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Wage Inequality *

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Abstract

The objective of this paper is to study why are some workers paid more than others. To do so we construct and quantitatively assess an equilibrium search model with on-the-job search, general human capital accumulation and two sided heterogeneity. In the model workers differ in abilities and firms differ in their productivities. The model generates a simple (log) wage variance decomposition that is used to measure the importance of firm and worker productivity differentials, frictional wage dispersion and workers’ sorting dynamics. We calibrate the model using a sample of young workers for the UK. We show that heterogeneity among firms generates a lot of wage inequality. Among low skilled workers job ladder effects are small, most of the impact of experience on wages is due to learning-by-doing. High skilled workers are much more mobile. Job ladder effects have sizeable impact.

Keywords: Job search, human capital accumulation, wage inequality, turnover.
JEL: J63, J64, J41, J42.

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1 Introduction

In this study we address the following question: why are some workers paid more than others? This is a classic question posed by many economists through the years. In recent times five reasons for such inequality have been proposed. First, differentials in workers’ abilities have long been recognized as an important source of wage inequality. Human capital theory, pioneered by Becker (1964), further explains why years of education and other relevant fixed factors play a role in explaining wage differences. Second, following early work by Mincer (1974), many have argued learning-by-doing implies workers accumulate more human capital while working. This implies that time in employment may well play an important role in wage inequality. Third, time spent in employment may contribute in another way to wage inequality. If workers search while employed, the longer a worker is employed the greater the probability she will find a higher paying job and this in itself can lead to inequality (e.g. Burdett, 1978, for early work in this area). Fourth, some have stressed there are productivity differences among firms and some firms may choose to offer higher wages. Several quite different theories reach this conclusion (see, for example, Lentz and Mortensen, 2008, Postel-Vinay and Robin, 2002, and Bartelsman and Doms, 2000). Finally, it has been argued by some that frictions in the labour market can by themselves generate wage dispersion among the employed even when workers and firms are identical (see, for example, Burdett and Mortensen, 1998, and Hornstein, Krusell and Violante, 2011). The objective of this study, then, is to assess the extent to which each of these factors contribute to overall wage inequality.

This paper considers equilibrium wage formation using a standard search framework (e.g. Burdett and Mortensen, 1998, (henceforth B/M) and Burdett, Carrillo-Tudela and Coles, 2011) where firms have different productivities and workers have different abilities. Employed workers also accumulate general human capital through learning-by-doing and engage in on-the-job search. Like Bagger et al. (2014), an important issue is to explain why wages, on average, increase with experience. An important difference, however, is that here we also provide a complete characterisation of the market distribution of employee wages. This characterisation is useful for we can then decompose overall wage inequality into its constituent parts. By calibrating the model to the data, we evaluate the relative importance of worker and firm heterogeneity, search (and sorting) and learning-by-doing as explanations for overall wage inequality.

A second contribution of the paper is that we directly address the issues raised by Hornstein et al. (2007, 2011). The $M/m$ ratio is defined as the ratio of the average wage earned to the lowest wage paid in the market among equally productive workers. For plausible parameter

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1. This approach necessarily abstracts from life-cycle effects on job search and wages. For recent work on this issue, see Bowlus and Liu (2013) and Menzio, Telyukova and Visschers (2012).
2. Bagger et al. (2014) use the offer-matching framework to structurally estimate how wages evolve at firms when workers’ productivity also evolves stochastically.
3. A different approach uses statistical models, based on Mincer’s (1974) original work, to try to identify the impact of experience on wages. Prominent examples include Topel (1991), Altonji and Williams (2007), Dustmann and Meghir (2005), among many others. The evidence from this body work, however, remains hotly debated. There are a few other papers which have investigated learning-by-doing within a search environment. Bunzel et al. (2000) provide an interesting early example. Fu (2009), Yamaguchi (2010) and Bagger et al. (2014) provide more recent examples. Manning (2003), Rubinstein and Weiss (2007), Barlevy (2008) and Bowlus and Liu (2013) estimate a wage process similar to the one identified here but do not consider equilibrium.
values, Hornstein et al. (2011) explain why most search models generate a reservation wage that is too high to match the observed \( Mm \) ratio in the US economy. Here we show why an equilibrium search framework with on-the-job search and learning-by-doing generates realistic \( Mm \) values. The simple reason being that when experience is valuable, unemployed workers are willing to accept low starting wages.\(^4\) Although generating empirically relevant \( Mm \) ratios is not difficult, the Hornstein et al. (2007, 2011) approach remains important for it not only provides relevant information for calibrating the model, it provides a coherent empirical framework for analysing wage dispersion. Indeed our calibration approach follows closely their methodology.\(^5\)

To quantitatively assess the model we use labour market histories of a sample of young workers drawn from the British Household Panel Survey (BHPS). We choose to evaluate the model on young workers as it is precisely at this stage of a worker’s labour market history that job mobility is most common. As there are strong differences on returns to education across workers, we divide the sample into three different educational or skill groups and analyze them separately. After controlling for several observable characteristics that are unrelated to the model, the calibration shows that the contribution of labour market experience in accounting for wage dispersion is sizeable, but decreases with workers’ skills and the differences among skill groups arise due to the importance of sorting dynamics and the returns to on-the-job search. Low skilled workers are more likely to move from one job to another via unemployment whereas medium and high skilled workers are more likely to move from job to job via on-the-job search. For high and medium skilled workers, firm productivity differentials explain 39 and 48 percent of overall wage dispersion and differences in earned piece rates explain 19 and 23 percent, while worker ability differentials explains the remainder 30 and 42 percent, respectively. For low skilled workers, differences in earned piece rates explain around 30 percent, while worker ability and firm productivity differentials explain 23 and 51 percent, respectively. These findings show that the importance of worker ability differences in explaining wage dispersion is positively correlated with workers’ skills, while the importance of firm productivity differentials are negatively correlated with workers’ skills.

The rest of the paper is outlined as follows. Section 2 describes the model. Section 3 defines and characterizes the equilibrium. Section 4 describes the data, the calibration procedure and present the main results. All proofs are relegated to an Appendix.

2 The Model

Time is continuous with an infinite horizon and we only consider steady state. There is a continuum of firms and workers, each of measure one. Any worker’s life is described by the exponential distribution with parameter \( \phi > 0 \). To keep the population of workers constant, \( \phi \) also describes the inflow of new labour market entrants.

A worker when entering the labour market has initial ability \( \varepsilon \) which is considered a random draw from an exogenous distribution \( A(\cdot) \) with support \([\underline{\varepsilon}, \bar{\varepsilon}]\). Learning-by-doing implies a

\(^4\)Indeed some college interns would seem to work for no pay in return for job experience.

\(^5\)Also see Ortego-Marti (2012) and Tjaden and Wellschmied (2014) for related work on the \( Mm \) approach.
worker’s ability increases at rate $\rho$ when working, where $0 < \rho < \phi$. Assuming an unemployed worker’s productivity remains constant through time, a type $\varepsilon$ worker with $x$ years experience has productivity $y = \varepsilon e^{\rho x}$.

