slow recovery were characterized by a deep and prolonged decline in job creation, particularly among firms less than five years old, an increase in financial volatility, and a decline in bank lending. The recession occurred during a period when job and worker reallocation in the US had been declining for nearly 25 years (Foster, Grim, and Haltiwanger, 2014).

The net decline in bank lending in all loan categories–including to consumers, to firms, and for real estate purposes–was a novel feature of the Great Recession compared to previous post-Volker (or post-1979) recessions, as was the large decline in job creation by smaller or younger firms. Most recessions, by contrast, are characterized by sharp increases in job destruction coupled with slow job creation (see Foster, Grim, and Haltiwanger 2014). Unemployment remained persistently high for two years following the onset of the recovery and weak job creation was an important factor[1].

Foster, Grim, and Haltiwanger (2014) demonstrate that the process of labor reallocation also changed during the Great Recession and decline in new firm entry was an important feature. In this paper, we argue that disruption in lending to new or young firms as a result of the financial crisis reduced employment through a decline in job creation. This fact may have contributed to the ongoing decline in new firm formation and allocation dynamics discussed in Foster, Grim, and Haltiwanger (2014). We focus on the firm credit channel, taking a step back in the chain linking the dynamics of job flows to aggregate outcomes, to consider the impact of financial shocks on credit dynamics and the transmission through credit to unemployment.

In an environment where firms and banks establish long term relationships, we discover a direct and an indirect channel for disruption in credit flows to impact employment that works through two mechanisms: changes in the distribution of bank-firm credit contracts and changes in the inefficiency wedge[2]. Both channels induce a reallocation process for credit among banks and firms. The direct channel works through forces that cause separations of bank-firm relationships, either exogenously or endogenously through changes in the productivity cutoff at which banks will provide credit to firms (that are heterogeneous in their productivity). This channel directly impacts output and employment. The indirect channel works through the intermediation wedge which depends on the probability of the continuation of a credit contract and the number of active credit contracts. Deviation in this wedge from its steady state value results in changes in total factor productivity, employment and output.

We use a search and matching framework with heterogeneous firms to model frictions in the credit market and include wage rigidities in the labor market and price rigidities in the market for final goods to mimic some of the aggregate features of the slow recovery. Our model gives rise to an amplification mechanism for aggregate shocks which we call an 'inefficiency wedge' and which impacts productivity directly. We consider

[1] See Kudlyak and Sánchez (2017) for a discussion of the weakness in large firm job creation following the Great Recession

[2] Mehrotra and Sergeyev (2016) establish that the consumer credit channel, discussed in Mian and Sufi (2011), primarily reduces employment via lower demand through job destruction.

two types of financial shock, the first is an increase in the exogenous separation rate governing the probability banks and firms remain matched under the same credit contract. This type of shock could empirically correspond to a sharp decrease in bank lending due to factors such as a shift in capital adequacy, reductions in interbank lending, or other exogenous shifts in bank balance sheets unrelated to collateral constraints. Our second type of financial shock increases the bargaining strength of firms versus banks. Changes in the relative bargaining power between banks and firms when negotiating conditions of a loan contract could reflect parties' beliefs about the risk of a breakdown of negotiations or asymmetric information that reduces the bargaining strength of one of the parties[3]. As the result of a financial shock, bank surplus falls but so does aggregate joint surplus causing banks to exit the lending market, resulting in an increase in credit tightness. The increase in credit tightness reduces overall lending and the number of firms producing as the productivity cutoff rises. Among producing firms, labor demand falls, further contributing to a rise in unemployment rate.

Under an exogenous bank-firm separation shock, we have four main findings: first, our framework generates an inefficiency wedge that varies with the level of financial frictions and directly influences total factor productivity through a selection effect, second, following a financial shock, reservation productivity rises resulting in fewer firms producing (e.g., decline in active firms), and but among those firms that produce they use more labor. The increase in the cutoff or reservation productivity should result in an increase in total factor productivity as only the most productive firms get loans and produce. But the inefficiency wedge mitigates this increase resulting in a small reduction in TFP. The extensive margin effect reduces employment as fewer firms hire workers and produce while the intensive margin effect increases employment as those firms that do produce hire more labor. However, the extensive margin effect is much larger, so that the total effect is an overall increase in unemployment. Third, lending frictions exacerbate the effect of the shock by reducing the continuation probability for bank-firm relationships. A decline in the continuation probability results in the severance of firm-bank relationships causing firms and banks to continue to search for credit partners, to cease production, in the case of firms, or exit the market in the case of banks. This feature of the credit market increases the persistence of the shock. Fourth, the intensive margin effect mentioned above is caused by movements in the Nash bargained marginal cost and affects only firms that have survived the separation shocks. Firms that remain matched with banks and able to produce, will hire more workers because the marginal costs of doing so have fallen.

Alternatively, when we allow Nash bargaining shares for firms and banks to be time-varying and consider a reduction in banks' bargaining power, we observe a similar chain of events but with two differences. A shock that reduces bank bargaining power results in a similar set of movements in firm, bank, credit and macro aggregate variables as an exogenous separation shock: the resulting reduction in bank surplus causes

[3]For instance, Binmore, Rubinstein, and Wolinsky 1986 argue that the bargaining share, η, reflects asymmetries between the two parties or differing beliefs in the likelihood of a breakdown in bargaining.

banks to exit the loan market and rely on excess reserves held at the central bank. This action increases credit tightness and significantly reduces the number of bank-firm credit contracts. This in turn reduces the number of firms able to produce directly while also increasing the threshold productivity level for those firms able to obtain a credit contract and produce. Through both mechanisms employment is reduced. Two differences with the exogenous shock emerge however, one, reservation productivity rises significantly more under varying bargaining shares, meaning that firms that are in credit contracts are more productive thus there is a more moderate decline in total factor productivity. But, in this case the marginal cost of hiring labor rises, meaning that the intensive margin effect moves in the same direction–to reduce employment–as the extensive margin effect. Thus the increase in unemployment is larger. Additionally, the fall in net credit creation is smaller–there is less credit destruction but also less credit creation, so the overall the decline in net credit is less pronounced. Overall, unemployment is worse in this scenario but the decline in GDP is smaller and the decline in inflation is larger.

The main transmission mechanism in this paper–the sorting of firms by a productivity cutoff generated through changes in the cost of finance–shares similarities with the one developed in Petrosky-Nadeau (2013). The inefficiency wedge which arises endogenously in our model is also related to the intermediation wedge in Ajello (2016) which depends in part on slow wage and price adjustment. In our case, slow wage adjustment interacts with the reservation productivity level–and hence affects the distribution of firms in credit contracts–through the slow adjustment of marginal costs. In the next section we discuss the large literature on the relationship between credit and employment and our contribution.

2 Related Literature

The mechanisms highlighted in this paper are consistent with recent empirical evidence from papers such as Iyer, Peydr, da Rocha-Lopes, and Schoar (2014), Boustanifar (2014), Huber (2018)and Chodorow-Reich (2014), that use detailed bank-firm data, banking reforms, or carefully matched labor and credit data to consider linkages between financial shocks and bank lending or bank lending and employment. These papers focus on the impact of a financial shock on credit supply. Iyer, Peydr, da Rocha-Lopes, and Schoar (2014) use Portuguese loan-level data to determine that financial shocks caused banks that relied more on interbank borrowing to reduce their lending much more than less reliant banks. Firms in relationships with affected banks were unable to get access to the credit they needed. Boustanifar (2014) uses U.S. banking reforms to study the relationship between the availability of finance and employment. He provides evidence that easing of financing constraints had a direct effect on employment by financing the fixed costs of employment allowing firms to hire their first worker as well as additional workers. Chodorow-Reich (2014) finds that, given the longevity of banking relationships and difficulty of forming new relationships, pre-crisis clients of banks who were more exposed to the financial crisis had a lower probability of receiving a loan and for those firms that did acquire a loan, the interest spread was higher. Using data on German lending, Huber (2018) finds that decreases in lending by a large German bank reduced the growth of firms reliant on its loans and that this effect was persistent even after lending had returned to its baseline. In our paper, the transmission mechanisms that arise from our model qualitatively match this evidence. We find that following a financial shock, bank firm relationships separate and banks exit the credit market resulting in a decline in credit creation, a rise credit destruction, and an overall decline in lending declines as well as a significant increase in lending spreads. These effects are also persistent, lasting for approximately 10 quarters depending on how the shock is modeled. As a result of the credit market turmoil, firms who lose their banking relationship, are unable to employ workers and produce output, causing unemployment to rise significantly and persistently. We find that output is slower to return to baseline than employment even after credit conditions have returned to normal because of the lingering effect on total factor productivity due to the inefficiency wedge. These features are generated by supply side disruptions in the credit market and abstract from simultaneous demand side responses. The increase in spreads is caused by tightening in the credit markets due initially to the exogenous breaking of bank-firm relationships and then later due to an endogenous rise in the cutoff productivity rate. The increase in tightness and qualifications to receive a loan results in a strong persistent decline in bank lending. Both the deposit rate and the Nash bargained lending rate increase but the latter increases by more.

Chodorow-Reich (2014) finds that financial shocks transmitted through the credit supply channel had differential effects depending on the size of the firm: small to medium sized firms exposed to banks in poor health experienced significant employment declines compared with larger firms or firms that were not exposed. Heterogeneous effects of disturbances in credit markets by firm size are consistent with work by Adrian, Colla, and Shin (2012) who demonstrate that banking relationships may be less important for large firms with multiple sources of debt as well as equity finance[4]. Our paper highlights a mechanism credit shock propagation mechanism that reduces entry of firms–that is unrelated to firm balance sheets or collateral constraints–as well as job creation. The dynamics we highlight are relevant for understanding the relationships between credit and employment during normal and crisis periods and may provide insight into reallocation trends in credit and job markets.

Papers such as Craig and Haubrich (2013), Contessi and Francis (2013) and Herrera, Kolar, and Minetti (2014) provide empirical evidence on the patterns of credit reallocation across banks over the business cycle as well as the specific properties of loan creation and destruction complimenting the statistical evidence on job flows. Herrera, Kolar, and Minetti (2014) and Contessi, DiCecio, and Francis (2015) also consider the reallocation of credit across firms. The combination of the credit and job flows literature provides significant

[4]However, evidence in Greenstone, Mas, and Nguyen (2014) suggests that the decline in small business lending can account for at most 15 percent of the 6 percent decline in employment during the Great Recession.

evidence on the heterogeneous patterns of these flows at any phase in the business cycle and their relationships particularly during periods of low liquidity.

The theoretical literature linking credit availability to employment has become quite large. Important early contributions, such as Wasmer and Weil (2004) and den Haan, Ramey, and Watson (2003), modeled decentralized credit markets using search and matching frictions to understand the propagation of aggregate shocks through financial intermediation to output, employment, and investment. More recent papers such as Becsi, Li, and Wang (2013) build detailed models of financial intermediation incorporating lending markets characterized by search-and-matching frictions as well as informational frictions. Petrosky-Nadeau and Wasmer (2013) adds search in lending markets to a labor search framework to understand how financial frictions can increase volatility in vacancies and unemployment a feature typically missing in labor search models without other sources of frictions.

Although we do not model a decentralised labour market, we induce volatility in unemployment while retaining relatively smooth real wages by including wage rigidities in the form of Calvo staggered wage setting as described in Erceg, Henderson, and Levin (2000). We model unemployment similarly to Blanchard and Galí (2010), Galí (2011) and Galí, Smets, and Wouters (2012) as a reinterpretation of the labour market in the standard New Keynesian model. The existence of market power, wage rigidities, and a participation condition for the marginal supplier of each labor type produces a gap between aggregate labor demand and the labor force. This gap is related to the difference between the prevailing aggregate real wage and the average disutility of labor expressed in terms of consumption and also positively related to the unemployment rate. This wedge is similar to that described in Justiniano, Primiceri, and Tambalotti (2011). Credit conditions affect the marginal cost of labor as well as aggregate labor demand on both the extensive and intensive margin (discussed in detail below). The labor force is also affected by credit conditions via the aggregate marginal rate of substitution of the marginal supplier of labor.

We model the lending market similarly to Becsi, Li, and Wang (2013) but without information frictions in order to focus on the propagation of financial shocks to labor markets. Our framework, unlike Becsi, Li, and Wang (2013), does not generate credit rationing. In our model, banks obtain funds by raising retail deposits in a competitive market. Firms must finance their wage bill via external funding which can be obtained through a match with an unmatched bank. If a matched firm and bank choose to cooperate, a loan contract is agreed on and Nash bargaining determines how the joint surplus of the match is shared, generating a match-specific loan principal and a credit interest rate. The search and matching friction in the loan market produces an endogenous financial inefficiency wedge that appears as an additional input in the aggregate production function. This inefficiency wedge depends on the aggregate probability of continuation for a loan contract as well as on the mass and productivity distribution of active producing firms. Florian Hoyle, Limnios, and Walsh (2018) use a similar framework for the loan market but use it to investigate a channel system of interest rate control in an interbank market without nominal rigidities.

In our model, in equilibrium, there is a productivity threshold such that only those firms matched with a bank whose productivity level is above this threshold are able to produce. Thus, equilibrium is characterized by a distribution of actively producing firms as well as a distribution of match-specific loan rates. Like in Ajello (2016), financial intermediation frictions result in misallocation of credit and labor. Nominal wage and price rigidities then further slow efficient adjustment following the shock, prolonging the impact on employment and output. We disentangle the role of nominal rigidities on the transmission of the financial shock to employment and output to show that it is the presence of the inefficiency wedge, the role of reservation productivity, and credit market tightness in conjunction with nominal frictions that increases the amplitude and persistence of the financial shocks.

Financial shocks modeled as the breaking of bank-firm contracts, whether through an increase in separation rates or a change in Nash bargaining shares, initially reduce production because of the lack of funds to pay workers. This results in an increase in the measure of firms searching for a credit contract. Following the impact, depending on how the shock is modeled, banks exit the market further exacerbating the increase in credit market tightness and increasing the measure of firms without a credit contract. Since the financial shock also increases the cutoff productivity for banks to profitably provide loans, the number of producing firms declines and due to the influence of the inefficiency wedge, reduces total factor productivity even as the productivity of the remaining matched firms rises. Employment depends on the endogenous probability of firms finding a credit relationship, the number of firms that are actually producing during the period, the average productivity of those firms and the dispersion of wages generated in the labor market. Since in this model, the number of firms producing during the period is determined by the number of active loan contracts, unemployment fluctuations are related to credit conditions as well as to the stance of monetary policy which controls the deposit rate.

Aggregate output and employment are also indirectly influenced by the bank's opportunity cost of funds (equal to the deposit rate) which affects the level of economic activity at both the extensive and intensive margins. A rise in the deposit rate increases the threshold level of the idiosyncratic productivity of firms that generate a positive joint surplus. As a consequence, fewer firms are able to secure financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to the Nash bargained marginal cost, so that an increase in the deposit rate, ceteris paribus, reduces labor demand at each level of the real wage. This is the intensive margin effect. The Nash bargained marginal cost of labor arises from solving the bilateral bargaining problem between a bank and a firm over the loan rate and size. The existence of a cost channel implies that loan size determines production scale (and hiring) for all active firms while search frictions in the loan market imply that the relevant interest rate capturing the cost channel is not the negotiated loan rate but the opportunity cost of funds each bank faces. The intensive and extensive margin channels work to reduce aggregate output as the deposit rate rises, and symmetrically both channels increase output as the deposit rate falls. In addition, credit market conditions reflected in the probability of a firm matching with a bank affect the extensive margin; a rise in credit market tightness increases the reservation productivity and fewer firms obtain credit. Both interest costs measured by the deposit rate and credit conditions measured by credit market tightness matter for employment and output.

Our framework exhibits a cost channel of monetary policy, as discussed above, similar to Ravenna and Walsh (2006), in which firms must finance wage payments in advance of production. The standard implication of the cost channel is that the cost of labor is affected by the loan interest rate. However, when the loan rate is the outcome of a bargaining process, as it is in our model, the role of the loan rate is to split the surplus between the borrower (the firm) and the lender (the bank). In this context, the loan rate is irrelevant for the firm's employment decision which ultimately is a consequence of the conditions of the credit contract. Although a cost channel arises, it depends on the bank's opportunity cost of funds (determined by the deposit rate), not the interest rate charged on the loan.

In the next section, we provide some motivating empirical evidence on the business cycle properties of gross credit and job flows. We then develop a New Keynesian model with a decentralized lending market, price frictions in the final goods sector, and wage frictions in the intermediate goods sector as outlined above. We focus on the responses of credit and labor markets to two types of financial shocks. We then dissect the role of nominal frictions in producing the direction, amplitude, and persistence of the results and discuss our conclusions.

3 Empirical evidence on gross credit flows, unemployment and financial shocks

In this section we present some empirical regularities on the relationship between job flows and credit. We discuss a set of unconditional moments and the business cycle properties of gross credit and job flows. We use these empirical characteristics to motivate the model we develop subsequently.

3.1 Cyclical properties of gross credit and job flows

Although we primarily consider net job flows in this paper by focusing on employment and unemployment, the data on gross job flows provides insight into the factors driving unemployment and the relationship between credit flows and unemployment through changes in job creation. We use quarterly data on manufacturing job flows from Faberman (2012) updated with data from the Business Economic Dynamics (BED) database and quarterly data on job flows in all non-governmental sectors also from the BED. For credit flows, we use a measure of credit availability derived from information in the Reports of Income and Condition. The

Reports of Income and Condition, known as the Call Reports, must be filed each quarter by every bank and savings institution overseen by the Federal Reserve (i.e., those who hold a charter with the Federal Reserve). These reports contain a variety of information from banks' income statements and balance sheets. We use quarterly reported total loans and lending to commercial and industrial enterprises to create measures of credit creation and destruction. Quarterly Call Report data is available beginning in 1979Q1. We use an additional 20 quarters of historical data collected and organised in Craig and Haubrich (2013). Lending data is then linked to data from the National Information Center (NIC) on mergers and acquisitions during this period. Using the M&A data from the NIC we can remove bias that might arise from counting lending activity at both the acquired and acquiring bank (See Contessi and Francis 2013 for a full discussion of how these data are compiled).[5]

In order to determine 'gross credit flows' we use a technique first adapted from the labor literature in Dell'Ariccia and Garibaldi (2005) for credit flows. Define $l_{i,t}$: as total loans for bank i in quarter t. Let $g_{i,t}$ be the credit growth rate for bank i between t and $t-1$, adjusted for mergers or acquisitions. Then we can define:

$$POS_t = \sum_{i|g_{i,t}\geq 0}^{N} \alpha_{i,t} g_{i,t}$$

$$NEG_t = \sum_{i|g_{i,t}<0}^{N} \alpha_{i,t} \left| g_{i,t} \right|$$

where

$$\alpha_{i,t} = \left(\frac{0.5(l_{i,t} + l_{i,t-1})}{\sum\limits_{i=1}^{N} l_{i,t-1}} \right)$$

and $\frac{l_{i,t}+l_{i,t-1}}{2}$ is a measure of the average loan portfolio size of bank i between period t and $t-1$ and where $\sum\limits_{i=1}^{N} l_{i,t-1}$ is the loan portfolio of the banking system in the previous period.

Given these measures of credit creation (POS) and credit destruction (NEG), we define net lending as $NET = POS - NEG$. We use a similar accounting measure for gross job flows using data directly from the BED (1992Q2 - 2017Q1) and Faberman (2012) (for 1973Q1-1991:Q4).

Table (5) in Appendix A provides the means and standard deviations of lending and job flow variables for our entire sample (1973Q1 to 2017Q1) and three sub-periods–the Great Moderation, 1984Q1 to 2007Q2; the Great Recession, 2007Q3 to 2009Q2; and the post Recession period, 2009Q3 to 2017Q1.[6]

[5] We then remove from the data seven investment banks and credit card companies that acquired commercial bank charters during the 2008-09 recession and attendant financial crisis.

[6] In this paper we have focused on the behavior of credit and job flows during and following the Great Recession. Please see

Three features of these summary statistics are notable. First, the mean of loan creation plus loan destruction (SUM) for commercial & industrial loans is significantly larger during the Great Recession than during other reported sub-periods. The sum of loan creation and destruction is a measure of 'churning' or reallocation in the banking sector. Increases in reallocation (SUM) can come from expansion in credit availability or from an increase in credit destruction. Each of these possibilities has different implications for reallocation. Simultaneously high credit creation and destruction produces significant reallocation even though high credit destruction implies more credit search or firm exit. But given credit availability, finding new contracts should be quicker. Low creation and high destruction, ceterus paribus, implies less reallocation and longer search times, while high creation and low destruction implies both less reallocation and less search. In our sample and for this time period, high reallocation is driven by an increase in loan destruction. During the Great Recession, lower credit creation than average but higher credit destruction; the ratio of credit destruction to creation was much higher for total lending and commercial & industrial lending than in previous periods or during the post recession period. This implied more firms were searching for credit.

If reallocation across banks translated into reallocation across firms that improved allocative efficiency, we would expect higher productivity growth following the recession. But we observe lower total factor productivity and average labor productivity (see figure 4)[7].

Second, the mean value for net loan creation during each subperiod is positive indicating credit growth in each period including the Great Recession. Net loan creation however is very low during the post Great Recession period due to very weak loan creation. Third, excess loan creation (the difference between the SUM and the absolute value of the NET) is the largest during the Great Moderation. Excess loan creation (EXC) measures credit reallocation in excess of what is required to accommodate a change in net credit. For example, if credit creation equals one and credit destruction equals zero, then SUM equals one as does NET, thus EXC equals zero meaning no additional reallocation of credit between banks occurred when credit creation increased. Following the Great Recession, EXC is much lower than it has been historically, implying little reallocation of funds across banks. Reallocation of credit–provided it is reallocated in a productivity enhancing manner–can have positive impacts on output and employment as noted above.

We find, third, that mean job creation is higher during the Great Moderation than any other sub-period, while job destruction is higher during the Great Recession than other sub-periods. Interestingly, the mean of net job creation is negative during both the Great Moderation and the Great Recession though it is significantly more negative during the Great Recession. (Davis and Haltiwanger, 1992) note that typically job destruction is high during recessions while job creation is less responsive causing recessions to be episodes

Contessi, DiCecio, and Francis (2015) for more detailed statistics and analysis of credit flows in general.

[7] Hsieh and Klenow (2017) argue that reallocation is not a significant feature in productivity growth. As discussed in comments by John Haltiwanger (available here: http://econweb.umd.edu/~haltiwan/Comments_on_Hsieh_Klenow_Aug_22_2017.pdf), they neglect important reallocation frictions which may be impacting business dynamism (for instance see Decker, Haltiwanger, Jarmin, and Miranda (2017)). Our data uncovers changes in reallocation in credit and labor markets over the last 40 years but we do not explicitly link these changes to frictions in this paper.

of increased reallocation which in some cases can be productivity enhancing. During the Great Recession, job reallocation was much lower than usual because job creation declined significantly. The decline in reallocation likely exacerbated the decline in output through its impact on productivity (see Foster, Grim, and Haltiwanger 2014).

Typically during recessions, credit creation slows and destruction sharply rises, particularly for commercial lending. Figure 1 displays quarterly net credit flows (credit creation less credit destruction) of commercial and industrial lending following the trough of the last four recessions. We focus on commercial & industrial lending since it is the most directly related to job creation. However, there are indirect effects on job creation through credit destruction of real estate lending and loans to individuals. The trough of the recession is dated using NBER dates and depicted as zero; movement in net credit following the trough is graphed for eight quarters. We also graph the average of the past four recessions which is strongly influenced by the 2007-09 recession. Each of the past three recessions–1990, 2001, and 2007– display a similar pattern with net credit continuing to contract even as GDP and other indicators rise. Net credit flows rebounded much faster following the recession in 1981 and displays a distinctly different pattern. The three more recent recessions follow the same pattern although the 2007-09 recession has the largest and most prolonged decline in credit. Figure 2 considers a separate measures credit flows. The top row depicts credit creation (left graph) and credit destruction (right graph). We find that during the Great Recession, credit creation continued to fall after the NBER dated trough (at zero, where credit creation is set to its mean of 3 percent during the past three recessions) unlike its behavior during any of the previous three recessions though its recovery looked similar to the post-1990 recession recovery. Credit destruction, similarly, was significantly larger in the first quarter following the trough of the 2007-09 recession and in that sense much different than any of the three previous recessions. In the bottom row, the left graph shows a measure of credit reallocation which is the sum of credit creation and destruction. The movements in reallocation for the 2007-09 recessions looked somewhat similar to the 1990 and 2001 recessions initially, but then displayed a long and persistent decrease that has yet to be reversed. The right graph in the bottom row provides another measure of credit reallocation–excess reallocation–which is the sum of creation and destruction less the absolute value of the net. Excess reallocation declined significantly through the trough of the 2007-09 recession and then recovered after approximately three quarters. (Contessi, DiCecio, and Francis, 2015) note that excess reallocation tends to rise during recessions although the Great Recession was an exception. Foster, Grim, and Haltiwanger (2014) find evidence that reallocation of input and output across producers declined during the Great Recession. They suggest that credit market distortions, which are amplified during financial crises, can influence reallocation more than productivity and other considerations. In these cases, reallocation may not be productivity enhancing depending on the initial allocation of credit and the distribution of productivity.

The reallocation of credit we focus on is reallocation across *banks* not between firms. This type of reallocation occurs because a bank changes its lending portfolio or due to bank entry or exit. It affects liquidity in the lending market and therefore firms' access to credit, particularly for firms in longer term relationships with banks. Although reallocation of credit across banks does not strictly translate into the reallocation of credit across firms, if we focus on smaller firms, for whom bank credit is a significant portion of their financing, or larger firms in significant banking relationships (particularly during periods of high risk aversion by lenders or opaque balance sheets of borrowers), these firms are typically reliant on their relationship with a given bank, see for instance Petersen and Rajan (1994). In this sense, credit reallocation across banks can provide some evidence for reallocation across firms and changes in access to credit. Herrera, Kolar, and Minetti (2014) and Contessi, DiCecio, and Francis (2015) provide detailed analysis about credit reallocation across firms as well as across banks and discuss the mechanisms by which inter-firm and inter-bank reallocation impacts aggregate outcomes at business cycle frequencies.

Net job flows typically demonstrate a pro-cyclical pattern with slower job creation during recessions and rising job destruction. During the Great Recession, job destruction increased by more than during previous recessions (percentage wise) and was a much larger contributor to the increase in unemployment than during previous recessions. The job creation rate was also depressed for much longer than usual, which reduced the post-recession job finding rate more significantly than following previous recessions. Figure 3 considers net job creation in manufacturing following the trough of the last four recessions. It is clear from the figure, that net job growth after the trough of the Great Recession displays a similar pattern to past recessions particularly compared with the 2001 recession. But since the decline in net creation was significantly larger, there was much more recovery required. At the trough, unemployment was still rising and full recovery in net job creation to pre-recession levels did not occur until approximately four quarters after the NBER dated trough of the 2007-09 recession.[8] We have used job flows in manufacturing as our focus in part because of the availability of historical data. The trends we observe in manufacturing creation–on a cyclical basis–were mirrored in other industries. We note however that manufacturing employs only 8.5 percent of the labor force in 2017 a large decline from the early 1970s when it employed 24 percent of the workforce.

