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Juan Aquino-Chávez

N.R. Ramírez-Rondán

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Juan Aquino-Chávez
Washington University in St. Louis

N.R. Ramírez-Rondán
Universidad del Pacífico

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Abstract

The measurement of the sources of economic growth is essential for understanding the long-term perspective of any economy. From an empirical viewpoint, the results from any growth-accounting exercise depend both on the functional form that summarizes the technology set and the factor share values. We estimate the physical capital's share in output implied by a Cobb-Douglas production function. Instead of growth rates, we analyze time series in levels to preserve the long-run information contained in the data. We also make use of the cross-section dimension (between countries) to overcome the low availability of long time series. The Fully Modified OLS (FMOLS) and Dynamic OLS (DOLS) estimators are used in a panel cointegration framework for 109 countries over the 1951-2014 period. For several measures of labor input, our physical capital's share estimates range between 0.46 and 0.56 for the largest set of countries. Our estimates of the physical capital's share in output vary significantly across regions.

JEL Classification: C23, E23, 047

Keywords: production function, factor shares, cointegration, panel data.

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Corresponding author: Juan Aquino-Chávez, Department of Economics, Washington University, 1 Brookings Dr., St. Louis, MO 63130, U.S.A. E-mail address: juan.aquino@wustl.edu.

1 Introduction

The measurement of the sources of economic growth (capital, labor and productivity) is essential for the long-term perspective of any economy. The results of growth-accounting exercises depend both on the specification of the production function and the factor share values. In general, studies in this area have calculated these shares from either national income data calculations or production function estimations specified in terms of growth rates. In standard growth accounting, factor shares are used to decompose growth over time (for a given country) into two components, one being explained by the growth in factor inputs and the another one summarizing unexplained factors (i.e. the Solow residual) which are usually attributed to productivity. Similarly, in the cross-country approach (based on a production function estimation), factor shares are used to decompose the variation in income across countries into a component explained by the variation in savings and rates of population growth and an unexplained component linked to international differences in productivity.

A first strand of this literature estimates factor shares from national income data, which includes total payments to labor (employee compensation) and capital (corporate profits), such that the factor shares are the fraction of income paid to each factor. In low-income countries, labor force is composed by employers, self-employed and informal employees although their corresponding income levels are incorrectly classified. Gollin (2002) proposed data-adjusting methods in order to account for self-employed income. Bernanke and Gürkaynak (2002) estimate the labor's share by assuming that all the economies in their sample lie on a balanced growth path. They replicate and update Gollin's calculation of the operating surplus of private unincorporated enterprises (OSPUE) measure and the labor force correction for their sample of countries. They calculate the labor's share for each of 54 countries for the 1980-1995 period and their results exhibit labor's shares ranging from 0.22 to 0.81. Nevertheless, the aforementioned OSPUE variable is not available for all countries with national income data and many low-income countries do not even have such data[1].

A second strand of the same literature estimates a production function in terms of growth rates by using gross domestic product data. Elias (1992) discusses some properties of a production function used to adjust the aggregate data of seven Latin American economies (Argentina, Brazil, Chile, Colombia, Mexico, Peru and Venezuela). The ordinary least squares (OLS) estimator is used after pooling time series across countries

[1] García-Verdú (2005) proposes a method for estimating the labor's and capital's factor shares by using cross-sectional household survey data that contains detailed information on household income by source in Mexico.

and an estimated physical capital's share value of 0.39 is reported[2]. In a similar fashion, Senhadji (2000) estimates a production function in levels and first differences for 88 countries grouped into regions (Africa, East Asia, South Asia, Middle East, Latin America and Industrial countries) for the 1960-1994 period. For the largest set of countries, the author reports a physical capital's share estimate equal to 0.55 for a model with human capital and equal to 0.52 for a model without human capital. On the other hand, when the analysis is performed for each region the estimates range from 0.28 to 0.72 for the first model and from 0.28 to 0.74 for the second model[3].

The main disadvantage of estimating a production function expressed in terms of rates of growth lies at the removal of the stochastic trend from each macroeconomic series. As a consequence, the nonstationary nature of these time series is not exploited as the first-difference operator removes all the long-run information contained in the data[4]. However, it is worth mentioning that the cointegration literature has proven the superiority of analyzing level rather first-difference equations under nonstationarity. Moreover, unlike the short-term counterpart, an explicit long-run relationship between the involved variables is proposed.