Firms have a constant returns to scale technology and are ex-ante heterogeneous with fixed productivity parameter $p$. Let $\Gamma(\cdot)$ denote the exogenous distribution of productivities across firms which, for ease of exposition we assume is differentiable [no mass points] with connected support $[p, \bar{p}]$.

A worker with productivity $y$ who is employed at a firm with productivity $p$ generates flow revenue $yp$. We assume a firm pays each of its employees the same piece rate $\theta$, and so a worker $y$ employed at firm $p$ paying piece rate $\theta$ earns flow wage $w = \theta yp$. Given the firm’s $p$ and $\theta$, however, it is convenient to define $z = p\theta$ as its corresponding wage rate paid, where $zy$ describes the wage paid to any employee $y$. We let $F(z)$ denote the fraction of firms which offer wage rate no greater than $z$, with support denoted $[z, \bar{z}]$. $F(\cdot)$, of course, is endogenously determined.

Each unemployed and employed worker receives job offers according to an exogenous Poisson process with parameters $\lambda_u$ and $\lambda_e$ respectively. Conditional on receiving a job offer, random matching implies $F(z)$ describes the probability the offered wage rate is no greater than $z$. If a worker rejects a job offer, there is no recall.

Each employed worker is displaced into unemployment at rate $\delta > 0$. While unemployed, a worker with productivity $y$ enjoys flow payoff $by$, where $b$ denotes home productivity.

Workers are risk neutral, discount the future at rate $r \geq 0$ and maximize expected discounted lifetime income. Firms do not discount the future and so maximize steady state flow profit.

### Optimal Search Strategies

For a given $F$, consider optimal worker behavior. Let $W^U(y)$ denote the maximum expected lifetime payoff of an unemployed worker with productivity $y$, let $W^E(y, z)$ denote the maximum expected lifetime payoff of a worker with productivity $y$ employed at a firm paying wage rate $z$.

The Bellman equation implies $W^E(\cdot)$ is defined recursively by:

$$(r + \phi)W^E(y, z) = zy + \rho \frac{\partial W^E}{\partial y} + \lambda_e \int_z^\bar{z} \left[ W^E(y, z') - W^E(y, z) \right] dF(z') + \delta \left[ W^U(y) - W^E(y, z) \right].$$

where the second term describes the increase in value through learning-by-doing. It is immediate that $W^E(\cdot)$ must be strictly increasing in $z$. Hence any employee $y$ quits to any outside offer $z' > z$. Thus all employees adopt the same quit strategy and so the rate an employee leaves a firm paying wage rate $z$ is

$$q(z) = \phi + \delta + \lambda_e (1 - F(z)).$$

As there is no human capital accumulation while unemployed, the Bellman equation describ-
ing \( W^U(y) \) is

\[(r + \phi)W^U(y) = by + \lambda_u \int_0^\infty \max[W^E(y, z') - W^U(y), 0]dF(z').\]

As \( W^E(y, z) \) is strictly increasing in \( z \), an unemployed worker accepts job offer \( z' \) if and only if \( W^E(y, z') \geq W^U(y) \). Thus an unemployed worker with productivity \( y \) adopts a reservation wage strategy, where the worker’s reservation wage rate \( z_R \) solves \( W^E(y, z_R) \geq W^U(y) \).

As all workers are risk neutral and income and learning-by-doing are both proportional to \( y \), the solution to the above Bellman equations takes the separable form:

\[ W^U(y) = \alpha^U y, \quad \text{and} \quad W^E(y, z) = \alpha^E(z)y, \]

where \( \alpha^U \) and \( \alpha^E(\cdot) \) are determined below. The unemployed worker’s reservation wage rate \( z_R \) is now given by \( \alpha^E(z_R) = \alpha^U \) and so is independent of worker productivity \( y \). Proposition 1 determines \( \alpha^U \) and \( \alpha^E(\cdot) \).

**Proposition 1:** Given \( F(\cdot) \), optimal job search implies

(i) \( \alpha^E(\cdot) \) is the solution to the initial value problem:

\[ \frac{d\alpha^E}{dz} = \frac{1}{q(z) + r - \rho}, \]

with \( \alpha^E(z) = \left( z + \delta \alpha^U \right) \left( r + \phi + \delta - \rho \right) \) at \( z = \tau \),

(ii) \( (\alpha^U, z_R) \) satisfy the pair of equations:

\[ \rho \alpha^U = b - z_R + (\lambda_u - \lambda_e) \int_{z_R}^{\infty} \frac{1 - F(z)}{q(z) + r - \rho} dz, \quad (1) \]

\[ (r + \phi)\alpha^U = b + \lambda_u \int_{z_R}^{\infty} \frac{1 - F(z)}{q(z) + r - \rho} dz. \quad (2) \]

Further \( \tau > b(r + \phi - \rho)/(r + \phi) \) implies a unique solution exists for \( \alpha^U, \alpha^E(\cdot) \) and that \( \alpha^U > 0 \) and \( z_R < \tau \).

Using (1) and (2), the reservation wage rate \( z_R \) is given by:

\[ (r + \phi)z_R = b(r + \phi - \rho) + [\lambda_u (r + \phi - \rho) - (r + \phi)\lambda_e] \int_{z_R}^{\infty} \frac{1 - F(x)}{q(x) + r - \rho} dx. \quad (3) \]

Given this characterisation of optimal worker behavior, we now consider optimal firm behavior.

**Firm Profits**

Consider now the optimal wage setting strategy of a firm \( p \) when \( F \) describes the distribution of wage rate offers made by all other firms and unemployed workers adopt reservation wage rate \( z_R \) given by (3). As a firm with productivity \( p < z_R \) can only make negative profit, it cannot be active in the labour market. Hence define \( p_0 = \max\{z_R, p\} \) which describes the lowest productivity firm which is active in the labour market. As \( 1 - \Gamma(p_0) \) describes the measure of
active firms, those with \( p \geq p_0 \), then

\[
\Gamma_0(p) = \frac{\Gamma(p) - \Gamma(p_0)}{1 - \Gamma(p_0)}
\]  

(4)
describes the distribution of firm productivities across active firms.

To characterise equilibrium, we first need to define three steady-state objects conditional on worker type \( \varepsilon \): (a) \( U_\varepsilon \) is the fraction of type \( \varepsilon \) workers who are unemployed, (b) \( N_\varepsilon(y) \) is the fraction of unemployed type \( \varepsilon \) workers with productivity no greater than \( y \), and (c) \( H_\varepsilon(y, z) \) is the joint distribution function describing the probability that an employed type \( \varepsilon \) worker has current ability no greater than \( y \), employed at wage rate no greater than \( z \).