Figure 4 provides measures of a set of aggregate variables for the last four recessions: total factor productivity, average labor productivity, real GDP, and the unemployment rate. TFP is shown in percentage change terms, while the log of average labour productivity is smoothed using a Baxter King filter, and log real GDP is detrended. In these figures, total factor and labor productivity initially recovered strongly following the trough of the Great Recession but declined following the initial recovery (in a pattern that is reflected by previous recessions as well), while real GDP remained below trend and the unemployment rate, which increased to near historic rates, returned to its pre-Recession level. These features of the Great Recession

[8] Fort, Haltiwanger, Jarmin, and Miranda (2013) investigate worker and job flow patterns across firms of different sizes and ages and find that job flow patterns exhibit significant heterogeneity across firm size and age. In this paper, we ignore differential patterns of job flows across firms to focus on aggregate employment and unemployment.

are consistent with a number of explanations, not all of which relate to credit availability. In this paper, we focus on the relationships among credit, unemployment, and productivity and the impact of price and credit frictions on these macro-aggregates[9].

4 The model economy

We use a New Keynesian general equilibrium model augmented with a banking sector characterized by search frictions and heterogeneous firms to study the relationship among credit, hiring, and productivity. We focus on the interplay between a decentralized market for credit and frictions in the labor market that generate the slow adjustment in wages and employment in this economy.

Our model economy is populated by households, banks, firms, and a central bank. The household supplies differentiated labor to firms, holds cash and bank deposits, and purchases final output in the goods market. Firms seek financing, hire labor financed by bank loans, and produce output. Banks accept deposits, sometimes hold reserves with the central bank, and finance the wage bill of firms.[10] Three features of the model are of special relevance. First, we assume households cannot lend directly to firms. While this type of market segmentation is taken as exogenous, it could be motivated by assuming informational asymmetries under which households do not have access to technology to monitor firms while banks do. This asymmetry also forces firms to make up-front payments to workers to secure labor. Second, lending activity involving firms and banks occurs in a decentralized market characterized by random matching. Third, we assume all payment flows must be settled at the end of each period. At the beginning of each period, financial shocks are realized and households deposit funds with a bank. The amount of deposits is sensitive to features in the model and responsive to financial shocks since these impact the interest paid on deposits too. Firms are subject to aggregate financial shocks and idiosyncratic productivity shocks. Both of these shocks determine whether it is profitable for a firm to operate and, if it is, at what scale. Prior to production, unmatched firms must seek lenders. Similarly, unmatched banks search for borrowers. After the loan market closes, matched firms and their workers produce and households consume, while unmatched banks deposit their funds with the central bank and receive an interest rate matching the interest rate on deposits. After markets close all net payment flows are settled. Therefore loans are not risky and there is no possibility of default. At the end of the period, banks receive repayment from firms and the bank transfers all its profits to the representative household. We detail each component of the economy below.

[9]See Kudlyak and Sánchez (2017) for a discussion about whether relatively larger or smaller firms were more adversely affected by shocks during this period and consequently to what extent credit availability could play a major role in explaining unemployment.

[10]Banks hold reserves with the central bank when they cannot find a project to fund. The process is explained below.

4.1 Firms

The production side of the model is characterized by a two-sector structure that distinguishes between intermediate and final good producers as in Walsh (2005). Firms in the intermediate good sector must have a credit relationship with a bank before production can occur. Only the subset of intermediate good producers obtaining funding will hire workers and produce. The market for intermediate goods is competitive. Each producing firm in the intermediate good sector hires a continuum of workers that includes each type of labor service offered by the household.

Firms in the final good sector purchase the intermediate good and costlessly transform it into a continuum of differentiated final goods sold to the household in a market characterized by monopolistic competition. We assume final goods firms face Calvo pricing restrictions.[11]

4.1.1 Final good producers

We assume there is a continuum of monopolistically competitive firms indexed by j, each producing a differentiated final good. All firms in the final good sector have access to the following technology:

$$Y_t^f(j) = X(j) \tag{1}$$

where $X_t(j)$ is the quantity of the single intermediate good used to produce the final good variety j. Final good producers purchase $X(j)$ from intermediate good producers in a competitive market at the common price P_t^I and sell their output directly to households as a differentiated final good. Each final good producer faces the following demand schedule obtained from the household decision problem

$$Y_t^f(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} C_t \tag{2}$$

where C_t is the aggregate demand for final or consumption goods.

Price setting Prices for final goods are sticky as in Calvo (1983). Let $1-\theta_p$ be the probability that a firm adjusts its price each period. The nominal total cost for a final good producer of variety j is $TC_t^n(j) = P_t^I X(j)$ with nominal marginal cost $MC_t^n(j) = P_t^I$. As usual, by symmetry, all intermediate good producers who set prices in period t will choose the same price, denoted by P_t^*, since they face an identical problem given by

$$\max_{P_t^*} E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} \left\{ P_t^* Y_{t+k|t}^f - TC_t^n\left(Y_{t+k|t}^f\right)\right\} \tag{3}$$

[11] The separation between final and intermediate good sectors simplifies the difficulty associated with having a producing firm set its output price and bargain with a bank simultaneously.

s.t

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\epsilon_p} C_{t+k} \quad \text{for } k = 0, 1, 2, ..., \tag{4}$$

where $\Delta_{t,t+k}$ is the household stochastic discount factor and $Y_{t+k|t}^f$ denotes the demand faced at $t+k$ for a firm that last reset its price in period t, which is consistent with the households' optimality condition with respect to each final good variety.

4.1.2 Intermediate good producers

We assume intermediate good producers must search for external funding in order to produce. A firm with financing operates a production technology and produces a homogeneous intermediate good indexed by z in a perfectly competitive market. Nominal total costs for an intermediate goods firm includes total labor cost, $R_t^l (j, \omega_{z,t}) W_t N_t (\omega_{z,t})$, plus the fixed cost of production, $P_t^I x^f$, where $R_t^l (j, \omega_{z,t})$ is the gross loan interest rate negotiated bilaterally between bank 'j'and firm 'z'.

In this subsection, we first describe the technology, labor demand and nominal profits for a firm that has obtained financing. In the next section, we describe the loan market and the decisions each intermediate goods producer must take when searching for external funds or after obtaining a bank loan.

Technology and labor demand If an intermediate goods producer is matched with a bank, it is endowed with the following technology:

$$y_t (\omega_{z,t}) = \xi^{pf} A_t \omega_{z,t} N_t (\omega_{z,t})^{\alpha} \tag{5}$$

where ξ^{pf} is a scale technology parameter, A_t is the aggregate productivity level, $\omega_{z,t}$ is a firm-specific idiosyncratic productivity level drawn from a uniform distribution function $G(\omega)$ with support $[\underline{\omega}\ \bar{\omega}]$, and $N_t (\omega_{z,t})$ is the firm's employment index given by

$$N_t (\omega_{z,t}) = \left(\int_0^1 N_t (i, \omega_{z,t})^{\frac{\varepsilon_w - 1}{\varepsilon_w}} di \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}} \tag{6}$$

where $N_t (i, \omega_z)$ is the demand for labor type i by firm z. Cost minimization, taking wages as given, implies the following demand for labor type i:

$$N_t (i, \omega_{z,t}) = \left(\frac{W_t (i)}{W_t} \right)^{-\varepsilon_w} N_t (\omega_{z,t}) \quad \text{for all } i \tag{7}$$

The aggregate demand for labor type i is obtained by aggregating $N_t(i, \omega_z)$ across all producing firms

$$\begin{aligned} N_t(i) &= \int_z N_t(i, \omega_z)\, dz \\ &= \left(\frac{W_t(i)}{W_t}\right)^{-\varepsilon_w} \int_z N_t(\omega_z)\, dz \end{aligned}$$

where $\int_z N_t(\omega_z)\, dz$ denotes the aggregate labor index of all producing intermediate goods firms during period t and W_t denotes the aggregate wage index.[12] Wages must be paid in advance of production to the household and can only be funded by external bank finance. The nominal loan each intermediate goods producer must obtain is given by their wage bill during period t :

$$L_t(\omega_{z,t}) = \int_0^1 W_t(i)\, N_t(i, \omega_{z,t})\, di = W_t N_t(\omega_{z,t}) \tag{8}$$

We assume loans are paid back to the bank with a gross interest rate $R_t^l(j, \omega_{z,t})$ at the end of the period and no default occurs. End of the period real profits, for an intermediate good producer with funding and idiosyncratic productivity, $\omega_{z,t}$, is:

$$\pi_t^I(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t^l(j, \omega_{z,t})\, l_t(j, \omega_{z,t}) \tag{9}$$

Where the loan principle expressed in real terms is the real wage bill of the firm and it is given by $l_t(j, \omega_{z,t}) = w_t N_t(\omega_{z,t})$. The loan contract requires the repayment of the total debt with the bank, including interest, $R_t^l(j, \omega_{z,t})\, l_t(j, \omega_{z,t})$ within the same period.

4.2 A decentralized loan market

We assume the process of finding a credit partner is costly in terms of time and resources. Intermediate good producers and banks face search and matching frictions that prevent instantaneous trading in the loan market, implying not all market participants will end up matched at a given point in time. Upon a successful match, bilateral Nash bargaining between the parties determines the firm's employment level and the way the match surplus is shared. We allow for both exogenous and endogenous destruction of credit matches, and a matching technology that determines the aggregate flow of new credit relationships over time as a function of the relative number of lenders and borrowers searching for credit partners.

We assume a continuum of banks and firms with the number of banks seeking borrowers varying endoge-

[12]Note that z indexes the mass of active intermediate good producers in the economy. The upper limit of z measures the total number of producing firms which is determined in equilibrium. As explained below, in equilibrium, the total number of intermediate good producers is less than one whenever there are search frictions in the loan market.

nously and being determined by a free entry condition to the loan market. We assume that banks have a constant returns to scale technology for managing loans so that we can treat each loan as a separate match between a bank and a firm. Each intermediate good producer is endowed with one project and is either searching for funding or involved in an ongoing credit contract with a bank. If a firm is matched with a bank, the bank extends the necessary funds to allow the firm to hire workers and produce.

4.2.1 The matching process

Firms searching for external funds, f_t, are matched to banks seeking for borrowers, b_t^u, according to the following constant returns to scale matching function

$$m_t = \mu f_t^{\nu} (b_t^u)^{1-\nu} \tag{10}$$

The function m_t determines the flow of new credit contracts during date t; μ is a scale parameter that measures the productivity of the matching function and $0 < \nu < 1$ is the elasticity of the match arrival with respect to the mass of searching firms.

Matching rates The variable $\tau_t = f_t / b_t^u$ is the measure of credit market tightness. The probability that an intermediate good producer with an unfunded project is matched with a bank seeking to lend at date t is denoted by p_t^f and is given by

$$p_t^f = \mu \tau_t^{\nu - 1} \tag{11}$$

Similarly, the probability that any bank seeking borrowers is matched with an unfunded entrepreneur at time t is denoted by p_t^b and is given by

$$p_t^b = \mu \tau_t^{\nu} \tag{12}$$

Since $\tau_t = p_t^b / p_t^f$, a rise in τ_t implies it is easier for a bank to find a borrower relative to a firm finding a lender and so corresponds to a tighter credit market. A higher τ_t reduces the expected time a bank must search for a credit partner, lowering the bank's expected pecuniary search costs. At any date the number of newly matched banks must equal the number of newly matched firms, or $p_t^b b_t^u = p_t^f f_t$.

Separations and the evolution of loan contracts Credit relationships may end exogenously with probability δ_t whose process is explained below. Contractual parties engaged in a credit relationship that survives this exogenous separation hazard can also decide to dissolve the contract depending on the realization of the productivity of the firm's project. The decision to endogenously dissolve a credit relationship is characterized by an optimal reservation policy with respect to $\omega_{z,t}$ and denoted by $\tilde{\omega}_t$. If the realization of $\omega_{z,t}$ is above the firm specific productivity cut-off, both parties agree to continue the credit relationship, allowing the entrepreneur to produce conditional on surviving the exogenous separation hazard. On the contrary, if the realization of $\omega_{z,t}$ is below $\tilde{\omega}_t$, both parties choose to dissolve the credit relationship. The probability of endogenous termination of a credit match is $\gamma_t(\tilde{\omega}_t) \equiv prob(\omega_{z,t} \leq \tilde{\omega}_t) = G(\tilde{\omega}_t)$ while the overall separation rate is $\delta_t + (1-\delta_t)\gamma_t(\tilde{\omega}_t)$. Existence and uniqueness of the optimal reservation policy $\tilde{\omega}_t$ are shown in appendix C.

Let f_{t-1}^m be the measure of intermediate good producers that enter period t matched with a bank. Of those, $(1-\delta_t) f_{t-1}^m$ firms survive the exogenous hazard and a fraction $\gamma_t(\tilde{\omega})$ of the survivals receive idiosyncratic productivity shocks that are less than $\tilde{\omega}_t$ and so do not produce. The number of intermediate good producers that actually produce in period t, therefore, is $(1-\delta_t)(1-\gamma_t(\tilde{\omega}_t))f_{t-1}^m$. The number of firms in a credit relationship at the end of period t, denoted by f_t^m, is given by the number of firms producing during time t plus all the new matches formed at time t. Then, the evolution of f_t^m is expressed as

$$f_t^m = \varphi_t(\tilde{\omega}_t) f_{t-1}^m + m_t \tag{13}$$

where $\varphi_t(\tilde{\omega}_t)$ is the overall continuation rate of a credit relationship defined to be:

$$\varphi_t(\tilde{\omega}_t) = (1-\delta_t)(1-\gamma_t(\tilde{\omega}_t)) \tag{14}$$

and $1-\varphi_t(\tilde{\omega}_t) = \delta_t + (1-\delta_t)\gamma_t(\tilde{\omega}_t)$ denotes the overall separation rate.

We normalize the total number of potential intermediate good producers in every period to one and assume that if a credit relationship is exogenously dissolved at time t, both parties start searching immediately during the period. If the credit relationship survives the exogenous separation hazard but then endogenously dissolves, both parties must wait until the next period to start searching for a credit partner again. This assumption implies that the number of firms seeking finance during period t, which we have denoted by f_t, is equal to the number of searching firms at the beginning of time t, $(1-f_{t-1}^m)$ plus the number of firms that started the period matched with a bank but were exogenously separated $(\delta_t f_{t-1}^m)$. Therefore,

$$f_t = 1-(1-\delta_t) f_{t-1}^m. \tag{15}$$

Notice that there are still some firms that have been endogenously separated but cannot search in period t. These firms are unmatched but waiting to search again next period.

Gross credit flows Our timing assumption implies that the fraction $p_t^f \delta_t f_{t-1}^m$ of matched intermediate good producers that were exogenously separated during time t, are able to find a new credit relationship within the same period of time. Credit creation, CC_t, is defined as the number of newly created credit relationships at the end of time t net of the number of exogenous credit separations that are successfully re-matched in a given period:

$$CC_t = m_t - p_t^f \delta_t f_{t-1}^m. \quad (16)$$

The credit creation rate, cc_t is

$$cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f \delta_t. \quad (17)$$

Credit destruction, CD_t, is defined as the total number of credit separations at the end of time t, $(1 - \varphi_t(\tilde{\omega}_t)) f_{t-1}^m$ net of the number of exogenous credit separations that are successfully re-matched in a given period:

$$CD_t = (1 - \varphi_t(\tilde{\omega}_t)) f_{t-1}^m - p_t^f \delta_t f_{t-1}^m. \quad (18)$$

The credit destruction rate, cd_t, is

$$cd_t = (1 - \varphi_t(\tilde{\omega}_t)) - p_t^f \delta_t. \quad (19)$$

The implied credit reallocation rate is defined by

$$cr_t = cc_t + cd_t, \quad (20)$$

and net credit growth rate is

$$cg_t = cc_t - cd_t. \quad (21)$$

4.2.2 Intermediate good producers and the loan market

As long as the credit contract prevails, the firm receives sufficient external funds to pay workers in advance of production each period. After selling its output to the final goods producers, the firm repays its debt with the bank and transfers all remaining profits to the household.[13]

Value functions If the intermediate good producer obtains financing its instantaneous real profit flow is $\pi_t^I(\omega_{z,t})$. Profits depend on the status of the intermediate good producer, that is, whether the firm is searching for external funds or it is producing. A firm searching for external funds obtains zero real profits since we assume there are no extra search costs when a producer is searching for funding. Under these assumptions, the firm's decision-making is characterized by two value functions: The value of being matched with a bank and able to produce at date t, denoted by $V_t^{FP}(\omega_{z,t})$ and the value of searching for external funds at date t, denoted by V_t^{FN}, both measured in terms of current consumption of the final good[14] . $V_t^{FP}(\omega_{z,t})$ is given by

[13] As in Fiore and Tristani (2013), we abstract from the endogenous evolution of net worth by assuming firms do not accumulate internal funds after repaying their debt.

[14] See appendix B for derivations.

$$V_t^{FP}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{(1-\varphi_t(\tilde{\omega}_{t+1}))V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right\}$$

where $\Delta_{t,t+1} = \frac{\beta\lambda_{t+1}}{\lambda_t}$ is the household stochastic discount factor. The value of producing is the flow value of current real profits plus the expected continuation value. At the end of the period, the credit relationship is dissolved with probability $1-\varphi_t(\tilde{\omega}_{t+1})$ and the firm must seek new financing. An intermediate good producer with funding at date t will continue producing at time $t+1$ if it survives both separation hazards which occurs with probability $\varphi_t(\tilde{\omega}_{t+1})$. In the latter case, only those firms receiving an idiosyncratic productivity realization $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$ will remain matched and produce during next period. Firms with $\omega_{z,t+1} < \tilde{\omega}_{t+1}$ endogenously separate from their bank and obtain V_{t+1}^{FN}.

The value of searching for external funds $\left(V_t^{FN}\right)$ for a firm at date t expressed in terms of current consumption is

$$V_t^{FN} = \mathrm{E}_t\Delta_{t,t+1}\left\{p_t^f\left[(1-\varphi_t(\tilde{\omega}_{t+1}))V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right] + \left(1-p_t^f\right)V_{t+1}^{FN}\right\} \tag{22}$$

We assume matches made in period t do not produce until $t+1$. Therefore a searching firm that is matched with a bank during period t will produce in period $t+1$ if it survives next period's separation hazards which occurs with overall probability $\varphi_t(\tilde{\omega}_{t+1})$. With probability $(1-p_t^f)$, the firm does not match and must continue searching for external funds during next period's loan market. The net surplus to a firm is defined as $V_t^{FS}(\omega_{z,t}) = V_t^{FP}(\omega_{z,t}) - V_t^{FN}$.

4.2.3 Banks and the loan market

Banks collect deposits from households and invest them in loans with firms. The deposit market is assumed to be a centralized competitive market. Due to the decentralized nature of the loan market, some banks end the period without any loans in their portfolio. In this case, the bank deposits its funds with the central bank as excess reserves and receives an interest rate matching the interest rate on deposits, leaving the bank with negative profits due to search costs. All uncertainty is revealed before loans are extended: loans are made and paid back during the same period. At the end of the period, the bank transfers all its profits to the representative household.

A bank can only form a credit relationship with one firm and vice versa until separation occurs. Bank

$j's$ balance sheet expressed in real terms is

$$\boldsymbol{\chi}_t(j)\, l_t(j, \omega_{z,t}) + (1 - \boldsymbol{\chi}_t(j)) \frac{ER_t(j)}{P_t} = \frac{D_t(j)}{P_t} \tag{23}$$

where $\boldsymbol{\chi}_t(j)$ is an indicator function taking the value of 1 if bank j extends a loan $l_t(j, \omega_{z,t})$ to a firm whose idiosyncratic productivity $\omega_{z,t}$ exceeds a cut-off level and 0 otherwise, $ER_t(j)$ represents nominal excess reserves held with the central bank in case $\boldsymbol{\chi}_t(j) = 0$ and $D_t(j)$ are household deposits. In equilibrium, there will be a measure of banks with positive loans and a measure of banks with excess reserves. Notice that when the bank extends a loan ($\boldsymbol{\chi}_t(j) = 1$) the bank balance sheet implies that the bank lends out all of its resources $l_t(j, \omega_{z,t}) = \frac{D_t(j)}{P_t}$ which means there is no credit rationing. This is due to the fact there is no default risk.

Bank Profits A bank searching for a borrower will incur a search cost $\frac{P_t^I}{P_t}\kappa$ measured in units of the final good and earn zero profits. The current flow of profits of a bank with household deposits $D_t(j)$ can be written as

$$\pi_t^b(j) = \boldsymbol{\chi}_t(j)\, R_t^l(j, \omega_{z,t})\, l_t(j, \omega_{z,t}) + (1 - \boldsymbol{\chi}_t(j)) \left(R_t^r \frac{ER_t(j)}{P_t} - \frac{\kappa}{\mu_t^p} \right) - R_t^d \frac{D_t(j)}{P_t} \tag{24}$$

where $R_t^l(j, \omega_{z,t})$ is the bilateral bargained gross loan rate between bank j and firm z, R_t^r is the gross interest rate on excess reserves and $R_t^d = 1 + i_t$ is the gross deposit rate. The problem of a bank is to maximize its current profits subject to its balance sheet. Optimality with respect to deposits requires that every period $\left(R_t^r - R_t^d\right) D_t(j) = 0$. Since household deposits are always positive in equilibrium, the bank will choose to collect deposits until the gross interest rate on excess reserves is equal to the gross interest rate on deposits, that is $R_t^r = R_t^d = R_t = 1 + i_t$. Substituting the bank's balance sheet and the optimality conditions with respect to $D_t(j)$ into the profit function yields

$$\pi_t^b(j) = \begin{cases} \pi_t^b(j, \omega_{z,t}) = \left(R_t^l(j, \omega_{z,t}) - R_t\right) l_t(j, \omega_{z,t}) & \text{if extends a loan to firm } \omega_{z,t} \\ -\frac{\kappa}{\mu_t^p} & \text{otherwise} \end{cases} \tag{25}$$

Where R_t reflects the opportunity cost of lending. The determination of $R_t^l(j, \omega_{z,t})$ is explained below as the result of Nash bargaining between the bank and the intermediate good producer. The loan size is given by the labor costs of firm z, that is $l_t(j, \omega_{z,t}) = w_t N_t(\omega_{z,t})$.

Bank Value functions Under the assumptions detailed above, the problem of a bank can be characterized by two value functions: The value of lending to a firm at date t, denoted by $V_t^{BL}(\omega_{z,t})$ and the value of searching for a potential borrower at date t, denoted by V_t^{BN}. Both value functions are measured in terms of current consumption of the final good and are given by

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{(1-\varphi_{t+1}(\tilde{\omega}_{t+1}))V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right\}$$

and

$$V_t^{BN} = -\frac{\kappa}{\mu_t^p} + \mathrm{E}_t\Delta_{t,t+1}\left\{\begin{array}{c} p_t^b\left[(1-\varphi_{t+1}(\tilde{\omega}_{t+1}))V_{t+1}^{BN} + \varphi_{t+1}(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right] \\ +\left(1-p_t^b\right)V_{t+1}^{BN}\end{array}\right\} \tag{26}$$

The value of extending a loan is the current value of real profits plus the expected continuation value. A bank that extends a loan to a firm with idiosyncratic productivity $\omega_{z,t}$ at date t will continue financing the same firm at time $t+1$ with probability $\varphi_t(\tilde{\omega}_{t+1})$. The credit relationship is severed at time $t+1$ with probability $\delta_t + 1 - \varphi_t(\tilde{\omega}_{t+1})$, in which case the bank obtains a future value of V_{t+1}^{BN}. The value of searching for a borrower at date t is given by the flow value of the search costs plus the continuation value. A searching bank faces a probability $1-p_t^b$ of not being matched during time t, obtaining a future value of V_{t+1}^{BN} and a probability p_t^b of being matched. If a searching bank ends up being matched with a firm at time t, then at the beginning of period $t+1$ it will face a probability of separation before extending the loan.

Free entry condition In equilibrium, free entry of banks into the loan market ensures that $V_t^{BN} = 0$ for all t. Using this in V_t^{BN}, the free entry condition can be written as

$$\frac{\kappa}{\mu_t^p p_t^b} = \mathrm{E}_t\Delta_{t,t+1}\left\{\varphi_{t+1}(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{zt+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right\} \tag{27}$$

Banks will enter the loan market until the expected cost of finding a borrower $\frac{\kappa}{\mu_t^p p_t^b}$ is equal to the expected benefit of extending a loan to a firm with idiosyncratic productivity $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$. As banks enter the market, the probability a searching bank finds a borrower will fall, up to the point where equality of the above condition is restored. Note that free entry of banks into the loan market modifies the value function $V_t^{BL}(\omega_{z,t})$ as follows

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{\varphi_{t+1}(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{BL}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right\} \tag{28}$$

The net surplus for bank extending a loan to a firm with productivity $\omega_{z,t}$ is

$$V_t^{BS}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \frac{\kappa}{\mu_t^p p_t^b} \tag{29}$$

4.2.4 Employment and the loan contract: Generalized Nash bargaining

At any point in time, a matched firm and bank that survive the exogenous and endogenous separation hazards engage in bilateral bargaining over the interest rate and loan size to split the joint surplus resulting from the match.[15] This joint surplus is defined as $V_t^{JS}(\omega_{z,t}) = V_t^{FS}(\omega_{z,t}) + V_t^{BS}(\omega_{z,t})$ and it is given by

$$V_t^{JS}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - w_t R_t N_t(\omega_{z,t}) + \left(1 - p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi_{t+1}(\tilde{\omega}_{t+1}) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}(\tilde{\omega}_{t+1})}. \tag{30}$$

Let $\overline{\eta}$ be the firm's share of the joint surplus. and $1 - \overline{\eta}$ the banks'. The Nash bargaining problem for an active credit relationship is

$$\max_{\left\{R_t^l(j,\omega_{z,t}), l_t(\omega_{z,t})\right\}} \left(V_t^{FS}(\omega_{z,t})\right)^{\overline{\eta}} \left(V_t^{BS}(\omega_{z,t})\right)^{1-\overline{\eta}} \tag{31}$$

where $V_t^{FS}(\omega_{z,t})$ and $V_t^{BS}(\omega_{z,t})$ are defined above. The first order conditions imply the following optimal sharing rule:

$$\overline{\eta} V_t^{BS}(\omega_{z,t}) = (1 - \overline{\eta}) V_t^{FS}(\omega_{z,t}) \tag{32}$$

and an employment condition that sets the marginal product of labor equal to a markup μ_t^p over the marginal cost of labor inclusive of the bank's opportunity cost when extending a loan to an intermediate good producer:

$$\alpha \xi^{pf} A_t \omega_{z,t} N_t^*(\omega_{z,t})^{\alpha-1} = \mu_t^p w_t R_t \tag{33}$$

Notice that $w_t R_t$ is expressed in terms of the final good and it has to be transformed back in terms of the intermediate good as it is the marginal product of labor. We refer to $\mu_t^p w_t R_t$ as the Nash bargained marginal cost of labor expressed in terms of the intermediate good to differentiate it with the actual real marginal cost of labor expressed in terms of the intermediate good of a producing firm which depends on the match-specific loan interest rate and it is given by $\mu_t^p w_t R_t^l(j, \omega_{z,t})$.

The loan interest rate derived here simply ensures the joint surplus generated by a credit relationship is divided optimally between the firm and the bank, with the relevant interest rate capturing the cost channel being R_t, the bank's opportunity cost of funds. As mentioned in the introduction, even though firms face different interest rates on bank loans, since the loan rate depends on the firms' idiosyncratic productivity realization, the interest cost relevant for labor demand is the same for all firms. In this sense, the Nash bargained loan rates, $R_t^l(j, \omega_{z,t})$, are non-allocative. The allocative role of prices in the credit market is accomplished through credit market tightness, τ_t.

[15] See Appendix B for derivations.

