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Abstract

We study the efficiency of equilibrium in a business cycle model where monopolistically competitive firms acquire costly information about aggregate fundamentals before making pricing and input decisions. We show that market power reduces the private value of information relative to its social value, causing too little investment in learning and inefficient cyclical fluctuations. Importantly, this is true even in an environment where the ex-post response to information is socially optimal. A leading example of this dichotomy between *ex-post* and *ex-ante* efficiency is an environment where firms choose labor input under uncertainty about aggregate productivity. When firms set nominal prices, on the other hand, their actions exhibit a inefficiently high sensitivity to private signals. The combination of this inefficiency in information use and market power makes the overall direction of the inefficiency in information acquisition ambiguous. Finally, we show that the standard full information policy response to market power-related distortions can reduce welfare under endogenous uncertainty. These results hold for different types of shocks (real and nominal) and for a general class of information acquisition technologies.

JEL Classification: D62, D82, E31, E32, E62

Keywords: Incomplete information, Costly information, Externalities, Business cycles, Optimal policy.

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1 Introduction

A large and growing literature in modern macroeconomics focuses on the role of dispersed information about fundamentals in understanding fluctuations in economic activity[1]. The main contribution of this paper is to highlight a new source of inefficiency in this class of models, one that arises when information is acquired endogenously. We show that incentives of monopolistically competitive firms to learn about aggregate shocks are typically not aligned with the social value of doing so. This leads to a suboptimal level of learning in equilibrium, distorting both the level of economic activity as well as its sensitivity to shocks. This distortion is distinct from those affecting incentives to use information and therefore, is present even when the information is used in a socially optimal fashion.

This inefficiency arises from 2 distinct sources. The first is that a firm with market power does not internalize all the benefits of better aligning its decisions with fundamentals. In other words, under imperfect competition, the *private* value of information, the change in expected profits, is less than the social value, which reflects changes in expected social surplus. As a result, in a *laissez faire* equilibrium, monopolistically competitive firms tend to make suboptimally low levels of investment in information. Importantly, this holds even if the sensitivity of actions to such information is the socially optimal one. A real business cycle version of our model - where firms make labor input decisions under uncertainty about aggregate productivity shocks - exhibits this combination of *ex-ante* inefficiency and *ex-post* efficiency.

The second source of inefficiency emerges when information is *used* suboptimally. In our general equilibrium environment, this occurs when firms set nominal prices under uncertainty (and let quantities be determined by realized demand conditions). We show that firms set prices that are 'too sensitive' to their private signals, because they do not internalize their contribution to overall uncertainty in the economy. More precise information exacerbates this inefficiency, partly (and in some cases, completely) offsetting the direct benefits of taking actions under better information. Private payoffs do not reflect this trade-off and therefore, tend to overvalue information.

When firms make quantity choices under uncertainty, only the first of these two sources is operational and the equilibrium features under-acquisition of information relative to the welfare maximizing benchmark. When firms set nominal prices, however, both channels are present, making the overall direction of the inefficiency ambiguous. For this case, we divide the parameter

[1] An inexhaustive reading list is Amador and Weill (2010), Angeletos and La'O (2009, 2012), Hellwig (2005), Hellwig and Venkateswaran (2009), Lorenzoni (2009, 2010), Mackowiak and Wiederholt (2009, 2011b), Moscarini (2004), Reis (2006), Roca (2010), Venkateswaran (2012) and Woodford (2003).

space into various regions depending on whether the equilibrium features too much or too little learning. When the elasticity of substitution between the firms' products is low, market power is high and the first effect tends to dominate and we have under-acquisition. The opposite happens when goods are highly substitutable or when the quality of information is low.

Our results have a number of implications. First, they show that conclusions about the efficiency of aggregate fluctuations derived under complete or exogenous information do not extend to an environment with costly learning. Second, policies aimed at correcting market power-related distortions have additional effects when information is endogenously chosen in equilibrium. When information is used in a socially optimal fashion, policies which align changes in social surplus with private payoffs always improve welfare. In particular, the standard complete information policy response to non-competitive behavior - a constant revenue subsidy - is also the optimal policy with endogenous information and achieves the constrained-efficient solution. However, this is no longer true with inefficiency in information use, as in the price-setting environment. Then, policies which correct market power alone can even reduce welfare. To understand this counter-intuitive finding, recall that market power partly offset the distortion in the private value of information arising from the inefficiency in information use. The constant subsidy, by giving firms a greater share of the total surplus, also creates incentives to invest a suboptimally high amount in learning. This can, under some circumstances, more than offset the beneficial effects of removing the noncompetitive distortion. It is important to note that this is an effect which arises only when information choice is modeled explicitly. We show the optimal policy in such an environment must be state-contingent and takes the form of a countercyclical revenue subsidy which addresses both sources of inefficiency. Finally, our results also have implications for the social value of public information. More public information can lead to a reduction in welfare because it crowds out private information production. Intuitively, this occurs when the equilibrium features an inefficiently low of private learning.

Though our focus is the fully articulated business cycle environment, we also show how our results apply to coordination games in general, using the beauty contest framework of Morris and Shin (1998) and others. A general insight emerges - what matters for information use is only the *relative* importance of the various components of the payoff function, but the value of information is also influenced by the *absolute* level of the payoff. We show that externalities in payoff functions (e.g. as in Morris and Shin (2002) or Angeletos and Pavan (2007)) can distort the level of social versus private payoffs and cause private and social values of information to diverge, even if they leave incentives to use such information undistorted[2].

It is important to note that our analysis imposes little structure on the learning technology beyond the existence of interior solutions. Our general specification encompasses several commonly used formulations (e.g. rational inattention, costly signals). Also, while we focus on private signals for most of our analysis, the sources of inefficiency highlighted are relevant to the acquisition of public information as well[3]. Finally, we focus on aggregate technology and nominal disturbances primarily to facilitate comparison with earlier work but our analysis can be easily extended to other types of shocks as well.

Related literature: This paper bears a direct connection to the body of work embedding heterogeneous information in business cycle models. One branch of this literature [4] takes the information structure as exogenous and derives implications for equilibrium responses. A second strand [5] endows agents with a learning technology and allows them to endogenously determine the extent of information, as in this paper. The main difference between this paper and this latter group is that we are concerned primarily with efficiency, while most of the other papers focus on other properties of equilibrium outcomes.

Two independent recent papers are important exceptions[6]. Colombo, Femminis and Pavan (2012) and Mackowiak and Wiederholt (2011b) investigate the efficiency of information choice in a quadratic utility framework[7]. As in this paper, Colombo, Femminis and Pavan (2012) also find that efficiency in information use does not imply optimal information choice and characterize the link between payoff externalities and efficiency. Working under the rational inattention paradigm,

[2]In an independent paper, Colombo, Femminis and Pavan (2012) arrive at the same result, using a general quadratic specification for payoffs.

[3]In Section 7.5, we illustrate this using the beauty contest model.

[4]For example, Woodford (2003), Moscarini (2004), Angeletos and Pavan (2007), Angeletos and La'O (2008, 2009), Nimark (2008), Hellwig (2008b, 2008a), Lorenzoni (2009, 2010), and Hellwig and Venkateswaran (2009). The large literature on noisy rational expectations models in asset pricing, including seminal work by Hellwig (1980) and Diamond and Verrecchia (1981), mostly falls under this category, as does the recent work on global games, following Morris and Shin (1998, 2002).

[5]For example, Mackowiak and Wiederholt (2009, 2011a) consider a setting where agents face a constraint on their ability to process information, while Hellwig and Veldkamp (2009), Gorodnichenko (2008) and Reis (2006) introduce explicit costs of planning or acquiring information. In the asset pricing context, Grossman and Stiglitz (1980), Ganguli and Yang (2009), Barlevy and Veronesi (2000) and Veldkamp (2006b, 2006a) all consider environments where information is chosen endogenously. Myatt and Wallace (2010) study a beauty contest setting where agents choose what signals to pay attention to.

[6]Chahrour (2012) also looks at the welfare implications of costly *public* signals.

[7]In an unpublished working-paper version of Hellwig and Veldkamp (2009), information acquisition is shown to be efficient in a beauty-contest model without externalities.

Mackowiak and Wiederholt (2011b) study the optimality of attention allocated to rare events. The insights from these papers, while related, are not directly applicable to the fully articulated microfounded environments that are the focus of this paper. Our analytical framework allows us to derive closed-form expressions for the objects of interest and thus allows us to capture all effects of information acquisition, without resorting to approximations[8]. This leads us to draw more robust conclusions about welfare and sets the stage for a quantitative evaluation. Our results - on the sources and magnitude of the inefficiency - are directly interpretable in terms of model primitives, viz. preference and technology parameters. To the best of our knowledge, our analysis is the first to provide a comprehensive picture of the incentives to acquire information in a standard macroeconomic environment.

Our work also complements earlier work on efficiency under exogenous information. Angeletos and Pavan (2007) show that information is used inefficiently in equilibrium when private and social incentives to coordinate are different. Hellwig (2005) and Roca (2010) analyze these incentives in a general equilibrium monetary model while Angeletos and La'O (2009) study them in a real business cycle context. Our unified framework helps clarify the underlying sources of suboptimality in information use in these papers. For example, we are able to bring out more transparently the role of price versus quantity choice in their results. More importantly, however, we focus on social and private incentives to *acquire* information, a distinct source of inefficiency in this class of models. Amador and Weill (2010) also study the efficiency when the extent of information is endogenously determined in equilibrium. However, this occurs through learning from endogenous objects and not, as in this paper, through the acquisition of costly information. Finally, our findings on the effects of market power on information choice also contribute to a broader agenda studying the efficiency implications of imperfect competition - see, for example, Bilbie et. al (2008) and the references therein.

The rest of the paper is organized as follows. In section 2, we use a simple model to show how imperfect competition distorts incentives to learn. Section 3 lays out the full model, which embeds information acquisition in a general equilibrium real business cycle model with aggregate shocks. The next 3 sections consider three commonly used versions of this environment. Section 4 is a real business cycle environment, where firms make labor input choices under imperfect information about aggregate productivity shocks. Sections 5 and 6 repeat the analysis under price-setting and nominal shocks respectively. Section 7 studies information choice in a reduced-form coordination

[8] Which rule out, for example, 'non-strategic' effects of uncertainty on payoffs in Colombo, Femminis and Pavan (2012).

game with quadratic payoffs. Section 8 contains a brief conclusion. Proofs are collected in the Appendix.

2 A Simple Example

The purpose of this section is to build intuition about the connection between market power and value of information. We study a simple environment where a single monopolist makes production choices under uncertainty. She is endowed with a technology that transforms the numeraire, denoted N, into final goods, denoted Q, according to

$$Q = AN^{\frac{1}{\delta}}, \qquad \delta > 1,$$

where A is a log-normally distributed technology shock, i.e. $a \equiv \ln A \sim N(0, \sigma_a^2)$. The profit of the monopolist is given by

$$\Pi = PQ - N,$$

where P is the price of the final good in terms of the numeraire. The monopolist faces a representative consumer with a utility function

$$C = \left(\frac{\theta}{\theta - 1}\right) Q^{\frac{\theta-1}{\theta}} - PQ, \qquad \theta > 1.$$

Optimization by the consumer implies

$$P = Q^{\frac{-1}{\theta}}.$$

The total social surplus is

$$U = \left(\frac{\theta}{\theta - 1}\right) Q^{\frac{\theta-1}{\theta}} - N.$$

Using the consumer's optimality condition, we can rewrite this as

$$U = \left(\frac{\theta}{\theta - 1}\right) PQ - N.$$

Thus, in this constant demand elasticity environment, there is a simple relationship between the consumer's utility and revenue. As $\theta \to \infty$, the difference vanishes, i.e. profits equal the social surplus.

When making her production decision, the monopolist faces uncertainty about the realization of the technology shock A. In particular, she only sees a noisy signal

$$s = a + e, \qquad e \sim N(0, \sigma^2),$$

and chooses input[9], N. Formally, her problem is

$$\begin{aligned}\Pi &= \max_N \quad \mathbb{E}\left[PQ \mid s\right] - N \\ &= \max_N \quad \mathbb{E}\left[\left(AN^{\frac{1}{\delta}}\right)^{\frac{\theta-1}{\theta}} \mid s\right] - N,\end{aligned}$$

where the operator $\mathbb{E}\left[\cdot|s\right]$ represents the expectation conditional on the signal s. The first order condition is

$$\frac{\theta-1}{\theta\delta}\mathbb{E}\left[A^{\frac{\theta-1}{\theta}} \mid s\right] N^{\frac{\theta-1}{\theta\delta}-1} = 1.$$

Standard results for conditional expectations of log-normal random variables imply

$$\log\ N = \kappa + \alpha \cdot s,$$

where

$$\begin{aligned}\alpha &= \frac{\delta(\theta-1)}{1-\theta+\theta\delta}\left(\frac{\sigma_a^2}{\sigma_a^2+\sigma_\varepsilon^2}\right), \\ \kappa &= \left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\log\frac{\theta-1}{\theta\delta} + \frac{1}{2}\left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\left(\frac{\theta-1}{\theta}\right)^2\left(\frac{\sigma_a^2\sigma^2}{\sigma_a^2+\sigma^2}\right).\end{aligned}$$

Before analyzing the value of information, it is instructive to examine the efficiency properties of this policy more closely. Consider the surplus-maximizing response function, i.e. the solution to

$$U = \max_N \quad \mathbb{E}\left[\left(\frac{\theta}{\theta-1}\right)PQ \mid s\right] - N.$$

It is easy to show that the solution takes the same form as the monopolist's policy, with

$$\begin{aligned}\alpha^* &= \alpha, \\ \kappa^* &= \kappa + \left(\frac{\theta\delta}{1-\theta+\theta\delta}\right)\ln\left(\frac{\theta}{\theta-1}\right) \quad > \quad \kappa.\end{aligned}$$

In other words, the elasticity of labor input with respect to the signal (and therefore, to the fundamental) is the socially optimal one but the monopolist chooses a suboptimally low average level of labor input. Thus, the monopolist uses information *efficiently* even though she finds it optimal to restrict production.

The private value of information to the monopolist is the sensitivity of the (ex-ante) expected profit to the variance of the noise in the signal. A straightforward application of the envelope theorem yields

$$\frac{\partial \mathbb{E}\Pi}{\partial \sigma^2} = -\frac{\alpha^2}{2}\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)\mathbb{E}N < 0.$$

[9] The results go through even if the monopolist had to choose prices instead of input.