We now compute \( \Omega(z; p) \) defined as steady state flow profit of a firm \( p \) which pays wage rate \( z \geq z_R \). The standard way of doing this is to integrate over the profits generated by those workers employed at the firm. With a zero discount rate, however, Burdett et al. (2011) show steady state flow profit is more easily obtained by integrating over the inflow of new hires times the expected lifetime profit of each hire. Consider then a firm with productivity \( p \) which pays wage rate \( z \geq z_R \). If it hires a new employee with productivity \( y \), the lifetime discounted profit from that hire is \((p - z)y/(q(z) - \rho)\) as the employee leaves the firm at rate \( q(z) \) and productivity \( y \) grows at rate \( \rho \) as long as the employment relationship survives. The steady state flow profit of a firm \( p \) is therefore:

\[
\Omega(z; p) = \int \left[ \lambda_\varepsilon U_\varepsilon \int_{y' = \varepsilon}^{\infty} \frac{(y - z)y'}{q(z) - \rho} dN_\varepsilon(y') + \lambda_\varepsilon (1 - U_\varepsilon) \int_{y' = \varepsilon}^{\infty} \int_{z' = z}^{z} \frac{(p - z)y'}{q(z) - \rho} dH_\varepsilon(y', z') \right] dA(\varepsilon).
\]

where the first term describes the profits generated by recruiting new workers from the unemployment pool, the second by attracting workers employed at firms paying a wage rate below \( z \).

For \( p \in [p_0, \bar{p}] \), define maximal profit as \( \overline{\Omega}(p) = \max_{z \geq z_R} \Omega(z; p) \). Let \( F_p(z) \) denote the distribution of wage rates offered by firms with productivity \( p \). We now formally define equilibrium.

A Market Equilibrium is a set \( \{z_R, U_\varepsilon, N_\varepsilon(\cdot), H_\varepsilon(\cdot, \cdot), F(\cdot), F_p(\cdot)\} \) for all \( \varepsilon \in [\underline{\varepsilon}, \bar{\varepsilon}] \) and \( p \in [p_0, \bar{p}] \) such that:

(i) the productivity distribution of active firms \( \Gamma_0 \) is given by (4) with \( p_0 = \max \{z_R, \bar{p} \} \);
(ii) the constant profit condition is satisfied for each firm type \( p \in [p_0, \bar{p}] \); i.e.,

\[
\Omega(z; p) = \overline{\Omega}(p) \quad \text{for } z \text{ where } dF_p(z) > 0;
\]
\[
\Omega(z; p) \leq \overline{\Omega}(p) \quad \text{for } z \text{ where } dF_p(z) = 0,
\]
(iii) where aggregation implies offer distribution

\[
F(z) = \int_{p_0}^{\bar{p}} F_p(z) d\Gamma_0(p);
\]
(iv) \( U_\varepsilon, N_\varepsilon(\cdot) \) and \( H_\varepsilon(\cdot, \cdot) \) are consistent with steady state turnover, and
(v) $z_R$ solves the conditions in Proposition 1.

Given a Market Equilibrium exists it is simple to show (and has been shown many times in
the literature) that

(a) $z = z_R$, and
(b) for each $p \in [p_0, p]$, $F_p(.)$ must be continuous [no mass points].

Standard turnover arguments further imply unemployment rate $U_\varepsilon = U$ where

$$U = \frac{\phi + \delta}{\phi + \delta + \lambda_u}$$

is the same for all ability types. Given this simplification, Lemma 1 now solves for the market
distributions $N_\varepsilon(.)$ and $H_\varepsilon(.)$.

**Lemma 1: A Market Equilibrium implies**

$$N_\varepsilon(y) = 1 - \frac{\lambda_u \delta}{(\phi + \lambda_u)(\phi + \delta)} \left( \frac{y}{\varepsilon} \right)^{\frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \delta + \lambda_u)}} \text{ for all } y \geq \varepsilon,$$

$$H_\varepsilon(y, z) = \frac{(\phi + \delta) F(z)}{q(z)} \left[ 1 - \left( \frac{y}{\varepsilon} \right)^{\frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \delta + \lambda_u)}} \right] - \frac{\delta \lambda_u F(z)}{\lambda_u \delta + \lambda_e (1 - F(z)) (\phi + \lambda_u)} \left[ \left( \frac{y}{\varepsilon} \right)^{\frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \delta + \lambda_u)}} - \left( \frac{y}{\varepsilon} \right)^{\frac{q(z)}{\rho}} \right]$$

for all $z \in [z, \overline{z}]$ and $y \geq \varepsilon$.

A Market Equilibrium requires that a firm of productivity $p \in [p_0, \overline{p}]$ chooses $z \leq p$ to
maximize $\Omega(z,p)$. Given $N_\varepsilon(.)$ and $H_\varepsilon(.,.)$ identified in Lemma 1 the next result solves for
$\Omega(z,p)$.

**Proposition 2: Steady state profits**

$$\Omega(z,p) = \tilde{\varepsilon} l(z)(p - z),$$

where $\tilde{\varepsilon} = E\{\varepsilon\}$ and

$$l(z) = \frac{\lambda_u \phi (\phi + \delta - \rho)(\phi + \delta + \lambda_e - \rho)}{[q(z) - \rho]^2 [\phi (\phi + \delta + \lambda_u) - \rho (\phi + \lambda_u)]}.$$ 

As $\phi > \rho$ by assumption, a little algebra establishes that $l(.) > 0$ and is increasing in $z$ for all
$z \geq \tilde{z}$. As $l'(.) > 0$, it is straightforward to show that equilibrium implies more productive firms
offer a strictly higher $z$. We let $z = \zeta(p)$ denote the equilibrium wage rate offer strategy of firm
$p$ in a market equilibrium and the above implies $\zeta(.)$ is strictly increasing. The offer distribution
$F$ thus solves $F(z) = \Gamma_0(\zeta(p))$. It is now straightforward to obtain a closed form solution for $\zeta$.

**Proposition 3: A Market Equilibrium implies**

$$\zeta(p) = p - [q(p) - \rho]^2 \int_{\tilde{z}}^{p} \frac{dx}{[q(x) - \rho]^2}. \quad (7)$$
Note, the offered $\zeta(p)$ described in (7) is derived for a given $z_R$ and $p_0$. Showing an equilibrium exists requires showing that $z_R$ solves the conditions in Proposition 3 and that $p_0 = \max\{p, z_R\}$. Given $p_0$ and noting that in equilibrium $F(\zeta(p)) = \Gamma_0(p)$, (3) and (7) imply that $z_R$ solves $T(z_R; p_0) = 0$, where

$$T(z_R; p_0) \equiv (r + \phi)z_R - b(r + \phi - \rho) - [\lambda_u(r + \phi - \rho) - (r + \phi)\lambda_c]\int_{p_0}^p \left[\frac{p_0 - z_R}{(\phi + \delta + \lambda_c - \rho)^2} + \int_x^x \frac{ds}{(q(s) - \rho)^2}\right]\beta(x)dx,$$

and

$$\beta(x) = \frac{2(q(x) - \rho)\lambda_c(1 - \Gamma_0(x))\Gamma_0'(x)}{q(x) + r - \rho} > 0.$$

Denote $z_R(p_0)$ the solution to $T(z_R; p_0) = 0$ for any $p_0$. Since $p_0 = \max\{p, z_R(p_0)\}$, however, there are two possible cases. First $p_0 = p$ if and only if $p > z_R(p)$. Otherwise, $p_0 > p$ and some firms will not be active in the labour market. Given these results, we can now establish existence and uniqueness of equilibrium.