Two problems emerge in the usage of long-run estimation methods. The first problem is given by the potential endogeneity of the explanatory variables (capital and labor). The former is a typical argument against the estimation of production functions for calculating factor shares through Two-Step Ordinary Least Squares as proposed by Engle and Granger (1987). Nonetheless, this potential problem is corrected by the maximum-likelihood methodology developed by Johansen (1988) and the Fully Modified and Dynamic Ordinary Least Squares (FMOLS and DOLS, respectively) developed by Phillips and Hansen (1990), Hansen (1992), Saikkonen (1991) and Stock and Watson (1993). Moreover, the two latter methodologies correct the possible autocorrelation between error terms.

The second problem consists of the low availability of sufficiently sized samples, especially in developing countries, which makes the estimates very sensitive to changes in the sample size. Nevertheless, the cross-section dimension (between countries) in the data can be exploited as in the pioneer developments by Levin et al. (2002) and Im et al. (2003) for unit root testing in a panel data framework. Additionally, developments by Kao (1999), McCoskey and Kao (1998) and Pedroni (2004) also allow to test for cointegration in a

[2] Elias (1992) considers a production function model with a deterministic trend and dummy variables.

[3] Senhadji (2000) considers a production function with constant returns to scale and employs the fully modified OLS estimator.

[4] The first-difference operator eliminates low frequencies and therefore emphasizes the short-term fluctuations contained in the data.

panel data environment. In this regard, panel cointegration has been proposed in order to exploit the cross-section dimension. Authors such as Banerjee (1999), Phillips and Moon (2000) and Baltagi and Kao (2000) provide an overview of developments in this area.

The panel cointegration approach has been widely used in empirical macroeconomics to estimate long-run relationships such as the purchasing power parity (Pedroni, 2001), the demand for money (Mark and Sul, 2003), the analysis of international R&D spillovers (Kao et al., 1999) and the estimation of a production function that incorporates urbanization (McCoskey and Kao, 1998), among others. In a work related to ours, Marrocu et al. (2001) estimate a long-run production function across 20 regions and 17 sectors for the Italian economy during the 1970-1994 period. The authors report physical capital's and labor's share estimates equal to 0.52 and 0.47, respectively, for the whole dataset and equal to 0.60 and 0.59, respectively, for all the regions. In a similar fashion, they report capital's and labor's share estimates equal to 0.61 and 0.18, respectively, for all the sectors.

The Penn World Table (Feenstra et al., 2015) recently included comparable macroeconomic series such as real gross domestic product, labor, physical capital stock, human capital, price of capital and price of consumption, among others, for 182 countries and territories over the 1950-2014 period. In this paper we make use of the latest update to estimate via FMOLS and DOLS in a panel cointegration framework, a Cobb-Douglas production function exhibiting constant returns to scale by using alternative measures of labor and capital and by analyzing a worldwide set of countries as well as regional groups.

The remaining sections of this paper are organized as follows. Section 2 describes the production function to be estimated and the dataset to be employed. Section 3 discusses both the estimation methodology and results. Section 4 concludes.

2 Model Specification and Data

2.1 Functional Form

The literature on economic growth has traditionally assumed a Cobb-Douglas production function that implies a unit elasticity of substitution between capital and labor. Given this assumption, the interpretation of the weights on physical capital and labor as the corresponding shares in output requires only the additional constant-returns-to-scale assumption. Although assumed for the sake of simplicity, a Cobb-Douglas function remains a valid description of the production process and related data are available across countries and over time for estimation purposes[5].

[5] The parameter describing the physical capital's share in output is typically set to the benchmark value of $1/3 \approx 0.33$ as suggested by the national income accounts of some industrial countries.