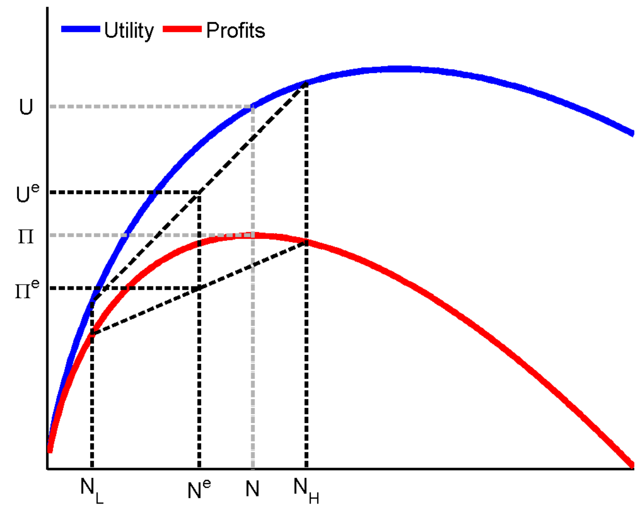

Figure 1: Profits and Utility

where $\mathbb{E}$ takes expectations over the realizations of the aggregate shocks and the signals. $\mathbb{E}N$ is the unconditional expectation of input. The derivative is negative, i.e. profits decline with poorer information. Analogously, the social value is the change in expected total surplus i.e. $\frac{\partial \mathbb{E}U}{\partial \sigma^2}$. We can show that social surplus is proportional to profits, i.e.

$$\begin{aligned} \mathbb{E}U &= \underbrace{\left[\frac{\left(\frac{\theta}{\theta-1}\right)\frac{\theta\delta}{\theta-1} - 1}{\frac{\theta\delta}{\theta-1} - 1} \right]}_{>\,1} \mathbb{E}\Pi \\ \Rightarrow \frac{\partial \mathbb{E}U}{\partial \sigma^2} &= \left[\frac{\left(\frac{\theta}{\theta-1}\right)\frac{\theta\delta}{\theta-1} - 1}{\frac{\theta\delta}{\theta-1} - 1} \right] \frac{\partial \mathbb{E}\Pi}{\partial \sigma^2} \quad < \quad \frac{\partial \mathbb{E}\Pi}{\partial \sigma^2}. \end{aligned}$$

Thus, noisier signals lead to a greater loss of utility compared to profits. The source of the difference, the $\frac{\theta}{\theta-1}$ in the numerator, is simply the constant of proportionality between consumer surplus and revenue. Intuitively, the effect on profits from better information underestimates the change in social surplus because revenues do not capture all the utility gained by the consumer. Only in the limiting case of infinite demand elasticity does the private value coincide with the social value.

Figure 1 provides a graphical illustration for this intuition. The profit maximizing choice for a given level of the technology shock under perfect information, N, leads to a full information profit of Π. The corresponding social surplus is denoted U. Under imperfect information, the firm chooses a scale of production that is lower on average than under full information[10] The expected profit drops to Π^e while the social surplus drops to U^e. Since the utility curve is steeper than the profit curve at N^*, the private loss from less information ($\Pi - \Pi^e$) underestimates the social loss ($U - U^e$).

In the general equilibrium environments studied in the rest of the paper, the underlying demand structure will be more complicated, but this basic intuition carries over almost exactly. Firms internalize the effects of better information on prices and therefore, attach a lower value to their own learning and therefore, tend to acquire less than the socially optimal amount of information. However, other equilibrium linkages can cause information to be used inefficiently, which can overwhelm this channel for underinvestment in learning.

3 A Business Cycle Model

In this section, we lay out a microfounded business cycle model with dispersed information. The fully-articulated and flexible specification will allow us to examine the efficiency of equilibrium information choice under various assumptions about the nature of shocks (real vs. nominal) and decisions (prices vs. quantity). These cases will be examined in detail in the next 3 sections.

Time is discrete, $t = 0, 1, 2....$ The economy is populated by a continuum of entrepreneurs and a final good producer. The entrepreneurs or firms as we will sometimes refer to them in our exposition, each have access to a technology, which transforms labor into a differentiated intermediate good. These technologies, are located on a continuum of informationally-separate islands, with one firm per island. Firms make two decisions - an *ex-ante* information choice, modeled as the precision of a private signal about an aggregate shock and an *ex-post* production/pricing choice.

Preferences and Technology: Entrepreneur i enjoys a per-period utility according to[11]

$$C_{it} - N_{it} - \upsilon(\sigma_{ei}^2),$$

where C_{it} is consumption of final goods and N_{it} the labor input[12]. The last term reflects the cost

[10]To keep the graph simple, we approximate the firm's decision under uncertainty with only two levels of labor input - N_L and N_H. The exact distribution is normal, centered at N^e.

[11]The assumption of linearity is not crucial for any of our results, but simplifies the expressions considerably.

[12]We model the entrepreneur as choosing how much of his own effort to commit to production. This backyard production specification is only for simplicity. In earlier versions of this paper, we worked with explicit labor markets on each island and our results go through almost exactly. .

of acquiring private information. The agent is subject to a budget constraint

$$P_t C_{it} = P_{it} Y_{it} \ .$$

Production of intermediate goods is described by a decreasing returns to scale production function:

$$Y_{it} = A_t N_{it}^{\frac{1}{\delta}} \ ,$$

where $\delta > 1$ and A_t is aggregate productivity.

The final good is a CES composite of the intermediate goods

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di \right)^{\frac{\theta}{\theta-1}} \ ,$$

where the parameter θ is the elasticity of substitution between intermediate goods. Throughout the paper, we will assume that $\theta > 1$.

Finally, aggregate variables are linked by the following quantity equation:

$$P_t Y_t = M_t \ ,$$

where M_t is the (exogenous) level of nominal demand.

In the following three sections, we study in detail 3 versions of this general framework:

- Quantity (labor input) choice with aggregate productivity shocks
- Price choice with aggregate productivity shocks
- Price choice with aggregate nominal shocks

4 Model I: Quantity choice with productivity shocks

In this version, the only source of aggregate uncertainty is the level of aggregate technology A_t. Nominal demand is constant, i.e. $M_t = M \quad \forall t$. Note that under complete information, this is the canonical real business cycle model, with monopolistic competition replacing the standard representative firm assumption[13].

Firms observe a private signal about the aggregate productivity shock and choose labor input. Then, production takes place, the firms sell their output and buy the final good for consumption. Figure 4 shows the timing of events in each period.

[13] Angeletos and La'O (2009) study a similar environment with dispersed but exogenous information. The main modeling difference is that they have many firms on each island, a feature that is easy to incorporate into our setup.

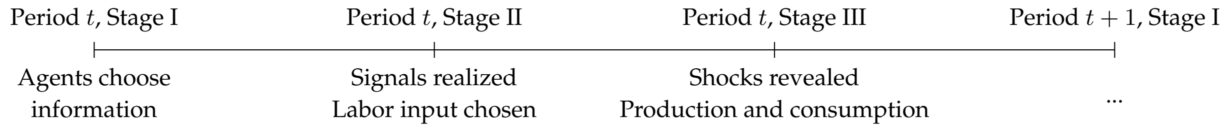

Figure 2: Timeline of Events

We will show that information about the aggregate shock is used efficiently in this environment, but the incentives to learn are suboptimally low. As a result, the *laissez-faire* equilibrium with endogenous information exhibits inefficient fluctuations, even though the same economy under the assumption of exogenous information does not. The intuition is similar to the simple example in the previous section - imperfect substitutability leads to a wedge between the private value of information and its the social value. As a result, agents in equilibrium expend a suboptimally low level of effort in information acquisition. Only in a limiting case, as goods becomes perfect substitutes, does the equilibrium achieve efficiency.

Aggregate productivity is log-normally distributed, i.e. $\log A_t \equiv a_t \sim N\left(0, \sigma_a^2\right)$. For simplicity, we focus on the case where this is an i.i.d shock, but our results go through for more general stochastic processes as well[14].

Information structure: Before choosing labor input, each agent observes a private signal s_{it} about the current productivity shock:

$$s_{it} = a_t + e_{it} \ ,$$

where $e_{it} \sim N\left(0, \sigma_{ei}^2\right)$. The variance of the noise term, σ_{ei}^2 is the variance chosen *ex-ante* by the firm.

Optimality: The competitive firm producing the final good solves :

$$\max \quad P_t Y_t - \int_0^1 P_{it} Y_{it} di \ ,$$

$$Y_t = \left(\int_0^1 Y_{it}^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}} \ ,$$

where P_{it} is the price of intermediate good i. Optimality yields the usual demand function for good i

$$Y_{it} = \left(\frac{P_{it}}{P_t}\right)^{-\theta} Y_t \ . \tag{1}$$

[14] For example, if a_t is an $AR(1)$ process, our results go through exactly with the aggregate shock now interpreted as the current innovation to the aggregate productivity level.

Substituting from the budget constraint, we can write the intermediate producer's objective in Stage II as follows:

$$\Pi_{it} = \max_{N_{it}} \ \mathbb{E}_{it}\left(\frac{P_{it}}{P_t} A_t N_{it}^{\frac{1}{\delta}} - N_{it}\right) \ ,$$

where the operator $\mathbb{E}_{it}(\cdot)$ represents the expectation conditional on firm i's information $\mathcal{I}_{it}$, i.e. $\mathbb{E}_{it}(\cdot) \equiv \mathbb{E}_t(\cdot\,|\mathcal{I}_{it})$.

Substituting from the demand function (1),

$$\Pi_{it} = \max_{N_{it}} \ \mathbb{E}_{it}\left[\left(\frac{Y_{it}}{Y_t}\right)^{\frac{-1}{\theta}} A_t N_{it}^{\frac{1}{\delta}} - N_{it}\right], \tag{2}$$

The solution is to choose an input level that equates expected marginal revenue to marginal cost

$$\mathbb{E}_{it}\left[\left(\frac{\theta-1}{\delta\theta}\right) Y_t^{\frac{1}{\theta}} A_t^{\frac{\theta-1}{\theta}} N_{it}^{\frac{\theta-1-\theta\delta}{\delta\theta}}\right] = 1. \tag{3}$$

Rearranging,

$$N_{it}^{\frac{1+\theta\delta-\theta}{\delta\theta}} = \frac{\theta-1}{\delta\theta}\mathbb{E}_{it}\left[Y_t^{\frac{1}{\theta}-\gamma} A_t^{\frac{\theta-1}{\theta}}\right]. \tag{4}$$

Information acquisition: In the first stage of each period, before signals are realized, each agent chooses the extent of information to acquire, taking as given choices of other firms in the economy. The unconditional expectation of profits is defined as:

$$\hat{\Pi}_{it}\left(\sigma_{ei}^2, \sigma_e^2\right) \equiv \mathbb{E}\Pi_{it} \ , \tag{5}$$

where $\mathbb{E}$ takes expectations over the realizations of the aggregate shocks and the signals

The problem of the agent in the first stage can then be written as:

$$\max_{\sigma_{ei}^2} \quad \hat{\Pi}_{it}\left(\sigma_{ei}^2, \sigma_e^2\right) - \upsilon\left(\sigma_{ei}^2\right) \ ,$$

where $\upsilon(\cdot)$ is the cost of information[15] as a function of the noise in the signal with $\upsilon'(\cdot) < 0, \ \upsilon''(\cdot) > 0$. Our focus in this paper is on differences in the *value* of information to private agents and to the planner, so we wish to impose as little structure as possible on the *cost* of information. The

[15]For example, under the rational inattention paradigm, as in Sims (2003), this would be determined by the cost of information processing capacity, which is defined as the extent of reduction in entropy about the fundamental shock. Alternatively, if information choice takes the form of deciding how many signals to acquire, the function $\upsilon(\cdot)$ is interpreted as the total cost of acquiring a basket of signals with the same informational content as a single signal with precision σ_e^2.

only additional assumption we make is that the solution to the above information choice problem (and later, that of the planner) lies in the interior[16], i.e. is characterized by :

$$\frac{\partial \hat{\Pi}}{\partial \sigma_{ei}^2} - \upsilon'(\sigma_{ei}^2) = 0 \ . \tag{6}$$

4.1 Equilibrium

A equilibrium is (i) a set of information choices for each firm (ii) island-specific labor inputs as functions of the signal on the island (iii) aggregate consumption and output as functions of the aggregate state such that: (a) the labor input is optimal, given island-specific information and wages and the functions in (iii) above, (b) taking the behavior of aggregates in (iii) as given, the information choice in (i) solves the Stage I problem, (c) markets clear and (d) the functions in (iii) are correct.

We focus on symmetric equilibria, where all agents acquire the same amount of information in stage I and follow the same strategies in stage II. The characterization of the equilibrium in stage II essentially follows the same procedure as in Angeletos and La'O (2009). We begin with a conjecture that, in equilibrium, firms follow a symmetric labor input policy of the form[17]

$$n_{it} = k_2 + \alpha s_{it} \ , \tag{7}$$

where k_2 and α are coefficients to be determined in equilibrium. The former determines the (unconditional) average level of (the log of) employment, while the latter is the elasticity. The details of this guess-and-verify approach are in the Appendix. The expressions for the response coefficients are given in the following result.

Proposition 1 *In a symmetric equilibrium, labor input is given by (7), with*

$$\alpha = \left(\frac{\delta}{\delta - 1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2 + \left(\frac{1+\delta\theta-\theta}{\delta\theta-\theta}\right)\sigma_e^2}\right] \ , \tag{8}$$

$$k_2 = \left(\frac{\theta\delta}{1+\theta\delta-\theta}\right)\log\left(\frac{\theta-1}{\delta\theta}\right) + \left[\frac{1-\theta+\theta\delta}{\theta\left(\delta-1\right)}\right]\frac{\alpha\sigma_e^2}{2} + \left[\frac{1}{\theta\left(\delta-1\right)}\right]\frac{\alpha^2\sigma_e^2}{2}. \tag{9}$$

where σ_e^2 is the variance of the error in agents' signals.

[16] A necessary condition is that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \hat{\Pi}}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} - \frac{\partial^2 \upsilon}{\partial \hat{\sigma}_e^2 \partial \hat{\sigma}_e^2} < 0 \ .$$

[17] Hereafter, variables in small cases denote variables in logs, i.e. $x \equiv \log(X)$

The expression for α has an intuitive interpretation. The first part $\frac{\delta}{\delta-1}$ is simply the full information elasticity of employment to a productivity shock. Under incomplete information, this is downweighted by the second part, an adjusted signal-to-noise ratio. The adjustment essentially increases the weight of the noise (by a factor $\frac{1+\delta\theta-\theta}{\delta\theta-\theta} > 1$), reflecting the well-known effect of strategic complementarities. In other words, firms in this economy have an incentive to coordinate their actions (due to the imperfect substitutability of the goods they produce). Since the informational friction dampens the overall response of the economy to the fundamental, agents find it optimal to respond less than one-for-one to their expectations of fundamental.