**Theorem 1:** There exists a unique Market Equilibrium.

## 3 Implications

In a Market Equilibrium wages are disperse as: (i) workers have disperse initial abilities; (ii) productivity differences arise in cross section as workers differ (by age and thus) by labour market experience; (iii) there are differences in wages paid due to search frictions and labour market sorting. The aim of the calibration is to quantify the relative importance of each of these factors when explaining wage inequality. We focus the discussion by commenting on four particular implications of the model.

The model identifies the following wage equation for worker $i$ with initial ability $\varepsilon_i$, experience $x_{it}$ at date $t$ and employment at firm $j$ paying wage rate $z_j$:

$$\log w_{ijt} = \rho x_{it} + \log z_j + \log \varepsilon_i. \quad (9)$$

In contrast to a large fraction of the empirical labour literature, this wage equation contains a firm fixed effect. From the worker’s perspective, that fixed effect $z_j$ is the realised outcome to a stochastic search process. As the implied quit process is not random (an employee at firm $j$ only quits to an outside offer paying a higher wage rate $z > z_j$), such turnover has important empirical implications both for individual wage dynamics and cross-sectional wage inequality.

### 1. Decomposing experience effects on wages

The classic explanation for why wages increase with experience is that there is learning-by-doing and so more experienced workers (being more productive) earn higher wages. But on-the-job search with no learning-by-doing also predicts that wages, on average, increase with experience (e.g. Burdett, 1978, Burdett and Mortensen, 1998). Conditional on experience, the
wage equation (9) implies expected log wage:

\[
E(\log w \mid x) = \rho x + E(\log z \mid x) + E(\log \epsilon).
\]  

(10)

It is straightforward to show \(E(\log z \mid x)\) is increasing and concave in experience. One important aim of the calibration is to evaluate how much of the observed impact of experience on wages is due to each of these processes.

2. Equilibrium sorting

Equilibrium implies more productive firms offer higher wage rates \(z\). As equilibrium search also implies \(E(\log z \mid x)\) is increasing in \(x\), there is positive assortative matching between the productivity of a firm and the average experience (and thus average productivity) of its workforce. Such sorting increases wage inequality. For example, older workers tend to earn more not only because they are more experienced and so more productive, they are also more likely to be employed at more productive firms which pay higher wage rates.

3. Variance decomposition of log wages

As experience effects \((x_i)\) and job search outcomes \((z_j)\) are both independent of initial ability \(\epsilon_i\), the wage equation (9) implies the following variance decomposition of log wages:

\[
\text{var}(\log w) = \rho^2 \text{var}(x) + \text{var}(\log z) + 2\rho \text{cov}(\log z, x) + \text{var}(\log \epsilon).
\]  

(11)

The first term describes the contribution of learning-by-doing and disperse labour market experiences in explaining wage inequality. The second and third terms describe how variations in wage rates across firms affect wage inequality. A perfectly competitive market would imply both of these terms are zero (the law of one price). Search frictions instead generate disperse wage rates \(z\), where the third term describes the added wage inequality generated by sorting (that higher productivity firms tend to employ more experienced workers). The last term attributes the wage dispersion that is otherwise not captured by the model as unobserved dispersion in worker abilities.

As \(z = \theta p\), one can further decompose the dispersion in firm wage rates as

\[
\text{var}(\log z) = \text{var}(\log \theta) + \text{var}(\log p) + \text{cov}(\log z, \log \theta).
\]  

(12)

\footnote{Integration by parts implies

\[
E(\log z \mid x) = \log z_R + \int_x^{z_R} \frac{1 - H(z \mid x)}{z} dz.
\]

As \(y = e^{\rho e^\delta x}\), each firm \(p\) uses strategy \(z = \zeta(p)\) and so \(F(\zeta(p)) = \Gamma_0(p)\), it can be shown that

\[
\frac{1}{\lambda u} \frac{\lambda u \delta + \lambda u (1 - F(z)) (\phi + \lambda u)}{\lambda u \delta + \lambda u (1 - F(z)) (\phi + \lambda u)}.
\]

\[
\partial H(z \mid x) / \partial x < 0 \quad \text{and} \quad \partial^2 H(z \mid x) / \partial x^2 > 0,
\]

it is easily established that \(E(\log z \mid x)\) is an increasing and concave function of experience.
The first term captures the variation in wage rates that arises when there is no firm heterogeneity (e.g., Burdett et al., 2011). The issue then is how important is firm heterogeneity in explaining wage dispersion? An important feature of the model is there is piece rate compression: although more productive firms pay higher wage rates, they do not increase wages so much that they increase the piece rate paid $\theta = z/p$. This is not entirely surprising as the perfectly competitive case implies perfect piece rate compression - that all firms pay the same wage rate regardless of productivity $p$. Taking piece rate compression into account, we must add the second and third terms in (12) together and so identify the net effect of firm heterogeneity on frictional wage inequality. It turns out this net effect is large: firm heterogeneity has a large impact on wage dispersion.

4. The Mm ratio

The Mm ratio is defined as the ratio of the average wage earned to the lowest wage paid in the market among equally productive workers. For plausible parameter values, Hornstein et al. (2011) explain why most search models generate a reservation wage that is too high to match the observed Mm ratio in the US economy. That paper does not, however, consider a model with both on-the-job search and learning-by-doing as done in Burdett et al. (2011). With both features present, the on-the-job search framework easily generates empirically relevant Mm ratios: unemployed workers are willing to accept low starting wages as experience is valuable.\(^8\) Here we use information on the Mm ratio to usefully calibrate the model. Using the approximation method described in Hornstein et al. (2011), Lemma 2 relates the Mm ratio to the fundamentals of the model.

**Lemma 2:** Given a market equilibrium, the Mm ratio can be well approximated as

$$Mm \approx \left[ 1 + \frac{\lambda_u (r + \phi - \rho) - (r + \phi) \lambda_e}{(r + \phi)(r + \phi + \delta + \lambda_e - \rho)} \right] / \left[ \frac{r + \phi - \rho}{r + \phi} \frac{b}{z^M} + \frac{\lambda_u (r + \phi - \rho) - (r + \phi) \lambda_e}{(r + \phi)(r + \phi + \delta + \lambda_e - \rho)} \right],$$

where $z^M$ is the average $z$ earned by employed workers.

4 Quantitative Analysis

We calibrate the model using simulated methods of moments to match salient features of the UK labour market using the British Household Panel Survey (BHPS).

Data

The BHPS is an annual survey of individuals, age 16 years or more, in a nationally representative sample of about 5,500 households. Approximately 10,000 individuals are interviewed

\(^8\)Hornstein et. al (2011) adopt as baseline calibration values $\lambda_u = 0.43$, $\delta = 0.03$, $b/z^M = 0.4$, $r = 0.0041$, $\phi = 0.0021$. In addition, they set $\lambda_e = 0.13$ to quantify the Mm ratio obtained from a model with on-the-job search, but without learning-by-doing. They set $\rho = 0.0017$ to quantify the Mm ratio obtained from a model with learning-by-doing, but without on-the-job search. When using all these parameters values in the model presented here we obtain an $Mm = 1.49$, a ratio which is within the bounds of the estimated Mm ratios, presented in Hornstein et al. (2007).
each year. It started in 1991 and was subsumed by the new and bigger survey “Understanding Society” in 2010. The BHPS contains socioeconomic information, including information about household organization, the labour market, income and wealth, housing, health and socioeconomic values. Using this information one is able to reconstruct the labour market histories of individuals since leaving full-time education. Maré (2006) provides a comprehensive guide on how to derive consistent histories that summarize individual’s transitions between employment, unemployment and non-participation; transitions between jobs; occupational and industry changes; actual and potential work experience; wages and hours worked; and several socioeconomic characteristics that are standard in household survey data.