We consider a family of Cobb-Douglas production functions of the form

$$Y_{it} = F(K_{it}, L_{it}, A_{it}) = K_{it}^{\alpha} L_{it}^{1-\alpha} A_{it}, \tag{1}$$

$t = 1, \dots, T$, $i = 1, \dots, N$. In equation (1), Y, K, L and A represent aggregate output, physical capital, labor and total factor productivity, respectively. Also T and N are the number of years and countries, respectively. We further assume that the productivity level A obeys

$$A_{it} = \exp(a_i + \rho t + u_{it}). \tag{2}$$

In (2), a_i is a country-specific fixed effect, ρ is the growth rate of productivity and u_{it} is a disturbance (error) term. In this setup, the productivity level is also driven, besides the fixed effect and the error term, by a linear trend. Omitting such a trend would imply that, if the variables cointegrated, the productivity is a stationary series and this is not a plausible assumption[6]. Substituting (2) into (1) leads to

$$Y_{it} = K_{it}^{\alpha} L_{it}^{1-\alpha} \exp(a_i + \rho t + u_{it}), \tag{3}$$

which, since constant returns to scale and perfect competition in factor markets are assumed, can be written as

$$\frac{Y_{it}}{L_{it}} = \left(\frac{K_{it}}{L_{it}}\right)^{\alpha} \exp(a_i + \rho t + u_{it}), \tag{4}$$

or, equivalently,

$$\log(Y_{it}/L_{it}) = a_i + \rho t + \alpha \log(K_{it}/L_{it}) + u_{it}, \tag{5}$$

where, for a sake of exposition, log denotes natural logarithm. In (5), the parameter that represents the physical capital's share in output is given by α. Once provided with an estimate of α, the resulting labor's share is given by $1 - \alpha$.

2.2 Data

Depending on the specific measures for capital and labor to be employed, we consider several sets of countries and territories. The number cross-sectional units ranges from 81 and 109 and the study period spans from 1960 to 2014. Data are obtained from Penn World Table 9.0 updated as of August 19, 2016 by Feenstra et al. (2015). We focus on

[6]Low-income economies may have had a poor productivity performance, in this case the parameter ρ, the growth rate of productivity, should be zero.

balanced panel data for two reasons. First, the theory behind most of the related tests and estimators is developed for the case of balanced data. Second, 55 years per country constitute enough information to identify a stable long-run relationship. Table 1 shows the constructions and definitions of the variables to be used in the estimation process.

Table 1: Definitions

Variable	Definition
Output	Real gross domestic product (GDP) at constant 2005 national prices (in millions of 2005 US dollars).
Physical capital	Capital stock at constant 2005 national prices (in millions of 2005 US dollars). It is estimated by accumulating and depreciating past investments using the perpetual inventory method.
Population	Total population (in millions).
Employment	Number of people in an employee engagement (in millions).
Human capital	Index of human capital per person, based on years of schooling (Barro y Lee, 2010) and returns to education (Psacharopoulos, 1994).
Quality of capital	Ratio of price level of household consumption to price level of capital formation, both being USA price levels with 2005=1.

Note: for details on the construction of each variable across countries, see Feenstra et al. (2015).

In all of our estimates, the output measure is given by the real Gross Domestic Product (GDP). For the case of labor, we consider four measures: total population, employment, human-capital-adjusted population (which includes a measure of quality of labor) and human-capital-adjusted employment. We do not consider average annual hours worked by people engaged since there are not enough observations for all countries.

A first measure of stock of physical capital is constructed by using the perpetual inventory method. A second measure consists of the stock of capital adjusted by its quality. Jorgenson and Griliches (1967) construct an index of quality of capital by using a weighted average of the investment in machines/technology and buildings/infrastructure. Nonetheless, disaggregated series of investment by category are not available. Greenwood and Jovanovic (2001) propose an alternative measurement of quality of capital, related to the relative price of investment in terms of consumption[7].

In principle, adequate measures of capital and labor should also include measures of factor utilization such as the unemployment rate and the rate of capacity utilization in

[7]Quality appears in the equation of accumulation of capital, which means that quality appears as a technological progress specific to investment. The idea is that, when the quality of capital increases, more goods can be produced from physical capital by giving up one unit of output or consumption.

order to correct for the effective use of labor and capital, respectively. However, these series are only available for a small number of countries and years.

3 Methodology and Results

We first consider four worldwide samples containing from 81 to 109 countries and territories for the 1960-2014 period, which depend on the variables that were included. On the one hand, when we used population (adjusted by human capital) as a measure of labor 109 (103) countries were available. On the other hand, when we used either employment or employment adjusted by human capital as the measure of labor, only 83 countries were available. Additionally, the largest sample was divided into six regions (see Appendix A for details on the classification) which implies that the assumption of identical technologies across regions was relaxed.

Unlike the four measures of labor described above, the only measure of capital we consider is the one obtained from the perpetual inventory method. We do not adjust this variable by quality since its related estimates provide no plausible results. Namely, the ratio of price level of household consumption to price level of capital formation is unstable for most of the countries and produces nonsense indicators[8].