Finally, we characterize the information acquisition decision in stage I. We begin by noting that the maximized stage II profit function, equation (5), depends on both the information choices of the agent herself as well as everybody else in the economy. The latter enter payoffs through the aggregate response coefficients, α and k_2. Ex-ante expected profits, conditional on a choice of individual error variance σ_{ei}^2, are obtained by taking expectations over the realization of the random variable $\mathbb{E}_{it}(a_t)$.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \text{argmax}_{\sigma_{ei}^2} \ \hat{\Pi}\left[\sigma_{ei}^2, \alpha(\sigma_e^2), k_2(\sigma_e^2)\right] - \upsilon\left(\sigma_{ei}^2\right),$$

where we make explicit the dependence of α and k_2 on σ_e^2 according to the equilibrium relationships (8)-(9).

In the Appendix we show that, under the assumption of an interior solution, the optimality condition associated with this problem is

$$-\frac{1}{2}\left(\frac{\theta-1}{\delta\theta}\right)\hat{\Pi}\alpha^2 = \upsilon'(\sigma_e^2), \tag{10}$$

where $\hat{\Pi}$ is the unconditional expected profit and α is the equilibrium response coefficient. Since both these objects are themselves functions of σ_e^2, this is a fixed point relation in σ_e^2 and completes the characterization of equilibrium.

4.2 Efficiency in Information Use

We now turn to its efficiency properties. We begin by showing that information use is optimal[18]. To achieve this, we compare the equilibrium coefficients α and k_2 to those chosen by a planner, who is interested in maximizing household utility. Importantly, the planner is assumed to be

[18]This subsection is an application of the welfare results in Angeletos and La'O (2009).

information-constrained, i.e. cannot pool information across islands but is free to choose how agents respond to the signals. We exploit log-normality and restrict attention to symmetric log-linear policy rules of the form:

$$n_{it} = \tilde{k}_2 + \tilde{\alpha}\ s_{it}\ . \tag{11}$$

Then, it is straightforward to derive the aggregate labor input, consumption and welfare are:

$$\begin{aligned} N_t &= \exp\left[\tilde{k}_2 + \tilde{\alpha} a_t + \frac{1}{2}\tilde{\alpha}^2\sigma_e^2\right], \\ C_t &= Y_t = \exp\left[\left(1+\frac{\tilde{\alpha}}{\delta}\right)a_t + \frac{\tilde{k}_2}{\delta} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\frac{\tilde{\alpha}^2}{\delta^2}\sigma_e^2\right], \end{aligned}$$

and the corresponding ex-ante expectations

$$\begin{aligned} \hat{N}\left(\tilde{k}_2,\tilde{\alpha}\right) &= E(N_t) = \exp\left[\tilde{k}_2 + \frac{1}{2}\tilde{\alpha}^2(\sigma_a^2+\sigma_e^2)\right], \\ \hat{C}\left(\tilde{k}_2,\tilde{\alpha}\right) &= E(C_t) = \exp\left[\frac{1}{2}\left(1+\frac{\tilde{\alpha}}{\delta}\right)^2\sigma_a^2 + \frac{\tilde{k}_2}{\delta} + \frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\frac{\tilde{\alpha}^2}{\delta^2}\sigma_e^2\right], \\ \mathbb{U} &= \hat{C} - \hat{N}. \end{aligned} \tag{12}$$

The efficient use of information is characterized by coefficients α^* and k_2^* that maximize utility, i.e.

$$(\alpha^*, k_2^*) = \text{argmax}_{\tilde{k}_2,\tilde{\alpha}} \quad \hat{C}\left(\tilde{k}_2,\tilde{\alpha}\right) - \hat{N}\left(\tilde{k}_2,\tilde{\alpha}\right).$$

The optimality conditions of this problem are

$$\begin{aligned} C^* &= \delta N^*, \\ C^*\left[\frac{1}{\delta}\left(1+\frac{\alpha^*}{\delta}\right)\sigma_a^2 + \left(\frac{\theta-1}{\theta\delta}\right)\frac{\alpha^*}{\delta}\sigma_e^2\right] &= N^*\alpha^*\left(\sigma_a^2+\sigma_e^2\right). \end{aligned} \tag{13}$$

Using the first equation, the second condition can be rewritten as

$$\left[\frac{1}{\delta}\left(1+\frac{\alpha^*}{\delta}\right) - \frac{\alpha^*}{\delta}\right]\sigma_a^2 = \frac{\alpha^*}{\delta}\sigma_e^2\left[1-\frac{\theta-1}{\theta\delta}\right]. \tag{14}$$

The two sides of this equation reflect the trade-off faced by the planner. A stronger response to the signal makes actions better aligned with the fundamental, but also increases the inefficient variation in them. The planner sets α to equate the marginal benefit from the former channel to the marginal cost from the latter.

Agents in equilibrium face a similar trade-off. Consider the ex-ante profit of firm which takes as given the responses of other firms (α, k_2) and chooses the coefficients of its own best response

$(\hat{\alpha}, \hat{k}_2)$

$$\exp\left\{\left(\frac{\theta-1}{\theta\delta}\right)\hat{k}_2+\frac{1}{\theta\delta}k_2+\frac{1}{2}(\theta-1)\frac{\alpha^2}{\theta^2\delta^2}\sigma^2\right\}$$
$$\exp\left\{\frac{1}{2}\left[\left(\frac{\theta-1}{\theta}\right)\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]^2\sigma_a^2+\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\}$$
$$-\exp\left\{\hat{k}_2+\frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2+\sigma_e^2\right)\right\}.$$

The associated optimality conditions are

$$\begin{aligned} C_i &= \left(\frac{\theta}{\theta-1}\right)\delta N_i\,, \\ \left\{\frac{1}{\delta}\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2\left(\frac{\theta-1}{\theta}\right)+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\}C_i &= N_i\hat{\alpha}\left(\sigma_a^2+\sigma_e^2\right). \end{aligned}$$

Using the first equation and invoking symmetry, the second condition becomes

$$\left(\frac{\theta-1}{\theta}\right)\frac{1}{\delta}\left(1+\frac{\alpha}{\delta}\right)\sigma_a^2+\frac{\alpha}{\delta}\left(\frac{\theta-1}{\theta}\right)\left(\frac{\theta-1}{\theta\delta}\right)\sigma_e^2=\left(\frac{\theta-1}{\theta}\right)\frac{\alpha}{\delta}\left(\sigma_a^2+\sigma_e^2\right).$$

Re-arranging,

$$\left(\frac{\theta-1}{\theta}\right)\left[\frac{1}{\delta}\left(1+\frac{\alpha}{\delta}\right)-\frac{\alpha}{\delta}\right]\sigma_a^2=\left(\frac{\theta-1}{\theta}\right)\frac{\alpha}{\delta}\sigma_e^2\left[1-\frac{\theta-1}{\theta\delta}\right]. \tag{15}$$

Comparing (14) and (15), we see that the private benefits and costs of a stronger response are proportional to those faced by the planner, with a scaling factor $\frac{\theta-1}{\theta}$. In other words, the trade-off faced by agents in equilibrium is the same as the social tradeoff. Note that this only applies to the choice of the response coefficient α. The usual monopoly inefficiency of restricted production applies here as well, but only as a distortion to the average level of activity k_2. Formally, the following result shows that the equilibrium α coincides with the corresponding socially optimal coefficient, but k_2 is inefficiently low. Moreover, the difference between k_2 and k_2^* is invariant to the information structure and vanishes in the competitive limit as $\theta \to \infty$.

Proposition 2 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\alpha^* = \alpha, \tag{16}$$
$$k_2^* = k_2 + \frac{\delta}{\delta-1}\log\left(\frac{\theta}{\theta-1}\right), \tag{17}$$

where α and k_2 are as defined in Proposition 1.

Thus, the distortion caused by imperfect competition takes the form of a constant scaling down of labor input, but does not distort the elasticity of aggregate employment with respect to the shock. This result has an important implication - when information is exogenous, the average level of activity in this economy is inefficiently low, but fluctuations are constrained efficient.

4.3 Efficiency of Information Choice

Next, we show that, despite the optimal response to signals *ex-post*, the ex-ante information acquisition decision is inefficient. Our benchmark is the level of information that maximizes *ex-ante* utility in a symmetric equilibrium, i.e.

$$\max_{\sigma_e^2} \quad \mathbb{U}\left(\sigma_e^2\right) - \upsilon\left(\sigma_e^2\right) \ ,$$

where $\mathbb{U}$ is the expected utility characterized in (12).

We restrict attention to the case where the solution to the above problem is interior, i.e. characterized by the first-order condition[19]:

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \frac{\partial \upsilon}{\partial \sigma_e^2}. \tag{18}$$

Comparing (18) to (6), it is easy to see that information choice is efficient if, and only if, the marginal value to the planner, $\partial \mathbb{U}/\partial \sigma_e^2$ coincides with the private value to the firm, $\partial \hat{\Pi}/\partial \sigma_{ei}^2$.

The next proposition presents the main result of this section. It shows that the intuition from the simple example in Section 2 goes through in this richer general equilibrium environment as well. In any symmetric equilibrium, there is a constant wedge between the private value of information by firms and its social value.

Proposition 3 *In a symmetric equilibrium, the private value of information is always less than its social value, i.e.*

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = \left(1 + \frac{\delta}{(\theta-1)(\delta-1)}\right)\left(\frac{\partial \hat{\Pi}}{\partial \sigma_{ei}^2}\right)_{\sigma_e^2 = \sigma_{ei}^2} < \ 0 \qquad \forall\ \sigma_e^2 \in \mathbb{R}^+. \tag{19}$$

Therefore, the level of information acquired in equilibrium is inefficiently low.

From (19), it is easy to see that the inefficiency is related to the elasticity of substitution, θ. The intuition is similar to the simple example - with imperfect substitutability, marginal revenue is strictly less than the marginal surplus. Therefore, more information, or equivalently better alignment of actions with fundamentals, causes a smaller improvement in profits relative to total utility. As a result, the monopolist attaches a lower value to learning than the social planner and therefore, acquires less than the socially optimal level of information. The extent of this underacquisition is

[19] As with the equilibrium information choice, we also need to assume that the cost function is sufficiently convex, i.e.

$$\frac{\partial^2 \mathbb{U}}{\partial \sigma_e^2 \partial \sigma_e^2} - \frac{\partial^2 \upsilon}{\partial \sigma_e^2 \partial \sigma_e^2} < 0 \ .$$

decreasing in θ. As goods become more and more substitutable, the difference between marginal revenue and marginal surplus shrinks. In the perfectly competitive limit, as $\theta \to \infty$, the gap between the social value and the private value of information vanishes[20].

The implications for efficiency of equilibrium outcomes are immediate. When information is exogenous, the average level of activity is inefficiently low but cyclical fluctuations are constrained efficient. However, this is no longer true when information is endogenous. Too little information is acquired in equilibrium and through its effects on k_2 and α, this suboptimality influences both the average level of activity in equilibrium as well as the elasticity with respect to the shock. The sign of the effect on the former is in general ambiguous, but the response coefficient α is lower, i.e. the sensitivity of employment (and therefore, of output) to the technology shock is inefficiently muted. This is a novel source of inefficiency in this class of models - one that is absent both under the canonical full information assumption as well as under exogenous information (e.g. Angeletos and La'O 2009).

A less obvious implication relates to the social value of public information. Suppose entrepreneurs also had access to a free public signal about aggregate productivity in this environment. It is straightforward to show that this reduces the value of private signals information, leading to lower investments in information. In other words, public information crowds out private information. Since the latter was already being produced at an inefficiently low level, this is detrimental to welfare and in some cases, can overwhelm the direct benefit of more information[21]. Note that this effect is not present when information is exogenous. In that case, public information can reduce welfare only if it is *used* inefficiently. With endogenous information, however, the social value of public information can be negative, even in the absence of *ex-post* inefficiencies.

Finally, we turn to the policy implications of our inefficiency result.

4.4 Optimal Policy

In this subsection, we show that constrained efficiency is restored in this environment if policy is used to correct the average level distortion in production. In particular, a constant revenue

[20] Note that market power and imperfect substitutability are controlled by the same parameter θ. One can easily extend this framework to parameterize these two forces separately. For example, in Angeletos and La'O (2009), a continuum of firms on each island produce differentiated inputs, which are bundled together to produce a final good. Imperfect substitutability of these island-specific final goods leads to aggregate demand linkages, while the differentiated nature of inputs gives firms on each island market power. Our efficiency results extend to this environment as well, with this latter parameter playing the role of θ.

[21] Colombo, Femminis and Pavan (2012) find a similar result with in a quadratic utility framework.

subsidy, equal to the markup not only removes the firm's incentives to underproduce, but also leads it to invest the socially optimal amount in information production.

Given an arbitrary revenue subsidy Λ, the problem of the firm becomes:[22]

$$\Pi_{it} = \max_{N_{it}} \quad \mathbb{E}_{it}\left[\Lambda P_{it} Y_{it} - N_{it}\right] \ ,$$

It is easy to show that the level distortion in activity is removed, i.e. k_2 equals k_2^*, if the subsidy satisfies

$$\Lambda = \frac{\theta}{\theta - 1} \ .$$

More interestingly, this subsidy also aligns marginal revenue with the change in total surplus and therefore, equates the private marginal value of information to the social value, leading to both *ex-post* and *ex-ante* efficiency.

Proposition 4 *A symmetric equilibrium with a constant revenue subsidy $\Lambda = \frac{\theta}{\theta-1}$ is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

Thus, in a real business cycle environment with endogenous information, policies aimed at correcting market-power related inefficiencies have an additional benefit - they also remove the wedge between private and social value of information, eliminating inefficiencies (in both the average level and the fluctuations) arising from suboptimal information choice. This conclusion, however, depends crucially on the fact that information is efficiently used - as the price-setting model in the following section will highlight.

5 Model II: Price-setting with productivity shocks

In this section, we modify the environment in the previous section and assume that entrepreneurs set nominal prices (as opposed to choosing labor input) after observing private signals of aggregate productivity[23]. Formally, the intermediate goods producers now choose nominal prices for their products and commit to producing any amount demanded at that price[24]. The en-

[22] In addition, a lump sum transfer $\tau_R \int P_{it} Y_{it} di$ is subtracted from the budget constraints.