We construct individual labour market histories following Maré’s (2006) procedure, considering only white male workers. To focus on young workers we consider those individuals that were originally sampled in 1991 and were between 16 and 30 years of age at that time. We construct their entire employment history since leaving full-time education using retrospective work history information and follow these workers over time until 2004 (or earlier if they left the sample before). We then stratify the sample of workers into three educational or skill groups. We consider workers to be low skilled if they reported having no qualification, other qualifications, apprenticeship, CSE, commercial qualifications or no O-levels. Medium skilled workers are those who reported having O-level or equivalent qualifications. High skilled workers are those that achieved A-levels, nursing qualifications, teaching qualifications, university degree or higher and other higher qualifications.\(^9\) We further restrict attention to paid (dependent) full-time employment spells in the private sector and unemployment spells that lasted at least one month. To keep the sample as homogeneous as possible we only consider those employment and unemployment spells that occur before an individual reported he became (if at all) self-employed, a civil servant, worked for the central or a local government or the armed forces, long-term sick or entered retirement. We also dropped those individuals that re-entered full-time education or had a spell in government training.

These restrictions leave us with a sample of 1,867 individuals, where 486 are considered low skilled, 658 medium skilled and 723 high skilled. We assume that an individual changed jobs if he changed employer. A change in employer is identified when the worker declared a change in his 2-digit occupation and 2-digit industry. In principle, this could underestimate the number of jobs an individual holds during his working life as he can change jobs within the same employer. However, to be consistent with the theory, we consider job-to-job transitions as employer-to-employer transitions.\(^10\) We consider as our earnings variable the real hourly (gross) wage of these individuals.\(^11\) We trim the wage data by 5 percent on each side to reduce measurement

\(^9\)See Dustmann and Pereira (2008) for a similar classification using the BHPS. The main difference is that we consider those workers with nursing qualifications, teaching qualifications and A-levels as high skilled workers. We do this to have somewhat an even number of workers in each skill group.

\(^10\)Since we do not count spells that shorter than a month a transition in which the individual changed employer but experienced an intervening spell of unemployment of less than a month is considered a direct job-to-job transition. If the individual experiences an unemployment spell longer than a month, then he is considered unemployed. See Jolivet et al. (2006) for a similar assumption.

\(^11\)Following Dustmann and Pereira (2008), we construct real hourly wages by dividing monthly (gross) earnings by 4.33 weeks and then by the average number of hours worked in a week in full-time jobs. We also take into account overtime hours and use the CPI to deflate nominal wages.
error and to consider all jobs that pay above the national minimum wage, introduced in the UK in 1999. It is worth pointing out that wage data is only available as from 1991, the first wave of the BHPS. Overall there are 12,091 spells in the sample, where 3,434 of those are associated with low skilled workers, 4,382 are associated with medium skilled workers and 4,275 are associated with high skilled workers.

4.1 Calibration

**Calibration**

We consider the reference time period as a month. Following Hornstein et al. (2011), we let $r = 0.0041$ and fix $\phi = 0.0021$ so that workers participate in the labour market, on average, for 40 years. We approximate the firm productivity and worker ability distributions using (truncated) Weibull distributions. Let $\kappa_1, \kappa_2$ and $p$ describe the shape, scale and location parameters, respectively, of the productivity distribution among firms; and $\alpha_1, \alpha_2$ and $\xi$ describe the shape, scale and location parameters, respectively, of the worker ability distribution.

This parameterization leaves a set of 13 parameters, $\Psi = \{\delta, \lambda_u, \lambda_e, \rho, \mu, \nu, \kappa_1, \kappa_2, b, \xi, \zeta, \alpha_1, \alpha_2\}$ to be estimated. To do this, we minimize the sum of squared distances between a set of simulated moments from the model and their counterparts in the data, using the identity matrix as weighting matrix.

<table>
<thead>
<tr>
<th>Moments</th>
<th>Low Skilled Data</th>
<th>Low Skilled Model</th>
<th>Medium Skilled Data</th>
<th>Medium Skilled Model</th>
<th>High Skilled Data</th>
<th>High Skilled Model</th>
</tr>
</thead>
<tbody>
<tr>
<td>Average durations (months)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unemp spell</td>
<td>12.35</td>
<td>12.15</td>
<td>8.42</td>
<td>8.32</td>
<td>5.87</td>
<td>5.84</td>
</tr>
<tr>
<td>Employment spell</td>
<td>41.63</td>
<td>43.77</td>
<td>68.63</td>
<td>68.26</td>
<td>71.00</td>
<td>70.50</td>
</tr>
<tr>
<td>Job spell</td>
<td>35.68</td>
<td>37.03</td>
<td>43.26</td>
<td>43.90</td>
<td>45.76</td>
<td>44.58</td>
</tr>
<tr>
<td>Returns to experience (%)</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2 years</td>
<td>7.67</td>
<td>6.65</td>
<td>7.92</td>
<td>7.07</td>
<td>9.12</td>
<td>8.19</td>
</tr>
<tr>
<td>4 years</td>
<td>14.62</td>
<td>12.86</td>
<td>14.94</td>
<td>13.75</td>
<td>17.35</td>
<td>15.88</td>
</tr>
<tr>
<td>6 years</td>
<td>20.85</td>
<td>18.62</td>
<td>21.06</td>
<td>20.06</td>
<td>24.67</td>
<td>23.05</td>
</tr>
<tr>
<td>8 years</td>
<td>26.36</td>
<td>23.94</td>
<td>26.27</td>
<td>25.98</td>
<td>31.09</td>
<td>29.71</td>
</tr>
<tr>
<td>10 years</td>
<td>31.15</td>
<td>28.81</td>
<td>30.59</td>
<td>31.51</td>
<td>36.61</td>
<td>35.86</td>
</tr>
<tr>
<td>Wage Dispersion</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mm ratio</td>
<td>1.48</td>
<td>1.44</td>
<td>1.52</td>
<td>1.55</td>
<td>1.51</td>
<td>1.45</td>
</tr>
<tr>
<td>mean(log $w$)</td>
<td>0</td>
<td>-5.71E-03</td>
<td>0</td>
<td>9.51E-03</td>
<td>0</td>
<td>-2.14E-02</td>
</tr>
<tr>
<td>var(log $w$)</td>
<td>0.089</td>
<td>0.088</td>
<td>0.082</td>
<td>0.079</td>
<td>0.094</td>
<td>0.098</td>
</tr>
<tr>
<td>Skewness(log $w$)</td>
<td>0.027</td>
<td>-0.114</td>
<td>0.013</td>
<td>-0.095</td>
<td>-0.049</td>
<td>-0.265</td>
</tr>
<tr>
<td>Kurtosis(log $w$)</td>
<td>3.196</td>
<td>2.618</td>
<td>2.987</td>
<td>2.671</td>
<td>3.004</td>
<td>3.013</td>
</tr>
<tr>
<td>log($\overline{w}$)</td>
<td>-0.991</td>
<td>-0.954</td>
<td>-0.968</td>
<td>-1.018</td>
<td>-1.287</td>
<td>-1.217</td>
</tr>
<tr>
<td>log($\overline{w}$)</td>
<td>1.048</td>
<td>0.887</td>
<td>0.904</td>
<td>0.960</td>
<td>1.024</td>
<td>1.044</td>
</tr>
</tbody>
</table>