In a similar fashion to the cointegration literature for time series, the panel cointegration approach involves three steps:

- test for the order of integration of each variable by using panel unit root tests,
- test whether the variables cointegrated or not, and
- estimate the model and make statistical inference.

3.1 Panel Unit Root Tests

Levin et al. (2002) developed one of the first unit root tests for panel data. The authors use an augmented version of the Dickey-Fuller framework in order to test for the presence of a common unit root against the alternative of stationarity among the cross-sections and let the two dimensions (N and T) to grow in an independent way. Im et al. (2003) complement these by developing new tests based on group statistics of the Lagrange Multiplier (LM) type. The main feature of the latter tests consists of their flexible formulation that does

[8] Several developing countries experienced periods of high inflation during the 1980 decade (for example Argentina, Bolivia and Peru reached an inflation rate of 3080, 11750 and 7841 per cent, respectively) and this lead to an important distortion in price variables.

not impose the stationarity of all the cross-sections under the alternative hypothesis. Similarly, Breitung (2001) studied the local power of panel unit root tests.

An alternative approach was developed by Choi (2001) and Maddala and Wu (1999). These authors proposed tests of the Fisher type that average the values of the significance levels associated to the unit root tests obtained from each cross-section. Such tests exhibit some desirable properties such as their applicability to unbalanced panels and the relaxation of the cross-section independence hypothesis. In a similar fashion to the unit root literature for time series, several other tests were proposed for panel unit root testing. Baltagi and Kao (2000) provide a survey of this literature.

Table 2 shows the results of the panel unit root testing applied to the variables involved in the estimation process, both in levels and first differences. Most of the tests indicate that the series are integrated of order one or, equivalently, contain a unit root. The test by Levin et al. (2002) is the only one that indicates that the variable $\log(Y/L)$ ($\log(K/L)$) is stationary when employment (population) and human-capital adjusted employment (population) are used as measures of labor. Nonetheless, the remaining tests in Table 2 exhibit superior properties, such as allowing for individual unit root processes, to the test by Levin et al. (2002).

3.2 Panel Cointegration Tests

The second step of our analysis consists of testing whether the variables of interest are cointegrated. For this purpose, several tests were proposed. The first panel cointegration test was developed by Kao (1999), who proposed residual-based tests for the null of no cointegration and derived critical values in an analogous way to the residual two-step procedure by Engle and Granger (1987).

Pedroni (1999, 2004) first proposed four tests, three include a nonparametric correction and one includes a parametric correction in an Augmented Dickey-Fuller (ADF) fashion. The author also proposed three tests constructed upon the average of their components. As a consequence, the asymptotic results are derived from the limiting distribution of the average of numerators and denominators rather than the average of the entire statistics[9].

In a similar way to the Fisher-type panel unit root tests, Maddala and Wu (1999) proposed an alternative approach for cointegration testing. Specifically, the authors developed tests constructed from individual tests across cross-sections (Johansen's cointegration trace and maximum eigenvalue tests). For a review of other tests proposed within this literature, see Baltagi and Kao (2000).

[9] These tests allow to include regressions containing fixed effects and time trends.

Table 2: Panel unit root tests (p-values)

	$\log(Y/L)$ in levels			
Method	Pop. as labor	Pop.×HC as labor	Emp. as labor	Emp.×HC as labor
Null hypothesis: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	1.0000	1.0000	1.0000	0.9973
ADF - Fisher Chi-square	0.8629	0.9835	0.8793	0.5299
PP - Fisher Chi-square	0.9999	1.0000	0.8013	0.6975
Null hypothesis: Unit root (assumes common unit root process)				
Levin, Lin y Chu t-stat	0.5465	0.4702	0.0003	0.0000
Breitung t-stat	1.0000	1.0000	1.0000	1.0000

	$\log(Y/L)$ in first differences			
Method	Pop. as labor	Pop.×HC as labor	Emp. as labor	Emp.×HC as labor
Null hypothesis: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	0.0000	0.0000	0.0000	0.0000
ADF - Fisher Chi-square	0.0000	0.0000	0.0000	0.0000
PP - Fisher Chi-square	0.0000	0.0000	0.0000	0.0000
Null hypothesis: Unit root (assumes common unit root process)				
Levin, Lin y Chu t-stat	0.0000	0.0000	0.0000	0.0000
Breitung t-stat	0.0000	0.0000	0.0000	0.0000