[23] Lorenzoni (2009) studies a similar environment with exogenous dispersed information, but does not address questions of efficiency.

[24] Whether firms compete by choosing prices or quantities is a matter of some debate. See Aiginger(1999) for a survey. One of the studies cited in that paper describes a survey of Austrian manufacturing on their main strategic variable. About 38% of the 930 firms surveyed said they produce a specific quantity, thereafter permitting demand to decide price conditions while the remaining said they set prices leaving competitors and the market to determine quantity sold.

trepreneur's problem is now:

$$\max_{P_{it}} \; E_{it} \left(\frac{P_{it}}{P_t}\right)^{1-\theta} Y_t - \left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} \frac{Y_t}{A_t}\right]^{\delta} . \tag{20}$$

As with quantity choice, optimality equates expected marginal revenue to expected marginal cost:

$$(\theta - 1)\, P_{it}^{-\theta}\, E_{it}\left[P_t^{\theta-1} Y_t\right] = \theta\delta P_{it}^{-\theta\delta-1} E_{it}\left[P_t^{\theta\delta} \frac{Y_t^{\delta}}{A_t^{\delta}}\right] . \tag{21}$$

A comparison (21) and the optimality condition under labor input choice, equation (3) in the previous section, reveals an important difference between the two environments. When a firm chooses its labor input under uncertainty, its marginal cost is known, or more generally, unaffected by the actions of other agents. This is not the case under price setting - the marginal cost to a firm from changing its own price depends on the aggregate price level P_t. Therefore, the sensitivity of average prices to the shock affects each firm's uncertainty about its own marginal cost. As we will see, this additional interaction leads to an externality and will lead to both *ex-post* and *ex-ante* inefficiencies.

The solution strategy follows the same guess-and-verify procedure as in the previous section. We begin with a conjecture that individual prices are set according to:

$$p_{it} = k_2 + \alpha s_{it} \; . \tag{22}$$

The expressions for the response coefficients in a symmetric equilibrium are derived in the Appendix and collected in the following result.

Proposition 5 *In a symmetric equilibrium, firms follow a pricing rule of the form (22), with*

$$\begin{aligned} \alpha &= \left(\frac{-\delta}{\delta - 1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2 + \left(\frac{1+\delta\theta-\theta}{\delta-1}\right)\sigma_e^2}\right], \\ (\delta - 1)k_2 &= \ln\left(\frac{\theta\delta}{\theta - 1}\right) + (\delta - 1)\, m + \frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2 - \delta(\theta - 1)^2 + 1 - \theta\right] + \frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2 - \alpha^2\right], \end{aligned}$$

where σ_e^2 is the variance of the error in agents' signals.

As with quantity choice, the adjustment to the signal-to-noise ratio in the expression for α reveals a coordination motive - strategic complementarities further dampen the response of the aggregate price level to the shock.

5.1 Efficiency in information use

We define the socially optimal response as the utility-maximizing choice of an information-constrained planner, who is free to set the response coefficients α and k_2 but is subject to all the other equilibrium constraints. In particular, given a cross-sectional distribution of prices $\{P_{it}\}$, the aggregate price level P_t and output Y_t are determined by the zero-profit condition of the final goods producer and the quantity equation respectively.

Proposition 6 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\begin{aligned} \alpha^* &= \left(\frac{-\delta}{\delta-1}\right)\left[\frac{\sigma_a^2}{\sigma_a^2+\theta\left(\frac{1+\delta\theta-\theta}{\delta-1}\right)\sigma_e^2}\right], \\ (\delta-1)k_2^* &= -\ln\left(\frac{\theta}{\theta-1}\right)+(\delta-1)\,k_2(\alpha^*), \end{aligned} \tag{23}$$

where $k_2(\alpha^*)$ denotes is the equilibrium level coefficient in Proposition 5 with α^* replacing α.

Thus, the equilibrium features prices that are too responsive[25] to signals, i.e. $\alpha^* > \alpha$. In other words, information used suboptimally when monopolistically competitive firms set prices and optimally when the choice variable is labor input instead. The intuition is related to the marginal cost uncertainty mentioned earlier. Firms do not take into account their contribution to the uncertainty faced by other firms in the economy and as a result, set prices that are too responsive to private signals[26].

The level coefficient also is suboptimal - but now it includes not only the usual markup distortion but also the effects of the inefficient sensitivity to information. Importantly, the latter persist even as the former disappears, e.g as θ tends to infinity.

5.2 Efficiency of information choice

Not surprisingly, the *laissez-faire* choice of signal precision in this environment is suboptimal. Recall that information choice was suboptimal in the quantity choice model, even without *ex-post* inefficiencies in its effect on actions, so the results in Proposition 6 make *ex-ante* efficiency even

[25] Hellwig (2005) arrives at a similar result in an environment with monetary shocks.

[26] What is crucial for the efficiency results is not that the marginal cost is uncertain but that it depends on the actions of other firms. If firms committed to an output level (Y_{it}) in advance, marginal cost would be uncertain (because A_t is not known), but this is unaffected by the behavior of other agents. As a result, information is still used efficiently. Similarly, in the single firm environment of Section 2, responses to signals are efficient even with price choice.

less likely. The characterization of the socially optimal information choice follows the same procedure as section 4.3. Information choice is efficient if, and only if, the marginal social value of learning, $\partial\mathbb{U}/\partial\sigma_e^2$ coincides with the private value, $\partial\hat{\Pi}/\partial\sigma_{ei}^2$.

There is one additional complication. Unlike the quantity choice environment, the social value of information is not always positive (even though information is always privately valuable). This is because changing information now has two effects on welfare. The first, or direct effect, is simply the value of better alignment with fundamentals. The second, or indirect, effect arises because the (inefficient) response coefficient changes with the level of information. Formally, as we show in the Appendix, the social value can be decomposed as follows,

$$\begin{aligned}\frac{\partial\mathbb{U}}{\partial\sigma_e^2} &= -\mathbb{U}\left(\frac{\theta\delta\left(1-\theta+\theta\delta\right)}{2\left(\delta-1\right)}\alpha^2+\frac{\delta\left(\theta-1\right)\left(1-\theta+\theta\delta\right)\sigma^2}{\left(\delta-1\right)}\alpha\frac{d\alpha}{d\sigma^2}\right)\\ &= -\mathbb{U}\frac{\theta\delta\left(1-\theta+\theta\delta\right)}{\left(\delta-1\right)}\left[\frac{\alpha^2}{2}+\left(\frac{\theta-1}{\theta}\right)\sigma^2\alpha\frac{d\alpha}{d\sigma^2}\right].\end{aligned}$$

The two terms inside the square brackets represent the two effects. The first, the direct one, implies that information is socially valuable, while the second term, the indirect effect, goes in the opposite direction since $\alpha\cdot d\alpha/d\sigma^2<0$. The social value of information can be negative when the latter outweighs the former. Holding other parameters fixed, this is more likely when θ is high. Intuitively, higher θ makes realized production levels more sensitive to price differences, making marginal cost uncertainty particularly damaging. In the other direction, as we approach unit elasticity, $\theta\to1$, the indirect effect becomes arbitrarily small[27] and the equilibrium responses coincide with the planner's.

Obviously, if the social value of information is negative at the equilibrium choice of σ_e^2, information choice is trivially inefficient (increasing the noise in the signals will raise utility *and* save on information costs). The more interesting case is when both the profit-maximizing and utility-maximizing information choices are in the interior, i.e. we are in the region where the social value is positive,i.e. $\partial\mathbb{U}/\partial\sigma_e^2<0$. This will always be the case if information is sufficiently cheap. Formally, we assume that the cost function $v(\cdot)$ is such that both the equilibrium and the socially optimal level of information satisfy the following condition:

Assumption 1 $(\theta-2)\left(1-\theta+\theta\delta\right)\sigma_e^2\le\left(\delta-1\right)\theta\sigma_a^2$.

Conditional on being in this region[28], the utility maximizing choice is characterized by equating $\partial\mathbb{U}/\partial\sigma_e^2$ to the marginal cost. The next proposition shows that information acquisition is typi-

[27] As $\theta\to1$, expenditure shares are close to constant, so the strategic linkage becomes very weak.

[28] Note that this always holds for $\theta<2$. If $\theta>2$, then we need σ_e^2 to be sufficiently low.

cally inefficient, though the direction is ambiguous[29]. The result essentially divides the parameter space into two regions, depending on whether the equilibrium exhibits too much or too little information production. Only for a non-generic combination of parameters does the equilibrium choice of σ_e^2 coincide with the utility maximizing level.

Proposition 7 *Suppose $\theta > 2$ and the conditions of Assumption 1 are met. Then, there is over-acquisition of information in equilibrium if the following condition holds at the equilibrium σ_e^2:*

$$\sigma_e^2 > \frac{\theta\,(\delta-1)}{(1-\theta+\theta\delta)}\left[\frac{(1-\theta+\theta\delta)+\delta}{\theta\,(\theta-1)\,(\delta-1)+2\,(1-\theta+\theta\delta)\,(\theta-2)}\right]\sigma_a^2\ .$$

If inequality is reversed, there is underacquisition.

The underlying intuition is not hard to see. There are two sources of inefficiency in information choice, working in opposite directions. As in the quantity choice environment, imperfect substitutability gives firms pricing power, which implies that do not appropriate all the benefits of better information, pushing them towards underacquisition. However, the excess sensitivity of equilibrium responses to information makes increasing it less valuable from a social point of view. The combination of these two forces leaves us with the ambiguous finding in the proposition. Higher θ or σ_e^2 exacerbates the inefficiency in information use, strengthening the second channel and making over-acquisition more likely.

A full quantitative investigation is beyond the scope of this paper, but it is easy to verify that reasonable calibrations of incomplete information monetary models easily satisfy this condition. In other words, the empirically relevant region of the parameter space seems to be one where the equilibrium information acquisition is more than the social optimal level. As an illustrative case, set $\theta = 4, \delta = 1.5$. Then, the noise in private signals only needs to be one-sixth as volatile as aggregate productivity for the above condition to hold. In other words, we need only a very modest departure from full information to see over-acquisition of information in equilibrium.

The implications of this finding for the constrained efficiency of fluctuations as well as the social value of public information are similar to that under the labor input choice model of the previous section. However, the presence of both sources of inefficiency has important implications for policy. We turn to this issue in the following subsection.

[29]For brevity, we only present results for the case where $\theta > 2$ (the empirically relevant case for most macroeconomic models).

5.3 Optimal Policy

Consider the constant subsidy aimed at correcting the monopoly distortion studied in section 4.4 Recall that, with labor input choice, this policy restored constrained efficiency. However, when firms set prices instead, it can have unintended consequences and even reduce welfare. In other words, the desirability of subsidies to correct underproduction might hinge on the nature of firms' decision variable firms - prices or quantities.

To see why the revenue subsidy can be detrimental to welfare, note that aligning marginal revenue with the marginal social surplus increases the private value of information and leads to more learning. However, this additional investment in information acquisition can be socially suboptimal. To put it differently, without the policy, the two sources of inefficiency partially offset each other. If only one of them is removed, the economy bears the full brunt of the other, which could more than overcome the direct benefits of the policy. In other words, incomplete policy responses can do more harm than good.

Figure 3 illustrates such a case. The top panel depicts information choice in equilibrium, where the marginal cost of information (υ') intersects the marginal benefit (π'). The corresponding level of welfare is shown in the bottom panel. Without the subsidy (the solid lines), the equilibrium features over-acquisition of information (note that the variable on the x-axis is precision, the inverse of σ_e^2). In fact, at the equilibrium choice, the social value is negative ! The subsidy raises the private value of information and therefore, leads to more learning. The removal of the monopoly distortion to average production raises utility for all levels of information (the direct effect of removing the markup distortion), but the new equilibrium is associated with a lower level of welfare than without the subsidy (the point D in the bottom panel compared to C).

Finally, we characterize the optimal policy in this environment. For ease of comparison with the quantity choice model, we consider a revenue subsidy of the form

$$\Lambda\ A_t^{\delta\tau}\ ,$$

where the policy parameter τ is the sensitivity of the subsidy to fundamentals[30]. We begin by deriving the values of Λ and τ that lead to *ex-post* efficiency, for a given level of noise in signals σ_e^2.

Proposition 8 *Given σ_e^2, equilibrium allocations coincide with the choices of the planner, i.e.* $(\alpha, k_2) =$

[30] In the quantity choice model, the optimal policy set $\tau = 0$, i.e. the subsidy was invariant to the realization of the fundamental.

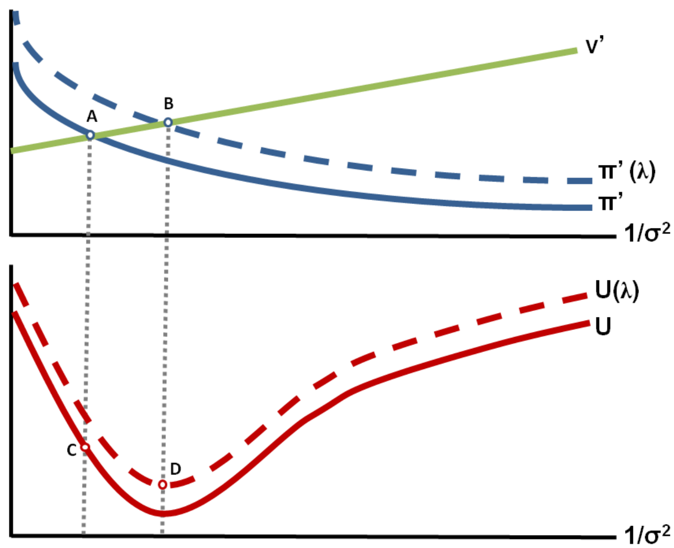

Figure 3: Effect of subsidy on welfare

(α^*, k_2^*) *if the subsidy coefficients satisfy*

$$\tau = \frac{\alpha^*}{\alpha^{eq}} - 1 \quad < \ 0\ , \tag{24}$$
$$\Lambda = \left(\frac{\theta}{\theta - 1}\right) \exp\left\{\frac{\sigma_a^2 \delta\tau^* \left(2\alpha^* - \delta\tau^*\right)}{2}\right\}.$$

The optimal policy is a state-contingent revenue subsidy - decreasing in the technology shock A_t. This countercyclicality dampens the effect of the shock on firm's profits and therefore, reduces the firms' incentives to adjust prices in response to an expected shock, fixing the excess sensitivity problem in equilibrium responses. The level coefficient Λ has the usual markup correction, with an adjustment for level effects arising from the state-contingent part.