**Targeted Moments** We target 15 moments based on the main characteristics of the labour market to which the model is directly related. Table 1 describes these moments decomposed by
skill group (low, medium, high) as described above.\footnote{The targeted moments are compared with the corresponding moments from model simulations for different values of the parameters in \( \Psi \) until the loss functions described above is minimised. For each model simulation run, we set \( b = 0.4z^M \), following Hornstein et al. (2011), to jointly recover the values of \( z_R \), \( z^M \) and \( b \) using (8) and (19). Further, the simulated data is constructed such that it has the same structure as the BHPS for consistent measurement. See the Appendix for further details of the simulation procedure.}

To identify wage dispersion from the data, we build on the approach of Hornstein et al. (2007). For each skill group, a regression of log wages is run on a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies, dummies for cohort effects and a time trend. The resulting log wage residual is then used as our measure of wage dispersion. The empirical wage experience profile is identified by regressing log wages on a quadratic on experience, a quadratic on tenure, a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies, dummies for cohort effects and a time trend.\footnote{One potential worry with the above specification is that workers’ unobservable characteristics might be biasing the estimated returns to general experience because, for example, more able workers could be more likely to receive outside offers than less able workers in the data. The results of Dustmann and Pereira (2008), however, suggest that any potential bias of this sort is very small. These authors estimate returns to experience for the UK using the BHPS by skill/education categories. When controlling for worker and job match (unobservable) fixed effects, their estimated experience effects hardly change across specification and estimation methods. See also Williams (2009).} To obtain the empirical \( Mm \) ratio we follow the procedure of Hornstein et al. (2007), which involves controlling for worker fixed effects in the experience regression. In the Appendix, we discuss this procedure in more detail. The calibration requires not only that the model is consistent with the observed log wage residuals across workers (by skill group) but also with the estimated experience effects and the empirical \( Mm \) ratio. A minor difficulty with this approach is that the mean log wage residual is zero for each skill group. Below we will find that the high skilled learn more quickly, have longer employment spells, are employed in more productive firms, spend less time in unemployment, etc. But the calibration requires each skill group has the same average log wage residual. It thus compensates by attributing a higher mean worker ability \( \varepsilon \) to the lowest skill group. Clearly computing mean ability by skill group requires also taking into account the occupational and location dummies used in the original regression. This is not straightforward as one must adopt a theory of how different ability workers select into different occupations (lumberjack or accountant) and locations (Sherwood Forest or London). Fortunately this issue does not otherwise distort our results as equilibrium market behaviour is independent of the assumed distribution of abilities \( \varepsilon \). In essence the unobserved worker ability distribution captures the variation in the data (by skill group) that is not otherwise explained by the model.

Key features of the data described in Table 1 find the low skilled group (compared to the high skilled group) has longer average unemployment spells (one year compared to 6 months), shorter average employment spells (3.5 years compared to 6 years) but average job spells which are not so different (3 years compared to 3.75 years). This latter statistic arises as high skilled workers have much higher quit rates. Such turnover provides direct information on the transition parameters \( \delta \), \( \lambda_u \) and \( \lambda_e \). As the parameters that govern worker turnover, human capital accumulation and the firm productivity distribution, determine the shape of the wage-experience profile, information on the average wage-experience profile also helps tie down parameter values. We also incorporate
information on the distribution of (residual) log wages.

Table 1 shows the fit of the model is very good and has no difficulty in matching the empirical $Mm$ ratios or the cross-sectional wage distribution. What is of central interest now is understanding how the labour market differs across skill groups and identifying the impact of job search and learning-by-doing on (i) average wage profiles by experience and (ii) wage inequality.

<table>
<thead>
<tr>
<th>Parameters</th>
<th>Low Skilled</th>
<th>Medium Skilled</th>
<th>High Skilled</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\lambda_u$</td>
<td>0.082</td>
<td>0.121</td>
<td>0.171</td>
</tr>
<tr>
<td>$\delta$</td>
<td>0.021</td>
<td>0.012</td>
<td>0.012</td>
</tr>
<tr>
<td>$\lambda_e$</td>
<td>0.010</td>
<td>0.038</td>
<td>0.039</td>
</tr>
<tr>
<td>$\rho$</td>
<td>0.0018</td>
<td>0.0016</td>
<td>0.0020</td>
</tr>
<tr>
<td>$\bar{p}$</td>
<td>3.049</td>
<td>6.950</td>
<td>9.810</td>
</tr>
<tr>
<td>$\kappa_1$</td>
<td>0.5908</td>
<td>0.5040</td>
<td>0.403</td>
</tr>
<tr>
<td>$\kappa_2$</td>
<td>0.4018</td>
<td>0.4033</td>
<td>0.403</td>
</tr>
<tr>
<td>$\bar{b}$</td>
<td>25.176</td>
<td>25.222</td>
<td>28.349</td>
</tr>
<tr>
<td>$\bar{e}$</td>
<td>0.808</td>
<td>2.701</td>
<td>3.889</td>
</tr>
<tr>
<td>$\alpha_1$</td>
<td>1.222</td>
<td>0.906</td>
<td>1.770</td>
</tr>
<tr>
<td>$\alpha_2$</td>
<td>2.320</td>
<td>1.564</td>
<td>1.563</td>
</tr>
<tr>
<td>$\bar{z}$</td>
<td>0.560</td>
<td>0.174</td>
<td>0.116</td>
</tr>
<tr>
<td>$z_R$</td>
<td>1.405</td>
<td>4.284</td>
<td>6.718</td>
</tr>
<tr>
<td>urate</td>
<td>0.217</td>
<td>0.105</td>
<td>0.077</td>
</tr>
<tr>
<td>Pr $EE$</td>
<td>0.004</td>
<td>0.011</td>
<td>0.011</td>
</tr>
</tbody>
</table>

Parameters

Table 2 describes the calibrated parameter values for each skill group. Those values present a picture that is similar to the one identified in Steward (2007). Relative to the high skilled group, low skilled workers spend more time in unemployment and there is little upward job mobility through job-to-job transitions. As they spend long periods in unemployment, low skilled worker also accumulate less human capital through learning-by-doing and so earned wages are likely to remain low into the longer term. In contrast, high skilled workers can more quickly find work while unemployed, enjoy much greater job security and enjoy a much greater chance of receiving (and quitting to) an attractive outside offer.