	$\log(K/L)$ in levels			
Method	Pop. as labor	Pop.×HC as labor	Emp. as labor	Emp.×HC as labor
Null hypothesis: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	0.8364	0.6943	1.0000	1.0000
ADF - Fisher Chi-square	0.0001	0.0166	0.9996	0.9987
PP - Fisher Chi-square	1.0000	1.0000	1.0000	1.0000
Null hypothesis: Unit root (assumes common unit root process)				
Levin, Lin y Chu t-stat	0.0000	0.0000	0.0195	0.0009
Breitung t-stat	1.0000	1.0000	1.0000	1.0000

	$\log(K/L)$ in first differences			
Method	Pop. as labor	Pop.×HC as labor	Emp. as labor	Emp.×HC as labor
Null hypothesis: Unit root (assumes individual unit root process)				
Im, Pesaran and Shin W-stat	0.0000	0.0000	0.0000	0.0000
ADF - Fisher Chi-square	0.0000	0.0000	0.0000	0.0000
PP - Fisher Chi-square	0.0000	0.0000	0.0000	0.0000
Null hypothesis: Unit root (assumes common unit root process)				
Levin, Lin y Chu t-stat	0.0000	0.0003	0.0008	0.0000
Breitung t-stat	0.0000	0.0000	0.0000	0.0000

Note: HC, Pop and Emp stand for human capital, population and employment, respectively. The specification of the unit root tests includes individual effects and individual linear trends.

Table 3 shows the panel cointegration testing results. For most of the cases, we reject the null of no cointegration, except for the case in which human-capital adjusted employment is used as a measure of labor. Nevertheless, the Johansen-type Fisher panel cointegration test indicates that we cannot reject the null of at most one cointegration equation in all the cases. Therefore, there is strong evidence supporting the cointegration of the variables of interest and the fact that our estimates do not constitute spurious relationships.

Table 3: Panel cointegration tests (p-values)

	Cointegration equation			
Method	Pop. as labor	Pop.×HC as labor	Emp. as labor	Emp.×HC as labor
Pedroni residual cointegration test				
Null hypothesis: no cointegration				
Alternative hypothesis: common AR coefs. (within-dimension)				
Panel v-Statistic	0.0001	0.1167	0.0005	0.1733
Panel rho-Statistic	0.0034	0.0943	0.0754	0.6883
Panel PP-Statistic	0.0012	0.0007	0.0184	0.3499
Panel ADF-Statistic	0.0007	0.0016	0.0162	0.4356
Alternative hypothesis: individual AR coefs. (between-dimension)				
Panel rho-Statistic	0.3643	0.9411	0.5136	0.8588
Panel PP-Statistic	0.0248	0.0022	0.0199	0.2552
Panel ADF-Statistic	0.0003	0.0000	0.0010	0.2030
Kao residual cointegration test				
Null hypothesis: no cointegration				
Panel ADF-Statistic	0.000	0.0000	0.0000	0.0000
Johansen Fisher panel cointegration test				
Null hypothesis: number of cointegration equations				
Trace test				
None	0.0000	0.0000	0.0000	0.0000
At most 1	0.5862	0.2253	0.1062	0.1217
Max-eigenvalue test				
None	0.0000	0.0000	0.0000	0.0000
At most 1	0.5862	0.2253	0.1062	0.1217

Note: HC, Pop and Emp stand for human capital, population and employment, respectively.

3.3 Estimation results

The final step of our analysis consists on panel cointegration estimation and inference. On the one hand, Kao and Chiang (2001) found that the bias-corrected OLS estimator does not improve over the OLS estimator. Pedroni (2000, 2001) proposed panel FMOLS estimators that extend those by Phillips and Hansen (1990) and Phillips (1995). On the other hand, Kao and Chiang (2001) proposed a panel DOLS estimator that extends the work of Saikkonen (1991) and Stock and Watson (1993). For further details, see Baltagi and Kao (2000).

Table 4 reports the estimation results for worldwide and regional sets of countries and for different estimation methods and measures of labor. The results indicate that, in general, the capital's share ranges from 0.30 to 0.67. For the worldwide sample capital's share ranges from 0.46 to 0.56. When the employment is adjusted by human capital as the most reasonable measure of labor, the capital's share ranges from 0.46 to 0.65[10].