The optimal loan principal negotiated between credit partners is

$$l_t^*(j,\omega_{z,t}) = \left(\frac{\alpha\xi^{pf}A_t\omega_{z,t}}{\mu_t^p w_t^\alpha R_t}\right)^{\frac{1}{1-\alpha}} \tag{34}$$

with an equilibrium loan interest rate

$$R_t^l\left(j,\omega_{z,t}\right) = \frac{1}{l_t^*(j,\omega_{z,t})}\left((1-\overline{\eta})\left(\frac{y_t^*(\omega_{z,t})-x^f}{\mu_t^p}\right)+\overline{\eta}\left(R_t w_t N_t^*(\omega_{z,t})-\frac{\kappa p_t^f}{\mu_t^p p_t^b}\right)\right) \tag{35}$$

The loan interest rate divides the joint surplus of a credit match in such a manner that a fraction $1-\overline{\eta}$ of the firm profits relative to the loan principal is obtained by the bank while a fraction $\overline{\eta}$ of the bank's opportunity cost of lending net of search costs and relative to the loan principal is obtained by the firm.

The above conditions imply that firm z with $\omega_{z,t} \geqslant \tilde{\omega}_t$ will produce $y_t^*(\omega_{z,t})$ units of the intermediate good and employ $N_t^*\left(\omega_{z,t}\right)$ workers, given by:

$$y_t^*(\omega_{z,t}) = \left(\xi^{pf}A_t\omega_{z,t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu_t^p w_t R_t}\right)^{\frac{\alpha}{1-\alpha}} \tag{36}$$

$$N_t^*\left(\omega_{z,t}\right) = \left(\frac{\alpha\xi^{pf}A_t\omega_{z,t}}{\mu_t^p w_t R_t}\right)^{\frac{1}{1-\alpha}} \tag{37}$$

Notice that the credit contract implies that in equilibrium, there will be a distribution in the size of firms such that more productive firms will be able to obtain a greater amount of lending, hire more workers and become larger firms, conditional on surviving.

4.2.5 The optimal reservation policy: Endogenous separations

The joint surplus of a credit relationship can be written explicitly as a function of the idiosyncratic productivity shock $\omega_{z,t}$ in order to facilitate the characterization of the loan market equilibrium as follows

$$V_t^{JS}(\omega_{z,t}) = \frac{1}{\mu_t^p}\left((1-\alpha)\left(\xi^{pf}A_t\omega_{z,t}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu_t^p w_t R_t}\right)^{\frac{\alpha}{1-\alpha}} - x^f + \frac{\kappa}{p_t^b}\left(\frac{1-\overline{\eta}p_t^f}{1-\overline{\eta}}\right)\right) \tag{38}$$

The optimal reservation policy with respect to the idiosyncratic productivity shock implies that

$$\begin{aligned} &if \quad \omega_{i,t} \leq \tilde{\omega}_t \quad \Longrightarrow V_t^{JS}(\omega_{i,t}) \leq 0 \\ &if \quad \omega_{i,t} > \tilde{\omega}_t \quad \Longrightarrow V_t^{JS}(\omega_{i,t}) > 0. \end{aligned}$$

Since the joint surplus is increasing in the firm's idiosyncratic productivity, there is an unique threshold level

$\tilde{\omega}_t$ defined by

$$V_t^{JS}(\tilde{\omega}_t) = 0 \tag{39}$$

such that the joint surplus is negative for any firm facing an idiosyncratic productivity $\omega_{i,t} < \tilde{\omega}_t$. The optimal threshold level $\tilde{\omega}_t$ is

$$\widetilde{\omega}_t = \left(\frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(\mu_t^p w_t R_t)^\alpha}{\xi p^f A_t} \right) \left[x^f - \left(\frac{1-\overline{\eta} p_t^f}{1-\overline{\eta}} \right) \frac{\kappa}{p_t^b} \right]^{1-\alpha} \tag{40}$$

Since $\tilde{\omega}_t$ is independent of i, the cutoff value is the same for all firms and banks. Moreover, it is decreasing in aggregate productivity A_t so that a positive aggregate productivity shock means the number of credit matches that separate endogenously falls and more matched firms produce. The cutoff value is increasing in the Nash bargained marginal cost of labor $(\mu_t^p w_t R_t)$, and the firm's fixed cost of production (x^f).

The bank's opportunity cost of funds R_t influences the level of economic activity at both the extensive and intensive margins. A rise in R_t increases the threshold level of the idiosyncratic productivity, $\tilde{\omega}_t$, of firms that obtain a positive joint surplus. As a consequence, fewer firms are able to secure financing and produce. This is the extensive margin effect. Conditional on producing, firms equate the marginal product of labor to the Nash bargained marginal cost, $\mu_t^p w_t R_t$, so that an increase in R_t, ceteris paribus, reduces labor demand at each level of the real wage. This is the intensive margin effect. Both channels work to reduce aggregate output as R_t rises. In addition, credit market conditions reflected in p_t^f (the probability of a firm matching with a bank) and p_t^b (the probability of a bank matching with a firm) directly affect the extensive margin; a rise in τ_t (a credit tightening) increases $\tilde{\omega}_t$ and fewer firms obtain credit. Both interest costs measured by R and credit conditions measured by τ matter for employment and output.[16]

Finally, the evolution of credit market tightness is obtained by using the free entry condition, the Nash bargaining sharing rule, and the definition of the joint surplus of a credit relationship, and it is given by the following Euler equation for τ_t:

$$\frac{\kappa}{\mu_t^p \mu \tau_t^\varphi} - \mathrm{E}_t \Delta_{t,t+1} \varphi_{t+1}(\tilde{\omega}_{t+1}) \left(1 - \overline{\eta} \mu \tau_{t+1}^{\varphi-1}\right) \frac{\kappa}{\mu_{t+1}^p \mu \tau_{t+1}^\varphi} = (1-\overline{\eta}) \mathrm{E}_t \Delta_{t,t+1} \frac{1}{\mu_{t+1}^p} \left((1-\alpha) \frac{Y_{t+1}^I}{f_t^m} - \varphi_{t+1}(\tilde{\omega}_{t+1}) x^f \right) \tag{41}$$

Where Y_t^I denotes the aggregate production of intermediate goods defined below. The aggregate dynamics for τ_t are determined by the point where the average current cost of searching for productive borrowers is equal to the average expected benefit of extending a loan. The latter has two components: (i) The average expected output produced by all active intermediate firms net of the average fixed cost of production (right hand side of equation (41)) and (ii) The expected average savings in search costs in $t+1$ conditional on

[16] See Appendix B for derivations of the loan market equilibrium.

surviving the credit separation hazards (The second term of the left hand side of equation (41)).

4.3 Households

Each household has a continuum of members. Following Galí (2011), each household member is represented by the unit square and indexed by $(i, j) \in [0, 1]^2$. Where i denotes the type of labor service in which a given household member is specialized and j determines the dis-utility from work for each household member. The dis-utility from work is given by $\chi_t j^{\overline{\varphi}}$ if employed and zero otherwise with χ_t being an exogenous preference shifter. As is standard in the unemployment literature, we assume full risk sharing of consumption among household members (see for example, Andolfatto 1996). Utility from consumption is separable and logarithmic in a CES index of the quantities consumed of the different goods available. Given separability of preferences between consumption and dis-utility from work, full risk sharing implies $C_t\,(i, j) = C_t \quad \forall i, j$, where $C_t\,(i, j)$ is the consumption for a household member specialized in labor type i and having dis-utility of work $\chi_t j^{\overline{\varphi}}$.

Each household member has the following period utility function: $U(C_t, j) = \log C_t - \mathbf{1}_t(i, j)\chi_t j^{\overline{\varphi}}$, where $\mathbf{1}_t\,(i, j)$ is an indicator function taking the value of one if the corresponding household member is employed and zero otherwise. Aggregating across all household members yields the household period utility function denoted by $U\,(C_t, N_t\,(i)\,, \chi_t)$ and given by:

$$U\,(C_t, N_t\,(i)\,, \chi_t) = \log C_t - \chi_t \int_0^1 \frac{N_t\,(i)^{\overline{\varphi}+1}}{1+\overline{\varphi}} di \tag{42}$$

where $N_t\,(i)$ is the fraction of household members specialized in labor type i who are employed during the period. In other words, $N_t\,(i)$ is the employment rate or aggregate demand during period t among workers specialized in labor type i.

The household problem

We assume the household enters the period with money holdings given by M_{t-1} and deposits a fraction of its money holdings, denoted by D_t, in the bank. Each household receives a nominal lump-sum transfer T_t from the government. Employed household members are paid their labor income in advance. The household uses its labor income and money holdings, net of deposits, to buy a continuum of final goods subject to the CIA constraint and the sequence of budget constraints. The representative household is assumed to maximize:

$$E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \log C_t - \chi_t \int_0^1 \frac{N_t\,(i)^{\overline{\varphi}+1}}{1+\overline{\varphi}} di \right\} \tag{43}$$

subject to the following cash in advance constraint:

$$P_t C_t \leq M_{t-1} + T_t - D_t + \int\limits_0^1 W_t(i) N_t(i) \, di \tag{44}$$

and end of period money holdings given by:

$$M_t = (1 + i_t) D_t + T_t + \Pi_t^b + \Pi_t^I + \Pi_t^f + M_{t-1} - D_t + \int\limits_0^1 W_t(i) N_t(i) \, di - P_t C_t \tag{45}$$

where i_t is the nominal net interest rate on deposits, Π_t^b, Π_t^I, and Π_t^f are nominal profits transferred respectively by banks, intermediate and final good producers. When choosing C_t, the household takes as given the distribution of wages $\{W_t(i)\}_{\forall i}$ as well as employed household members for each labor type $\{N_t(i)\}_{\forall i}$. The household optimality condition with respect to consumption is then given by the standard Euler equation

$$\frac{1}{C_t} = \beta E_t \left\{ \left(\frac{1 + i_t}{1 + \pi_{t+1}} \right) \frac{1}{C_{t+1}} \right\} \tag{46}$$

where $\pi_t = \frac{P_t - P_{t-1}}{P_{t-1}}$ is the net inflation rate. In this case, the marginal utility of consumption is equal to: $\frac{1}{C_t} = \lambda_t + \mu_t$ where λ_t and μ_t are the multipliers associated with the budget constraint and the CIA constraint respectively. The stochastic discount factor is distorted by the nominal interest rate and it is given by $\Delta_{t,t+1} = \beta \left(\frac{1+i_t}{1+\imath_{t+1}} \frac{C_t}{C_{t+1}} \right)$.

Workers and wage setting Workers specialized in a given type of labor, reset their nominal wage with probability $1 - \theta_w$ each period. Following (Erceg, Henderson, and Levin, 2000), when re-optimizing wages during period t, workers choose a wage W_t^* in order to maximize household utility taking as given all aggregate variables. Household workers of type i face a sequence of labor demand schedules of the form:

$$N_t(i) = \left(\frac{W_t(i)}{W_t} \right)^{-\varepsilon_w} \int\limits_z N_t(\omega_z) \, dz \tag{47}$$

where W_t denotes the aggregate wage index given by $W_t = \left(\int\limits_0^1 W_t(i)^{1-\epsilon_w} di \right)^{\frac{1}{1-\epsilon_w}}$ and $\int\limits_z N_t(\omega_z) \, dz$ denotes aggregate labor demand across all *active* intermediate good producers indexed by z.

The wage setting optimization problem for the household workers specialized in labor type i is specified as

$$\max_{W_t^*} E_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \left\{ \log C_{t+k} - \chi_{t+k} \int\limits_0^1 \frac{N_{t+k|t}^{\overline{\varphi}+1}}{1 + \overline{\varphi}} dz \right\} \tag{48}$$

subject to

$$N_{t+k|t} = \left(\frac{W_t^*}{W_{t+k}}\right)^{-\varepsilon_w} \int\limits_z N_{t+k}(z)\, dz \tag{49}$$

and the CIA and budget constraints.[17]

Unemployment dynamics

Following Galí (2011), we use household welfare as the maximizing criterion, rather than individual utility, and take as given current aggregate labor market conditions. The worker indexed by (i, j) is then willing to work in period t if and only if the real wage is greater than or equal to the disutility of labor,$\chi_t j^{\overline{\varphi}}$, relative to the marginal value of income, $\lambda_t = \frac{1}{(1+i_t)C_t}$, that is:

$$\frac{W_t(i)}{P_t} \geq \frac{\chi_t j^{\overline{\varphi}}}{\lambda_t} = (1+i_t)C_t\chi_t j^{\overline{\varphi}} \tag{50}$$

Let $L_t(i)$ be the marginal supplier of type i labor. The marginal supplier of type i labor satisfies the above equation with equality, since she is indifferent between working or not working.The labor force or aggregate participation condition is obtained by integrating over all the marginal suppliers, $L_t = \int_0^1 L_t(i)di$. Then, the aggregate supply of labor is defined by

$$w_t = (1+i_t)\, C_t\chi_t\, (L_t)^{\overline{\varphi}} \tag{51}$$

where w_t denotes the average real wage of the economy. In the presence of wage rigidities, labor force dynamics are mostly driven by wealth effects, that is, by the inverse of the marginal value of income. The CIA constraint implies that the gross nominal interest rate acts as a consumption tax, affecting the marginal utility of consumption. Therefore, changes in C_t and i_t induce shifts in the labor supply.

The unemployment rate is defined as

$$U_t = 1 - \frac{N_t}{L_t} \tag{52}$$

with N_t being aggregate employment which corresponds to the following index:

$$N_t = \int\limits_z \int_0^1 N_t(i, \omega_{z,t})\, didz \tag{53}$$

where as explained below, $N_t(i, \omega_{z,t})$ is the demand for labor type i by the intermediate producer z, who is characterized by idiosyncratic productivity $\omega_{z,t}$.

[17] Notice that $N_{t+k|t}$ denotes the quantity demanded in period $t+k$ of a labor type whose wage was last reset in period t.

4.4 Government

Central bank budget constraint There are no government bonds in this economy but the central bank pays the same interest rate as the banks' deposit rate on excess reserves. Therefore, the central bank's budget constraint is given by:

$$i_t ER_t + RCB_t = M_t - M_{t-1} \tag{54}$$

where RCB_t denotes the central bank transfers to the treasury and M_t is the money supply. Aggregate excess reserves, ER_t, are obtained by integrating across the measure of banks not able to extend loans to intermediate good producers within the period, that is

$$ER_t = \int_j \left(1 - \boldsymbol{\chi}_t\left(j\right)\right) \frac{ER_t\left(j\right)}{P_t} dj \tag{55}$$

where as explained above, $\boldsymbol{\chi}_t\left(j\right)$ is an indicator function taking the value of 1 if the bank extends a loan and 0 if the bank maintains its funds as excess reserves with the central bank.

Consolidated government budget constraint Combining the above two constraints for the government sector yields the following consolidated budget constraint:

$$M_t - M_{t-1} = P_t T_t + i_t ER_t \tag{56}$$

where the treasury budget constraint is defined as $RCB_t = P_t T_t$.

Monetary policy We assume that the central bank follows an exogenous growth rate for the nominal supply of money given by

$$M_t = \left(1 + \theta_t\right) M_{t-1} \tag{57}$$

where θ_t denotes the nominal money growth given by

$$\left(\frac{\theta_t}{\theta}\right) = \left(\frac{\theta_{t-1}}{\theta}\right)^{\rho_\theta} \exp\left(\in_t^\theta\right) \tag{58}$$

Notice that in this case, the nominal interest rate on deposits, R_t, will be an endogenous variable clearing the market for real money balances.

4.5 Aggregation and Market Clearing

Final goods sector Market clearing in the final goods market requires demand to equal supply for each final good which implies:

$$C_t(j) = Y_t^f(j) \quad \text{for all } j \tag{59}$$

Using the same CES aggregator for final consumption goods as the one used for final goods yields the following aggregate equilibrium condition:

$$C_t = Y_t^f \tag{60}$$

Aggregating individual production functions across all final good producers and taking into account that the demand schedule must be consistent with household optimization, gives us the following condition:

$$C_t \Delta_t^p = X_t \tag{61}$$

where $X_t = \int X(j)\, dj$ and $\Delta_t^p = \int \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} dj$ measures price dispersion.

Intermediate good sector Recall that firms and goods in the intermediate good sector are indexed by the idiosyncratic productivity of each active producer, $\omega_{z,t}$. The equilibrium condition in this market is given by

$$X_t(\omega_{z,t}) = y_t(\omega_{z,t}) \quad \text{for all } z \tag{62}$$

where the demand for each intermediate good is denoted by $X_t(\omega_{z,t})$ and comes from the final good producers. Aggregating across each of the z producers yields the following market clearing condition

$$\begin{aligned} X_t &= \int_z y_t(\omega_{z,t})\, dz \\ &\equiv Y_t^l \end{aligned}$$

where Y_t^l denotes the aggregate supply of intermediate goods and is given by the total number of producing firms $\varphi_t(\tilde{\omega}_t) f_{t-1}^m$ times their average output, that is

$$Y_t^I = \varphi_t(\tilde{\omega}_t) f_{t-1}^m E\left[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \widetilde{\omega}_t\right] \tag{63}$$

where $E\left[y_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \widetilde{\omega}_t\right] = \int\limits_{\widetilde{\omega}_t}^{\overline{\omega}} y_t^*(\omega_{z,t}) \frac{g(\omega)d\omega}{(1-\gamma_t)}$ is average output for all active intermediate good producers. Assuming that $g(\omega)$ is a uniform distribution allows us to explicitly calculate the truncated expectation of $y_t^*(\omega_{z,t})\ \forall \omega_{z,t} > \widetilde{\omega}_t$ and compute Y_t^I as

$$Y_t^I = (1-\delta_t)\alpha^{\frac{\alpha}{1-\alpha}} \left(\frac{\xi^{pf} A_t}{(\mu_t^p w_t R_t)^{\alpha}} \right)^{\frac{1}{1-\alpha}} \left(\frac{(\overline{\omega})^k - (\widetilde{\omega}_t)^k}{k(\overline{\omega} - \underline{\omega})} \right) f_{t-1}^m \tag{64}$$

where $k = \frac{2-\alpha}{1-\alpha}$. Note that Y_t^I depends directly on the measure of firms matched with a bank at the beginning of the period f_{t-1}^m, on the probability that a credit contract survives during the period but scaled by k: $(1-\delta_t)\left(\frac{(\overline{\omega})^k - (\widetilde{\omega}_t)^k}{k(\overline{\omega}-\underline{\omega})}\right)$, and on the aggregate productivity shock, A_t. Moreover, Y_t^I depends inversely on the Nash bargained real marginal cost of labor expressed in terms of the intermediate good, $\mu_t^p R_t w_t$.

Labor market The aggregate labor demand, defined as $N_t = \int\limits_z \int_0^1 N_t(i, \omega_{z,t})\, didz$, is a downward sloped schedule in the real wage-employment/labor force space since it directly depends on the Nash bargained marginal cost of labor which in turn depends on the real wage, the nominal deposit interest rate and the price mark-up. Following the same steps as for aggregating the intermediate good, Y_t^I, the double integral that defines N_t can be expressed as the total number of producing firms times their average labor demand:

$$\begin{aligned} N_t &= \varphi_t(\tilde{\omega}_t) f_{t-1}^m E\left[N_t^*(\omega_{z,t}) \mid \omega_{z,t} \geq \widetilde{\omega}_t\right] \\ &= (1-\delta_t) \left(\frac{\alpha \xi^{pf} A_t}{\mu_t^p w_t R_t} \right)^{\frac{1}{1-\alpha}} \left(\frac{(\overline{\omega})^k - (\widetilde{\omega}_t)^k}{k(\overline{\omega} - \underline{\omega})} \right) f_{t-1}^m \Delta_t^w \end{aligned}$$

Aggregating individual labor demands implies integrating along the index z. The upper limit of z is given by the fraction of active intermediate good producers which is an endogenous variable in our setting and is given by $\varphi_t(\tilde{\omega}_t) f_{t-1}^m$

The labor supply or aggregate labor force schedule is obtained by aggregating over all marginal suppliers of each labor type and it is given by

$$w_t = R_t C_t \chi_t (L_t)^{\overline{\varphi}}$$

Labor supply is a positively sloped schedule due to the presence of the aggregate labor market participation condition. As in Galí (2010), the unemployment rate corresponds to the horizontal gap between the labor supply and the labor demand schedules at the level of the prevailing average real wage. In this model, the position of the labor demand and supply schedules depends directly on R_t due to the presence of a working capital channel. More importantly, the position of the labor demand schedule directly depends on the cutoff productivity value, $\widetilde{\omega}_t$, the exogenous probability of credit separation δ_t, and on the measure of active intermediate good producers f^m_{t-1}, which reflects the impact of search frictions in the loan market on the labor market.

Aggregate technology and TFP for the intermediate good sector Combining the above equations for Y^I_t and N_t and letting $F_t = (1-\delta_t)\left(\frac{(\overline{\omega})^k-(\widetilde{\omega}_t)^k}{k(\overline{\omega}-\underline{\omega})}\right) f^m_{t-1}$, yields the following expression for the aggregate production function in the intermediate good sector:

$$Y^I_t = \xi^{pf} A_t \left(F_t\right)^{1-\alpha}\left(\frac{N_t}{\Delta^w_t}\right)^{\alpha} \tag{65}$$

where

$$N_t = \left(\frac{\alpha \xi^{pf} A_t}{\mu^p_t w_t R_t}\right)^{\frac{1}{1-\alpha}} F_t \Delta^w_t \tag{66}$$

Notice that F_t is the endogenous component of total factor productivity for the aggregate technology of the intermediate good sector and defined to be defined as $TFP_t = \xi^{pf} A_t (F_t)^{1-\alpha}$. F_t depends on the exogenous separation rate, the measure of producing firms and on credit conditions that are reflected on the reservation productivity. The assumption that $\omega_{z,t}$ follows a uniform distribution with support $[\underline{\omega}, \overline{\omega}]$ implies a total continuation rate given by $\varphi_t\left(\tilde{\omega}_t\right) = (1-\delta_t)\left(\frac{\overline{\omega}-\tilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right)$.

Deposit and loan markets The deposit market equilibrium implies that households have deposits in all active banks, therefore in the aggregate equilibrium $D_t = \int\limits_j D_t\left(j\right) dj$ must hold. Since all active intermediate good producers take loans to cover their wage bill, market clearing in the loan market requires

$$l^*_t\left(j, \omega_{z,t}\right) = w_t N^*_t\left(\omega_{z,t}\right) \quad \text{for all } z \tag{67}$$

Aggregating the above condition across all active intermediate good producers and taking into account the wage heterogeneity due to the wage rigidity assumption, yields the following expression for aggregate loans:

$$l_t = \frac{w_t N_t}{\Delta^w_t} \tag{68}$$

Aggregating $l_t\left(j, \omega_{z,t}\right) = d_t\left(j\right)$, where $d_t\left(j\right)$ denotes real deposits at bank j, across all active intermediate good producers and all lending banks yields an aggregate relation between loans and deposits:

$$l_t = \varphi_t\left(\tilde{\omega}_t\right) f^m_{t-1} d_t \tag{69}$$

Thus in the aggregate, loans are a fraction of deposits. The specific fraction is endogenous and given by the measure of active credit contracts during period t which is $\varphi_t(\tilde{\omega}_t) f_{t-1}^m$. By the same token, the aggregate level of excess reserves is the fraction of real deposits banks were not able to lend out to firms, that is

$$er_t = \left(1 - \varphi_t(\tilde{\omega}_t) f_{t-1}^m\right) d_t \tag{70}$$

Equilibrium in this economy also takes into consideration the aggregate balance sheet for banks

$$l_t + er_t + \xi^{bs} = d_t \tag{71}$$

where ξ^{bs} is a fixed residual that we use for calibration. It represents residual assets in the banking sector.

Household Intermediate good producers, final good producers, and banks transfer their profits to the household at the end of each period. The aggregate real transfer received by the household from banks and each type of firm is given by

$$\pi_t^b = \left(R_t^l - R_t\right) l_t - b_t^u \kappa \tag{72}$$

$$\pi_t^I = \frac{Y_t^I}{\mu_t^p} - R_t^l l_t - \varphi_t(\tilde{\omega}_t) f_{t-1}^m x^f \tag{73}$$

$$\pi_t^f = C_t \left(1 - \frac{\Delta_t^p}{\mu_t^p}\right) \tag{74}$$

where R_t^l is the average loan rate defined below.

Goods and Money Markets Taking into account all of the aggregate equilibrium conditions and budget constraints, the aggregate resource constraint in this economy is characterized by

$$C_t = Y_t^f = Y_t^I - \left(b_t^u \kappa + \varphi_t(\tilde{\omega}_t) f_{t-1}^m x^f\right) \tag{75}$$

Therefore equilibrium in the final goods market requires that consumption equals aggregate household income which, in turn, is equal to aggregate production of the intermediate good net of aggregate search and fixed costs. On the other hand, aggregating the CIA constraint, together with the government budget constraint, the aggregate balance sheet of banks as well as the aggregate equilibrium condition in the loan market, yields the following equilibrium condition for the real money balances market:

$$C_t = m_t - (1 + i_t) er_t \tag{76}$$

The above equilibrium condition implies the aggregate supply of real money balances is allocated to aggregate consumption as well as repaying excess reserves holdings.

Finally, we define the average spread of interest rates (average credit spread) as the difference between the average loan rate and the bank's opportunity cost of funds, given by the deposit rate:

$$\frac{R_t^l l_t - R_t l_t}{l_t} = \frac{1}{l_t}\left[\left((1-\overline{\eta})(1-\alpha)\right)\frac{1}{\mu_t^p}\frac{Y_t^I}{\varphi_t(\tilde{\omega}_t) f_{t-1}^m} - \left((1-\overline{\eta})\frac{x^f}{\mu_t^p} + \overline{\eta}\frac{\kappa}{\mu_t^p \tau_t}\right)\right] \tag{77}$$

where the terms $R_t^l l_t$ and $R_t l_t$ are obtained by computing the following conditional expectations:

$$R_t^l l_t = E\left[R_t^l(j, \omega_{z,t})\, l_t^*(\omega_{z,t}) \mid \omega_{it} \geq \widetilde{\omega}_t\right]$$

and

$$R_t l_t = E\left[R_t l_t^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t\right]$$

5 Computation and simulations

The non-linear system of equations that characterizes the dynamic equilibrium of the model is summarized in the non-stochastic steady state section in Appendix B and Appendix C. The vector of endogenous variables X_t is given by the following 41 variables classified according to the following groups:

1. Prices and real wages (11 variables):

$$X_{1,t} = \left[\Pi_t, \Pi_t^*, w_t, w_t^*, \widetilde{g}_{1,t}, \widetilde{g}_{2,t}, f_t^1, f_t^2, \mu_t^p, \Delta_t^p, \Delta_t^w\right] \tag{78}$$

2. Real variables (7 variables):

$$X_{2,t} = \left[Y_t^I, Y_t^f, C_t, N_t, U_t, L_t, \Delta_{t,t+1}\right] \tag{79}$$

3. "Monetary policy" variables (3 variables):

$$X_{3,t} = [m_t, er_t, R_t] \tag{80}$$

4. Credit market variables (15 variables):

$$X_{4,t} = \left[l_t, d_t, \tau_t, p_t^b, p_t^f, b_t^u, f_t^m, f_t, F_t, \widetilde{\omega}_t, \varphi_t(\widetilde{\omega}_t), cd_t, cc_t, cg_t, cr_t\right] \tag{81}$$

5. Auxiliary definitions for calibration purposes (4 variables):

$$X_{5,t} = [LS_t, FCS_t, \widehat{l}_t, \widehat{er}_t] \tag{82}$$

where LS_t denotes the labor share of GDP, FCS_t is the fixed cost of production share of GDP, $\widehat{l}_t$ is aggregate loans as a fraction of total deposits and $\widehat{er}_t$ is aggregate excess reserves as a fraction of total deposits.