Decomposing the experience effect on wages

The job-to-job transition rate is computed using

$$ Pr \, EE = (\phi + \delta) \left[ \left( 1 + \frac{\phi + \delta}{\lambda_e} \right) \ln \left( 1 + \frac{\lambda_e}{\phi + \delta} \right) - 1 \right], $$

The rate at which a high skilled worker receives a preferred outside offer is three times that of a low skilled worker.
Recall the expected log wage conditional on experience is
\[ E(\log w \mid x) = \rho x + E(\log z \mid x) + E(\log \varepsilon). \]

For the low skilled group, the calibration finds that 88% of the experience effect on wages is due to learning-by-doing. The rationale is twofold. First learning-by-doing rates are reasonably large (comparable in value to the learning-by-doing rates of the high skilled). Second the low skilled are unlikely to climb the job ladder: their job destruction rate (\( \delta = 0.021 \) per month) is not only high, it is 5 times greater than the (average) rate at which low skilled workers quit to preferred outside offers (\( P(EE) = 0.004 \) per month). In other words each unskilled worker, when employed, is far more likely to return to the unemployment pool than climb the job ladder.

For the high skilled group, the job ladder plays a more important role: the calibration finds 69% of the experience effect on wages is due to human capital accumulation. Although learning-by-doing continues to explain the larger part of wage growth by experience, on-the-job search plays a more sizeable role.

This interpretation of the data is entirely consistent with the findings of Bagger et al. (2014) and Menzio, Telyukova and Visschers (2012) who suggest that, among young workers, human capital accumulation plays a more important role than job-to-job transitions in explaining average wage differentials. This does not mean, however, that search plays little role in explaining wage inequality.

### The variance decomposition of wage inequality by skill group

Table 3 describes the variance decomposition implied by (11):

<table>
<thead>
<tr>
<th>Skill Group</th>
<th>( \text{var}(\log w) )</th>
<th>( \text{var}(\log \varepsilon) )</th>
<th>( \rho^2 ) var(( x ))</th>
<th>var(( \log z ))</th>
<th>2( \rho \text{cov}(x, \log z) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Skill</td>
<td>0.088</td>
<td>0.016</td>
<td>0.015</td>
<td>0.055</td>
<td>0.002</td>
</tr>
<tr>
<td></td>
<td>Total=100%</td>
<td>18.2%</td>
<td>17.1%</td>
<td>62.5%</td>
<td>2.3%</td>
</tr>
<tr>
<td>Medium Skill</td>
<td>0.079</td>
<td>0.018</td>
<td>0.011</td>
<td>0.043</td>
<td>0.007</td>
</tr>
<tr>
<td></td>
<td>Total=100%</td>
<td>22.8%</td>
<td>13.9%</td>
<td>54.4%</td>
<td>8.9%</td>
</tr>
<tr>
<td>High Skill</td>
<td>0.098</td>
<td>0.030</td>
<td>0.018</td>
<td>0.042</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>Total=100%</td>
<td>30.6%</td>
<td>18.4%</td>
<td>42.9%</td>
<td>8.2%</td>
</tr>
</tbody>
</table>

Column 1 describes the total variation in log wages for each skill group, the remaining columns decompose that variation using (11). Not surprisingly unobserved worker heterogeneity [column 2] explains a large chunk of the observed wage dispersion, ranging from 18% for the unskilled group to 31% for the high skilled group. Dispersion in labour market experience with learning-by-doing [column 3] also contributes a significant amount to wage inequality, ranging between 14% to 18% depending on skill groups. In all cases, however, the largest contributor to wage inequality is the dispersion in wage rates paid by firms. We discuss this further below. The final column describes the effect of positive sorting on wage inequality. Reflecting that job
ladder effects are small for the low skilled, sorting only contributes 2.3% to total wage inequality for these workers. For the higher skill groups, sorting instead contributes around 8-9% to total wage variation.

Table 4: Variance decomposition of log \((z)\)

<table>
<thead>
<tr>
<th>Group</th>
<th>(\text{var}(\log z))</th>
<th>(\text{var}(\log \theta))</th>
<th>(\text{var}(\log p))</th>
<th>(\text{cov}(\log p, \log \theta))</th>
</tr>
</thead>
<tbody>
<tr>
<td>Low Skill</td>
<td>0.055</td>
<td>0.020</td>
<td>0.060</td>
<td>-0.024</td>
</tr>
<tr>
<td></td>
<td>62.5%</td>
<td>22.7%</td>
<td>68.2%</td>
<td>-27.3%</td>
</tr>
<tr>
<td>Medium Skill</td>
<td>0.043</td>
<td>0.014</td>
<td>0.045</td>
<td>-0.016</td>
</tr>
<tr>
<td></td>
<td>54.4%</td>
<td>17.7%</td>
<td>57.0%</td>
<td>-20.2%</td>
</tr>
<tr>
<td>High Skill</td>
<td>0.042</td>
<td>0.014</td>
<td>0.049</td>
<td>-0.021</td>
</tr>
<tr>
<td></td>
<td>42.9%</td>
<td>14.3%</td>
<td>50.0%</td>
<td>-21.4%</td>
</tr>
</tbody>
</table>

Table 4 now uses (12) to decompose the variation in firm wage rates \(z = \theta p\) and so identify the impact of firm heterogeneity on wage inequality. The first column reports the variation in firm wage rates as described in the previous table. Note the law of one price would imply zero variation (no firm fixed effects). The second column describes the wage rate dispersion that is attributable to pure non-competitive wage formation when firms are identical (e.g. Burdett et al., 2011). Without firm heterogeneity, search frictions would thus account for between 14-22% of observed wage inequality. Taking the last two columns together (where the last column describes piece rate compression), the added (net) effect of firm heterogeneity on total wage inequality is very large, being as high as 41% for the low skilled group (68%-27%) and 29% for the high skilled group (50%-21%). Thus firm heterogeneity has a very large impact on overall wage inequality, an effect which is missing in competitive markets.\(^{15}\)

5 Further Discussion

The framework developed here is nicely tractable and seems well suited to understanding the various causes of wage inequality. The empirical investigation presented yielded several new insights. Three seem particularly worthy of restating. First, heterogeneity among firms generates a lot of wage inequality. Second, among low skilled workers job ladder effects are small, most of the impact of experience on wages is due to learning by doing. Third, high skilled workers are much more mobile. Job ladder effects have sizeable impact.

In this paper we have analyzed wage differentials among workers assuming the economy is in steady state. An important question, however, is to what extend the employment patterns and the relative contributions of workers’ ability differentials, firm productivity differentials, and frictional wage dispersion change over the business cycle. Recent work by Coles and Mortensen (2012) suggests that such an extension is possible. We leave this important extension for future

\(^{15}\)Postel-Vinay and Robin (2002) also showed the importance of firm heterogeneity in explaining wage dispersion. Although we use a different data set and estimation method, our results are consistent with their findings.
research.