The estimates of the physical capital's share vary significantly across country groups where regions such as the Industrial countries and the East Asia, South Asia and Pacific report the highest values, whereas the Middle East and North Africa region reports the lowest values. Nonetheless, the estimates differ significantly across estimation methods (FMOLS and DOLS) and labor measures within the same region, excepting for the Industrial region. This so happens because the Industrial region contains more countries than other regions and the industrial countries therein display more stable series, making the estimates generally more robust. Notice that, when we consider the worldwide sample, the estimates are also quite robust since there are enough observations both across countries and years. In addition, it is worth noticing that the panel worldwide estimates are significantly higher than the usual value of 0.33 used in the economic growth literature.

Based on the notation in (1), the physical capital's share is given by $(\partial Y/\partial K) \times (K/Y) = (\partial F(K, L, A)/\partial K) \times (K/F(K, L, A)) = \alpha$. As argued by Senhadji (2000), under decreasing returns to capital its marginal product $(\partial Y/\partial K)$ is higher in developing countries. For the same reason, the capital-output ratio (K/Y) is lower in developing countries. Therefore, the product defining the share of physical capital can be either lower or higher for developing countries. This result is reflected in Table 4, where the Africa region possesses the lowest income per capita but has higher capital's share estimates than the Middle East and North Africa region. Also, the Industrial region with the highest income per capita exhibits similar estimates to those corresponding to the East Asia, South Asia and Pacific region.

Finally, we compare our results to the average of the individual cross-country estimates in a similar fashion to Senhadji (2000) who estimates a production function for 88 countries for the 1960-1994 period and reports regional averages. We estimate the production function for each country in the worldwide sample and Table 5 summarizes the estimation results by reporting the worldwide and regions averages. The estimates provide higher

[10] It is worth to mention that in Table 4 the capital's share estimates are statistically different from zero.

Table 4: Cobb-Douglas production function estimates

	Industrial		Latin America and Caribbean		Africa		Middle East and North Africa		East Asia, South Asia and Pacific		World	
	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS
Population as labor												
Capital's share	0.6221	0.6490	0.4546	0.4345	0.4510	0.3962	0.2945	0.2980	0.6508	0.6486	0.4957	0.4625
	(0.0370)	(0.0344)	(0.0589)	(0.0614)	(0.0314)	(0.0334)	(0.0777)	(0.0691)	(0.0230)	(0.0239)	(0.0197)	(0.0206)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		22		38		10		15		109	
Pop.$\times$HC as labor												
Capital's share	0.6201	0.6531	0.5538	0.5358	0.5185	0.4719	0.3632	0.3892	0.6678	0.6678	0.5433	0.5201
	(0.0401)	(0.0397)	(0.0550)	(0.0552)	(0.0329)	(0.0346)	(0.0756)	(0.0632)	(0.0246)	(0.0256)	(0.0205)	(0.0209)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		22		32		10		15		103	
Employment as labor												
Capital's share	0.6519	0.6407	0.4286	0.4084	0.4525	0.4086	0.4045	0.4262	0.5970	0.6009	0.5226	0.4903
	(0.0324)	(0.0310)	(0.0664)	(0.0670)	(0.0414)	(0.0407)	(0.0669)	(0.0604)	(0.0286)	(0.0309)	(0.0221)	(0.0222)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		16		18		10		13		81	
Emp.$\times$HC as labor												
Capital's share	0.6450	0.6381	0.5343	0.4973	0.4919	0.4614	0.4901	0.5181	0.6524	0.6542	0.5586	0.5361
	(0.0348)	(0.0335)	(0.0668)	(0.0682)	(0.0373)	(0.0369)	(0.0648)	(0.0576)	(0.0300)	(0.0327)	(0.0220)	(0.0219)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		16		18		10		13		81	

Note: HC, Pop and Emp stand for human capital, population and employment, respectively. Standard error in parentheses.