We solve the model using a standard perturbation method applied to a first order approximation around the non-stochastic steady-state of the model. Next we explain the calibration procedure for the unknown parameters of the model. We assume in steady state that the growth rate of real money balances is zero. Appendix B contains the computation of the non-stochastic steady state.

We calibrate the following subset of nine parameters: $x^f, \kappa, \mu, \xi^{pf}, \delta, \xi^{bs}, \chi, \alpha$ and ϵ_w to be consistent with specified steady-state targets for the following endogenous variables: $U, N, Y^f, \varphi(\tilde{\omega}), cd, FCS, LS, \frac{l}{d}$ and $\frac{er}{d}$. The strategy is explained in more detail in the next section.

5.1 Calibration

In order to compute the model's equilibrium, we must assign values to the following list of parameters:

- Preferences: $\beta, \overline{\varphi}, \chi$
- Technology: $A, \xi^{pf}, \alpha, x^f, [\underline{\omega}, \overline{\omega}]$
- Search technology and the loan market: $\mu, \nu, \delta, \kappa, \xi^{bs}, \overline{\eta}$
- Price and wage stickiness: $\theta_p, \theta_w, \epsilon_p, \epsilon_w$
- Monetary policy: θ_{ss}

We set the following subset of parameters according to convention: The subjective discount factor is set to $\beta = 0.99$ consistent with a steady-state real interest rate of 1 percent per quarter. We normalize the baseline level of technology to be $A = 1$ as well as the support for the idiosyncratic productivity to be $[\underline{\omega}, \overline{\omega}] = [0, 1]$. The parameters determining the degree of price and wage stickiness are set to imply an average duration of one year, that is $\theta_p = \theta_w = 0.75$. The latter is set consistent with much of the microeconomic evidence on wage and price setting [18] The elasticity of substitution among final goods is set to be $\epsilon_p = 9$, implying a steady sate price markup of $\mu^p = 1.125$ or 12.5% and the inverse of the Frisch labor supply elasticity is set to be $\overline{\varphi} = 5$ which corresponds to a Frisch elasticity of 0.2.

Additionally, we fix values for two of the six parameters related to the loan market: The firm's share in the Nash bargaining problem is assumed to be $\overline{\eta} = 0.35$ which is close to the 0.32 value used by Petrosky-Nadeau and Wasmer (2015). Petrosky-Nadeau and Wasmer (2015) calibrate the bank's share, $1 - \overline{\eta}$, by calculating the financial sector's share of aggregate value-added using data and the corresponding value added definition

[18]See, for example,Nakamura and Steinsson 2008.

from their model. We assume that the Hosios condition does not hold in steady state and set the elasticity of the matching function to $\nu = 0.7$. which is two times the firm's bargaining parameter. [19]

In the next table we summarize the parameter values described above:

Parameter	Description	Value
β	Discount rate	0.99
A	Baseline Technology	1.0
$[\underline{\omega}, \overline{\omega}]$	Support for idiosyncratic productivity	[0,1]
θ_p	Calvo parameter for price setting	0.75
θ_w	Calvo parameter for wage setting	0.75
ϵ_p	Elasticity of substitution among final goods	9.0
$\overline{\varphi}$	Inverse of Frisch Elasticity	5
$\overline{\eta}$	Firm's Nash bargaining share	0.35
ν	Matching function elasticity	0.7

Table 1: Parameters taken from the data and conventional values from the literature

A total of nine parameters of the model are calibrated to be consistent with a set of nine endogenous steady-state targets that we specify below. These parameters are classified as follows:

- Calibrated loan market parameters: The search cost faced by a bank κ, the scale parameter of the aggregate matching function μ, the exogenous probability of credit destruction δ and the residual term on the aggregate banks' balance sheet ξ^{bs}.
- Calibrated technology parameters: The elasticity of labor and the scale parameter in the aggregate production function for intermediate goods α and ξ^{pf} respectively as well as the fixed cost of producing the intermediate good x^f.
- Calibrated preference parameters: The preference shifter χ and the elasticity among labor types, ϵ_w.

In the next table, we report the steady state targets that we use to calibrate the above subset of parameters.

Parameter	Description	Value
U	Unemployment rate	0.05
N	Employment	0.59
Y^f	GDP	1
$\varphi(\tilde{\omega})$	Overall continuation rate	0.7
cd	Credit destruction rate	0.029
$\frac{\varphi(\tilde{\omega})f^m x^f}{Y^f}$	Fixed cost share of GDP	0.35
$\frac{wN}{Y^f}$	Labor share of GDP	2/3
$\frac{l}{d}$	Loan to deposits ratio	0.63
$\frac{er}{d}$	Excess reserves to deposits ratio	0.015

Table 2: Steady state targets

[19] We checked the model's robustness to the following range of values: $v \in [0.6, 0.8]$ and $\overline{\eta} \in [0.5, 0.8]$ for two reasons. First, when $v < 0.6$ the linear approximation of the dynamic equations of the model does not satisfy the Blanchard and Kahn 1980 rank condition. But second, if $\overline{\eta} < 0.5$, solving for the non-linear steady state yields imaginary roots. Both restrictions on the rage of values for ν and $\overline{\eta}$ may be an indication that there is no equilibrium in the loan market or that the loan market collapses such as in Becsi, Li, and Wang 2005.

Following Galí (2011) we target an unemployment rate of $U = 0.05$ and aggregate employment of $N = 0.59$ at the steady-state. Given this, equation SS3 above,implies an elasticity of substitution among labor types of $\varepsilon_w = 4.2743$ which in turn is associated with an average wage markup of 30 percent. We assume a 35 percent share of the fixed cost of production in GDP, $FCS = 0.35$, similar to Christiano, Eichenbaum, and Trabandt (2015). The steady-state overall continuation rate for a credit relationship is set at 70 percent,$\varphi\left(\tilde{\omega}\right) = 0.7.$, which is consistent with findings reported in Chodorow-Reich (2014) for banking relationships in the U.S. syndicated loan market using a sample that covers 2001 to June 2009 excluding loans to borrowers in finance, insurance or real estate and for which the purpose of the loan is not working capital or general corporate production. Specifically, Chodorow-Reich (2014) finds that after controlling for a bank's average market share, a bank that served as the prior lead lender for a private borrower has a 71 percent greater likelihood of serving as the new lead lender in the same loan contract. Given that the scale technology parameter, ξ^{pf}, is chosen in order to normalize the steady-state level of GDP to unity $\left(i.e, Y^f = 1 \text{ in equation SS4}\right)$, we can solve for x^f to be consistent with the steady-state target imposed on the fixed cost share of GDP, which is given by

$$\frac{\varphi\left(\tilde{\omega}\right) f^m x^f}{Y^f} = 0.35 \tag{SS9}$$

We target a steady-state loan to deposit ratio of $\frac{l}{d} = 0.63$ by using quarterly data on commercial and industrial loans as well as saving deposits for all U.S commercial banks during the great moderation period which is taken as between 1985 and 2007. The steady-state target for the loan to deposit ratio $\frac{l}{d}$, together with the steady-state target for $\varphi\left(\tilde{\omega}\right)$ explained above, allows us to obtain the steady-state level for the measure of firms in a credit relationship (f^m) by using the relationship between loans and deposits that arises when aggregating the balance sheet of those banks able to lend out their available funds to intermediate good producers. This condition at the steady-state is given by:

$$\frac{l}{d} = \varphi\left(\tilde{\omega}\right) f^m \tag{SS10}$$

Clearly, equations SS9 and SS10 together with the specified steady-state targets for $FCS, \varphi\left(\tilde{\omega}\right), \frac{l}{d}$ and Y^f are consistent with $f^m = 0.9$ and $x^f = 0.56$. Therefore, the steady-state of the model implies that 90 percent of firms (intermediate good producers) are in a credit contract with a bank. The parameter $x^f = 0.56$ is consistent with a steady-state involving a 35 percent fixed cost of production share of GDP, a 70 percent probability of overall continuation for a credit relationship, and a 90 percent measure of firms in a credit relationship.

We target the labor share of GDP at the steady-state of $LS = \frac{wN}{Y^f} = 2/3$. The latter definition together with the equilibrium condition in the loan market evaluated at the steady state, $l = wN$ yields a steady-state value for the real wage equal to $w = 1.13$ and aggregate real loans of $l = 2/3$. Then, given the steady-state target on the loan to deposit ratio, we obtain the steady-state value of aggregate real deposits to be $d = 1.06$. We use the average of all reserve balances with Federal Reserve banks during the great moderation period and the average of all saving deposits at U.S commercial banks during the same period of time in order to set the ratio of aggregate excess reserves to aggregate deposits to be 1.5 percent. The aggregate level of reserves consistent with the specified target is $\frac{er}{d} = 0.015$, and the steady state level of aggregate deposits obtained before is, $er = 0.0159$. The resource constraint of the economy implies consumption at the steady-state to be $Y^f = C = 1$ while the aggregate CIA constraint can be solved for the steady state level of real money balances m given our parameterization of $R = 1.0101$ and the steady state level of aggregate excess reserves that we have already obtained. Thus, at the steady-state, the supply of real money balances must be allocated to consumption and interest rate payments on excess reserves:

$$m = C + (R)\, er \tag{SS11}$$

with $m = 1.016$. The labor force equation evaluated at the steady-sate implies:

$$w = RC\chi\,(L)^{\overline{\varphi}} \tag{SS12}$$

given the parameterization of $\overline{\varphi}$ and the steady-state values for w,C,R and L obtained above, we can solve consistently for the preference shift parameter to be $\chi = 12.1074$. Notice that the calibration of χ is also consistent with the optimal price setting equation evaluated at the steady state, equation SS2 above, and therefore it is consistent with the steady-state level of employment we are targeting $(N = 0.59)$. The stochastic discount factor evaluated at the steady state yields $\Delta = \beta = 0.99$.

The aggregate balance sheet of banks evaluated at the steady-state allows us to obtain the residual term as a fraction of deposits as $\xi^{bs} = 1 - \dfrac{l}{d} - \dfrac{er}{d} = 0.3550$

Following Contessi and Francis (2013), we target an average quarterly credit destruction rate of 2.9 percent during the great moderation period. The credit destruction rate implied by the model and evaluated at the steady-state is

$$cd = 0.029 = 1 - \varphi\,(\tilde{\omega}) - \mu\tau^{\nu}\delta \tag{SS13}$$

Finally, the steady-state probability of continuation for a credit contract:

$$\varphi\,(\tilde{\omega}) = (1-\delta)\left(\frac{\overline{\omega} - \tilde{\omega}}{\overline{\omega} - \underline{\omega}}\right) \tag{SS14}$$

Given the above targets and parameter calibrations, equations SS4-SS8 together with equations SS13 and

SS14 can be solved for the following set of parameters $\kappa, \mu, \delta, \xi^{pf}, \alpha$ and the corresponding steady-state values for $\widetilde{\omega}$, and τ (See appendix C for the complete system of equations that characterize the steady-state).

The table below summarizes the calibrated parameters of the model solved to be consistent with the steady-state targets specified above.

Parameter	Description	Value
κ	Bank search costs	0.6697
μ	Matching function scale parameter	1.0564
δ	Exogenous probability of separation	0.2029
ξ^{pf}	Production function scale parameter	3.9482
α	Labor elasticity of production function	0.51
ξ^{bs}	Residual term on aggregate bank's balance sheet	0.3550
x^f	Fixed cost of production	0.5556
χ	Preference parameter for dis-utility of labor	12.1074
ϵ_w	Elasticity of substitution among labor types	4.2743

Table 3: Calibrated parameters to be consistent with steady state targets

The above results imply that $k = \frac{2-\alpha}{1-\alpha} = 3.0409$. The steady state values for some of the endogenous variables in the model are summarized in the following table:

Parameter	Value	Parameter	Value
Π	1	Π^*	1
Δ^w	1	Δ^p	1
Y^I	1.4854	Y^f	1
F	0.2355	C	1
b^u	0.2022	m	1.0160
f	0.2826	R	1.0101
p^f	0.9554	w	1.1299
p^b	1.3356	μ^p	1.1250
U	0.0500	L	0.6211
N	0.5900	$\varphi(\tilde{\omega})$	0.7000
f^m	0.9	$\tilde{\omega}$	0.1218
l	0.6667	d	1.0582
τ	1.3980	er	0.0159

Table 4: Steady state values

5.2 Equilibrium dynamics of financial shocks

In order to help understand the propagation mechanism of a financial shock in the model economy, we focus on two main equations that induce intensive and margin effects over employment and production as well as in the implications of these equations over the inefficiency wedge and different measures of aggregate productivity. The optimal hiring rule for all active credit matches and the reservation productivity level are key to understand the intensive and extensive margin effects produced by a financial shock. For clarity, we re-write both equations in terms of the Nash bargained marginal cost MC_t and credit market tightness τ_t:

$$MPL_t(\omega_{z,t}) = MC_t; \qquad \forall \omega_{z,t} > \widetilde{\omega}_t \tag{83}$$

$$\widetilde{\omega}_t = \frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(MC_t)^\alpha}{A_t} \left[x^f - \frac{\kappa}{1-\overline{\eta}} \left(\mu \tau_t^{1-v} - \overline{\eta} \mu \tau_t \right) \right]^{1-\alpha} \tag{84}$$

Recall that the marginal product of labor is given by $MPL_t(\omega_{z,t}) = \alpha \xi^{pf} A_t \omega_{z,t} N_t^*(\omega_{z,t})^{\alpha-1}$ and $MC_t = \mu_t^p w_t R_t$.

The first equation, equation(83) is the result of the Nash bargaining protocol over the joint surplus generated by a credit contract. Therefore, conditional on surviving, each credit contract will determine the loan principal and number of workers to hire consistent with the point where the marginal product of labor is equal to the Nash bargained real marginal cost of labor expressed in terms of the intermediate good. Changes in this equation generate an intensive margin effect since it holds only for those credit matches that have survived the exogenous as well as the endogenous separation hazards. In this case, the subset of surviving firms will modify their hiring decisions until equilibrium is restored.

The second equation (84) is obtained by setting the joint surplus for a credit contract to zero. The reservation productivity, $\widetilde{\omega}_t$, that results is a productivity threshold that determines the subset of firms able to obtain funds, hire workers, and produce during the period. This threshold productivity generates an extensive margin effect whenever it responds to aggregate macroeconomic shocks. The reservation productivity, $\widetilde{\omega}_t$, has two main determinants: MC_t and τ_t. In a partial equilibrium setting, an increase in MC_t will raise the reservation productivity taking as given credit market tightness, τ_t. By the same token, our benchmark calibration implies that holding MC_t constant, an increase in τ_t will produce an increase in the reservation productivity. The main transmission mechanism of aggregate shocks in this economy occurs through changes in $\widetilde{\omega}_t$ which ultimately changes as a result of movements in the joint surplus of a credit match, $V_t^{JS}(\omega_{z,t})$. Fluctuations in MC_t and τ_t affect $V_t^{JS}(\omega_{z,t})$: An increase in MC_t holding τ_t constant reduces the joint surplus of an active credit contract while a tighter credit market (higher τ_t), holding MC_t constant will also end up reducing this joint surplus. General equilibrium effects will determine all the variables simultaneously.

5.2.1 Productivity measurements: Credit inefficiency wedge, labor and firm productivity

Intensive and extensive margin effects of a financial shock are reflected in changes in the dynamics of τ_t and in the reallocation of credit among banks and firms: cr_t, cc_t and cd_t. This reallocation process will impact different measures of aggregate productivity such as the endogenous component of total factor productivity, labor productivity and aggregate output per firm. In this section, we discuss the impacts of credit inefficiency on productivity measures.

Recall that F_t is the endogenous component of technology and it depends on credit market conditions as well as on the reservation productivity level, $\tilde{\omega}_t$. This term is given by:

$$F_t = (1-\delta_t)\left(\frac{(\overline{\omega})^k - (\widetilde{\omega}_t)^k}{k\,(\overline{\omega}-\underline{\omega})}\right) f^m_{t-1} \tag{85}$$

Throughout this section, we define the credit inefficiency wedge as the endogenous component of technology unrelated to employment, given by: $F_t^{1-\alpha}$. In the case without credit market frictions, $F_t = 1$, aggregate output in the intermediate sector would be: $Y_t^I = \xi^{pf} A_t(\frac{N_t}{\Delta_t^w})^\alpha$. In this latter case, the only inefficiency that appears after aggregation is related to the presence of wage rigidities, Δ_t^w, which measures wage dispersion. But under the assumption of search and matching frictions in the loan market, the inefficiency wedge depends on the aggregate probability of continuation of a credit contract, φ, as well as on the mass of active credit contracts, f^m_{t-1}. Both of these features depend ultimately on the common reservation productivity threshold, $\widetilde{\omega}_t$. Conditions in the credit market affect the inefficiency wedge, potentially amplifying any aggregate shock hitting the economy.

On the other hand, labor productivity in the intermediate good sector is given by:

$$LP_t = \frac{Y_t^I}{N_t} = \frac{\xi^{pf} A_t}{(\Delta_t^w)^\alpha}\left(\frac{F_t}{N_t}\right)^{1-\alpha} \tag{86}$$

In our model, labor productivity is also affected by the credit wedge. If the loan market was a Walrasian centralized market then $F_t = 1$ and credit conditions do not affect labor productivity. Credit market frictions generate inefficient fluctuations in labor productivity, employment and intermediate output as well as final output due to the effects on F_t of changes in the the probability of continuation of the credit contract and the set of actively producing firms. This of course translates into inefficient fluctuations in the unemployment rate given the interaction of wage rigidities, market power, and the labor force participation condition that characterizes the labor market. Financial shocks are propagated and amplified by the endogenous response of this credit inefficiency wedge term.

We can define total factor productivity, TFP_t, as the term in the aggregate production function for the intermediate good sector not associated with the labor input, that is:

$$TFP_t = \xi^{pf} A_t\,(F_t)^{1-\alpha} \tag{87}$$

From equation (87), we obdserve that the inefficiency associated with the presence of credit frictions also affects the evolution of total factor productivity. In our setting, if $F_t \neq 1$, TFP is affected by aggregate shocks through the inefficiency wedge, otherwise it is exogenous. Therefore, total factor productivity is subject to inefficient endogenous fluctuations that amplify and propagate aggregate shocks to the macroeconomy.

Another measure of productivity is intermediate output per active firm, FP_t, which is given by:

$$FP_t = \frac{Y_t^I}{\varphi_t(\tilde{\omega}_t) f_{t-1}^m} \tag{88}$$

Our simulations show that under the benchmark calibration, TFP_t, LP_t and FP_t do not necessarily move in the same direction when the economy faces a financial shock. Notice that FP_t will be higher whenever $\varphi_t(\tilde{\omega}_t)$ is lower. The latter occurs when $\tilde{\omega}_t$ increases. On the other hand, F_t also falls as $\tilde{\omega}_t$ is higher. In our model, financial shocks are associated with persistent reductions in the overall continuation rate for a credit contract, fewer producing firms and a higher reservation productivity. Thus, FP_t will be higher during a financial shock while TFP_t lower. This result is indicative of a cleansing effect (survival of more productive firms) together with an inefficiency effect (lower credit reallocation) during a financial shock.

6 The effects of a financial shock

In our theoretical model, there are a variety of ways we could understand a financial shock. We model the shock first as affecting the exogenous probability of matched firm-bank separations. This mechanism corresponds to banks exogenously failing to renew lending contracts with firms due to a shock that impacts the bank balance sheet directly. The second method we use to model a financial shock impacts the firm's Nash bargaining share which directly reduces the joint surplus of a credit contract. We allow Nash bargaining shares to be time varying, and then observe the effects of a one period increase in the firm's share after which firms and banks expect the share to decrease to its steady state value. This could be understood as an sharp decline in bargaining faith or an increase in asymmetric information in the sense that firms are not as informed about bank liquidity as previously.

6.1 Exogenous separation shock

Figures **5-7** illustrate the dynamic responses of the credit market, labor market, and other aggregate variables to a financial shock, defined as an unexpected persistent increase in the exogenous separation rate for credit contracts, δ_t. The overall continuation rate of a credit contract is defined as $\varphi_t(\tilde{\omega}_t) = (1-\delta_t)(1-\gamma_t(\tilde{\omega}_t))$ where its exogenous component δ_t follows the following AR(1) process:

$$\delta_t - \delta = \rho_\delta (\delta_{t-1} - \delta) + \varepsilon_t^\delta \tag{89}$$

Our calibration procedure is consistent with a steady-state value for δ of 0.2029. In steady state, 90 percent of our firms are in a credit relationship and the steady state continuation rate of credit contracts is 70 percent. For our financial shock, we assume a very persistent shock with $\rho_\delta = 0.9$. [20] In figure 5, we

[20] We calibrate ρ_δ and the standard deviation of ε_t^δ to match the standard deviation of commercial and industrial credit creation and destruction rates for the 2007 - 2017 period.

depict the impact of the financial shock. The exogenous separation rate rises, returning slowly to baseline after approximately 16 quarters. As a result the joint surplus for bank-firm pairs falls immediately. This results in more banks and firms searching. However since the net surplus falls by more for banks than for firms and since banks can exit the market, the credit market tightens significantly from the perspective of firms (see below).

After a negative financial shock, there is a sharp increase in the number of intermediate good producers searching for funds, f_t. As noted, because banks can exit the market, when the shock causes the expected value of searching for a project/borrower to be temporarily negative, the measure of firms searching for funds is larger than the measure of banks searching for borrowers, inducing an increase in credit market tightness, τ_t (top left hand graph in figure 6). Moreover, intermediate good producers separated from their previous credit contract are not able to exit the market, but must continue searching in order to attempt production in the future. These two conditions cause credit market conditions to tighten from the perspective of borrowers, exhibited by a decline in the firm's finding rate, p_t^f (see figure 6, top right hand panel). When banks exit the loan market, the central bank must automatically increase excess reserves, er_t, to compensate for the fact that banks have no remaining assets to pay interest on households' deposits (see figure 7, top right hand).

The transmission of the financial shock is further reinforced by a decline in the joint surplus of the remaining bank-firm matches which is a consequence of the large persistent increase in the reservation productivity level $\widetilde{\omega}_t$. The reservation productivity level endogenously adjusts upward (see figure 6, top right panel), causing a further reduction in firm-bank matches as some of those matches which survived the increase in exogenous separation become endogenously separated as the set of firms with sufficiently high productivity shrinks. In other words, the bank-firm partners that enter the period matched having survived the exogenous financial shock, are faced with a second source of instability: since the joint surplus of a credit relationship falls, banks raise the productivity threshold at which providing funds becomes profitable. The increase in reservation productivity induces an even more pronounced and persistent fall in the overall continuation rate, $\varphi_t(\tilde{\omega}_t)$, causing more pairs to separate. The dynamics associated with the increase in the reservation productivity level produce an extensive margin effect–this is a selection effect that reduces the subset of firms able to obtain external funds, hire workers, and produce. The tighter credit conditions are also reflected in a persistent decline of the mass of firms engaged in a credit contract, f_{t-1}^m and a significant reduction in the aggregate amount of loans, l_t (figure 7 top right). These new credit conditions translate to a persistent increase in the average interest rate spread (over the deposit rate) as well as an increase in the average loan rate (see figure 7, top left panel).

However, the financial shock also generates an intensive margin effect that partially off-sets the extensive margin effect.[21] The intensive margin effect is related to the credit contracts -firm and bank pairs- that

[21]In the next section we discuss a financial shock where the intensive and extensive margin effects work in the same direction. Also, see Florian Hoyle and Francis 2019 for a discussion of a contractionary monetary policy shock, in a similar credit

survive the financial shock but adjust their existing loan contract by changing the conditions characterizing their bilateral bargaining protocol. Specifically, a financial shock reduces the Nash bargained marginal cost of labor expressed in terms of the intermediate good, MC_t for all active intermediate good producers, inducing a small recovery in aggregate labor demand–demand from firms who remain in active credit contracts–but which is not sufficient to offset the much larger negative extensive margin effect of the financial shock (see figure 6, bottom left panel). The persistent decline in the average real wage (figure 6, bottom left) is indicative of the much larger negative extensive margin effect on labor demand in relationship to the smaller intensive margin effect.

For labor market quantities, a negative financial shock generates a sharp and prolonged decline in employment and total factor productivity (see figure 7, bottom left panel) and a large increase in the unemployment rate. These responses are shaped by the sum of the intensive and extensive margin effects described above. The steep and persistent declines in total factor productivity–both initially and through quarter 2–are due to the fact output falls by more than employment and the fact that both labor and total factor productivity are negatively effected by the increase in credit inefficiency (decline in $F(t)$) due to the tightening of the credit market. The large and persistent increase in the unemployment rate is explained by the interaction between wage rigidities and the search and matching frictions of the loan market; as the credit market tightens, fewer firms are able to produce putting downward pressure on employment. Nominal wages are slow to adjust to the new environment, further reducing the demand for labor. The reduction in the demand for labor results in lower aggregate output.

In the credit market, the resulting tighter conditions are reflected in the response of gross credit flows. After a negative financial shock, the credit creation rate decreases persistently while the credit destruction rate increases. Aggregate loans thus decline and deposits increase (figure 7, top right panel). Excess reserves must increase by more than deposits to balance the money market since banks who exit or whose contracts separate have no funds to pay interest on deposits. The decline in creation is larger than the rise in destruction, implying a decline in the credit reallocation rate. Notably excess reallocation falls significantly and persistently, requiring more than 16 quarters to return to baseline. This is consistent with observed declines in excess reallocation during and beyond the Great Recession described in table 5.

Considering the impact on prices of an exogenous increase in the credit separation rate, we find no decline in the inflation rate associated with an increase in the short-lived increase in the price mark-up, μ_t^p which is due to the fact that real marginal costs decline but prices are sticky. The real supply of money increases causing the nominal interest rate to fall below zero for the first two quarters (see figure 7, top left graph). Following the impact of the shock, the fall in real money demand (proxied by the fall in consumption) more than compensates for the increase in the real money supply so that the nominal interest rate returns to zero.

environment, where the intensive and extensive margin effects also work in the same direction.

6.2 The effects of fluctuations in bargaining power as a financial shock

Random fluctuations in the relative bargaining power between banks and firms can be thought as another source of financial shocks since the dynamics of credit conditions (namely market tightness in the credit market, τ_t), the set of firms able to obtain external funds (determined by the productivity cut-off parameter, $\tilde{\omega}_t$), and as a consequence, total factor productivity, depend directly on the parameters of the search process and bargaining protocol assumed for the loan market. When allowing for random fluctuations in $\overline{\eta}_t$, the individual and average joint surplus to a credit contract as well as the bargained loan interest rate, among several other variables that characterize the aggregate equilibrium in the loan market, will depend on the expected relative bargaining power between contractual parties: $\frac{\mathbb{E}_t\overline{\eta}_{t+1}}{1-\mathbb{E}_t\overline{\eta}_{t+1}}$. Exogenous shocks to this new term, induce a propagation mechanism from the lending market to the rest of the economy similar to the exogenous separation shock explained in the previous section. One motivation for including this shock as a financial disruption comes from the bargaining literature and the well known equivalence between the static axiomatic and the dynamic strategic approach to bargaining. Binmore, Rubinstein, and Wolinsky (1986) show that in an strategic asymmetric Nash bargaining game, $\overline{\eta}$ reflects beliefs concerning the likelihood of a breakdown of negotiations. The main implication is that the higher is party i's estimate of the probability of a breakdown, the lower is its bargaining power. In our context, when a bank estimates a higher risk of breakdown of a loan negotiation, its bargaining power will be lower. We think of the $\overline{\eta}_t$ shock as a reduced form that shifts beliefs of possible breakdown of a negotiation.