More importantly, this policy also implements the socially optimal level of information acquisition, as the following result shows. Formally,

Proposition 9 *A symmetric equilibrium under the policy described in Proposition 8 (evaluated at the socially optimal σ_e^2), is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

In other words, the general insight from the quantity choice model goes through here as well - fixing *ex-post* inefficiencies in equilibrium responses also aligns private and social benefits from learning, leading ex-ante efficiency. Unlike the quantity choice model however, this requires policy to be state-contingent.

6 Model III: Price choice with nominal shocks

In this section, we show that the results from the previous section apply to learning about aggregate nominal shocks as well. In particular, firms adjust their prices by too much in response to expected changes in money supply. The combination of this inefficiency and market power once again leads to an ambiguous sign on the inefficiency in information choice.

The environment is identical to that of the previous section except that productivity is now constant (i.e. $A_t = A$) but aggregate nominal demand is stochastic. In particular, M_t is an iid[31], log-normally distributed random variable, i.e. $\log M_t \equiv m_t \sim N\left(0, \sigma_m^2\right)$. Intermediate goods producers choose nominal prices for their products and commit to producing any amount demanded at that price. Before setting prices, each firm observes a private signal s_{it} about the current monetary shock:

$$s_{it} = m_t + e_{it} \ ,$$

where $e_{it} \sim N\left(0, \sigma_{ei}^2\right)$. The variance of the noise term, σ_{ei}^2 is the variance chosen in stage I by the firm.

As before, the competitive firm producing the final good operates after the monetary shock is realized. Therefore, the problem of this firm remains the same, i.e. demand for intermediate goods is given by 1. Intermediate goods producer's choose prices prior to the realization of the monetary shock.

The intermediate producer's problem is:

$$\max_{P_{it}} \ E_{it} \left(\frac{P_{it}}{P_t}\right)^{1-\theta} Y_t - \left[\left(\frac{P_{it}}{P_t}\right)^{-\theta} \frac{Y_t}{A}\right]^{\delta} .$$

As before, we guess (and verify) that equilibrium prices follow:

$$p_{it} = k_2 + \alpha s_{it} \ . \tag{25}$$

The response coefficients in a symmetric equilibrium are collected in the following result.

Proposition 10 *In a symmetric equilibrium, firms follow a pricing rule of the form (25), with*

$$\alpha = \left[\frac{\sigma_m^2}{\sigma_m^2 + \left(\frac{1-\theta+\theta\delta}{\delta-1}\right)\sigma_e^2}\right] , \tag{26}$$

$$k_2 = \frac{1}{(\delta-1)} \ln \frac{\theta\delta}{\theta-1} - \delta a + \frac{\left(\delta^2-1\right)(1-\alpha)^2}{2(\delta-1)}\sigma_m^2 + \frac{\delta\theta(1-\theta+\theta\delta)}{2(\delta-1)}\alpha^2\sigma_e^2 + \frac{(\delta-1)(\theta-1)}{2(\delta-1)}\alpha^2\sigma_e^2 \tag{27}$$

[31] Again, for simplicity, we assume that nominal demand is iid, though it is straightforward to extend the analysis to richer stochastic processes.

where σ_e^2 is the variance of the error in agents' signals.

A symmetric stationary equilibrium can thus be represented as a fixed point problem in σ_e^2:

$$\sigma_e^2 = \text{argmax}_{\sigma_{ei}^2} \;\; \hat{\Pi}\left[\sigma_{ei}^2, \alpha(\sigma_e^2), k_2(\sigma_e^2)\right] - \upsilon\left(\sigma_{ei}^2\right) \;\; ,$$

where we make explicit the dependence of α and k_2 on σ_e^2 according to the equilibrium relationships (26)-(27).

6.1 Efficiency in Information Use

The socially efficient response function takes the same form as (25) with the coefficients given in the following result.

Proposition 11 *For a given σ_e^2, the planner's optimal response coefficients are:*

$$\begin{aligned} \alpha^* &= \left[\frac{\sigma_m^2}{\sigma_m^2 + \theta\left(\frac{1-\theta+\theta\delta}{\delta-1}\right)\sigma^2}\right] \\ (\delta-1)k_2^* &= \ln\left(\frac{\theta}{\theta-1}\right) + (\delta-1)\,k_2(\alpha^*). \end{aligned}$$

where the dependence of $k_2(\alpha^*)$ denotes the equilibrium level coefficient with α^* replacing α.

Again, the equilibrium features prices that are too responsive to signals, i.e. $\alpha^* < \alpha$. The intuition is very similar to the productivity shocks case - firms do not fully internalize the effect of their pricing decisions on the marginal cost uncertainty faced by other firms. As a result, they react too much to private signals, relative to the planner's solution.

6.2 Efficiency in Information Choice

Next, we compare the amount of information acquired in equilibrium to the utility-maximizing level. As with productivity shocks case discussed in section 5, the presence of direct and indirect effects means that the marginal social value of information is not always positive. We restrict attention to the region where this value is indeed positive. A sufficient condition is

Assumption 2 $(\theta-2)\,(1-\theta+\theta\delta)\,\sigma_e^2 \leq (\delta-1)\,\theta\sigma_m^2$.

Conditional on being in this region, whether the social planner acquires more or less information than the equilibrium depends only on the marginal value to the planner, $\partial\mathbb{U}/\partial\sigma_e^2$, versus the private value to the firm, $\partial\hat{\Pi}/\partial\hat{\sigma}_e^2$.

The following result mirrors Proposition 7 and shows that the equilibrium can feature both under- and over-acquisition. Again, as before, we only present results for the case where $\theta > 2$

Proposition 12 *Suppose $\theta > 2$ and the conditions of Assumption 2 are met. Then, there is over-acquisition of information in equilibrium if the following condition holds:*

$$\sigma_e^2 \geq \left[\frac{\delta\theta\left(\delta - 1\right)}{\left(1 - \theta + \theta\delta\right)\left[2\left(\theta - 1\right) + \left(\theta - 2\right)\theta\delta\right]} \right] \sigma_m^2 \, .$$

In the spirit of the simple numerical illustration following Proposition 7, suppose $\theta = 4, \delta = 1.5$. Then, the condition in the above result amounts to requiring that the variance of noise in signals be at least five percent of the variance of money supply, a small deviation from full monetary neutrality.

6.3 Optimal Policy

We conclude our discussion of this version of the model by characterizing optimal policy in this environment. In line with section 5, we consider revenue subsidies of the form,

$$\Lambda \; M_t^{(1-\delta)\tau} \; .$$

The following proposition characterizes the policy coefficients that correct both the sources of inefficiency in the equilibrium response functions.

Proposition 13 *Given σ_e^2, equilibrium allocations coincide with the choices of the planner, i.e. $(\alpha, k_2) = (\alpha^*, k_2^*)$ if the subsidy coefficients satisfy*

$$\begin{aligned} \tau &= \frac{\alpha^*}{\alpha^{eq}} - 1 \qquad < \quad 0, \\ \Lambda &= \left(\frac{\theta}{\theta - 1}\right)^{\frac{1}{\delta - 1}} \exp\left\{\frac{\sigma_m^2 \tau^* \left(2\left(1 - \alpha^*\right) + \left(1 - \delta\right)\tau^*\right)}{2}\right\}. \end{aligned} \tag{28}$$

As we would expect, this policy also removes the wedge between private and social value of information, ensuring that signal precisions in equilibrium are socially optimal. Formally,

Proposition 14 *A symmetric equilibrium with the policy described in Proposition 28 is constrained efficient, i.e. it attains the optimal allocation of the information constrained planner.*

7 A Beauty Contest Model

In this section, we study information choice in a beauty contest model, in the spirit of the global games literature, see Morris and Shin (1998, 2002)[32]. Though more abstract than the micro-founded environments of the previous sections, this setup will allow us to both demonstrate the applicability of our main results to coordination games more generally as well as to draw connections to earlier work on the efficiency properties of economies with dispersed information. We show that social and private value of information are, in general, different and so information acquisition in equilibrium is typically inefficient relative to a socially optimal benchmark. This inefficiency can arise be due to the suboptimal *use* of information, but it can be present even when information is used efficiently.

7.1 Payoffs and Information

There is a continuum of agents, indexed by $i \in [0, 1]$. The game in played in two stages. In stage I, agents choose how much private information (measured by the precision of a private signal about an aggregate fundamental) to acquire subject to a cost function. In stage II, signals are realized and agent i chooses an action $x_i \in \mathbb{R}$ to maximize expected the following *private* payoff function:

$$\Pi_i = \max_{x_i} \; -\mathbb{E}_i \left[\phi \left(x_i - a \right)^2 + \psi \left(x_i - \bar{x} \right)^2 \right],$$

where $\bar{x} \equiv \int_0^1 x_i di$ is the average action of all agents, A is the underlying aggregate state and $\mathbb{E}_i (\cdot) \equiv \mathbb{E} (\cdot | \mathcal{I}_i)$ is the expectation operator conditional on agent i's information set $\mathcal{I}_i$. The random variable a represents an aggregate state, which is normally distributed with mean zero and variance σ_a^2.

The payoff function for agent i has two components. The first component is linked to the (squared) deviation between the underlying state θ and agent i's action x_i. The second part is the squared distance between i's action and the average action of all the other agents in the economy, denoted $\bar{x}$. The two components capture the idea that an agent's payoff depends not only on fundamentals but also on actions of other agents (a feature that was present in the business cycle environments studied earlier). The parameters ϕ and ψ index the relative importance of these two components in private payoffs. For ease of exposition, we focus on the case where these two weights are positive, though this is not essential for our results.

[32] It is also possible to demonstrate our main results in the more general quadratic payoff structure, as in Angeletos and Pavan (2007).

Before choosing x_i, each agent has access to a private signal s_i about the fundamental:

$$s_i = a + e_i \ ,$$

where $e_i \sim N\left(0, \sigma_{ei}^2\right)$. This variance $\hat{\sigma}_e^2$ is the result of choices made in stage I by the agent. The noise term e_i is independent of a and independent across the population, i.e. $\mathbb{E}\left(e_i \, e_j\right) = 0$ for $i \neq j$. The agent's information set consists only of the common prior and this private signal.

Let $\hat{\Pi}(\cdot)$ denote the expected payoff in stage II (prior to the realization of the signals s_i):

$$\hat{\Pi}_i\left(\sigma_{ei}^2, \sigma_e^2\right) \ \equiv \ \mathbb{E}\left(\Pi_i\right),$$

where $\mathbb{E}\left(\cdot\right)$ is the expectation operator prior to the realization of signals, σ_{ei}^2 is the variance of the agent's own private signal and σ_e^2 is the variance of the signals of all the other agents in the economy[33]. The problem of the agent in the first stage can then be written as:

$$\max_{\hat{\sigma}_e^2} \ \ \hat{\Pi}_i\left(\sigma_{ei}^2, \sigma_e^2\right) - \upsilon\left(\sigma_{ei}^2\right) \ \ ,$$

where $\upsilon\left(\cdot\right)$ is the cost of information as a function of the noise in the signal. For now, we impose only that $\upsilon'(\cdot) < 0, \upsilon''(\cdot) > 0$.

7.2 Equilibrium

We start with the equilibrium in stage II. The agent's maximization problem directly yields the following first order condition:

$$x_i = \frac{\phi}{\phi + \psi}\mathbb{E}_i\left(a\right) + \frac{\psi}{\phi + \psi}\mathbb{E}_i\left(\bar{x}\right) \ \ .$$

We conjecture (and verify) that, in a symmetric equilibrium, the average action is linked to the realization of the fundamental θ according to this linear relationship:

$$\bar{x} = \alpha a \ \ .$$

Given this conjecture,

$$\begin{aligned}
x_i &= \frac{\phi + \psi\alpha}{\phi + \psi}\mathbb{E}_i\left(a\right) \\
x_i &= \left(\frac{\phi + \psi\alpha}{\phi + \psi}\right)\left(\frac{\sigma_a^2}{\sigma_a^2 + \sigma_{ei}^2}\right)\left(a + e_i\right) = \hat{\alpha}\left(a + e_i\right) \ \ , \qquad (29) \\
\Longrightarrow \hat{\alpha} &= \left(\frac{\phi + \psi\alpha}{\phi + \psi}\right)\left(\frac{\sigma_a^2}{\sigma_a^2 + \sigma_{ei}^2}\right).
\end{aligned}$$

[33]We restrict attention to symmetric equilibria, where all agents make the same information acquisition choices.

The conjecture is verified when

$$\hat{\alpha} = \alpha \ ,$$

which leads to the following result.

Proposition 15 *The unique symmetric equilibrium is given by $x_i = \alpha^{eq}\ s_i$, where*

$$\alpha^{eq} = \frac{\phi\sigma_a^2}{\phi\sigma_a^2 + (\phi+\psi)\sigma_e^2} \ . \tag{30}$$

Information Acquisition: Next, we turn to the ex-ante information acquisition decision in stage I. Recall that each agent chooses the precision of her private signals, subject to a cost function $\upsilon(\cdot)$. At the optimum, each agent equates the marginal value of more information to its cost. In a symmetric equilibrium, the envelope theorem implies that this *private* marginal value of information is the same for all agents and is given by:

$$\frac{\partial \hat{\Pi}_i}{\partial \hat{\sigma}_\epsilon^2} = -\left(\phi+\psi\right)\left(\alpha^{eq}\right)^2 \ . \tag{31}$$

Under the assumption of an interior optimum[34], the optimality condition in stage I becomes:

$$-\left(\phi+\psi\right)\left(\alpha^{eq}\right)^2 = \upsilon'(\sigma_e^2) \ .$$

Noting that λ^{eq} is in turn a function of the (symmetric) information choice, the above condition is a fixed point in σ_e^2 and completes the characterization of the equilibrium with endogenous information acquisition.

7.3 Welfare

Next, we study the efficiency properties of the equilibrium characterized in the previous subsection. The first step is to define a social welfare criterion, which is assumed to take the same form as private payoffs:

$$\begin{aligned} W &= U - \int \upsilon\left(\sigma_{ei}^2\right) di \\ &= -\mathbb{E}\left[\phi^* \int_0^1 \left(x_i - a\right)^2 di + \psi^* \int_0^1 \left(x_i - \bar{x}\right)^2 di\right] - \int \upsilon\left(\sigma_{ei}^2\right) di. \end{aligned}$$

where ϕ^* and ψ^* are both positive. Thus, welfare is declining in the average deviations from the fundamental θ and the cross-sectional dispersion in actions, but with weights that are potentially

[34]We assume that $\upsilon(\cdot)$ is such that the optimum is reached at an interior point.

different from the ones in private payoffs. These differences arise due to externalities, e.g. as in Morris and Shin (2002).