Table 5: Cobb-Douglas production function (average of countries) estimates

	Industrial		Latin America and Caribbean		Africa		Middle East and North Africa		East Asia, South Asia and Pacific		World	
	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS	FMOLS	DOLS
Population as labor												
Capital's share	0.7086	0.6963	0.6082	0.6155	0.4262	0.3774	0.5103	0.5199	0.5973	0.5954	0.5595	0.5387
	(0.0386)	(0.0391)	(0.0432)	(0.0423)	(0.0308)	(0.0316)	(0.0580)	(0.0557)	(0.0331)	(0.0350)	(0.0176)	(0.0178)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		22		38		10		15		109	
Pop.×HC as labor												
Capital's share	0.7995	0.8145	0.7148	0.7025	0.4900	0.4565	0.6220	0.6373	0.5716	0.5715	0.6370	0.6268
	(0.0336)	(0.0347)	(0.0423)	(0.0413)	(0.0311)	(0.0321)	(0.0607)	(0.0578)	(0.0358)	(0.0370)	(0.0172)	(0.0174)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		22		32		10		15		103	
Employment as labor												
Capital's share	0.6930	0.6731	0.7577	0.7655	0.5105	0.4639	0.5597	0.5382	0.5380	0.5162	0.6293	0.6030
	(0.0270)	(0.0291)	(0.0564)	(0.0588)	(0.0348)	(0.0371)	(0.0568)	(0.0563)	(0.0335)	(0.0352)	(0.0180)	(0.0189)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		16		18		10		13		81	
Emp.×HC as labor												
Capital's share	0.6820	0.6727	0.8098	0.8153	0.5586	0.5404	0.6652	0.6612	0.5474	0.5123	0.6603	0.6443
	(0.0302)	(0.0322)	(0.0474)	(0.0487)	(0.0310)	(0.0337)	(0.0590)	(0.0603)	(0.0410)	(0.0434)	(0.0177)	(0.0185)
Period	1960-2014		1960-2014		1960-2014		1960-2014		1960-2014		1960-2014	
Countries	24		16		18		10		13		81	

Note: HC, Pop and Emp stand for human capital, population and employment, respectively. Standard error in parentheses.

values than the panel estimates: the average estimates for the worldwide sample deliver higher values (from 0.06 to 0.10) than the panel worldwide estimations. Notice that, since the time span consists of only 55 years, the country estimations provide sensitive results which constitutes a caveat of the time series approach.

Even though most of the literature has adopted the Cobb-Douglas production function, this assumption can be locally relaxed in order to allow for a more general Constant Elasticity of Substitution (CES) production function. In this regard, we further estimated a CES production function model and show that the Cobb-Douglas function could still be a valid description of the production process (see Appendix B for details).

4 Conclusion

In this paper we estimated the physical capital's share in aggregate output based on a Cobb-Douglas production function model. The data was obtained from the Penn World Table 9.0 with a worldwide sample of 109 countries and territories for the 1960-2014 period.

We used the Fully Modified OLS (FMOLS) and Dynamic OLS (DOLS) estimators in a panel cointegration approach. For different measures of labor, we find a physical capital's share estimates ranging from 0.46 to 0.56.

Our estimates of the physical capital's share vary significantly across regions, with the Industrial region and the East Asia, South Asia and Pacific region exhibiting the highest values. Also, the Middle East and North Africa region exhibit the lowest values. In the Industrial region and the worldwide sample, the estimates remain robust across estimation methods and labor measures.

Finally, when we compare our results with the average of the individual cross-country estimates, the physical capital's share estimates in overall deliver higher values than the panel estimation, reflecting the fact that they produce upward bias in the estimates due to the few observations in the individual country regressions.

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Appendix A. Regional Groups

Table A.1: Regional groups

Industrial	Latin America and Caribbean	Africa		Middle East and North Africa	East Asia, South Asia and Pacific
Australia	Argentina	Benin	Lesotho	Algeria	Bangladesh
Austria	Barbados	Botswana	Madagascar	Cyprus	China
Belgium	Bolivia	Burkina Faso	Malawi	Egypt	Fiji
Canada	Brazil	Burundi	Mali	Iran	Hong Kong
Denmark	Chile	Cabo Verde	Mauritania	Israel	India
Finland	Colombia	Chad	Mauritius	Jordan	Indonesia
France	Costa Rica	Cameroon	Mozambique	Malta	Korea
Germany	Dominican R.	Central Afr. R.	Namibia	Morocco	Malaysia
Greece	Ecuador	Comoros	Niger	Tunisia	Nepal
Iceland	El Salvador	Congo	Nigeria	Syria	Pakistan
Ireland	Guatemala	Côte d'Ivoire	Rwanda		Philippines
Italy	Haiti	D.R. of Congo	Senegal		Singapore
Luxembourg	Honduras	Ethiopia	Seychelles		Sri Lanka
Japan	Jamaica	Gabon	South Africa		Taiwan
Netherlands	Mexico	Gambia	Tanzania		Thailand
New Zealand	Nicaragua	Ghana	Togo		
Norway	Panama	Guinea	Uganda		
Portugal	Paraguay	Guinea-Bissau	Zambia		
Spain	Peru	Kenya	Zimbabwe		
Sweden	Trinidad and T.				
Switzerland	Uruguay				
Turkey	Venezuela				
United Kingdom					
United States					