In this section we extend the model economy to incorporate exogenous shifts in the bargaining share of firms, $\overline{\eta}_t$. The time varying Nash bargaining problem for an active credit relationship is:

$$\max_{\left\{R_t^l(j,\omega_{z,t}),l_t(\omega_{z,t})\right\}} \left(V_t^{FS}(\omega_{z,t})\right)^{\overline{\eta}_t}\left(V_t^{BS}(\omega_{z,t})\right)^{1-\overline{\eta}_t} \tag{90}$$

where $\overline{\eta}_t$ follows the following AR(1) process:

$$\overline{\eta}_t-\overline{\eta}=\rho_{\overline{\eta}}\left(\overline{\eta}_{t-1}-\overline{\eta}\right)+\varepsilon_t^{\overline{\eta}} \tag{91}$$

Recall that our calibration procedure assumes a steady-state value of $\overline{\eta}=0.35$ as in Petrosky-Nadeau and Wasmer (2015) parametrization. Similar to the case of the δ shock, we calibrate $\rho_{\overline{\eta}}$ and the standard deviation of $\varepsilon_t^{\overline{\eta}}$ to match the standard deviation of commercial and industrial credit creation and destruction rates for 2007 - 2017 period.

The optimal solution of the bargaining problem yields a time varying sharing rule given by $\overline{\eta}_t V_t^{BS}(\omega_{z,t}) = (1-\overline{\eta}_t)V_t^{FS}(\omega_{z,t})$. In contrast with the fixed bargaining power case, the optimal sharing rule has an exogenous shifter that affects the allocation of the joint surplus in addition to any endogenous changes in $V_t^{BS}(\omega_{z,t})$ and $V_t^{FS}(\omega_{z,t})$ that may occur in equilibrium. Consistent with this rule, the match-specific joint surplus depends directly on the expected relative bargaining power in the following way:

$$V_t^{JS}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j, \omega_{z,t}) + \left(1 + \left(1 - p_t^f\right)\left(\frac{E_t\overline{\eta}_{t+1}}{1 - E_t\overline{\eta}_{t+1}}\right)\right)\frac{\kappa_t}{\mu_t^p p_t^b} \tag{92}$$

Then the productivity threshold level is:

$$\widetilde{\omega}_t = \left(\frac{1}{\alpha^\alpha (1-\alpha)^{1-\alpha}} \frac{(\mu_t^p w_t R_t)^\alpha}{\xi^{pf} A_t}\right)\left[x^f - \left(\frac{1 - p_t^f \mathbb{E}_t\overline{\eta}_{t+1}}{1 - \mathbb{E}_t\overline{\eta}_{t+1}}\right)\frac{\kappa_t}{p_t^b}\right]^{1-\alpha} \tag{93}$$

The steady state remains the same as in the model with constant bargaining shares since the process for $\overline{\eta}_t$ is expressed as deviations from the same steady state we have calibrated. When $\overline{\eta}_t = \overline{\eta}$ the equations that characterize the general equilibrium are the same as what we had before. In this sense, the old equilibrium conditions are nested in the new equilibrium conditions.[22]

Notice that the process for $\overline{\eta}_t$ implies that an exogenous increase in $\overline{\eta}_t$ will induce an increase in the term: $\frac{\mathbb{E}_t\overline{\eta}_{t+1}}{1-\mathbb{E}_t\overline{\eta}_{t+1}}$. Our baseline calibration implies that as a result the joint surplus of a credit contract falls at the individual (match-specific) as well as at the aggregate level. As expected, the latter effect implies a higher $\widetilde{\omega}_t$ as well that reinforces the initial shock. The time varying sharing rule also implies a bargained loan rate that depends on the relative expected bargained share given by:

$$R_t^l(j, \omega_{z,t}) l_t(j, \omega_{z,t}) = (1 - \overline{\eta}_t)\left(\frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} + \left(\left(1 - p_t^f\right)\frac{\mathbb{E}_t\overline{\eta}_{t+1}}{1 - \mathbb{E}_t\overline{\eta}_{t+1}}\right)\frac{\kappa_t}{\mu_t^p p_t^b}\right) + \overline{\eta}_t\left(R_t w_t N_t(\omega_{z,t}) - \frac{\kappa_t}{\mu_t^p p_t^b}\right) \tag{94}$$

According to Drautzburg, Fernandez-Villaverde, and Guerron-Quintana (2018) wages increase when workers gain more bargaining power. We would therefore expect that the bargained loan rate and the average loan rates would decrease when firms' bargaining power increases. However, in our case, productivity is idiosyncratic so individually bargained loan rates differ across firms. In addition, because more banks exit the market, the relative measure of firms increases and credit tightness increases significantly. This results in a higher average bargained loan rate than anticipated ceteris paribus.

In figure 8, we depict the shock and response of the bargaining surplus for banks and firms. Compared to the exogenous separation shock case, the majority of the decline in the aggregate joint surplus is driven by a decline in the net surplus for lending banks. This causes a disproportionate number of banks to exit and significant tightness in the credit market (see figure 9, top panels). Since there is no exogenous increase in separations, initially the measure of firms seeking funding does not increase appreciably. But the decline in net surplus drives up the productivity cut-off significantly, endogenously breaking bank-firm relationships and therefore causing an increase in the number of firms searching for funding within two quarters of the

[22] Please see appendix C for the derivations associated with this case.

initial shock since endogenously separated pairs must wait until next period to search again. Akin to the exogenous separation shock, the increase in the reservation productivity is associated with a persistent decline in the overall continuation rate of credit contracts and induces an extensive margin effect that reduces the number of producing firms and with it employed workers. The initial decline in credit creation is not as large as for the previous version of a financial shock but the steep decline in reallocation and excess reallocation is similar, though not as pronounced, as credit destruction rises by a similar percentage.

Real wages and the Nash bargained marginal cost of labor expressed in terms of the intermediate good (MC_t) however move differently. Real wages decline very slowly and not by much while MC_t rises (figure 9 bottom of panel 1) significantly for all surviving producing firms primarily due to a significant rise in the mark-up on intermediate goods. The increase in MC_t, induces an intensive margin effect: surviving active firms hire fewer workers, reducing aggregate labor demand at each level of the average real wage, and thus employment for all active production units. In the exogenous separation shock case, MC_t falls persistently through the simulation (16 quarters) and real wages fall significantly as well as persistently.

Similar to the exogenous separation case, in terms of macro-aggregates, average interest rate spreads as well as the average loan interest rate increase and lending declines significantly. In this case, total factor productivity declines similarly but unemployment increases substantially compared to the exogenous separation case. This is due to the fact that the intensive margin effect reduces hiring at firms that receive credit so that the intensive and extensive margin effects move in the same direction, that is to decrease employment. Output per firm however rises, even as total factor productivity falls (TFP falls because of the credit inefficiency wedge) because the distribution of firms has shifted toward more productive firms. Compared to the exogenous separation case, the impact of this effect is not as strong, but through the simulation, the overall effect is of similar magnitude. The resulting decline in output is also similar across both shocks.

6.3 Role of nominal frictions

In this section, we discuss the critical role nominal rigidities play for amplifying and propagating credit frictions. We examine the impulse response functions for the benchmark case (both wage and price frictions) and then re-simulate the model with either sticky wages or both sticky wages and prices turned off. We use an increase in the exogenous separation rate as a financial shock[23].

In figures **11-14** we show the impulse response functions for the aggregate response to a financial shock for three cases: our originally reported case (benchmark) with nominal wage and price frictions in addition to credit market search frictions (blue), the case of flexible wages, sticky prices, and credit market search frictions (black), the case of flexible prices, wages, and credit market search frictions (red). Since we model unemployment via slow wage adjustment with variable labor market participation, our model cannot generate

[23]Similar results are obtained for the bargaining shock. The results are available upon request.

unemployment once we remove from the sticky wage assumption.

The results suggest that sticky prices and wages are crucial for obtaining a strong and persistent response in credit markets to a financial shock. Under wage flexibility (red or black lines in the figures), a financial shock produces a persistent recession that is reflected only in GDP, consumption, and total factor productivity but that is only weakly reflected in credit market tightness, interest rates and credit destruction (see figures 11 and 14). First, considering credit market adjustment, we find that tightness increases by less than half when either wages are flexible or both prices and wages are flexible. This is primarily due to the fact that fewer firms are searching for funding since the reservation productivity doesn't increase as much, after the initial increase in the exogenous separation rate, fewer matched firm-bank pairs separate because of a rise in the reservation productivity and therefore more pairs remain matched. Reservation (or cut-off) productivity (that makes lending profitable for banks), depends on banks' surplus from matching and providing funds. Banks' surplus does not decline by as much because wages are able to adjust downwards so Nash bargained marginal costs decline which would lower the reservation productivity level if the deposit rate did not rise. Also, because the reservation productivity level is increasing in credit market tightness, since tightness does not increase as much, the increase in reservation productivity is also significantly lower.

As a result of this dampened effect on endogenous separations, credit destruction does not rise by as much so that most of the impact in the credit market is through the persistent fall in credit creation which is similar under sticky and flexible wages and prices. This means that although aggregate lending declines persistently in all three cases, the decline in the flexible price and wage case is approximately half of the decline in the case with nominal frictions.

Second, considering the response of GDP (and consumption which are equivalent since there is no capital in this model), with sticky prices but flexible wages, GDP falls by about half the amount as it does under sticky wages and prices. Even though the decline in GDP is much smaller under flexible wages and prices, it is persistent with GDP requiring 16 quarters to return to baseline. Financial shocks continue to produce a considerably persistent economic response due to the presence of credit frictions even without slow nominal adjustment.

Third, the biggest impact of nominal rigidities occurs in the labor market. Obviously without wage rigidity, there is no increase in unemployment. The very small decline in employment under sticky prices and flexible wages is generated by a decline in labor force participation rather than an increase in unemployment. When prices or wages are flexible, the Nash bargained real marginal cost and real wage adjust freely to the shock, reducing any impact on the labor market. However because of the cost channel, i.e., since the wage bill is financed, there is still an impact on labor demand, albeit small.

Notably, the presence of credit frictions in all three cases causes total factor productivity to decline by the same amount on impact with and without wage rigidities. This is due to the credit inefficiency wedge,

F_t in $TFP_t = \xi^{pf} A_t (F_t)^{1-\alpha}$. The decline in TFP is also persistent even without nominal rigidities though larger with wage rigidities due to the direct influence of wage setting behavior on labor productivity. After a financial shock, the fall in the marginal cost of labor is more pronounced in the case of wage and price rigidities. Therefore, the intensive margin effect, which offsets the extensive-margin-reduction in matched firm-bank pairs (in the exogenous separation version of the financial shock), is stronger in the case of price and/or wage flexibility as wages and prices are able to adjust downward.

7 Conclusion

The Great Recession and slow recovery was characterized by high and persistent unemployment and a decline in overall bank lending. The net decline in bank lending across all loan types was a novel feature of the Great Recession as it had not occurred in any previous post-Volker recession. These characteristics of the recession are suggestive of a relationship between bank credit and unemployment. We link credit flows to output and employment indirectly through search and matching frictions in the market for credit, embedded in an otherwise standard New Keynesian framework with wage and pricing frictions. We allow for endogenous credit destruction which then permits us to calculate movements in gross credit flows and match them to empirical credit behavior. Our model generates a 'credit inefficiency wedge' that arises as an endogenous component of aggregate technology unrelated to employment and acts like a productivity wedge. In the presence of credit frictions, the inefficiency wedge depends on the aggregate probability that credit contracts continue as well as on the number of active firms. Both of these factors depend ultimately on the reservation productivity threshold that separates producing from non-producing firms. Thus, in our model, credit tightness serves to amplify the effects of financial shocks on employment and output through the inefficiency wedge.

In this paper we make the following three contributions. First, we document a set of statistical properties of credit and labor markets. Second, in a general equilibrium model with heterogeneous firms, we provide a mechanism for understanding the relationship between credit and unemployment that generates qualitatively realistic movements in unemployment due to shocks in the credit market. We model credit market shocks in two ways, the first is as a shock to the separation rate of firm-bank pairs. In the second we allow bargaining power to be time varying and model financial shocks as shocks to the bargaining power of firms versus banks. We then test the strength of credit market frictions versus nominal frictions in generating our results and find that the impact of credit market frictions continues to be important particularly in terms of generating persistence effects of credit shocks even when assumptions about wage and price stickiness are removed. Third, our analysis of the impact on employment of financial shocks provides insights into movements in total factor productivity, labor productivity, and the productivity of individual firms.

The mechanisms in this paper–movements in credit market tightness and the productivity cutoff level–

that relate credit to unemployment, productivity and output in our model generate an intensive and extensive margin effect. Conditional on surviving, each credit contract will determine the loan principal and number of workers to hire consistent with the point where the marginal product of labor is equal to the Nash bargained real marginal cost of labor expressed in terms of the intermediate good. Changes in this relationship generate an intensive margin effect since it holds only for those credit matches that have survived the exogenous as well as the endogenous separation hazards. In this case, the subset of surviving firms will modify their hiring decisions until equilibrium is restored. Setting the joint surplus for a credit contract to zero provides us with the equilibrium reservation productivity, which is a productivity threshold that determines the subset of firms able to obtain funds, hire workers, and produce during the period. This threshold productivity generates an extensive margin effect whenever it responds to aggregate macroeconomic shocks. It is determined primarily by marginal costs and tightness in the credit market. The main transmission mechanism of aggregate shocks in this economy occurs through changes in the threshold or reservation productivity which ultimately changes as a result of movements in the joint surplus of a credit match. For an exogenous separation shock, the intensive and extensive margin effects move in opposite directions: the intensive margin effect (firms that are able to match with banks and produce, hire more workers) reduces the impact of the shock on employment. But the extensive margin is much larger and outweighs this effect. By contrast, when we model financial shocks as shocks to the firms' bargaining power, the intensive and extensive margin effects move in the same direction, amplifying the effect of the shock on productivity and employment.

As noted by Galí (2011) and discussed extensively in relationship to the volatility puzzle, quantitatively realistic labor market frictions are unlikely to have a large effect on output, employment, and productivity–a result we also find for credit market frictions in the absence of nominal rigidities. But we find, as Galí suggests for labor market frictions, that credit market frictions provide a mechanism for wage and price rigidities to play a role in the propagation and persistence of financial and monetary policy shocks. On their own, credit frictions create significant persistence in lost output and a significant decline in credit creation as well as a smaller persistent decline in total factor productivity.

The mechanisms we uncover in this paper relating credit reallocation to employment are suggestive for the work on labour reallocation. For instance Decker, Haltiwanger, Jarmin, and Miranda (2017) suggest that the dampening of job churn and reduction in firm entry in the past two decades has important implications for aggregate productivity. Although our paper provides a step in understanding the way in which links between credit dynamics and employment exacerbate financial shocks, there are several ways in which we could extend it to capture additional features of the economy such as firm default. Undoubtedly part of the reluctance of banks to lend was related to increases in default risk that accompanied the Great Recession and weak lending growth in the ensuing years. Firm heterogeneity in terms of size or age is also an important factor to consider when studying the mechanisms involved in the propagation of financial disturbances. These issues are left for future work.

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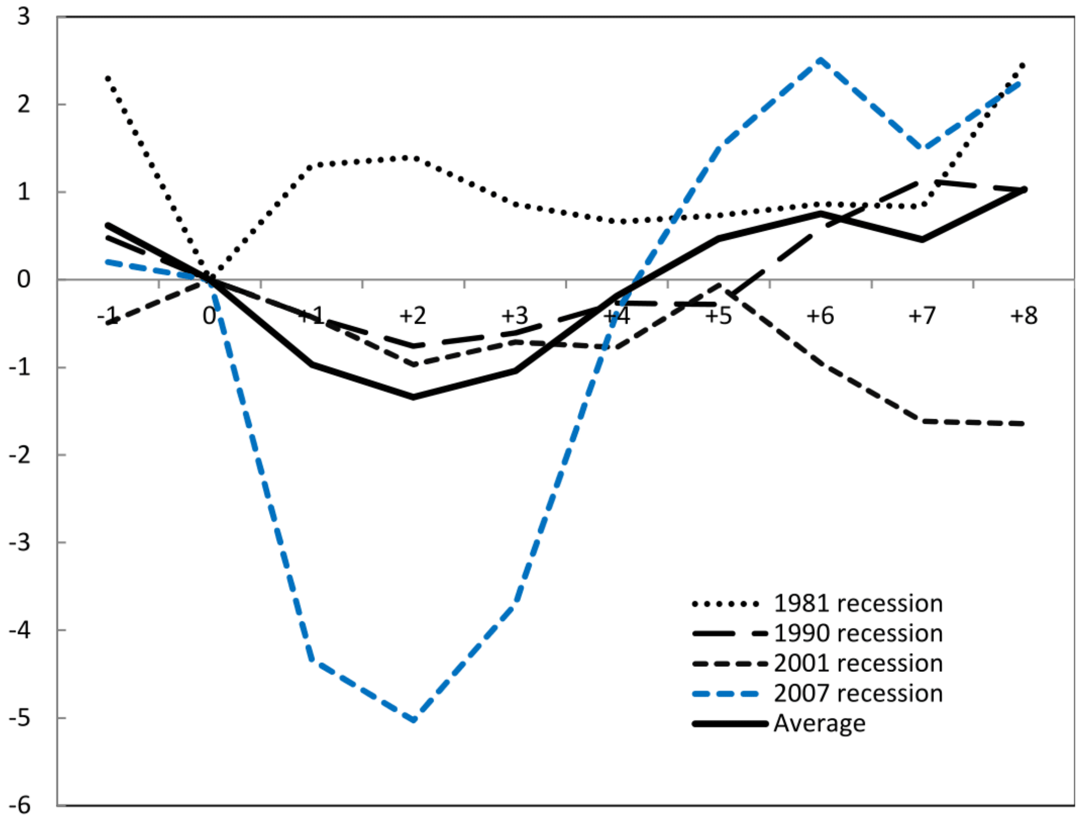
8 Appendix A: Tables and Figures

Table 5: Descriptive statistics for credit and employment

Period	1973Q1-2017Q1		1984Q1-2007Q2		2007Q3-2009Q2		2009Q3-2017Q1	
	Mean	SD	Mean	SD	Mean	SD	Mean	SD
Total loan creation	3.79	1.45	4.40	1.30	3.41	1.37	2.34	1.34
Total loan destruction	1.31	0.76	1.70	0.61	1.77	0.83	0.94	0.73
C&I loan creation	3.68	1.27	4.04	1.13	3.92	1.94	2.38	0.96
C&I loan destruction	1.53	0.94	1.92	0.73	2.17	1.18	1.28	1.10
Sum C&I lending	5.21	1.48	5.95	1.10	6.09	1.75	3.66	1.14
Net C&I lending	2.15	1.70	2.12	1.55	1.76	2.44	1.10	1.69
EXC C&I lending	2.87	1.62	3.75	1.37	3.44	1.68	1.99	0.88
Credit spread (BAA-AAA)	1.10	0.45	0.94	0.27	1.82	0.87	1.02	0.24
Excess bond premium*	0.04	0.49	0.0005	0.45	1.10	0.97	-0.13	0.24
Unemployment rate	6.39	1.56	5.71	1.04	6.28	1.72	7.27	1.83
Growth average labor productivity	0.37	0.60	0.45	0.54	0.83	0.72	0.13	0.48
Job creation	4.69	0.92	4.76	0.57	3.18	0.40	3.54	0.38
Job destruction	5.06	1.18	5.14	0.69	5.40	1.51	3.44	0.47
Sum JC + JD	9.75	1.78	9.91	1.01	8.58	1.18	6.98	0.56
Net JC-JD	-0.37	1.14	-0.39	0.77	-2.23	1.87	0.10	0.64
EXC job creation	8.94	1.64	9.30	1.01	6.35	0.80	6.46	0.59

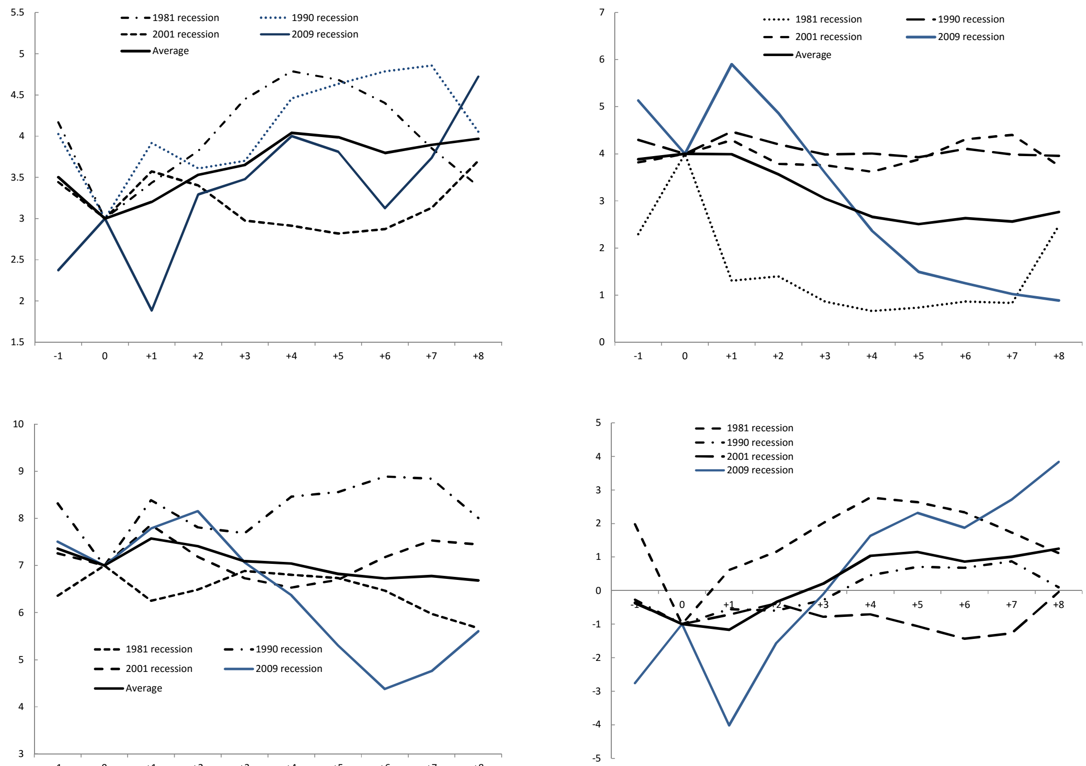
Note: Lending data is based on Reports of Income and Condition and calculations provided in Contessi, DiCecio, and Francis (2015). Job flows data are taken from Faberman (2012) and updated with the quarterly Business Employment Dynamics Surveys. The means and standard deviations of creation or destruction of credit or jobs are expressed in rates (percentage). The unemployment rate is seasonally adjusted and downloaded from the FRED data repository at the Federal Reserve, St Louis. Average Labor productivity growth is calculated from real GDP (seasonally adjusted) and hours of non-farm business employees and expressed in percentage terms. These data are also downloaded from FRED. SUM = Creation + Destruction, NET = Creation-Destruction, EXC = SUM - ABS(NET).

Figure 1: Net credit flows: Commercial lending



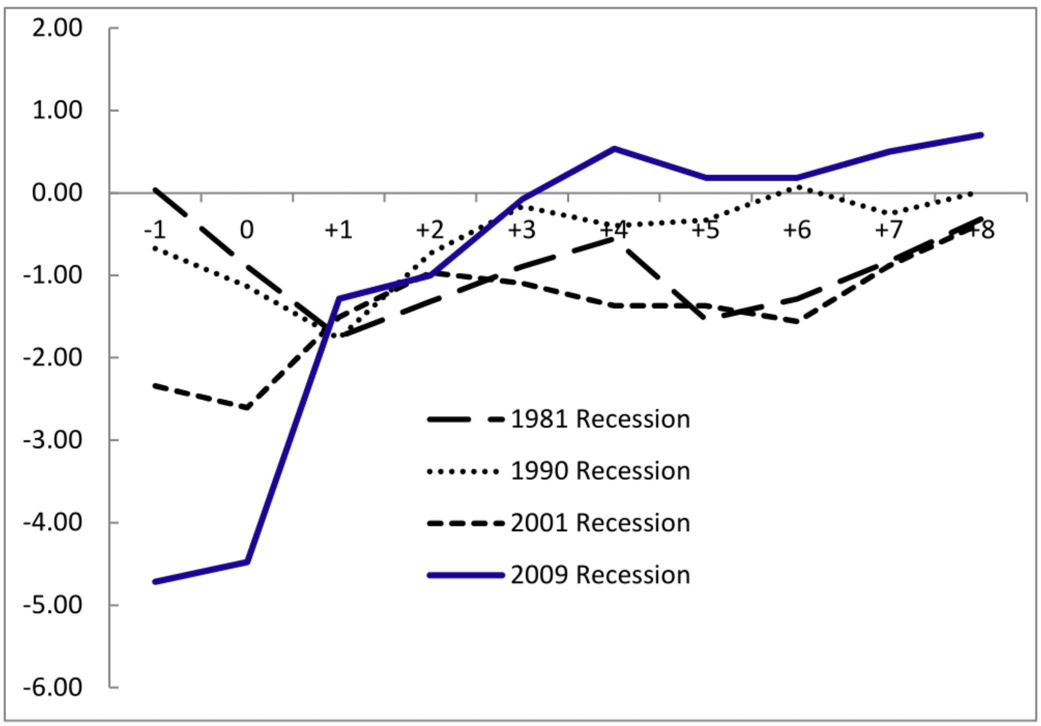
In percent; quarters from trough (located at 0). Net credit creation in commercial and industrial lending. Net credit creation is seasonally adjusted and smoothed with a moving average process. Authors' calculations based on Reports of Income and Condition.

Figure 2: Credit flows and reallocation: Commercial lending



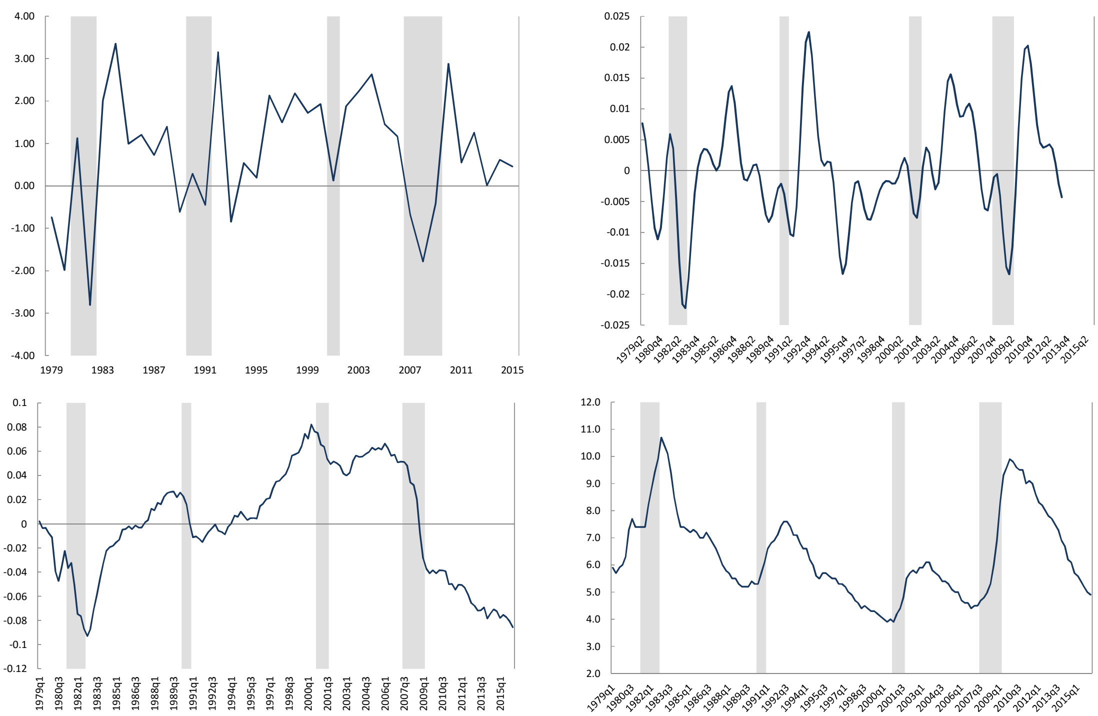
Top row: credit creation (NBER dated recession trough is at mean equal to 3 percent) and credit destruction (trough is at mean equal to 4 percent) in commercial and industrial lending. Bottom row: credit reallocation (which equals the sum of credit creation and destruction; trough is at mean equal to 7 percent) and excess credit reallocation (which equals the sum of credit creation and destruction less the absolute value of net credit creation; trough is at mean equal to -1 percent). All data are reported in percent, seasonally adjusted and smoothed with a moving average process. Authors' calculations based on Reports of Income and Condition.