At the symmetric equilibrium characterized in the previous subsection, welfare (before information acquisition costs) is

$$U = -\phi^*(\alpha^{eq} - 1)^2\sigma_a^2 - (\phi^* + \psi^*)\,(\alpha^{eq})^2\,\sigma_e^2\ .$$

The social value of information is given by

$$\frac{dU}{d\sigma_e^2} = -(\phi^* + \psi^*)\alpha^2 + \frac{dU}{d\alpha}\,\frac{d\alpha}{d\sigma_e^2}\ . \tag{32}$$

The information choice (assumed to be in the interior) that maximizes social welfare is one at which this marginal benefit is equal to the marginal cost of information $v'(\sigma_e^2)$. Recall that the equilibrium information choice was characterized by equating the private marginal value (31) to the marginal cost. Thus, the inefficiency in information choice is determined by the difference between social and private marginal values, i.e. $\frac{dU}{d\sigma_e^2}$ and $\frac{\partial\hat{\Pi}_i}{\partial\sigma_{\epsilon i}^2}$.

It is useful to first characterize the efficient use of information. This is modeled as the choice of an information-constrained planner who directly chooses agents' actions to maximize the above objective. We restrict attention to linear response functions of the form:

$$x_i = \alpha s_i\ .$$

The efficient use of information is then the solution to

$$\begin{aligned}\mathbb{U} &= \max_\alpha \quad -\phi^*(\alpha-1)^2\mathbb{E}a^2 + (\phi^*+\psi^*)\alpha^2\sigma_e^2\\ &= \max_\alpha \quad -(\phi^*(\alpha-1)^2\sigma_a^2 + (\phi^*+\psi^*)\alpha^2\sigma_e^2)\ .\end{aligned}$$

The first order condition[35] of the above problem is:

$$\frac{\partial\mathbb{U}}{\partial\alpha} : \phi^*(\alpha-1)\sigma_a^2 + (\phi^*+\psi^*)\alpha\sigma_e^2 = 0\ . \tag{33}$$

Re-arranging, we derive the following result:

Proposition 16 *The socially efficient linear response coefficient, denoted α^* is*

$$\alpha^* = \frac{\phi^*\sigma_a^2}{\phi^*\sigma_a^2 + (\phi^*+\psi^*)\sigma_e^2}. \tag{34}$$

[35]The second-order condition requires that $\phi^*\sigma_\theta^2 + (\phi^*+\psi^*)(\sigma_\theta^2+\sigma_e^2) \geq 0$.

Comparing the two response coefficients, α^{eq} and α^*, we see that equilibrium responses are efficient if, and only if, the agents attach the same relative weight to the two types of deviations in their private payoffs as the planner does. Formally,

Proposition 17 *For a given σ_e^2, equilibrium response is efficient if, and only if, the relative weights of the two components are equal in the private and social payoff function, i.e.*

$$\alpha^{eq} = \alpha^* \qquad \Leftrightarrow \qquad \frac{\phi}{\psi} = \frac{\phi^*}{\psi^*} \; .$$

This finding is an instance of a well-known feature of these models (see, for example, Angeletos and Pavan 2007) - differences between the social and private costs of dispersion and volatility can lead to information being used in a socially sub-optimal manner.

With some algebra, we can rewrite the social value of information in (32) as

$$\frac{dU}{d\sigma_e^2} = \frac{d\Pi}{d\sigma_{ei}^2} \left[\left(\frac{\phi^* + \psi^*}{\phi + \psi} \right) - 2\frac{\phi^*}{\phi} \left(\frac{\alpha^{eq}}{\alpha^*} - 1 \right) \right] . \tag{35}$$

This equation is key for understanding our inefficiency result. It shows that social and private marginal values of information can diverge for two reasons. First suppose $\alpha^{eq} = \alpha^*$, i.e. information use is socially optimal. Then, the second term inside the brackets is zero. Even so, information can contribute more (less) to social welfare than to private profits if $\phi^* + \psi^*$ is greater (smaller) than $\phi + \psi$. In other words, externalities can lead to inefficiencies through differences in the overall level of social welfare and private payoffs, even if they do not distort the relative importance of the two components of payoffs.

What happens when information use is inefficient ? For concreteness, consider the case where $\phi^* + \psi^* = \phi + \psi$ but $\alpha^{eq} > \alpha^*$, i.e. there are no level differences but agents respond too much to their signals, relative to the planner's solution. Then, the term inside the square bracket is less than 1, i.e. the social value of information is lower than the private value. Intuitively, more information has an additional effect - it makes the ex-post inefficiency more severe. The opposite happens when $\alpha^{eq} < \alpha^*$.

The information-constrained optimum in this economy, i.e. the outcome when the planner chooses both the amount of information and its ex-post use, is also easy to characterize. By the envelope theorem, the social marginal value of information in this allocation is given by

$$\frac{dU^*}{d\sigma_e^2} = -\left(\phi^* + \psi^*\right)\left(\alpha^*\right)^2 \; , \tag{36}$$

Equating this to the marginal cost yields fixed point relationship that defines the information-constrained optimum.

$$-\left(\phi^* + \psi^*\right)\left(\alpha^*\right)^2 = \upsilon'(\sigma_e^2) \; .$$

This level of information differs from the equilibrium one for the 2 reasons discussed earlier. The first is linked to the suboptimality in information use referred to earlier, i.e. to the fact that α^{eq} may not be equal to α^*. However, even if the equilibrium information use is efficient, i.e. $\alpha^{eq} = \alpha^*$, the private marginal value of information can still diverge from the socially optimal level because of a level effect, i.e. the difference between $\phi^* + \psi^*$ and $\phi + \psi$. To see this more clearly, note that we can rewrite this social value as follows:

$$\frac{dU^*}{d\sigma_e^2} = \underbrace{\frac{\partial\hat{\Pi}_i}{\partial\hat{\sigma}_e^2}}_{\text{Private value}} - \frac{1}{(\phi+\psi)} \underbrace{\left\{ \left[\left(\frac{\phi^* + \psi^*}{\phi + \psi} \right) - 1 \right] (\alpha^*)^2 + [(\alpha^*)^2 - (\alpha^{eq})^2] \right\}}_{\text{'Externalities'}} .$$

7.4 Implementing the Information-Constrained Optimum

In this subsection, we consider the nature of interventions that are necessary to correct the information-related inefficiencies in the equilibrium characterized above. Given a precision of signals, σ_e^2, we will show that efficiency in information use can be restored through a 'tax', which aligns the social and private weights attached to the two payoff components. However, in line with the general intuition behind the findings in the previous subsection, we will show that this, by itself, is not sufficient to align private incentives to acquire information with the social ones.

We start with the sub-optimal nature of information use. Formally, we consider a tax, τ, of the following form[36]

$$\Pi_i = \max_{x_i} \; - \mathbb{E}_i(\phi\tau\,(x_i - a)^2 + \psi\,(x_i - \bar{x})^2) \; .$$

For a given tax τ, the equilibrium response coefficient is:

$$\alpha^\tau = \frac{\phi\tau\sigma_a^2}{\phi\tau\sigma_a^2 + (\phi\tau + \psi)\sigma_e^2} \; .$$

Any response coefficient α can be implemented by setting the tax appropriately, i.e. by solving the following equation for τ,

$$\alpha = \frac{\phi\tau\sigma_a^2}{\phi\tau\sigma_a^2 + (\phi\tau + \psi)\sigma_e^2} \; .$$

In particular, to implement α^*, the socially optimal response, the tax is simply

$$\tau^* = \frac{\phi^*}{\psi^*}\,\frac{\psi}{\phi} \; .$$

The expression for the optimal tax rate is intuitive - it corrects the inefficiency in information use by aligning the relative weights of the two components in the private and social payoff functions.

[36]This formulation is not the only way to restore efficiency in use of information. The key point, however, is that correcting the inefficiency in information use is not sufficient to get the economy to the information-constrained optimum.

However, this correction by itself is not enough to align the private incentives to acquire information with those of the planner. The marginal private value of information under τ^*, is given by:

$$\frac{\partial \hat{\Pi}_i}{\partial \sigma_{ei}^2} = -\left(\phi \tau^* + \psi\right)\left(\alpha^*\right)^2 = -\frac{\psi}{\psi^*}(\phi^* + \psi^*)(\alpha^*)^2 \ .$$

Thus, the private marginal value of information is equal to the social marginal value if, and only if[37], $\psi = \psi^*$. In other words, even if payoffs are distorted by policy to achieve efficiency in the use of information, information choice still remains inefficient. In general, in order to restore efficiency along both these margins, we need 2 distinct forms of intervention - one which aligns the relative weights in private and social payoffs and another which corrects the level distortions. Here, we propose one such implementation. In addition to the τ policy discussed earlier, we employ another 'tax', denoted κ, which affects total payoffs. Then, the private payoff is :

$$\Pi_i = \max_{x_i} \ -\kappa \ \mathbb{E}_i(\phi \tau \left(x_i - a\right)^2 + \psi \left(x_i - \bar{x}\right)^2) \ .$$

We can then show that the following policy achieves the constrained-efficient allocation.

$$\begin{aligned} \tau &= \tau^* = \frac{\phi^*}{\psi^*} \ \frac{\psi}{\phi}, \\ \kappa &= \frac{\psi^*}{\psi} \ . \end{aligned}$$

7.5 Public Signals

While the analysis in this paper has focused on the acquisition of private information, the economic forces leading to inefficiency also affect incentives to learn through public signals. Here, we demonstrate this by extending the beauty contest model to include both public and private signals. The payoff structure is the same as before but agents' information set now also has the following additional signal:

$$S_i = a + \rho_i \ \epsilon \ \ ,$$

where $\epsilon \sim N\left(0, \hat{\sigma}_\epsilon^2\right)$ is a common noise term, while ρ_i reflects the extent to which agent i's signal is affected by that noise term. As $\rho_i \to \infty$, this signal becomes worthless from the perspective of forecasting a. In the other direction, as $\rho_i \to 0$, this becomes an arbitrarily precise signal of the fundamental.

[37]Note that this condition depends on the nature of tax that was introduced. If, for example, the distortion was a tax to the second component of the payoff function, then we need $\phi^* = \phi$ for the optimal tax to ensure efficiency in information acquisition as well, we need $\phi = \phi^*$.

The information cost is now a function of both the public and private information choices, i.e $\upsilon(\sigma_{ei}^2, \rho^2)$. One interpretation is that agent chooses both the precision and the degree of commonality in her information and could face potentially non-separable costs. As in the baseline model, we impose very little structure on this cost function, beyond monotonicity and curvature assumptions needed to ensure interior solutions to the optimization problems of the agent and the planner. The following proposition characterizes the equilibrium and socially optimal response functions. It confirms that efficiency in information use is obtained if the relative weights are the same in private and social payoffs.

Proposition 18 *1. There exists a pair of constants, α_1^{eq} and α_2^{eq} such that actions in a symmetric equilibrium are given by*

$$x_i = \alpha_1^{eq} s_i + \alpha_2^{eq} S_i \ .$$

2. There exist constants α_1^* and α_2^* such that the symmetric socially optimal response function is

$$x_i = \alpha_1^* s_i + \alpha_2^* S_i \ .$$

3. Given a symmetric information structure, i.e. with the same (σ_e^2, ρ^2) for all agents in the economy, the two sets of response coefficients are equal if, and only if, $\frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$.

We then turn to the choice of commonality, ρ^2. In general, inefficiencies in information use also drive a wedge between the social and private value of commonality. More interestingly, however, they can differ even when information is used optimally, i.e. $\frac{\phi}{\psi} = \frac{\phi^*}{\psi^*}$. To see this, note that the private marginal value of observing the public signal more precisely is

$$\frac{\partial \Pi}{\partial \rho^2} = -\phi \left(\alpha_2^{eq}\right)^2 \sigma_\epsilon^2 \ .$$

The social value is

$$\begin{aligned} \frac{\partial U}{\partial \rho^2} &= -\phi^* \left(\alpha_2^*\right)^2 \sigma_\epsilon^2 = -\phi^* \left(\alpha_2^{eq}\right)^2 \sigma_\epsilon^2 \ . \\ &= \left(\frac{\phi^*}{\phi}\right) \frac{\partial \Pi}{\partial \rho^2}. \end{aligned}$$

Again, as with private information choice, the *level* of social-versus-private payoffs matter for incentives to invest in information.

8 Conclusion

The preceding sections highlight a novel source of inefficiency in a class of business cycle models used widely in modern macroeconomics. *Ex-post* inefficiencies feed back into *ex-ante* incentives to invest in information, even when these inefficiencies leave responses to the information undistorted. This in turn leads to suboptimal levels of learning and through that, *ex-post* equilibrium outcomes that are constrained inefficient, both in terms of average levels and elasticity to fundamental shocks.

There are several directions for future work. With a view to maintaining analytical tractability, we have made several simplifying assumptions. For example, we focus exclusively on static decisions, but the channels we highlight also have implications for intertemporal decisions (e.g. through capital accumulation, pricing with nominal frictions etc.). Similarly, for expositional simplicity, we rule out additional shocks (aggregate or idiosyncratic) and other sources of information. Relaxing some of these assumptions will require the use of numerical methods, but will allow a quantitative evaluation of the inefficiency and the policy interventions necessary to correct it. On the theoretical side, exploring the connections between the payoff-linked inefficiencies in this paper with others identified by the literature (e.g. the inefficiency in Amador and Weill (2010)) is another interesting direction for future work.