Appendix B. The CES Production Function Model

As it is well known, the Cobb-Douglas production function constitutes a particular case of the CES production function. For this more general specification, the implied factor shares are not constant but depend on the factor-output ratio. In order to test for local misspecification, we consider a CES production function with two inputs given by

$$Y_{it} = \{\theta K_{it}^{-\delta} + (1-\theta)L_{it}^{-\delta}\}^{-\frac{1}{\delta}} A_{it}, \tag{6}$$

where θ and $1-\theta$ determine the optimal distribution of inputs and δ determines the (constant) elasticity of substitution (which equals $\sigma = (1+\delta)^{-1}$). Under the assumption of competitive markets, the capital's share is given by $\theta(K/Y)^{-\delta}$ and the labor's share is given by $(1-\theta)(L/Y)^{-\delta}$. Kmenta (1967) derived an approximation to the classical two-input CES production function

$$\log Y_{it} = \theta \log K_{it} + (1-\theta)\log L_{it} - \frac{\delta}{2}\theta(1-\theta)(\log K_{it} - \log L_{it})^2 + \log A_{it}, \tag{7}$$

by taking natural logs to both sides of (6) and computing a second-order Taylor expansion to $\log\{\theta K_{it}^{-\delta} + (1-\theta)L_{it}^{-\delta}\}$ around $\delta = 0$. Alternatively, the same expression can be obtained by computing a first-order Taylor expansion to the transformed CES function around $\delta = 0$ (Henningsen and Henningsen, 2011). Substituting (2) into (7) and defining $\beta = -\frac{\delta}{2}\theta(1-\theta)$ leads to

$$\log Y_{it} = a_i + \rho t + \theta \log K_{it} + (1-\theta)\log L_{it} + \beta(\log K_{it} - \log L_{it})^2 + u_{it}, \tag{8}$$

or, equivalently,

$$\log(Y_{it}/L_{it}) = a_i + \rho t + \theta \log(K_{it}/L_{it}) + \beta\{\log(K_{it}/L_{it})\}^2 + u_{it}. \tag{9}$$

Therefore, by estimating (9) and testing $H_0 : \beta = 0$ we can evaluate whether the Cobb-Douglas function constitutes a valid description of the production process. Table B.1 reports estimates of β for the worldwide sample, which indicate that $\{\log(K_{it}/L_{it})\}^2$ is not statistically significant and favor the Cobb-Douglas specification.

Table B.1: CES production function estimates

	World	
	FMOLS	DOLS
Population as labor		
Optimal of capital (θ)	0.2992	0.3549
	(0.1127)	(0.1572)
Parameter β	0.0105	0.0025
	(0.0060)	(0.0088)
Elasticity of substitution (σ)	1.1115	1.0226
Period	1960-2014	
Countries	109	
Pop.$\times$HC as labor		
Optimal of capital (θ)	0.2713	0.1378
	(0.1411)	(0.1684)
Parameter β	0.0154	0.0204
	(0.0080)	(0.0098)
Elasticity of substitution (σ)	1.1845	1.5215
Period	1960-2014	
Countries	103	
Employment as labor		
Optimal of capital (θ)	0.5981	0.5673
	(0.1554)	(0.1956)
Parameter β	-0.0033	-0.0056
	(0.0073)	(0.0093)
Elasticity of substitution (σ)	0.9735	0.9566
Period	1960-2014	
Countries	81	
Emp.$\times$HC as labor		
Optimal of capital (θ)	0.5334	0.4910
	(0.1773)	(0.2166)
Parameter β	0.0014	0.0019
	(0.0090)	(0.0110)
Elasticity of substitution (σ)	1.0116	1.0152
Period	1960-2014	
Countries	81	

Note: HC, Pop and Emp stand for human capital, population and employment, respectively. Standard error in parentheses.