Figure 3: Net job creation last two recessions



In percent; quarters from trough (located at 0). Net job creation in manufacturing. Authors' calculations based on data from Faberman (2012) and updated with recent Business Employment Dynamics data from the U.S. Census.

Figure 4: Aggregate data



Top row, from left, Total Factor Productivity (Based on data from Fernald (2012) update downloaded from http://www.frbsf.org/economic-research/indicators-data/total-factor-productivity-tfp/. Annual percentage change reported for business sector TFP.) and seasonally adjusted quarterly average labor productivity filtered with a Baxter King filter. Bottom row, from left, detrended logged seasonally adjusted quarterly real GDP and the unemployment rate. All data from the Federal Reserve, St Louis FRED data repository except for TFP as noted.

Figure 5: Financial shock modeled as exogenous separation shock

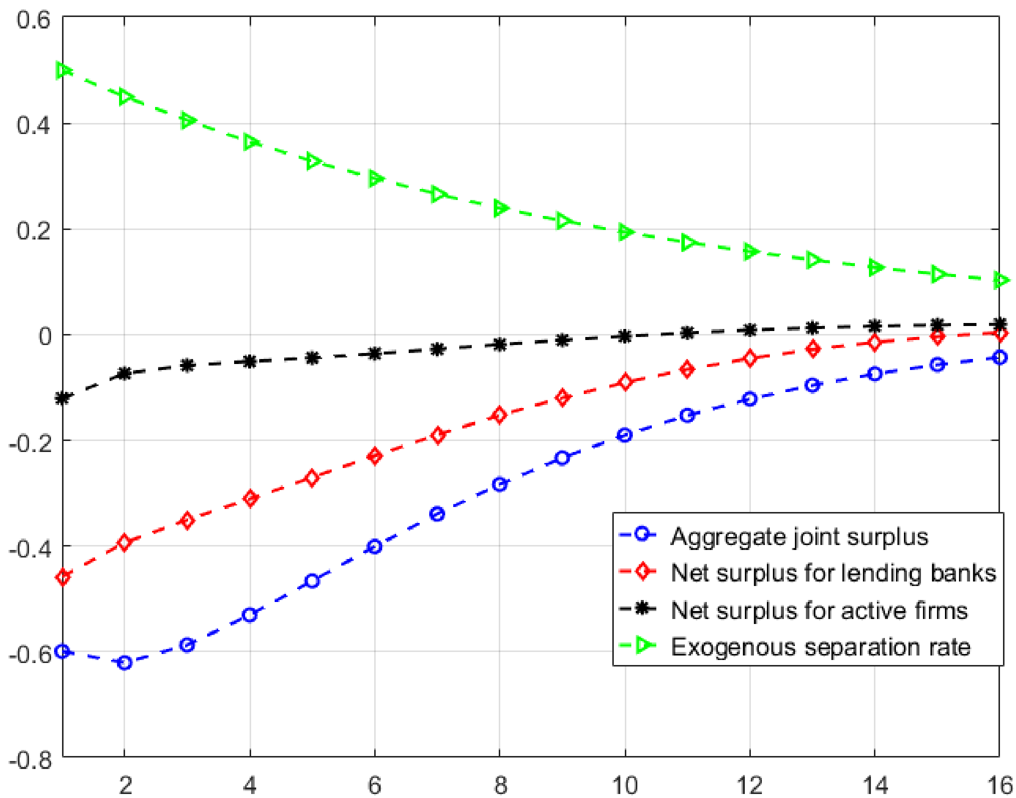


Figure 6: Model responses to a exogenous separation shock, panel 1

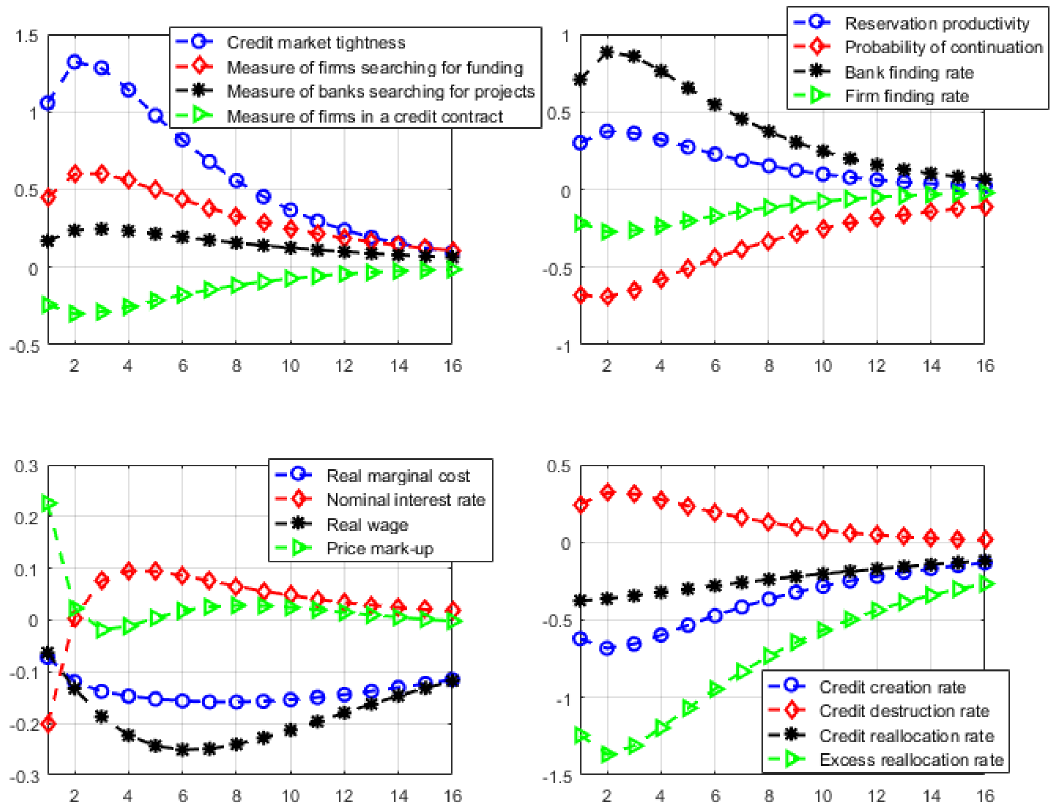


Figure 7: Model responses to a exogenous separation shock: panel 2

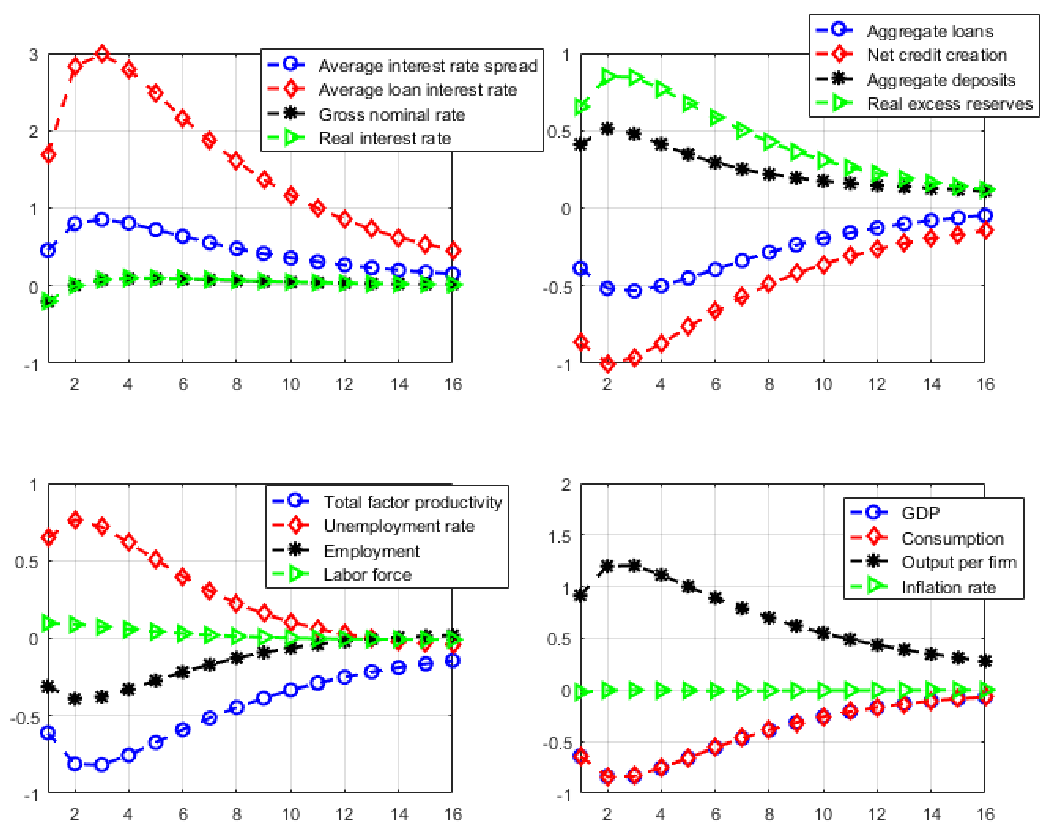


Figure 8: Financial shock modeled as bargaining shock

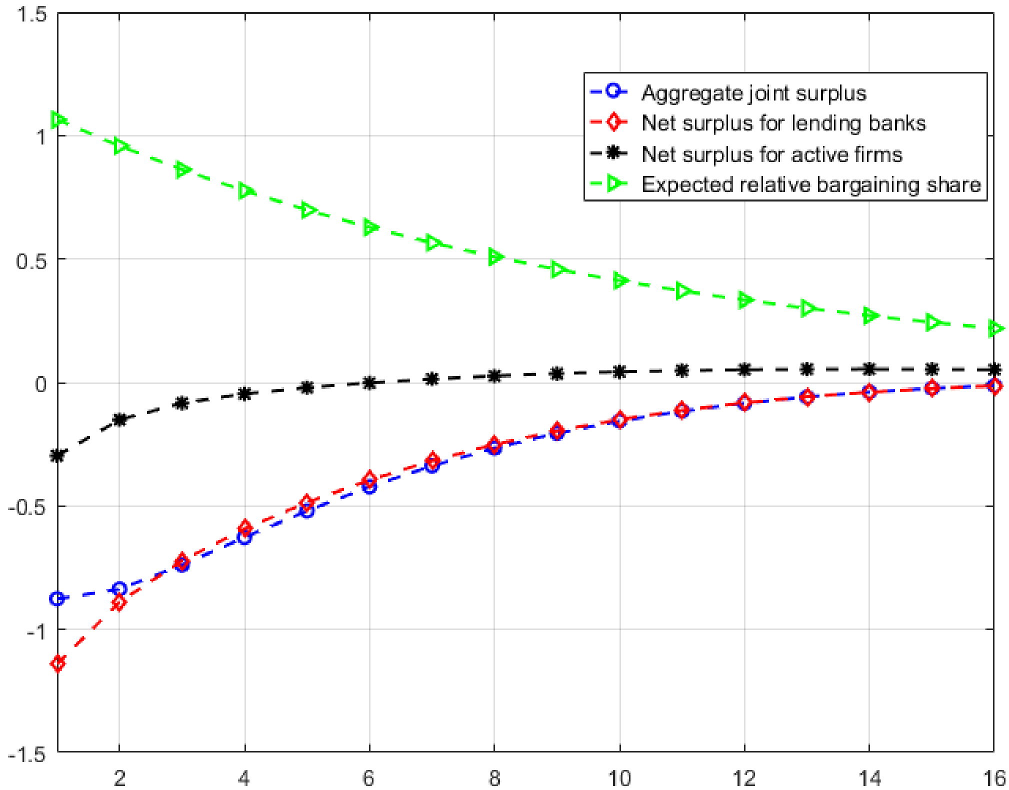


Figure 9: Model responses to bargaining shock, panel 1

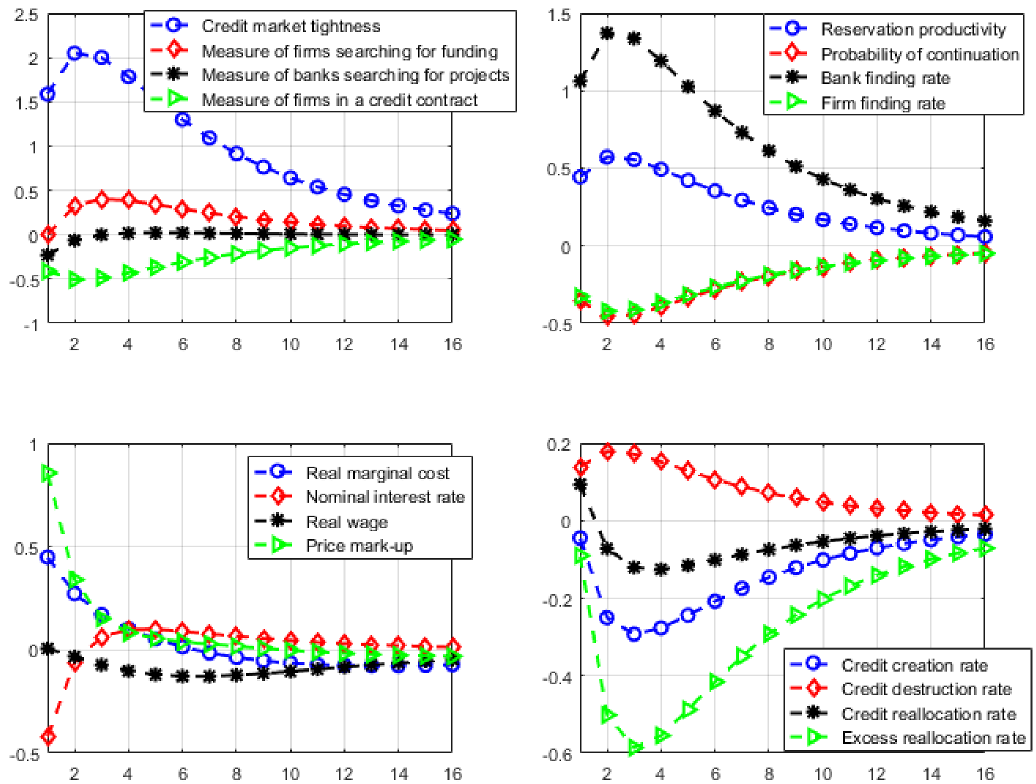


Figure 10: Model responses to bargaining shock, panel 2

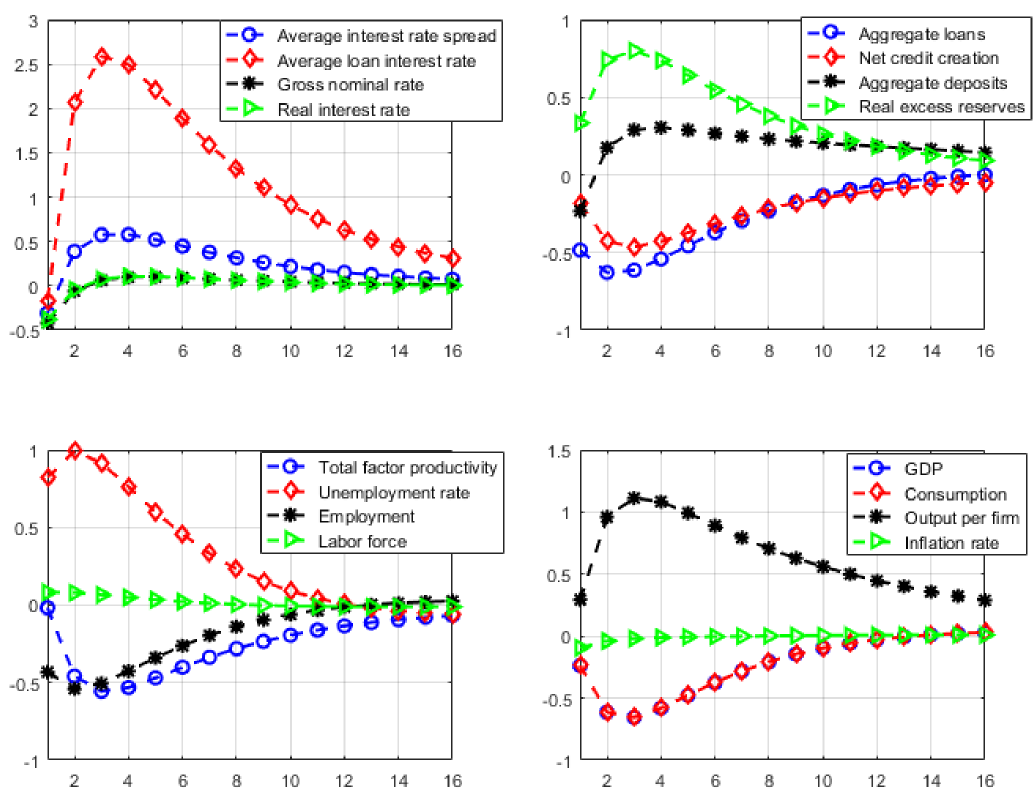


Figure 11: Model responses to a financial shock: The role of nominal rigidities, panel 1

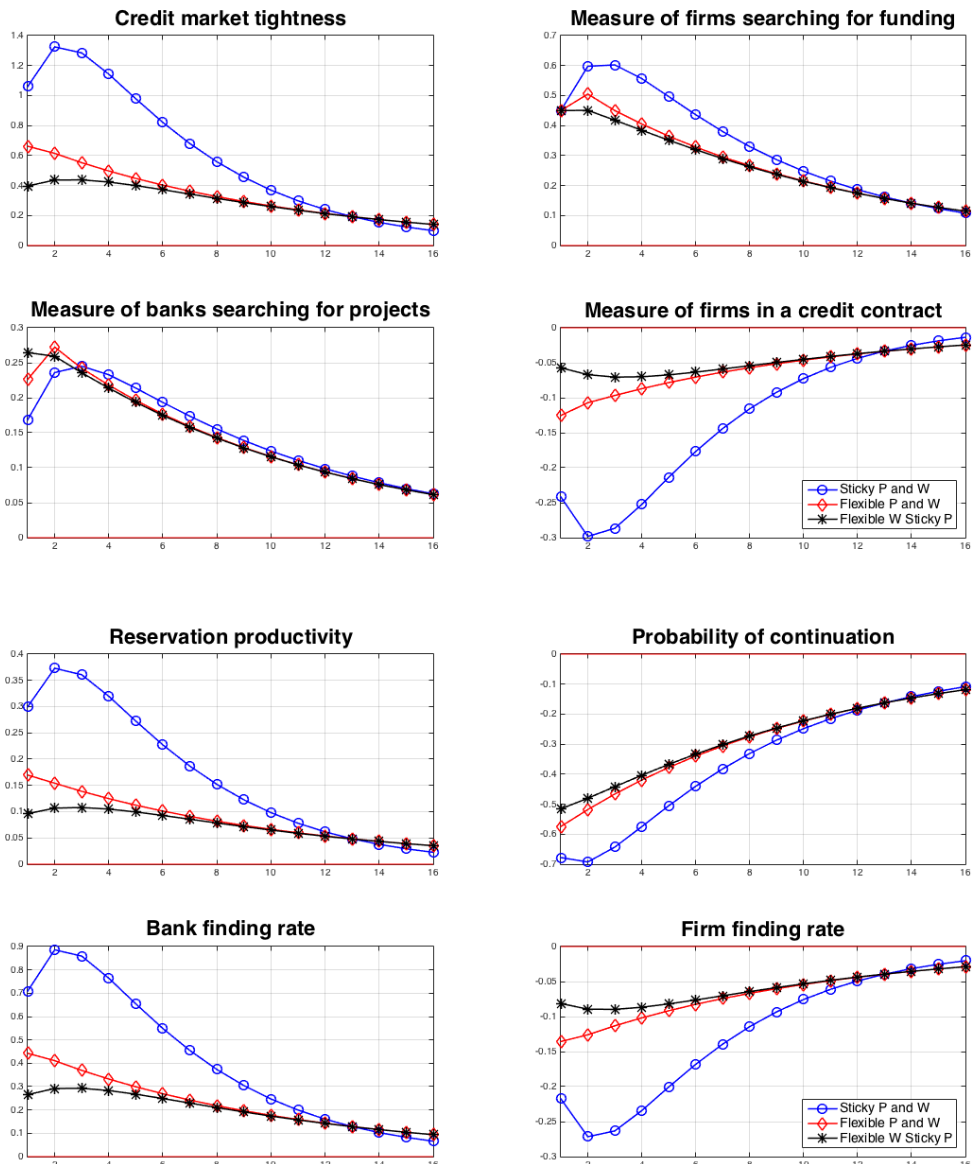


Figure 12: Model responses to a financial shock: The role of nominal rigidities, panel 2

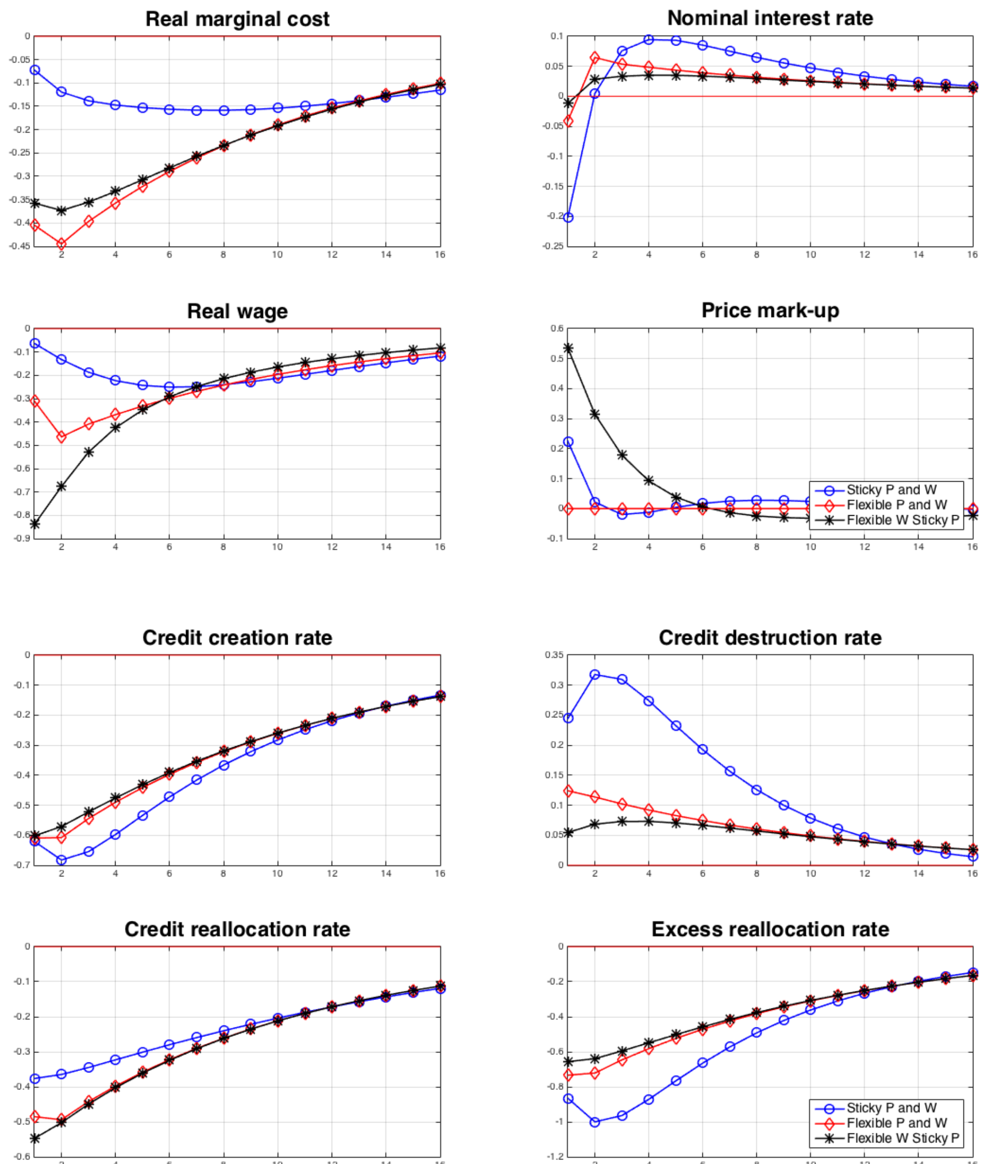


Figure 13: Model responses to a financial shock: The role of nominal rigidities, panel 3