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Appendix A Proofs of Results

A.1 Model I: Quantity choice with productivity shocks

A.1.1 Equilibrium

We solve for equilibrium by studying the problem of an individual entrepreneur i, who takes as given the information choices of all other entrepreneurs $j \neq i$ in the economy. In a symmetric equilibrium, she conjectures (correctly) that all other firms follow a log-linear policy rule:

$$n_{jt} = k_2 + \alpha s_{jt} \; .$$

This implies

$$\begin{aligned} y_{jt} &= a_t + \frac{n_{jt}}{\delta} = a_t + \frac{k_2}{\delta} + \frac{\alpha}{\delta} s_{jt} = \left(1 + \frac{\alpha}{\delta}\right) a_t + \frac{k_2}{\delta} + \frac{\alpha}{\delta} e_{jt} \; , \\ y_t &= \int y_{jt} \, dj = \left(1 + \frac{\alpha}{\delta}\right) a_t + \frac{k_2}{\delta} + \frac{1}{2}\left(\frac{\theta - 1}{\theta}\right) \frac{\alpha^2}{\delta^2} \sigma_e^2 \; , \\ \left(\frac{\theta-1}{\theta}\right) a_t + \frac{1}{\theta} y_t &= \left(\frac{\theta - 1}{\theta} + \frac{1}{\theta} + \frac{\alpha}{\theta\delta}\right) a_t + \frac{k_2}{\theta\delta} + \frac{1}{2}(\theta - 1) \frac{\alpha^2}{\theta^2\delta^2} \sigma_e^2 \; . \end{aligned}$$

We then guess (and verify) that i 's best response takes the form

$$n_{it} = \hat{k}_2 + \hat{\alpha} s_{it} \; .$$

Using this, we write i's objective function as

$$\begin{aligned} & \hat{K}_2^{\frac{\theta-1}{\theta\delta}} \exp\left\{\frac{1}{2}(\theta - 1)\frac{\alpha^2}{\theta^2\delta^2}\sigma_e^2\right\} E_{it}\left(A_t^{\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)} e_{it}^{\hat{\alpha}\frac{\theta-1}{\theta\delta}} K_2^{\frac{1}{\theta\delta}}\right) - \hat{K}_2 E_{it} A_t^{\hat{\alpha}} e_{it}^{\hat{\alpha}} \\ = \; & \exp\left\{\frac{\theta-1}{\theta\delta}\hat{k}_2 + \frac{1}{\theta\delta}k_2 + \frac{1}{2}(\theta-1)\frac{\alpha^2}{\theta^2\delta^2}\sigma_e^2\right\} \\ & \exp\left\{\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right) + \frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right] a_t + \hat{\alpha}\frac{\theta-1}{\theta\delta} e_{it}\right\} \\ & - \exp\left\{\hat{k}_2\right\} \exp\left\{\hat{\alpha} a_t + \hat{\alpha} e_{it}\right\} \; . \end{aligned}$$

The unconditional expectation is[38]

$$\underbrace{\frac{\exp\left\{\frac{\theta-1}{\theta\delta}\hat{k}_2+\frac{1}{\theta\delta}k_2+\frac{1}{2}\left(\theta-1\right)\frac{\alpha^2}{\theta^2\delta^2}\sigma_e^2\right\}}{\exp\left\{\frac{1}{2}\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]^2\sigma_a^2+\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_{ei}^2\right\}}}_{C}-\underbrace{\exp\left\{\hat{k}_2\right\}\exp\left\{\frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2+\sigma_{ei}^2\right)\right\}}_{N} \quad (37)$$

FOC:

$\hat{k}_2$

$$\left(\frac{\theta-1}{\theta\delta}\right)C=N\,. \quad (38)$$

$\hat{\alpha}$

$$\begin{aligned}\left\{\left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2\left(\frac{\theta-1}{\theta\delta}\right)+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_{ei}^2\right\}C &= \hat{\alpha}\left(\sigma_a^2+\sigma_{ei}^2\right)N \\ \left[\frac{\theta-1}{\theta}\left(1+\frac{\hat{\alpha}}{\delta}\right)+\frac{1}{\theta}\left(1+\frac{\alpha}{\delta}\right)\right]\sigma_a^2+\hat{\alpha}\left(\frac{\theta-1}{\theta\delta}\right)\sigma_{ei}^2 &= \hat{\alpha}\left(\sigma_a^2+\sigma_{ei}^2\right)\,.\end{aligned}$$

In a symmetric equilibrium, $\sigma_{ei}^2=\sigma_e^2$, $\hat{k}_2=k_2$ and $\hat{\alpha}=\alpha$. These conditions then become

$$\begin{aligned}\left(\frac{\theta-1}{\theta\delta}\right)\exp\left\{\frac{1}{\delta}k_2+\frac{1}{2}\left(1+\frac{\alpha}{\delta}\right)^2\sigma_a^2+\frac{1}{2}\alpha^2\left(\frac{\theta-1}{\theta\delta}\right)^2\sigma_e^2\right\} &= \exp\left\{k_2+\frac{1}{2}\hat{\alpha}^2\left(\sigma_a^2+\sigma_e^2\right)\right\}\,, \\ \left(1+\frac{\alpha}{\delta}-\alpha\right)\sigma_a^2 &= \alpha\left(1-\frac{\theta-1}{\theta\delta}\right)\sigma_e^2\,.\end{aligned}$$

Rearranging, we get the expressions in Proposition 1.

The expression for the private value of information on the left hand side of (10) is obtained by a direct application of the envelope theorem to (37) along with (38).

$$\begin{aligned}\frac{\partial\hat{\Pi}}{\partial\sigma_{ei}^2} &= C\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)^2-N\frac{1}{2}\hat{\alpha}^2 \\ &= \frac{1}{2}\hat{\alpha}^2\left[\left(\frac{\theta\delta}{\theta-1}\right)\left(\frac{\theta-1}{\theta\delta}\right)^2-1\right]N \\ &= -\frac{1}{2}\hat{\alpha}^2\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)N \\ &= -\frac{1}{2}\hat{\alpha}^2\left(\frac{\theta-1}{\theta\delta}\right)\hat{\Pi}\,, \quad (39)\end{aligned}$$

where the last step makes use of the fact that, in equilibrium, $\hat{\Pi}=C-N=\left(\frac{1-\theta+\theta\delta}{\theta-1}\right)N$.

[38] We need to verify that firms have no incentive to change their response coefficients after seeing the signal, i.e. we should check that the response also maximizes conditional expected profits. This is easy to show under log-normality.

A.1.2 Efficiency in information choice

Expected utility is given by

$$\begin{aligned}\mathbb{U} &= \exp\left[\frac{1}{2}\left(1+\frac{\alpha}{\delta}\right)^2\sigma_a^2+\frac{k_2}{\delta}+\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\frac{\alpha^2}{\delta^2}\sigma_e^2\right]-\exp\left[k_2+\frac{1}{2}\alpha^2\sigma_a^2+\frac{1}{2}\alpha^2\sigma_e^2\right]\\ &= C^*\exp\left(\frac{k_2-k_2^*}{\delta}\right)-N^*\exp\left(k_2-k_2^*\right),\end{aligned}$$

where k_2^* is the optimal response coefficient and (C^*, N^*) the corresponding unconditional expectations of consumption and labor input. Using (13), we get

$$\begin{aligned}\mathbb{U} &= \left[\delta\exp\left(\frac{k_2-k_2^*}{\delta}\right)-\exp\left(k_2-k_2^*\right)\right]N^*\\ &= \left[\delta\exp\left(\frac{k_2-k_2^*}{\delta}\right)-\exp\left(k_2-k_2^*\right)\right]\frac{\mathbb{U}^*}{\delta-1},\end{aligned}$$

where $\mathbb{U}^* = C^* - N^*$.Then,

$$\frac{\partial\mathbb{U}}{\partial\sigma_e^2}=\left[\delta\exp\left(\frac{k_2-k_2^*}{\delta}\right)-\exp\left(k_2-k_2^*\right)\right]\frac{1}{\delta-1}\frac{\partial\mathbb{U}^*}{\partial\sigma_e^2}.$$

The term in the square bracket is independent of σ_e^2. The envelope theorem implies,

$$\begin{aligned}\frac{\partial\mathbb{U}^*}{\partial\sigma_e^2} &= C^*\frac{1}{2}\left(\frac{\theta-1}{\theta}\right)\frac{\alpha^2}{\delta^2}-N^*\frac{1}{2}\alpha^2\\ &= -\frac{1}{2}\alpha^2\left(1-\frac{\theta-1}{\theta\delta}\right)N^*,\end{aligned}$$

Substituting,

$$\begin{aligned}\frac{\partial\mathbb{U}}{\partial\sigma_e^2} &= -\frac{1}{2}\alpha^2\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)\left[\delta\exp\left(\frac{k_2-k_2^*}{\delta}\right)-\exp\left(k_2-k_2^*\right)\right]\frac{N^*}{\delta-1}\\ &= -\frac{1}{2}\alpha^2\left(\frac{1-\theta+\theta\delta}{\theta\delta}\right)\frac{1}{\delta-1}\mathbb{U}\\ &= -\frac{1}{2}\alpha^2\left[\frac{1-\theta+\theta\delta}{(\theta-1)(\delta-1)}\right]\mathbb{U}\left(\frac{\theta-1}{\theta\delta}\right)\\ &= -\frac{1}{2}\alpha^2\left[1+\frac{\delta}{(\theta-1)(\delta-1)}\right]\mathbb{U}\left(\frac{\theta-1}{\theta\delta}\right).\end{aligned} \tag{40}$$

Since $\mathbb{U} = \hat{\Pi}$, we have the result in Proposition 3.

A.1.3 Policy

To see that the constant revenue subsidy also aligns private and social values of information, note that the only change in the derivation of the private value of information above is in (39). Since the level distortion to output is not present under this subsidy, $\hat{\Pi} = (\delta-1)N$ instead of $\hat{\Pi} = \left(\frac{1-\theta+\theta\delta}{\theta-1}\right)N$. Then, it is easy to see that the resulting expression for private value is identical to the social value in (40).

A.2 Model II: Price setting with productivity shocks

A.2.1 Equilibrium

As with the quantity choice, we begin with the problem of entrepreneur i, who believes (correctly) that everybody else is acting according to

$$p_{jt} = k_2 + \alpha s_{jt} = k_2 + \alpha a_t + \alpha e_{jt} \ .$$

The corresponding aggregate relationships are

$$\begin{aligned}
p_t &= k_2 + \alpha a_t + \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \ , \\
y_t &= m - k_2 - \alpha a_t - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \ , \\
y_{jt} &= -\theta\left(p_{jt} - p_t\right) + y_t = -\alpha\theta e_{jt} + \frac{\theta}{2}(1-\theta)\alpha^2\sigma^2 + m - k_2 - \alpha a_t - \frac{1}{2}(1-\theta)\alpha^2\sigma_e^2 \\
&= -\alpha\theta\varepsilon_{jt} + m - k_2 - \alpha a_t - \frac{1}{2}\alpha^2\sigma_e^2(\theta-1)^2 \ , \\
n_{jt} &= \left(y_{jt} - a_t\right)\delta = -\alpha\theta\delta\varepsilon\backslash e_{jt} + \delta m - \delta k_2 - \delta(\alpha+1)a_t - \frac{1}{2}\delta\alpha^2\sigma_e^2(\theta-1)^2 \ , \\
n_t &= \delta m - \delta k_2 - \delta(\alpha+1)a_t - \frac{1}{2}\delta\alpha^2\sigma_e^2(\theta-1)^2 + \frac{1}{2}\alpha^2\theta^2\delta^2\sigma_e^2 \\
&= \delta m - \delta k_2 - \delta(\alpha+1)a_t + \frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta - (\theta-1)^2\right] \ .
\end{aligned}$$

We then solve for i's optimal policy

$$p_{it} = \hat{k}_2 + \hat{\alpha}s_{it} \ .$$

The objective function can be written as

$$\begin{aligned}
& \hat{K}_2^{1-\theta}K_2^{\theta-2}\exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]ME_{it}\left[A_t^{\alpha(\theta-2)+\hat{\alpha}(1-\theta)}e_{it}^{\hat{\alpha}(1-\theta)}\right] - \\
& \hat{K}_2^{-\theta\delta}K_2^{\delta(\theta-1)}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right]M^\delta E_{it}A_t^{\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}}e_{it}^{-\theta\delta\hat{\alpha}} \\
= \ & M\exp\left\{(1-\theta)\hat{k}_2 + (\theta-2)k_2\right\}\exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right] \\
& \exp\left\{\left[\alpha(\theta-2) + \hat{\alpha}(1-\theta)\right]a_t + \hat{\alpha}(1-\theta)e_{it}\right\} \\
& -M^\delta\exp\left\{-\theta\delta\hat{k}_2 + \delta(\theta-1)k_2\right\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\
& \exp\left\{\left[\alpha\delta(\theta-1) - \delta - \theta\delta\hat{\alpha}\right]a_t - \theta\delta\hat{\alpha}e_{it}\right\} \ .
\end{aligned}$$

The unconditional expectation is

$$\underbrace{\begin{array}{c} M\exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]\exp\left\{(1-\theta)\hat{k}_2+(\theta-2)k_2\right\} \\ \exp\left\{\frac{1}{2}\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right]^2\sigma_a^2+\frac{1}{2}\hat{\alpha}^2(1-\theta)^2\sigma_{ei}^2\right\}\end{array}}_{C}$$

$$-\underbrace{\begin{array}{c} M^\delta\exp\left\{-\theta\delta\hat{k}_2+\delta(\theta-1)k_2\right\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\ \exp\left\{\frac{1}{2}\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\hat{\alpha}^2\sigma_{ei}^2\right\}\end{array}}_{N}$$

FOC

$\hat{k}_2$:

$$(\theta-1)C=\theta\delta N\ ,$$

$\hat{\alpha}$:

$$C\left\{\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right](1-\theta)\sigma_a^2+\hat{\alpha}(1-\theta)^2\sigma_{ei}^2\right\}=-N\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]\theta\delta\sigma_a^2+\theta^2\delta^2\hat{\alpha}\sigma_{ei}^2\ .$$

In a symmetric equilibrium, the FOC for k_2 becomes

$$\begin{aligned}
&& \frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2+m-k_2+\frac{1}{2}\left[-\alpha\right]^2\sigma_a^2+\frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2 \\
&=& \ln\left(\frac{\theta\delta}{\theta-1}\right)+\delta m-\delta k_2-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2+\frac{1}{2}\left[\delta(1+\alpha)\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2 \\
\Longrightarrow(\delta-1)\,k_2 &=& \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-\alpha^2\right] \\
&& +\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2-(\theta-2)(1-\theta)-(1-\theta)^2\right] \\
\Longrightarrow(\delta-1)\,k_2 &=& \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2+1-\theta\right]+\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-\alpha^2\right]\ .
\end{aligned}$$

The FOC for $\hat{\alpha}$ simplifies to,

$$\begin{aligned}
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[\alpha(\theta-2)+\alpha(1-\theta)\right](1-\theta)\sigma_a^2+\alpha(1-\theta)^2\sigma_e^2\right\} &=& \left[-\alpha\delta(\theta-1)+\delta+\theta\delta\alpha\right]\theta\delta\sigma_a^2+\theta^2\delta^2\alpha\sigma_e^2 \\
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\alpha(\theta-1)\sigma_a^2+\alpha(\theta-1)^2\sigma_e^2\right\} &=& \left[\delta\left(\alpha+1\right)\right]\theta\delta\sigma_a^2+\theta^2\delta^2\alpha\sigma_e^2 \\
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\alpha\sigma_a^2+\alpha(\theta-1)\sigma_e^2\right\} &=& \left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[\delta\left(\alpha+1\right)\right]\sigma_a^2+\alpha\sigma_e^2\theta\delta\right\} \\
\alpha\sigma_a^2+\alpha(\theta-1)\sigma_e^2 &=& \left[\alpha\delta+\delta\right]\sigma_a^2+\theta\delta\alpha\sigma_e^2 \\
-\delta\sigma_a^2 &=& \alpha\left[\left(\delta-1\right)\sigma_a^2+(1-\theta+\theta\delta)\sigma_e^2\right]\ ,
\end{aligned}$$

Rearranging yields the expressions in Proposition 5.