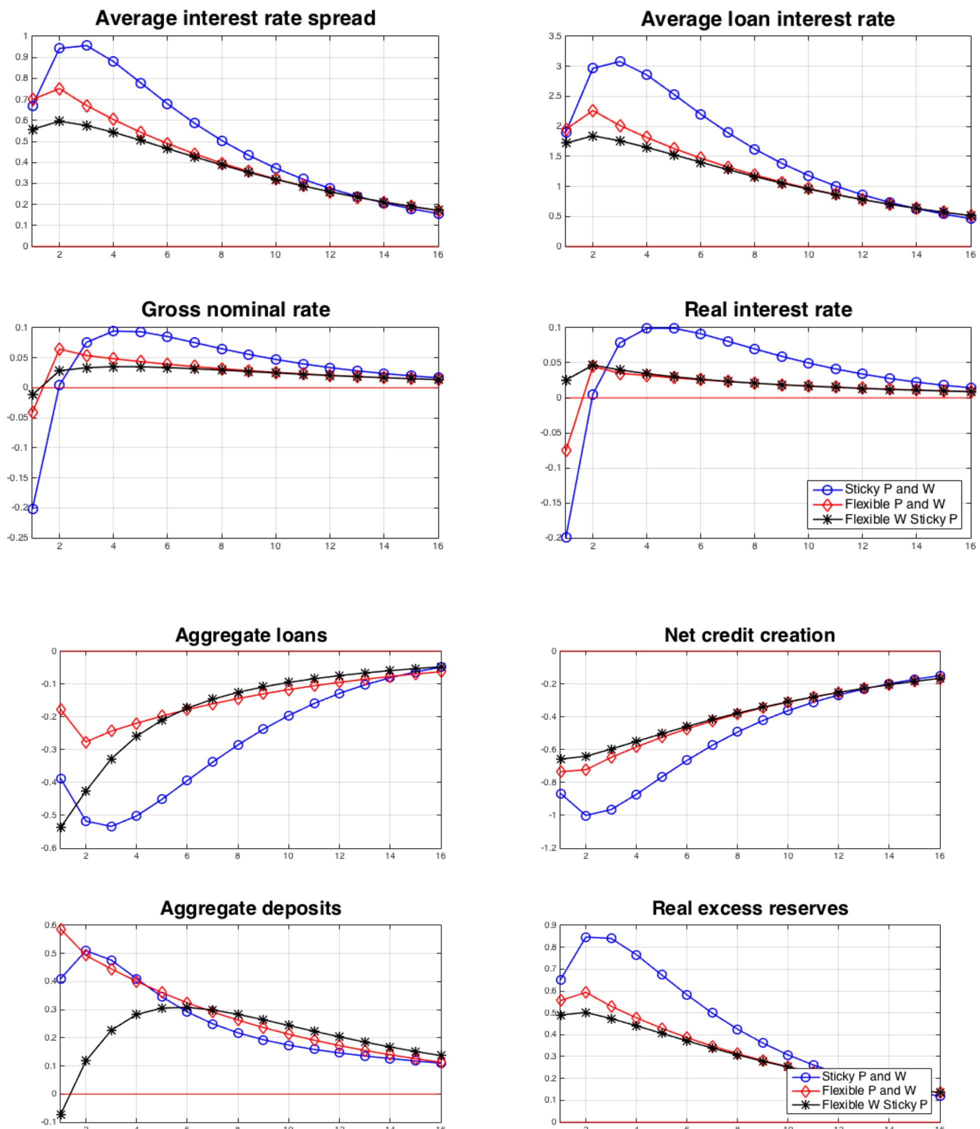
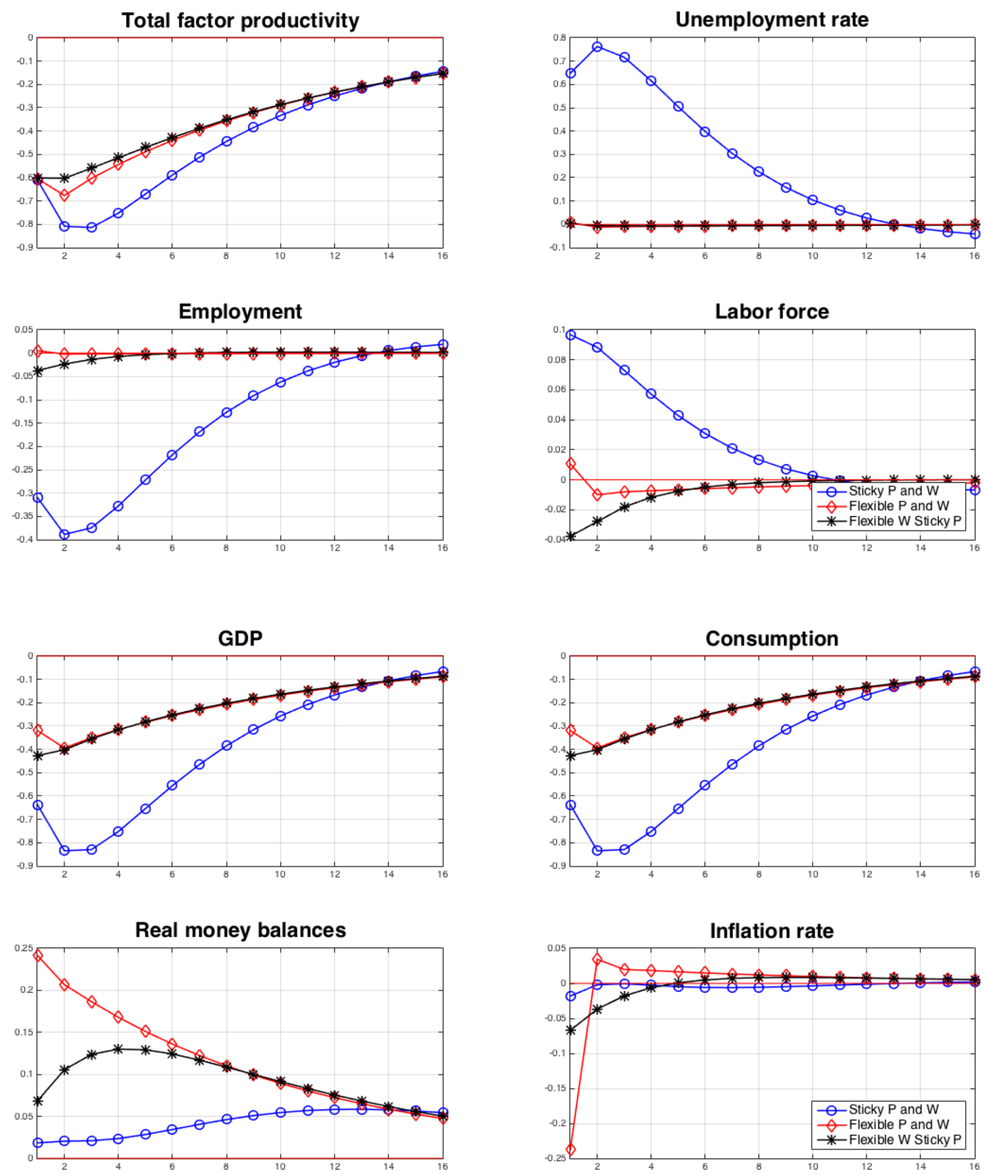


Figure 14: Model responses to a financial shock: The role of nominal rigidities, panel 4



9 On-line Appendix B: Technical details

9.1 Household problem

Aggregate household consumption is given by the standard CES aggregator:

$$C_t = \left(\int_0^1 C_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{95}$$

Optimization over consumption by households implies the following demand schedule for each differentiated good j, $C_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} C_t$ such that $\int_0^1 P_t(j) C_t(j) dj = P_t C_t$, where P_t is the final goods price index (i.e, the retail price index) and is determined as $P_t = \left(\int_0^1 P_t(j)^{1-\epsilon_p} dj \right)^{\frac{1}{1-\epsilon_p}}$.

9.1.1 Wage setting first order conditions

The first-order condition associated to the wage setting problem is given by

$$E_t \sum_{k=0}^{\infty} (\beta\theta_w)^k \left\{ \left(\frac{W_t^*}{P_{t+k}} - \frac{\varepsilon_w}{(\varepsilon_w - 1)} MRS_{t+k|t} \right) \left(\frac{N_{t+k|t}}{C_{t+k}} \right) \right\} = 0 \tag{96}$$

where $MRS_{t+k|t}$ denotes the marginal rate of substitution between consumption and employment for a type i worker whose wage is reset during period t, given by $MRS_{t+k|t} = C_{t+k}\chi_{t+k} \left(N_{t+k|t}\right)^{\overline{\varphi}}$. Note that the productivity distribution of producing firms will influence the wage setting equation and therefore the real wage. Under Calvo wage setting, the evolution of the aggregate real wage index,w_t, satisfies the following equation:

$$w_t^{1-\epsilon_w} = \theta_w \left(w_{t-1} \frac{1}{\Pi_t} \right)^{1-\epsilon_w} + (1 - \theta_w) (w_t^*)^{1-\epsilon_w} \tag{97}$$

The recursive formulation of the wage setting optimality condition, expressed in terms of the average real wage of the economy, w_t, is given by the following set of three equations:

$$f_{1,t} = \left(\frac{\varepsilon_w}{\varepsilon_w - 1} \right) f_{2,t} \tag{98}$$

$$f_{1,t} = (w_t^*)^{1-\varepsilon_w} (w_t)^{\varepsilon_w} \frac{N_t}{C_t \Delta_t^w} + \beta\theta_w E_t \left(\frac{1}{\Pi_{t+1}} \right)^{1-\varepsilon_w} \left(\frac{w_t^*}{w_{t+1}^*} \right)^{1-\varepsilon_w} f_{1,t+1} \tag{99}$$

$$f_{2,t} = \chi_t \left(\frac{w_t^*}{w_t} \right)^{-\varepsilon_w(1+\overline{\varphi})} \left(\frac{N_t}{\Delta_t^w} \right)^{(1+\overline{\varphi})} + (\beta\theta_w) E_t \left(\frac{1}{\Pi_{t+1}} \right)^{-\varepsilon_w(1+\overline{\varphi})} \left(\frac{w_t^*}{w_{t+1}^*} \right)^{-\varepsilon_w(1+\overline{\varphi})} f_{2,t+1} \tag{100}$$

where w_t^* is the optimal real wage, $\Pi_t = 1 + \pi_t$ is the gross inflation rate, $f_{1,t}$ and $f_{2,t}$ are auxiliary variables and Δ_t^W is the wage dispersion index given by $\Delta_t^W = \int\limits_0^1 \left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} dj$ which can be written recursively as:

$$\Delta_t^w = \theta_w \left(\frac{w_{t-1}}{w_t}\frac{1}{\Pi_t}\right)^{-\epsilon_w} \Delta_{t-1}^w + (1-\theta_w)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w}. \tag{101}$$

9.2 Production

9.2.1 Final good producers price setting

The resulting first order condition for the price setting problem of the final goods producers implies,

$$E_t \sum_{k=0}^{\infty} (\theta_p)^k \Delta_{t,t+k} Y_{t+k|t}^f \left\{ \frac{P_t^*}{P_{t+k}} - \frac{\epsilon_p}{\epsilon_p - 1}\frac{P_{t+k|t}^I}{P_{t+k}} \right\} = 0 \tag{102}$$

Final good producers obtain nominal profits at the end of the period of $\Pi_t^f(j) = P_t(j) Y_t^f(j) - P_t^l X_t(j)$. The monopolistic structure together with Calvo nominal price rigidites implies the following aggregate price index P_t for the final good:

$$P_t^{1-\epsilon_p} = \theta_p (P_{t-1})^{1-\epsilon_p} + (1-\theta_p)(P_t^*)^{1-\epsilon_p} \tag{103}$$

The recursive formulation of the optimal price setting equation is

$$g_{1,t} = \left(\frac{\epsilon_p}{\epsilon_p - 1}\right) g_{2,t} \tag{104}$$

$$g_{1,t} = \beta C_t \Pi_t^* + \theta_p E_t \left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right) g_{1,t+1} \tag{105}$$

$$g_{2,t} = \beta \frac{1}{\mu_t^p} C_t + \theta_p E_t g_{2,t+1} \tag{106}$$

where $\Pi_t^* = \frac{P_t^*}{P_t}$, $\mu_t^p = \dfrac{P_t}{P_t^l}$ is the mark-up of final good over intermediate good prices, $g_{1,t}$ and $g_{2,t}$ are auxiliary variables. Notice that up to a first order approximation the above three equations imply that deviations of the inflation rate with respect to steady state inflation are inversely related to deviations of the the mark-up with respect to its steady state.

9.3 Value functions for credit search problem

9.3.1 Intermediate good firms

The value function for a producing intermediate good firm during period t is

$$V_t^{FP}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{\delta_t V_{t+1}^{FN} + (1-\delta_t)\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN})dG(\omega)\right\} \tag{107}$$

The value of producing is the flow value of current real profits plus the expected continuation value. At the end of the period, the credit relationship is exogenously dissolved with probability δ_t, and the firm must seek new financing. With probability $(1-\delta_t)$, the firm survives the exogenous separation hazard. In the latter case, only those firms receiving an idiosyncratic productivity realization $\omega_{z,t+1} \geq \tilde{\omega}_{t+1}$ will remain matched and produce during next period. Firms with $\omega_{z,t+1} < \tilde{\omega}_{t+1}$ endogenously separate from their bank and obtain V_{t+1}^{FN}.

The value of searching for external funds at date t is

$$V_t^{FN} = p_t^f \mathrm{E}_t\Delta_{t,t+1}\left[\delta_t V_{t+1}^{FN} + (1-\delta_t)\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN})dG(\omega)\right] + \left(1-p_t^f\right)V_{t+1}^{FN} \tag{108}$$

where p_t^f is the probability of matching with a bank. Notice that we assume matches made in period t do not produce until $t+1$. With probability $(1-p_t^f)$, the firm does not match and must continue searching for external funds during next period's loan market.

Under Nash bargaining, the reservation productivity level $\tilde{\omega}_t$ that triggers endogenous separation is determined by the point at which the joint surplus of the match is equal to zero. The probability of endogenous separation is $\gamma_{t+1}(\tilde{\omega}_{t+1}) = G(\tilde{\omega}_{t+1}) = prob\,(\omega_{z,t+1} \leq \tilde{\omega}_{t+1})$. Given the existence and uniqueness of $\tilde{\omega}_{t+1}$, the integral term on the expected continuation value is

$$\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}), V_{t+1}^{FN})dG(\omega) = \gamma_{t+1}V_{t+1}^{FN} + (1-\gamma_{t+1}(\tilde{\omega}_{t+1}))\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}. \tag{109}$$

Therefore, the firm value functions can be written as

$$V_t^{FP}(\omega_{z,t}) = \pi_t^I(\omega_{z,t}) + \mathrm{E}_t\Delta_{t,t+1}\left\{(1-\varphi_t(\tilde{\omega}_{t+1}))V_{t+1}^{FN} + \varphi_t(\tilde{\omega}_{t+1})\int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}(\tilde{\omega}_{t+1})}\right\}$$

and

$$V_t^{FN} = \mathrm{E}_t \Delta_{t,t+1} \left\{ p_t^f \left[\left(1 - \varphi_t\left(\tilde{\omega}_{t+1}\right)\right) V_{t+1}^{FN} + \varphi_t\left(\tilde{\omega}_{t+1}\right) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FP}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}\left(\tilde{\omega}_{t+1}\right)} \right] + \left(1 - p_t^f\right) V_{t+1}^{FN} \right\} \tag{110}$$

Let the surplus to a producing firm be defined as $V_t^{FS}(\omega_{z,t}) = V_t^{FP}(\omega_{z,t}) - V_t^{FN}$, then the intermediate producer surplus of being in a credit relationship can be written as

$$V_t^{FS}(\omega_{z,t}) = \pi_t^I\left(\omega_{z,t}\right) + \left(1 - p_t^f\right) \mathrm{E}_t \Delta_{t,t+1} \varphi_t\left(\tilde{\omega}_{t+1}\right) \int_{\tilde{\omega}_{t+1}}^{\overline{\omega}} V_{t+1}^{FS}(\omega_{z,t+1}) \frac{dG(\omega)}{1 - \gamma_{t+1}\left(\tilde{\omega}_{t+1}\right)} \tag{111}$$

9.3.2 Banks

The value of lending is:

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b\left(\omega_{z,t}\right) + \mathbb{E}_t \Delta_{t,t+1} \left\{ \delta_{t+1} V_{t+1}^{BN} + \left(1 - \delta_{t+1}\right) \int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{BL}(\omega_{z,t+1}), V_{t+1}^{BN}) dG(\omega) \right\}$$

and the value of searching (screening) for projects is:

$$V_t^{BN} = -\frac{\kappa_t}{\mu_t^p} + \mathbb{E}_t \Delta_{t,t+1} \left\{ \begin{array}{c} p_t^b \left[\delta_{t+1} V_{t+1}^{BN} + \left(1 - \delta_{t+1}\right) \int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{BL}(\omega_{z,t+1}), V_{t+1}^{BN}) dG(\omega) \right] \\ + \left(1 - p_t^b\right) V_{t+1}^{BN} \end{array} \right\}$$

9.3.3 Free entry condition

- The free entry condition is given by:

$$V_t^{BN} = 0 \quad \forall t$$

- Banks will enter the lending market up to the point where the expected cost of finding a borrower (finding a profitable project to fund) is equal to the expected profit of extending a loan to a firm given that the potential credit contract survived the exogenous separation hazard. Therefore:

$$\frac{\kappa_t}{\mu_t^p p_t^b} = \mathbb{E}_t \Delta_{t,t+1} \left(1 - \delta_{t+1}\right) \int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{BL}(\omega_{z,t+1}), 0) dG(\omega)$$

- The value of lending under free entry:

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \underbrace{\mathbb{E}_t\Delta_{t,t+1}(1-\delta_{t+1})\int_{\underline{\omega}}^{\overline{\omega}}\max(V_{t+1}^{BL}(\omega_{z,t+1}),0)dG(\omega)}_{\frac{\kappa_t}{\mu_t^p p_t^b}}$$

$$V_t^{BL}(\omega_{z,t}) = \pi_t^b(\omega_{z,t}) + \frac{\kappa_t}{\mu_t^p p_t^b}$$

- **Net surplus to bank for extending a loan:**

$$\begin{aligned} V_t^{BS}(\omega_{z,t}) = & \quad V_t^{BL}(\omega_{z,t}) - V_t^{BN} \\ = & \quad \pi_t^b(\omega_{z,t}) + \frac{\kappa_t}{\mu_t^p p_t^b} \end{aligned}$$

9.3.4 Joint surplus of a credit contract

The joint surplus for a credit contract is

$$\begin{aligned} V_t^{JS}(\omega_{zt}) = & \quad V_t^{BS}(\omega_{zt}) + V_t^{FS}(\omega_{zt}) \\ = & \quad \pi_t^b(\omega_{zt}) + \frac{\kappa_t}{\mu_t^p p_t^b} + \pi_t^I(\omega_{zt}) \\ & + \left(1-p_t^f\right)\mathbb{E}_t\Delta_{t,t+1}(1-\delta_{t+1})\int_{\underline{\omega}}^{\overline{\omega}}\max(V_{t+1}^{FP}(\omega_{zt+1}) - V_{t+1}^{FN},0)dG(\omega) \end{aligned}$$

where:

$$\pi_t^I(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t^l(\omega_{z,t})l_t(j,\omega_{z,t})$$

and

$$\pi_t^b(\omega_{it}) = \left(R_t^l(j,\omega_{z,t}) - R_t\right)l_t(j,\omega_{z,t})$$

thus

$$\pi_t^b(\omega_{zt}) + \pi_t^I(\omega_{zt}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t^l(\omega_{z,t})l_t(j,\omega_{z,t}) + \left(R_t^l(j,\omega_{z,t}) - R_t\right)l_t(j,\omega_{z,t})$$

$$\pi_t^b(\omega_{zt}) + \pi_t^I(\omega_{zt}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j,\omega_{z,t})$$

and the joint surplus can be written as:

$$V_t^{JS}(\omega_{zt}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j,\omega_{z,t}) + \frac{\kappa_t}{\mu_t^p p_t^b} + \left(1 - p_t^f\right) \mathbb{E}_t \Delta_{t,t+1} \left(1 - \delta_{t+1}\right) \int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{zt+1}) - V_{t+1}^{FN}, 0) dG(\omega)$$

Since there exists a unique cutoff value for the idiosyncratic productivity given by ω_{t+1}^* then the integral term in the above equation can be written as:

$$\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}, 0) dG(\omega)$$

$$= \underbrace{\int_{\underline{\omega}}^{\omega_{z,t+1}^*} \underbrace{\max(\underbrace{V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}}_{<0}, 0) dG(\omega)}_{=0}}_{=0} + \int_{\omega_{t+1}^*}^{\overline{\omega}} \underbrace{\max(\underbrace{V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}}_{>0}, 0) dG(\omega)}_{V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}}$$

thus

$$\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}, 0) dG(\omega) = \int_{\omega_{t+1}^*}^{\overline{\omega}} \left(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}\right) dG(\omega)$$

Since the above integral is a truncated expectation, then it can be written as:

$$\int_{\underline{\omega}}^{\overline{\omega}} \max(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}, 0) dG(\omega) = \left(1 - \gamma_{t+1}\left(\omega_{z,t+1}^*\right)\right) \int_{\omega_{t+1}^*}^{\overline{\omega}} \left(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}\right) \frac{dG(\omega)}{1 - \gamma_{t+1}\left(\omega_{t+1}^*\right)}$$

The joint surplus can be written as:

$$V_t^{JS}(\omega_{zt}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j,\omega_{z,t}) + \frac{\kappa_t}{\mu_t^p p_t^b} + \left(1 - p_t^f\right) \mathbb{E}_t \Delta_{t,t+1} \left(1 - \delta_{t+1}\right) \int_{\omega_{t+1}^*}^{\overline{\omega}} \left(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}\right) dG(\omega)$$

or

$$V_t^{JS}(\omega_{zt}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j,\omega_{z,t}) + \frac{\kappa_t}{\mu_t^p p_t^b} + \left(1 - p_t^f\right) \mathbb{E}_t \Delta_{t,t+1} \varphi_{t+1}\left(\omega_{t+1}^*\right) \int_{\omega_{t+1}^*}^{\overline{\omega}} \left(V_{t+1}^{FP}(\omega_{z,t+1}) - V_{t+1}^{FN}\right) \frac{dG(\omega)}{1 - \gamma_{t+1}\left(\omega_{t+1}^*\right)}$$

where:

$$\varphi_{t+1}\left(\omega_{t+1}^{*}\right)=\left(1-\delta_{t+1}\right)\left(1-\gamma_{t+1}\left(\omega_{t+1}^{*}\right)\right)$$

Notice that the net surplus to a firm can be written as:

$$V_t^{FS}(\omega_{zt})=\pi_t^I\left(\omega_{zt}\right)+\left(1-p_t^f\right)\mathbb{E}_t\Delta_{t,t+1}\left(1-\delta_{t+1}\right)\int_{\omega_{t+1}^{*}}^{\overline{\omega}}\left(V_{t+1}^{FP}(\omega_{z,t+1})-V_{t+1}^{FN}\right)dG(\omega)$$

or

$$V_t^{FS}(\omega_{zt})=\pi_t^I\left(\omega_{zt}\right)+\left(1-p_t^f\right)\mathbb{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\omega_{t+1}^{*}\right)\int_{\omega_{t+1}^{*}}^{\overline{\omega}}\left(V_{t+1}^{FP}(\omega_{z,t+1})-V_{t+1}^{FN}\right)\frac{dG(\omega)}{1-\gamma_{t+1}\left(\omega_{t+1}^{*}\right)}$$

By applying the same steps as above, it is possible to write the free entry condition in terms of ω_{t+1}^{*} :

$$\frac{\kappa_t}{\mu_t^p p_t^b}=\mathbb{E}_t\Delta_{t,t+1}\left(1-\delta_{t+1}\right)\int_{\omega_{t+1}^{*}}^{\overline{\omega}}V_{t+1}^{BL}(\omega_{z,t+1})dG(\omega)$$

or

$$\frac{\kappa_t}{\mu_t^p p_t^b}=\mathbb{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\omega_{t+1}^{*}\right)\int_{\omega_{t+1}^{*}}^{\overline{\omega}}V_{t+1}^{BL}(\omega_{z,t+1})\frac{dG(\omega)}{1-\gamma_{t+1}\left(\omega_{t+1}^{*}\right)}$$

Characterizing loan market equilibrium Partial equilibrium in the loan market can be characterized by a system of two equations in two unknowns: credit market tightness τ_t and the reservation productivity level $\widetilde{\omega}_t$. The evolution of credit market tightness is obtained by using the free entry condition, the Nash bargaining sharing rule, and the definition of the joint surplus of a credit relationship, and it is given by the following equation:

$$\frac{\kappa}{\mu_t^p\mu\tau_t^{\varphi}}-\mathrm{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)\left(1-\overline{\eta}\mu\tau_{t+1}^{\varphi-1}\right)\frac{\kappa}{\mu_{t+1}^p\mu\tau_{t+1}^{\varphi}}=\left(1-\overline{\eta}\right)\mathrm{E}_t\Delta_{t,t+1}\frac{1}{\mu_{t+1}^p}\left(\left(1-\alpha\right)\frac{Y_{t+1}^I}{f_t^m}-\varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)x^f\right)\tag{112}$$

The second equation is given by the optimal reservation productivity level, $\widetilde{\omega}_t$ written as a function of τ_t:

$$\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\xi^{pf}A_t\widetilde{\omega}_t\right]^{\frac{1}{1-\alpha}}=\left(\mu_t^p w_t R_t\right)^{\frac{\alpha}{1-\alpha}}\left[x^f-\left(\frac{1-\overline{\eta}\mu\tau_t^{\nu-1}}{1-\overline{\eta}}\right)\frac{\kappa}{\mu\tau_t^{\nu}}\right]\tag{113}$$

At the steady state, the equations for τ_t and $\widetilde{\omega}_t$ become

$$\kappa\left(1-\beta\varphi\left(\tilde{\omega}\right)\left(1-\overline{\eta}\mu\tau^{\nu-1}\right)\right)=\mu\tau^{\nu}\left(1-\overline{\eta}\right)\beta\left(\left(1-\alpha\right)\frac{Y^I}{f^m}-\varphi\left(\tilde{\omega}\right)x^f\right)\tag{114}$$

and

$$\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}\xi^{pf}A\widetilde{\omega}\right]^{\frac{1}{1-\alpha}}=\left(\mu^{p}wR\right)^{\frac{\alpha}{1-\alpha}}\left[x^{f}-\frac{\kappa}{1-\overline{\eta}}\left(\frac{1-\overline{\eta}\mu\tau^{\nu-1}}{\mu\tau^{\nu}}\right)\right] \tag{115}$$

respectively.

9.4 Summarizing the non-linear equilibrium conditions

The system of non-linear equations that characterize the aggregate equilibrium of the model economy is: The vector of endogenous variables is summarized as:

$$X_t=\left[X_{1,t},X_{2,t},X_{3,t},X_{4,t},X_{5,t}\right] \tag{116}$$

Euler equation:

$$\frac{1}{C_t}=\beta E_t\left\{\left(\frac{R_t}{\Pi_{t+1}}\right)\frac{1}{C_{t+1}}\right\} \tag{D2}$$

CIA constraint:

$$C_t=m_t-R_t er_t \tag{D3}$$

Wage setting equation:

$$f_{1,t}=\left(\frac{\varepsilon_w}{\varepsilon_w-1}\right)f_{2,t} \tag{D4}$$

$$f_{1,t}=\left(w_t^*\right)^{1-\varepsilon_w}\left(w_t\right)^{\varepsilon_w}\frac{N_t}{C_t\Delta_t^w}+\beta\theta_w E_t\left(\frac{1}{\Pi_{t+1}}\right)^{1-\varepsilon_w}\left(\frac{w_t^*}{w_{t+1}^*}\right)^{1-\varepsilon_w}f_{1,t+1} \tag{D5}$$

$$f_{2,t}=\chi_t\left(\frac{w_t^*}{w_t}\right)^{-\varepsilon_w(1+\overline{\varphi})}\left(\frac{N_t}{\Delta_t^w}\right)^{(1+\overline{\varphi})}+\left(\beta\theta_w\right)E_t\left(\frac{1}{\Pi_{t+1}}\right)^{-\varepsilon_w(1+\overline{\varphi})}\left(\frac{w_t^*}{w_{t+1}^*}\right)^{-\varepsilon_w(1+\overline{\varphi})}f_{2,t+1} \tag{D6}$$

Aggregate wage index in real terms:

$$1=\theta_w\left(\frac{w_{t-1}}{w_t}\frac{1}{\Pi_t}\right)^{1-\epsilon_w}+\left(1-\theta_w\right)\left(\frac{w_t^*}{w_t}\right)^{1-\epsilon_w} \tag{D7}$$

Wage dispersion:

$$\Delta_t^w=\theta_w\left(\frac{w_{t-1}}{w_t}\frac{1}{\Pi_t}\right)^{-\epsilon_w}\Delta_{t-1}^w+\left(1-\theta_w\right)\left(\frac{w_t^*}{w_t}\right)^{-\epsilon_w} \tag{D8}$$

Price setting equation:

$$g_{1,t}=\beta C_t\Pi_t^*+\theta_p E_t\left(\frac{\Pi_t^*}{\Pi_{t+1}^*}\right)g_{1,t+1} \tag{D9}$$

$$g_{2,t}=\beta\frac{1}{\mu_t^p}C_t+\theta_p E_t g_{2,t+1} \tag{D10}$$

$$g_{1,t}=\left(\frac{\epsilon_p}{\epsilon_p-1}\right)g_{2,t} \tag{D11}$$

Aggregate price index in terms of inflation rates:

$$1 = \theta_p \left(\frac{1}{\Pi_t}\right)^{1-\epsilon_p} + (1-\theta_p)\left(\Pi_t^*\right)^{1-\epsilon_p} \tag{D12}$$

Price dispersion:

$$\Delta_t^p = \theta_p \left(\frac{1}{\Pi_t}\right)^{-\epsilon_p} \Delta_{t-1}^p + (1-\theta_p)\left(\Pi_t^*\right)^{-\epsilon_p} \tag{D13}$$

Unemployment:

$$U_t = 1 - \frac{N_t}{L_t} \tag{D14}$$

Aggregate labor supply:

$$w_t = R_t C_t \chi_t \left(L_t\right)^{\overline{\varphi}} \tag{D15}$$

Resource constraint:

$$\Delta_t^p C_t = Y_t^f \tag{D16}$$

Aggregate final good :

$$Y_t^f = Y_t^I - \left(b_t^u \kappa + \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m x^f\right) \tag{D17}$$

Aggregate bank's balance sheet:

$$l_t + er_t + \xi = d_t \tag{D18}$$

Consistency of aggregate loans and deposits:

$$l_t = \varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m d_t \tag{D19}$$

Aggregate equilibrium in the loan market:

$$l_t = \frac{w_t N_t}{\Delta_t^w} \tag{D20}$$

Aggregate production function for the intermediate good sector:

$$Y_t^I = A_t \xi^{pf} \left(F_t\right)^{1-\alpha} \left(\frac{N_t}{\Delta_t^w}\right)^{\alpha} \tag{D21}$$

Aggregate employment:

$$N_t = \left(\frac{\alpha A_t \xi^{pf}}{\mu_t^p w_t R_t}\right)^{\frac{1}{1-\alpha}} F_t \Delta_t^w \tag{D22}$$

Credit friction input (credit miss-allocation "input") F_t :

$$F_t = (1-\delta_t)\left(\frac{(\overline{\omega})^k - (\widetilde{\omega}_t)^k}{k\left(\overline{\omega}-\underline{\omega}\right)}\right) f^m_{t-1} \tag{D23}$$

Credit market tightness:

$$\tau_t = \frac{f_t}{b^u_t} \tag{D24}$$

Measure of firms in a credit relationship:

$$f^m_t = \varphi_t\left(\tilde{\omega}_t\right) f^m_{t-1} + p^f_t f_t \tag{D25}$$

Measure of firms searching for credit:

$$f_t = 1 - (1-\delta_t) f^m_{t-1} \tag{D26}$$

Overall continuation rate for a credit contract:

$$\varphi_t\left(\tilde{\omega}_t\right) = (1-\delta_t)\left(\frac{\overline{\omega}-\tilde{\omega}_t}{\overline{\omega}-\underline{\omega}}\right) \tag{D27}$$

Reservation productivity:

$$\left[\alpha^\alpha (1-\alpha)^{1-\alpha} A_t \xi^{pf} \widetilde{\omega}_t\right]^{\frac{1}{1-\alpha}} = \left(\mu^p_t w_t R_t\right)^{\frac{\alpha}{1-\alpha}}\left[x^f - \left(\frac{1-\overline{\eta}p^f_t}{1-\overline{\eta}}\right)\frac{\kappa}{p^b_t}\right] \tag{D28}$$

Evolution of credit market tightness:

$$\frac{\kappa}{\mu^p_t p^b_t} - \mathrm{E}_t \Delta_{t,t+1}\varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)\left(1-\overline{\eta}p^f_{t+1}\right)\frac{\kappa}{\mu^p_{t+1}p^b_{t+1}} = (1-\overline{\eta})\,\mathrm{E}_t\Delta_{t,t+1}\frac{1}{\mu^p_{t+1}}\left((1-\alpha)\frac{Y^I_{t+1}}{f^m_t} - \varphi_{t+1}\left(\tilde{\omega}_{t+1}\right)x^f\right) \tag{D29}$$

Stochastic discount factor:

$$\Delta_{t,t+1} = \beta\left(\frac{R_t}{R_{t+1}}\frac{C_t}{C_{t+1}}\right) \tag{D30}$$

Matching rate for firms:

$$p^f_t = \mu\tau_t^{\nu-1} \tag{D31}$$

Matching rate for banks:

$$p^b_t = \mu\tau_t^{\nu} \tag{D32}$$

Gross real interest rate:

$$1 + r_t = \frac{R_t}{\mathrm{E}_t\Pi_{t+1}} \tag{D33}$$

Credit destruction rate:

$$cd_t = 1 - \varphi_t\left(\tilde{\omega}_t\right) - p_t^f \delta_t \tag{D34}$$

Credit creation rate:

$$cc_t = \frac{m_t}{f_{t-1}^m} - p_t^f \delta_t \tag{D35}$$

Credit reallocation rate:

$$cr_t = cc_t + cd_t \tag{D36}$$

Net credit growth rate:

$$cg_t = cc_t - cd_t \tag{D37}$$

Labor share of GDP:

$$LS_t = \frac{w_t N_t}{Y_t^f} \tag{D38}$$

Fixed cost of production share of GDP:

$$FCS_t = \frac{\varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m x^f}{Y_t^f} \tag{D39}$$

Money growth rule:

$$m_t = \left(\frac{1+\theta_t}{\Pi_t}\right) m_{t-1} \tag{D1}$$

where

$$\theta_t - \theta^{ss} = \rho\left(\theta_{t-1} - \theta^{ss}\right) + \xi_t \tag{117}$$

Definition of aggregate loans as a fraction of deposits:

$$\widehat{l}_t = \frac{l}{d} \tag{D40}$$

Definition of aggregate excess reserves as a fraction of deposits:

$$\widehat{er}_t = \frac{er}{d} \tag{D41}$$

Average loan interest rate spread:

$$\begin{aligned} R_t^l l_t - \mathbb{E}\left[R_t l_t^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t\right] \quad = \quad & \left(\left(1-\overline{\eta}_t\right)\left(1-\alpha\Delta_t^W\right)\right) \frac{1}{\mu_t^p} \frac{Y_t^I}{\varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m} \\ & - \left(1-\overline{\eta}_t\right) \frac{x^f}{\mu_t^p} + \left(\left(1-\overline{\eta}_t\right)\left(1-p_t^f\right) \frac{\mathbb{E}_t \overline{\eta}_{t+1}}{1-\mathbb{E}_t \overline{\eta}_{t+1}} - \overline{\eta}_t\right) \frac{\kappa_t}{\mu_t^p p_t^b} \end{aligned} \tag{118}$$

Aggregate joint surplus of a credit contract:

$$V_t^{JS} = \frac{1}{\mu_t^p}\left(\left(1-\alpha\right)Y_t^I - \varphi_t\left(\omega_t^*\right)f_{t-1}^m x^f + \varphi_t\left(\omega_t^*\right)f_{t-1}^m\left(\frac{1-p_t^f\mathbb{E}_t\overline{\eta}_{t+1}}{1-\mathbb{E}_t\overline{\eta}_{t+1}}\right)\frac{\kappa_t}{p_t^b}\right) \tag{D43}$$

9.5 The complete steady state

Given the calibration strategy defined in the main text, the steady state of the model can be written as the following two blocks of equations:

1. **The recursive sub-system:**

$$\mu^p = \frac{\epsilon_p}{\epsilon_p - 1} \tag{1}$$

$$U = 1 - \frac{N}{L} \tag{2}$$

$$1 = \frac{\beta R}{\Pi} \tag{3}$$

$$R = 1 + i \tag{4}$$

$$1 + r = 1 + i \tag{5}$$

$$R\frac{\varepsilon_w - 1}{\varepsilon_w} = \left(1-U\right)^{\overline{\varphi}} \tag{6}$$

$$\widehat{l} + \widehat{er} + \widehat{\xi}^{bs} = 1 \tag{7}$$

$$\widehat{l} = \varphi\left(\tilde{\omega}\right)f^m \tag{8}$$

$$\Delta = \beta \tag{9}$$

$$FCS = \frac{\varphi\left(\tilde{\omega}\right)f^m x^f}{Y^f} \tag{10}$$

$$l = LS \tag{11}$$

$$l = wN \tag{12}$$

$$\widehat{l} = \frac{l}{d} \tag{13}$$

$$\widehat{\xi}^{bs} = \frac{\xi^{bs}}{d} \tag{14}$$

$$\widehat{er} = \frac{er}{d} \tag{15}$$

$$C = Y^f \tag{16}$$

$$C = m - (1 + i)\, er \tag{17}$$

$$w = RC\chi\, (L)^{\overline{\varphi}} \tag{18}$$

$$T = \tag{19}$$

$$w = w^* \tag{20}$$

2. **The simultaneous sub system:**

$$Y^f = Y^I - \left(b^u\kappa + \varphi\,(\tilde{\omega})\, f^m x^f\right) \tag{1}$$

$$Y^I = A\xi^{pf}\,(F)^{1-\alpha}\, N^\alpha \tag{2}$$

$$N = \left(\frac{\alpha A \xi^{pf}}{\mu^p w R}\right)^{\frac{1}{1-\alpha}} F \tag{3}$$

$$F = (1-\delta)\left(\frac{(\overline{\omega})^k - (\widetilde{\omega})^k}{k\left(\overline{\omega} - \underline{\omega}\right)}\right) f^m \tag{4}$$

$$\tau = \frac{f}{b^u} \tag{5}$$

$$f^m = \frac{p^f f}{1 - \varphi\left(\tilde{\omega}\right)} \tag{6}$$

$$f = 1 - (1-\delta) f^m \tag{7}$$

$$\varphi\left(\tilde{\omega}\right) = (1-\delta)\left(\frac{\overline{\omega} - \tilde{\omega}}{\overline{\omega} - \underline{\omega}}\right) \tag{8}$$

$$\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha} A \xi^{pf} \widetilde{\omega}\right]^{\frac{1}{1-\alpha}} = \left(\mu^p w R\right)^{\frac{\alpha}{1-\alpha}} \left[x^f - \left(\frac{1 - \overline{\eta} p^f}{1 - \overline{\eta}}\right) \frac{\kappa}{p^b}\right] \tag{9}$$

$$\left(1 - \beta \varphi\left(\tilde{\omega}\right)\left(1 - \overline{\eta} p^f\right)\right) \frac{\kappa}{p^b} = \left(1 - \overline{\eta}\right) \beta \left(\left(1-\alpha\right) \frac{Y^I}{f^m} - \varphi\left(\tilde{\omega}\right) x^f\right) \tag{10}$$

$$p^f = \mu \tau^{\nu - 1} \tag{11}$$

$$p^b = \mu \tau^{\nu} \tag{12}$$

$$cd = 1 - \varphi\left(\tilde{\omega}\right) - p^f \delta \tag{13}$$

The recursive sub-system solve for the following variables and calibrated parameters:

$$R, \Pi, \Pi^*, \Delta^p, \Delta^w, \mu^p, \varepsilon_w, L, \widehat{\xi}^{bs}, f^m, x^f, l, d, er, \chi, C, i, r, m, \Delta, T, w, w^*$$

While the simultaneous sub-system solve for :

$$\kappa, \mu, \xi^{pf}, \delta, \alpha, Y^I, b^u, F, \widetilde{\omega}, \tau, f, p^f, p^b$$

The simultaneous system of equations can be reduced to the following set:

$$Y^f = \left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{\mu^p wR}\right)^{\frac{\alpha}{1-\alpha}} (1-\delta) \left(\frac{(\overline{\omega})^k - (\widetilde{\omega})^k}{k\left(\overline{\omega} - \underline{\omega}\right)}\right) f^m - \left(\left(\frac{1-(1-\delta) f^m}{\tau}\right)\kappa + \varphi\left(\tilde{\omega}\right) f^m x^f\right) \tag{1}$$

$$N = \left(\frac{\alpha A\xi^{pf}}{\mu^p wR}\right)^{\frac{1}{1-\alpha}} (1-\delta) \left(\frac{(\overline{\omega})^k - (\widetilde{\omega})^k}{k\left(\overline{\omega} - \underline{\omega}\right)}\right) f^m \tag{2}$$

$$f^m = \frac{\mu\tau^{\nu-1}}{1-\varphi\left(\tilde{\omega}\right) + (1-\delta)\,\mu\tau^{\nu-1}} \tag{3}$$

$$\left[\alpha^\alpha (1-\alpha)^{1-\alpha} A\xi^{pf}\widetilde{\omega}\right]^{\frac{1}{1-\alpha}} = (\mu^p wR)^{\frac{\alpha}{1-\alpha}} \left[x^f - \left(\frac{1-\overline{\eta}\mu\tau^{\nu-1}}{1-\overline{\eta}}\right)\frac{\kappa}{\mu\tau^\nu}\right] \tag{4}$$

$$\left(1-\beta\varphi\left(\tilde{\omega}\right)\left(1-\overline{\eta}\mu\tau^{\nu-1}\right)\right)\frac{\kappa}{\mu\tau^\nu} = (1-\overline{\eta})\,\beta \left((1-\alpha)\frac{\left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu^p wR}\right)^{\frac{\alpha}{1-\alpha}}(1-\delta)\left(\frac{(\overline{\omega})^k - (\widetilde{\omega})^k}{k(\overline{\omega}-\underline{\omega})}\right) f^m}{f^m} - \varphi\left(\tilde{\omega}\right) x^f\right) \tag{5}$$

$$\varphi\left(\tilde{\omega}\right) = (1-\delta)\left(\frac{\overline{\omega} - \tilde{\omega}}{\overline{\omega} - \underline{\omega}}\right) \tag{6}$$

$$cd = 1 - \varphi\left(\tilde{\omega}\right) - \mu\tau^{\nu-1}\delta \tag{7}$$

Given our calibration strategy, the unknowns to be determined in the above system of equations are:

$$\widetilde{\omega}, \tau, \delta, \alpha, \kappa, \xi^{pf}, \mu$$

9.6 The non-stochastic steady state

We assume in steady state that the growth rate of real money balances is zero. This assumption together with the Euler equation evaluated at the steady state implies a gross inflation rate of $\Pi = 1$ and a gross nominal interest rate of $R = \frac{1}{\beta}$.[24] Thus the price and wage index equations together with the price and wage dispersion equations evaluated at the steady state with zero net inflation imply no relative price and wage distortions. This implies $\Pi^* = 1, \Delta^p = 1,\ w^* = w$ and $\Delta^w = 1$. Similarly, the optimal price setting equation evaluated at the steady state yields the following constant markup of final goods over intermediate goods prices:

[24] If the model is closed with a Taylor rule instead of a money growth rule then at the steady state the gross nominal interest rate is given by $R = \left(\frac{1}{\beta}\right)(\Pi)^{\phi_\pi}$ while the Euler equation implies $R = \frac{\Pi}{\beta}$. Since $\phi_\pi > 1$ then at the steady state $\Pi = 1$ and $R = \frac{1}{\beta}$., see appendix D for a version of the model as a cashless economy and a Taylor rule

$$\mu^{p}=\frac{\epsilon_{p}}{\epsilon_{p}-1} \tag{SS1}$$

By the same token, the wage setting equation evaluated at the steady state yields a constant markup of the real wage over the aggregate marginal rate of substitution:

$$w=\left(\frac{\varepsilon_{w}}{\varepsilon_{w}-1}\right)MRS \tag{SS2}$$

where $MRS=C\chi N^{\overline{\varphi}}$ is the aggregate marginal rate of substitution in steady state.

Equation SS2 together with the aggregate labor force equation and the unemployment rate definition evaluated at the steady state, imply the following relationship between the unemployment rate, the gross nominal deposit rate and the elasticity among labor types ε_w:

$$R\left(\frac{1}{1-U}\right)^{\overline{\varphi}}=\left(\frac{\varepsilon_{w}}{\varepsilon_{w}-1}\right) \tag{SS3}$$

Notice that if we parameterize $\overline{\varphi}$ and target a particular value for the unemployment rate as well as for the gross deposit rate at the steady-state, we obtain a value for the ε_w parameter. Combining the resource constraint together with the aggregate CIA constraint implies: $Y^f=C=m-R\left(er\right)$ where Y^f is given by

$$Y^{f}=\left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu^{p}wR}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\delta\right)\left(\frac{\left(\overline{\omega}\right)^{k}-\left(\widetilde{\omega}\right)^{k}}{k\left(\overline{\omega}-\underline{\omega}\right)}\right)f^{m}-\left(\left(\frac{1-\left(1-\delta\right)f^{m}}{\tau}\right)\kappa+\varphi\left(\tilde{\omega}\right)f^{m}x^{f}\right) \tag{SS4}$$

Aggregate labor demand, together with the aggregate 'credit' input denoted by F, implies that the following equation must hold at the steady state:

$$N=\left(1-\delta\right)\left(\frac{\alpha A\xi^{pf}}{\mu^{p}wR}\right)^{\frac{1}{1-\alpha}}\left(\frac{\left(\overline{\omega}\right)^{k}-\left(\widetilde{\omega}\right)^{k}}{k\left(\overline{\omega}-\underline{\omega}\right)}\right)f^{m} \tag{SS5}$$

Finally, the equilibrium in the loan market, evaluated at the steady-state, can be reduced to the following set of equations:

$$\left[\alpha^{\alpha}\left(1-\alpha\right)^{1-\alpha}A\xi^{pf}\widetilde{\omega}\right]^{\frac{1}{1-\alpha}}=\left(\mu^{p}wR\right)^{\frac{\alpha}{1-\alpha}}\left[x^{f}-\left(\frac{1-\overline{\eta}\mu\tau^{\nu-1}}{1-\overline{\eta}}\right)\frac{\kappa}{\mu\tau^{\nu}}\right] \tag{SS6}$$

$$\left(1-\beta\varphi\left(\tilde{\omega}\right)\left(1-\overline{\eta}\mu\tau^{\nu-1}\right)\right)\frac{\kappa}{\mu\tau^{\nu}}=\left(1-\overline{\eta}\right)\beta\left(\left(1-\alpha\right)\frac{\left(A\xi^{pf}\right)^{\frac{1}{1-\alpha}}\left(\frac{\alpha}{\mu^{p}wR}\right)^{\frac{\alpha}{1-\alpha}}\left(1-\delta\right)\left(\frac{\left(\overline{\omega}\right)^{k}-\left(\widetilde{\omega}\right)^{k}}{k\left(\overline{\omega}-\underline{\omega}\right)}\right)f^{m}}{f^{m}}-\varphi\left(\tilde{\omega}\right)x^{f}\right) \tag{SS7}$$

$$\left(1-\varphi\left(\tilde{\omega}\right)+\left(1-\delta\right)\mu\tau^{\nu-1}\right)f^{m}=\mu\tau^{\nu-1} \tag{SS8}$$

Given our calibration strategy, the steady-state of the model can be partitioned in two blocks. The first block of equations can be solved recursively and consists of equations D1-D20 evaluated at the steady-state. The second block of equations constitute a simultaneous system of equations that incorporate equations SS4-SS8 together with the definition of $\varphi\left(\tilde{\omega}\right)$ and cd (equations D27 and D34 evaluated at the steady-state).

10 (Online) Appendix C: Derivations for time varying Nash bargaining shares

We use time variation in Nash bargaining shares as a type of financial shock. When we allow Nash bargaining shares to change, many of the equilibrium conditions change. In this section, we detail the equations that change and provide some intuition for interpreting the changes.

10.1 Nash bargaining with time varying shares

The Nash bargaining problem is

$$\max_{\left\{R_t^l(\omega_{z,t}),l_t(\omega_{z,t})\right\}}\left(V_t^{FS}(\omega_{z,t})\right)^{\overline{\eta}_t}\left(V_t^{BS}(\omega_{z,t})\right)^{1-\overline{\eta}_t}$$

where $\overline{\eta}_t$ follows the following exogenous process:

$$\overline{\eta}_t-\overline{\eta}=\rho_{\overline{\eta}}\left(\overline{\eta}_{t-1}-\overline{\eta}\right)+\epsilon_t$$

and subject to the following constraints and definitions similar to the model with constant bargaining shares:

$$V_t^{FS}(\omega_{z,t})=\underbrace{\frac{y_t(\omega_{z,t})-x^f}{\mu_t^p}-R_t^l(\omega_{z,t})l_t(j,\omega_{z,t})}_{\pi_t^I(\omega_{z,t})}$$
$$+\left(1-p_t^f\right)\mathbb{E}_t\Delta_{t,t+1}\varphi_{t+1}\left(\omega_{t+1}^*\right)\int_{\omega_{t+1}^*}^{\overline{\omega}}\max(V_{t+1}^{FP}(\omega_{z,t+1})-V_{t+1}^{FN},0)\frac{dG(\omega)}{1-\gamma_{t+1}\left(\omega_{t+1}^*\right)}$$

$$V_t^{BS}(\omega_{z,t})=\underbrace{\left(R_t^l(j,\omega_{z,t})-R_t\right)l_t(j,\omega_{z,t})}_{\pi_t^b(\omega_{z,t})}+\frac{\kappa_t}{\mu_t^p p_t^b}$$

where:

$$y_t(\omega_{z,t}) = \xi^{pf} A_t \omega_{z,t} N_t \left(\omega_{z,t}\right)^{\alpha}$$

and

$$l_t\left(\omega_{z,t}\right) = w_t N_t(\omega_{z,t}) \quad \text{for all } \omega_{z,t} \geq \omega_t^*$$

The solution is characterized by a time-varying optimal sharing rule for $V_t^{JS}(\omega_{z,t})$ and an optimal hiring rule for $N_t^*(\omega_{z,t})$:

$$V_t^{FS}(\omega_{z,t}) = \overline{\eta}_t V_t^{JS}(\omega_{z,t})$$

$$V_t^{BS}(\omega_{z,t}) = \left(1 - \overline{\eta}_t\right) V_t^{JS}(\omega_{z,t})$$

$$\alpha \xi^{pf} A_t \omega_{z,t} N_t^*(\omega_{z,t})^{\alpha - 1} = \mu_t^p w_t R_t.$$

Using the optimal sharing rule and the free entry condition for banks, the joint surplus of a credit match can be written in as a function of the expected relative bargaining power, $\frac{\mathbb{E}_t \overline{\eta}_{t+1}}{1 - \mathbb{E}_t \overline{\eta}_{t+1}}$:

$$V_t^{JS}(\omega_{z,t}) = \frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} - R_t l_t(j, \omega_{z,t}) + \left(1 + \left(1 - p_t^f\right) \frac{\mathbb{E}_t \overline{\eta}_{t+1}}{1 - \mathbb{E}_t \overline{\eta}_{t+1}}\right) \frac{\kappa_t}{\mu_t^p p_t^b}$$

The loan contract is also characterized by the match-specific bargained loan interest rate $R_t^l(j, \omega_{z,t})$, that depends on $\frac{\mathbb{E}_t \overline{\eta}_{t+1}}{1 - \mathbb{E}_t \overline{\eta}_{t+1}}$ as well:

$$R_t^l(j, \omega_{z,t}) = \frac{1}{l_t(j, \omega_{z,t})} \left[\begin{array}{c} \left(1 - \overline{\eta}_t\right) \left(\frac{y_t(\omega_{z,t}) - x^f}{\mu_t^p} + \left(\left(1 - p_t^f\right) \frac{\mathbb{E}_t \overline{\eta}_{t+1}}{1 - \mathbb{E}_t \overline{\eta}_{t+1}} \right) \frac{\kappa_t}{\mu_t^p p_t^b} \right) \\ + \overline{\eta}_t \left(R_t w_t N_t(\omega_{z,t}) - \frac{\kappa_t}{\mu_t^p p_t^b} \right) \end{array} \right]$$

10.2 Aggregation

In this section, we present the main aggregate variables that are affected when extending the model to consider time-varying bargaining shares. When $V_t^{JS}(\omega_{z,t}^*) = 0 \quad \forall t$, we can solve for the reservation productivity which is common for all active credit contracts and it is given by:

$$\widetilde{\omega}_t = \left(\frac{1}{\alpha^{\alpha} \left(1 - \alpha\right)^{1 - \alpha}} \frac{\left(\mu_t^p w_t R_t\right)^{\alpha}}{\xi^{pf} A_t} \right) \left[x^f - \left(\frac{1 - p_t^f \mathbb{E}_t \overline{\eta}_{t+1}}{1 - \mathbb{E}_t \overline{\eta}_{t+1}} \right) \frac{\kappa_t}{p_t^b} \right]^{1 - \alpha}$$

On the other hand, the average joint surplus is:

$$
\begin{aligned}
E\left[V_t^{JS}(\omega_{z,t})\,|\omega_{z,t} \geq \omega_t^*\right] &= \int_{\omega_t^*}^{\overline{\omega}} V_t^{JS}(\omega_{z,t}) \frac{dG(\omega)}{1-\gamma_t\left(\omega_t^*\right)} \\
&= \frac{1}{\mu_t^p}\left((1-\alpha)\frac{Y_t^I}{\varphi_t\left(\omega_t^*\right) f_{t-1}^m} - x^f + \left(\frac{1-p_t^f \mathbb{E}_t \overline{\eta}_{t+1}}{1-\mathbb{E}_t \overline{\eta}_{t+1}}\right)\frac{\kappa_t}{p_t^b}\right)
\end{aligned}
$$

Then, the free entry condition becomes the loan creation equation that determines the dynamics of credit market tightness given the definitions of p_t^f and p_t^b :

$$
\begin{aligned}
&\frac{\kappa_t}{\mu_t^p p_t^b} - \mathbb{E}_t \Delta_{t,t+1} \varphi_{t+1}\left(\omega_{t+1}^*\right)\left(1-\overline{\eta}_{t+1}\right)\left(\frac{1-p_{t+1}^f \mathbb{E}_{t+1}\overline{\eta}_{t+2}}{1-\mathbb{E}_{t+1}\overline{\eta}_{t+2}}\right)\frac{\kappa_{t+1}}{\mu_{t+1}^p p_{t+1}^b} \\
&= \mathbb{E}_t \Delta_{t,t+1}\left(1-\overline{\eta}_{t+1}\right)\frac{1}{\mu_{t+1}^p}\left((1-\alpha)\frac{Y_{t+1}^I}{f_t^m} - \varphi_{t+1}\left(\omega_{t+1}^*\right) x^f\right)
\end{aligned}
$$

and the average loan interest rate spread is:

$$
\begin{aligned}
R_t^l l_t - \mathbb{E}\left[R_t l_t^*(\omega_{it}) \mid \omega_{it} \geq \widetilde{\omega}_t\right] &= \left((1-\overline{\eta}_t)\left(1-\alpha \Delta_t^W\right)\right)\frac{1}{\mu_t^p}\frac{Y_t^I}{\varphi_t\left(\tilde{\omega}_t\right) f_{t-1}^m} \\
&\quad - (1-\overline{\eta}_t)\frac{x^f}{\mu_t^p} + \left((1-\overline{\eta}_t)\left(1-p_t^f\right)\frac{\mathbb{E}_t\overline{\eta}_{t+1}}{1-\mathbb{E}_t\overline{\eta}_{t+1}} - \overline{\eta}_t\right)\frac{\kappa_t}{\mu_t^p p_t^b}
\end{aligned}
$$