The private value of information is a direct application of the envelope theorem:

$$\begin{aligned}
\frac{\partial \hat{\Pi}}{\partial \sigma_{ei}^2} &= \frac{1}{2}\hat{\alpha}^2(1-\theta)^2 C - \frac{1}{2}\theta^2\delta^2\hat{\alpha}^2 N \\
&= -\frac{1}{2}\hat{\alpha}^2\left\{-(1-\theta)^2\left(\frac{\theta\delta}{\theta-1}\right)+\theta^2\delta^2\right\}N \\
&= -\frac{1}{2}\hat{\alpha}^2\theta\delta(1-\theta+\theta\delta)N \\
&= -\frac{1}{2}\hat{\alpha}^2\theta\delta\,(\theta-1)\,\hat{\Pi}\,.
\end{aligned}$$

A.2.2 Efficiency in information use

The planner's problem to pick the optimal response coefficients:

$$\max_{\alpha,k_2} \quad \exp\left[m-k_2+\frac{1}{2}\alpha^2\sigma_a^2-\frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right]-\exp\left[\delta m-\delta k_2+\delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2+\frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta-(\theta-1)^2\right]\right].$$

The optimality conditions:

$$\begin{aligned}
\underbrace{\exp\left[m-k_2+\frac{1}{2}\alpha^2\sigma_a^2-\frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right]}_{C^*} &= \delta \quad \underbrace{\exp\left[\delta m-\delta k_2+\delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2+\frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta-(\theta-1)^2\right]\right]}_{N^*}, \\
\alpha\sigma_a^2+\alpha(\theta-1)\sigma_e^2 &= \delta(\alpha+1)\sigma_a^2+\alpha\sigma_e^2\left[\theta^2\delta-(\theta-1)^2\right] \\
&= \delta(\alpha+1)\sigma_a^2+\alpha\sigma_e^2\left[\theta(2-\theta+\theta\delta)-1\right] \\
&= \delta\alpha\sigma_a^2+\alpha\sigma_e^2\left[\theta(1-\theta+\theta\delta)+\theta-1\right]+\delta\sigma_a^2 \\
\alpha\sigma_a^2 &= \delta\alpha\sigma_a^2+\alpha\sigma_e^2\left[\theta(1-\theta+\theta\delta)\right]+\delta\sigma_a^2\,.
\end{aligned}$$

Solving, we get the coefficients in Proposition 6.

A.2.3 Efficiency in information choice

Expected utility is given by

$$\mathbb{U}=\exp\left[m-k_2+\frac{1}{2}\alpha^2\sigma_a^2-\frac{1}{2}(1-\theta)\alpha^2\sigma_e^2\right]-\exp\left[\delta m-\delta k_2+\delta^2(\alpha+1)^2\frac{1}{2}\sigma_a^2+\frac{1}{2}\alpha^2\sigma_e^2\delta\left[\theta^2\delta-(\theta-1)^2\right]\right].$$

The social value of information is

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2}=C\left[-\frac{1-\theta}{2}\alpha^2+(\sigma_a^2-(1-\theta)\sigma_e^2)\alpha\frac{d\alpha}{d\sigma_e^2}\right]-N\left[\frac{1}{2}\alpha^2\delta\left[\theta^2\delta-(\theta-1)^2\right]+\delta^2(\alpha+1)\sigma_a^2\frac{d\alpha}{d\sigma_e^2}+\alpha\sigma_e^2\delta\left[\theta^2\delta-(\theta-1\right.\right.$$

After some algebra,

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2}=-\mathbb{U}\frac{\theta\delta\,(1-\theta+\theta\delta)}{(\delta-1)}\left[\frac{\alpha^2}{2}+\ \left(\frac{\theta-1}{\theta}\right)\sigma^2\alpha\frac{d\alpha}{d\sigma_e^2}\right].$$

Plugging in $d\alpha/d\sigma_e^2$ and rearranging,

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} = -\frac{\delta\left(1-\theta+\theta\delta\right)}{\delta-1}\left[\frac{\theta\left(\delta-1\right)\sigma_a^2-\left(\theta-2\right)\left(1-\theta+\theta\delta\right)\sigma_e^2}{\left(\delta-1\right)\sigma_a^2+\left(1-\theta+\theta\delta\right)\sigma_e^2}\right]\alpha^2\mathbb{U}\,.$$

It is easy to see that the conditions in Assumption (1), ensure

$$\frac{\partial \mathbb{U}}{\partial \sigma_e^2} \leq 0\,.$$

Finally, we compare this social value to the private value obtained above. A few lines of tedious algebra yield the result in Proposition 7, i.e. that there is over-acquisition of information if and only if:

$$\sigma_e^2 > \frac{\theta\left(\delta-1\right)}{\left(1-\theta+\theta\delta\right)}\left[\frac{\left(1-\theta+\theta\delta\right)+\delta}{\theta\left(\theta-1\right)\left(\delta-1\right)+2\left(1-\theta+\theta\delta\right)\left(\theta-2\right)}\right]\sigma_a^2$$

A.2.4 Policy

Given a revenue subsidy of the form

$$\Lambda\ A_t^{\delta\tau},$$

the unconditional expectation now becomes

$$\underbrace{\begin{array}{c} M\Lambda\exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]\exp\left\{(1-\theta)\hat{k}_2+(\theta-2)k_2\right\} \\ \exp\left\{\frac{1}{2}\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)\right]^2\sigma_a^2+\frac{1}{2}\hat{\alpha}^2(1-\theta)^2\sigma_{ei}^2\right\}\end{array}}_{C}$$

$$-\underbrace{\begin{array}{c} M^\delta\exp\left\{-\theta\delta\hat{k}_2+\delta(\theta-1)k_2\right\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right] \\ \exp\left\{\frac{1}{2}\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\hat{\alpha}^2\sigma_{ei}^2\right\}\end{array}}_{N}\,.$$

The FOC for $\hat{k}_2$:

$$\left(\theta-1\right)C = \theta\delta N\,,$$

and for α :

$$\left\{\left[\alpha(\theta-2)+\hat{\alpha}(1-\theta)+\delta\tau\right](1-\theta)\sigma_a^2+\hat{\alpha}(1-\theta)^2\sigma_{ei}^2\right\}C = \left\{-\left[\alpha\delta(\theta-1)-\delta-\theta\delta\hat{\alpha}\right]\theta\delta\sigma_a^2+\theta^2\delta^2\hat{\alpha}\sigma_{ei}^2\right\}N\,.$$

Using the FOC for $\hat{k}_2$ and invoking symmetry

$$\begin{aligned}
\left(\frac{\theta\delta}{\theta-1}\right)\left\{\left[-\alpha+\delta\tau\right](1-\theta)\sigma_a^2+\alpha(1-\theta)^2\sigma_e^2\right\} &= \left\{\delta(1+\alpha)\theta\delta\sigma_a^2+\theta^2\delta^2\hat{\alpha}\sigma_e^2\right\} \\
\left[\alpha-\delta\tau\right]\sigma_a^2+\alpha(1-\theta)\sigma_e^2 &= \delta(1+\alpha)\sigma_a^2+\theta\delta\hat{\alpha}\sigma_e^2 \\
-\delta\sigma_a^2(1+\tau) &= \alpha\left[\left(\delta-1\right)\sigma_a^2+(1-\theta+\theta\delta)\sigma_e^2\right] \\
\alpha &= \frac{-\delta\sigma_a^2(1+\tau)}{\left[\left(\delta-1\right)\sigma_a^2+(1-\theta+\theta\delta)\sigma_e^2\right]} \\
\alpha &= \left(1+\tau\right)\alpha^{eq}\,. \qquad (41)
\end{aligned}$$

Invoking symmetry in the FOC for k_2

$$
\begin{aligned}
& M\Lambda \exp\left[\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2\right]\exp\{-k_2\}\exp\left\{\frac{1}{2}\left[-\alpha+\delta\tau\right]^2\sigma_a^2+\frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2\right\} \\
= & \left(\frac{\theta\delta}{\theta-1}\right)M^\delta \exp\{-\delta k_2\}\exp\left[-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2\right]\exp\left\{\frac{1}{2}\left[-\delta(1+\alpha)\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2\right\}
\end{aligned}
$$

In logs,

$$
\begin{aligned}
& & m+\lambda+\frac{1}{2}(\theta-2)(1-\theta)\alpha^2\sigma_e^2-k_2+\frac{1}{2}\left[-\alpha+\delta\tau\right]^2\sigma_a^2+\frac{1}{2}\alpha^2(1-\theta)^2\sigma_e^2 \\
& = & \ln\left(\frac{\theta\delta}{\theta-1}\right)+\delta m-\delta k_2-\frac{1}{2}\delta(1-\theta)^2\alpha^2\sigma_e^2+\frac{1}{2}\left[\delta(1+\alpha)\right]^2\sigma_a^2+\frac{1}{2}\theta^2\delta^2\alpha^2\sigma_e^2 \\
(\delta-1)\,k_2 & = & \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2-(\theta-2)(1-\theta)-(1-\theta)^2\right] \\
& & +\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-(-\alpha+\delta\tau)^2\right]-\lambda \\
(\delta-1)\,k_2 & = & \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^2\sigma_e^2\left[\theta^2\delta^2-\delta(1-\theta)^2+1-\theta\right] \\
& & +\frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha)^2-(-\alpha+\delta\tau)^2\right]-\lambda\,. \qquad (42)
\end{aligned}
$$

Any response function (α, k_2) can be implemented by setting the policy parameters (τ, λ) to satisfy (41) and (42). In particular, to implement the socially optimal (α^*, k_2^*), we need

$$
\begin{aligned}
\tau^* & = & \frac{\alpha^*}{\alpha^{eq}}-1\,, \\
\lambda^* & = & \ln\left(\frac{\theta\delta}{\theta-1}\right)+(\delta-1)\,m+\frac{1}{2}\alpha^{*2}\sigma^2\left[\theta^2\delta^2-\delta(1-\theta)^2+1-\theta\right]+ \\
& & \frac{1}{2}\sigma_a^2\left[\delta^2(1+\alpha^*)^2-(-\alpha^*+\delta\tau^*)^2\right]-(\delta-1)\,k_2^* \\
& = & \ln\left(\frac{\theta}{\theta-1}\right)+\frac{\sigma_a^2\delta\tau^*\left(2\alpha^*-\delta\tau^*\right)}{2}\,.
\end{aligned}
$$

which proves Proposition 8.

To see that this also aligns private and social values, first note that the policy implements the socially optimal response by construction, so we can directly apply the envelope theorem to get

$$\frac{d\mathbb{U}}{d\sigma_e^2}=-\frac{1}{2}\alpha^2\theta\delta(1-\theta+\theta\delta)EN^*\,.$$

Now, recall that, in equilibrium, private value is

$$\frac{d\hat{\Pi}}{d\sigma_{ei}^2}=-\frac{1}{2}\hat{\alpha}^2\theta\delta(1-\theta+\theta\delta)EN\,.$$

Since $EN = EN^*$with efficient response functions,

$$\frac{d\hat{\Pi}}{d\sigma_{ei}^2}=\frac{d\mathbb{U}}{d\sigma_e^2}$$

establishing the result in Proposition 9.

A.3 Model III: Price setting with nominal shocks

The proofs for the results in this section are almost identical to those of section 5, so in the interest of brevity, we omit them.

A.4 A Beauty Contest Model

Proof of Proposition 15 Follows directly by setting $\hat{\alpha} = \alpha$ in (29) and solving.

Proof of Proposition 16 We solve (33) for α.

Proof of Proposition 17 Follows from the comparison of the expressions for α and α^*.

Proof of Proposition 18 We start with a conjecture about the average action,

$$\bar{x} = \alpha_1 a + \alpha_2 S \, .$$

Then, the optimality condition of the agent implies

$$x_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \alpha_1 \right) \mathbb{E}_i(a) + \frac{\psi}{\phi + \psi} \alpha_2 S \, .$$

Integrating over i,

$$x_i = \left(\frac{\phi}{\phi + \psi} + \frac{\psi}{\phi + \psi} \alpha_1 \right) \bar{\mathbb{E}}(a) + \frac{\psi}{\phi + \psi} \alpha_2 S \, .$$

Next, note that we can write

$$\mathbb{E}_i(a) = \delta_1 s_i + \delta_2 S \, ,$$

where $\delta_1 = \frac{\frac{1}{\sigma_e^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\rho^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$ and $\delta_2 = \frac{\frac{1}{\rho^2 \sigma_\epsilon^2}}{\frac{1}{\sigma_e^2} + \frac{1}{\rho^2 \sigma_\epsilon^2} + \frac{1}{\sigma_\theta^2}}$. This implies that the cross-sectional average expectation $\bar{\mathbb{E}}(a) = \delta_1 a + \delta_2 S$. Substituting in the expression for $\bar{a}$ yields a system of linear equations. The solution is

$$\alpha_1^{eq} = \frac{\phi \delta_1}{\phi + \psi(1 - \delta_1)} \qquad \alpha_2^{eq} = \frac{\phi + \psi}{\phi} \frac{\delta_2}{\delta_1} \alpha_1^{eq} \, .$$

The planner's optimality conditions for α_1^* and α_2^* are

$$\phi^*(\alpha_1^* + \alpha_2^* - 1)\sigma_a^2 + (\phi^* + \psi^*)\alpha_1 \sigma_e^2 = 0 \, ,$$

$$\phi^*(\alpha_1^* + \alpha_2^* - 1)\sigma_a^2 + \phi_2^{*\alpha *} \rho^2 \sigma_\epsilon^2 = 0 \, .$$

Solving yields

$$\alpha_1^* = \frac{\phi^* \delta_1}{\phi^* + \psi^*(1 - \delta_1)} \, , \qquad \alpha_2^* = \frac{\phi^* + \psi^*}{\phi^*} \frac{\delta_2}{\delta_1} \alpha_1^* \, .$$

Comparing the two sets of coefficients yields the last part of the proposition.