References

[1]


Appendix

A Proofs

Proof of Proposition 1:
Given the functional forms for $W^U$ and $W^E$, the Bellman equation describing $W^U$ is equivalent to

$$(r + \phi)\alpha^U = b + \lambda_u \int_{z_R}^{\bar{z}} [\alpha^E(z') - \alpha^U] dF(z')$$

(14)

and the Bellman equation for $W^E$ is equivalent to

$$(r + \phi + \delta)\alpha^E(z) = z + \rho \alpha^E(z) + \lambda_e \int_0^z [\alpha^E(z') - \alpha^E(z)] dF(z') + \delta \alpha^U,$$

(15)

which is a functional equation for $\alpha^E(z)$. Differentiating (15) with respect to $z$ yields the differential equation describing $\alpha^E$ and evaluating (15) at $z = z_R$ yields its boundary value $\alpha^E(z_R)$.

We now solve the conditions for $z_R$ and $\alpha^U$. First evaluate (15) at $z = z_R$. As $\alpha^E(z_R) = \alpha^U$ one obtains

$$(r + \phi)\alpha^U = z_R + \rho \alpha^U + \lambda_e \int_{z_R}^{\bar{z}} [\alpha^E(z') - \alpha^U] dF(z').$$

Comparing this equation with (14), integrating by parts and using the differential equation for $\alpha^E$ establishes (1) described in Proposition 1. Similarly, $\alpha^E(z_R) = \alpha^U$, integration by parts and using the differential equation for $\alpha^E$ then yields (2) in Proposition 1. Thus (1) and (2) describe a pair of equations for $(\alpha^U, z_R)$.

We now establish that a solution exists and is unique. First note that the equation described by (1) has slope

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(1)} = -\frac{1}{\rho} \left[ \frac{r + \phi + \delta + \lambda_u (1 - F(z_R)) - \rho}{q(z_R) + r - \rho} \right] < 0$$

and implies $\alpha^U = (b - \bar{z})/\rho$ at $z_R = \bar{z}$. On the other hand, the equation described by (2) has slope

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(2)} = -\frac{1}{r + \phi} \left[ \frac{\lambda_u (1 - F(z_R))}{q(z_R) + r - \rho} \right] < 0,$$

for $z_R < \bar{z}$ and zero otherwise and implies that $\alpha^U = b/(r + \phi)$ at $z_R = \bar{z}$. Note that

$$\left[\frac{d\alpha^U}{dz_R}\right]_{eqn(2)} > \left[\frac{d\alpha^U}{dz_R}\right]_{eqn(1)}$$

for all $z_R$ and hence (2) is always flatter than (1). Continuity of (1) and (2) and the restriction $\bar{z} > b(r + \phi - \rho)/(r + \phi)$ then guarantee there exists a single crossing between these two functions such that $\alpha^U > 0$ and $z_R < \bar{z}$.

Proof of Lemma 1:
Consider the pool of type $\varepsilon$ unemployed workers with productivity no greater that $y$. It is
straightforward to verify that steady-state turnover implies

\[ N_\varepsilon (y) = \frac{\phi (\phi + \delta + \lambda_u) + \delta \lambda_u H_\varepsilon (y, \bar{\varepsilon})}{(\phi + \lambda_u)(\phi + \delta)} \]

for all \( y \geq \varepsilon \). Next consider the pool of type \( \varepsilon \) employed workers who have productivity no greater than \( y \) and receive a payoff no greater than \( z \). The arguments in Burdett et al. (2011) imply that \( H_\varepsilon (\cdot, \cdot) \) satisfies the following partial differential equation,

\[ \frac{\partial H_\varepsilon (y, z)}{\partial y} + \frac{q(z)}{\rho y} H_\varepsilon (y, z) = \frac{(\phi + \delta) F(z) N_\varepsilon (y)}{\rho y}, \]

(16)

for \( z \in [\varepsilon, \bar{z}] \) and \( y \geq \varepsilon \). For a given \( z \), integrating over \( y \) using the integrating factor \( y^{q(z)/\rho} \) and noting that \( H_\varepsilon (\varepsilon, z) = 0 \) yields

\[ H_\varepsilon (y, z) = \frac{(\phi + \delta) F(z)}{\rho} y^{-\frac{q(z)}{\rho}} \int_\varepsilon^y y^{\frac{q(z)}{\rho} - 1} N_\varepsilon (y') dy' \]

for all \( y \geq \varepsilon, \ varepsilon \in [\varepsilon, \bar{z}] \).

Using these formulae we now solve for steady state \( N_\varepsilon (\cdot) \) and \( H_\varepsilon (\cdot, \cdot) \). In particular, using the above expression for \( N_\varepsilon (\cdot) \) and simplifying yields

\[ \frac{\partial H_\varepsilon (y, z)}{\partial y} = \frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \lambda_u)} \frac{1 - H_\varepsilon (y, z)}{y} \]

for all \( y \geq \varepsilon \). As this differential equation is separable and we have the boundary condition \( H_\varepsilon (\varepsilon, z) = 0 \), integration implies

\[ H_\varepsilon (y, z) = 1 - \left( \frac{y}{\varepsilon} \right)^{-\frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \lambda_u)}} \].

Using this and simplifying yields the expression for \( N_\varepsilon (\cdot) \). Using the latter to substitute out \( N_\varepsilon (\cdot) \) in the above expression for \( H_\varepsilon (y, z) \), direct integration and some algebra then establishes (5). ||

Proof of Proposition 2:

Consider a firm with productivity \( p \) offering \( z \geq z_R \). This firm’s steady state profit is given by

\[ \Omega(z; p) = \frac{p - z}{q(z) - \rho} \int_\varepsilon^\infty \left[ \lambda_u U \int_{y'=\varepsilon}^\infty y' dN_\varepsilon (y') + \lambda_c (1 - U) \int_0^\infty \int_{z'=\varepsilon}^\infty y' \frac{\partial^2 H_\varepsilon (y', z')}{\partial y' \partial z'} dz' dy' \right] \ dA(\varepsilon). \]

Next use the results in Lemma 1 to solve for the integrals in \( \Omega(z; p) \). Consider the first integral in the expression in brackets. Using the expression for \( N_\varepsilon \) one obtains

\[ \int_{y'=\varepsilon}^\infty y' dN_\varepsilon (y') = \varepsilon N_\varepsilon (\varepsilon) + \int_\varepsilon^\infty \left( \frac{(\phi (\phi + \delta + \lambda_u))}{\rho (\phi + \lambda_u)} \right) \lambda_u \delta \frac{1}{\phi + \lambda_u} \left( \frac{y'}{\varepsilon} \right)^{-\frac{\phi (\phi + \delta + \lambda_u)}{\rho (\phi + \lambda_u)}} dy'. \]
Integration and some algebra then establish that
\[ \int_{y' = \varepsilon}^{\infty} y' dN_\varepsilon(y') = \frac{\phi(\phi + \delta + \lambda_u)\varepsilon}{(\phi + \delta)} \left[ \frac{(\phi + \delta - \rho)}{\phi(\phi + \delta + \lambda_u) - \rho(\phi + \lambda_u)} \right]. \]

Next consider the second integral in \( \Omega(z; \varpi) \). Integrating over \( z' \) implies
\[ \int_{y' = \varepsilon}^{\infty} \int_{z' = \varpi}^{z} \frac{\partial^2 H_\varpi(y', z')}{\partial y' \partial z'} \, dz' \, dy' = \int_{y' = \varepsilon}^{\infty} y' \left[ \frac{\partial H_\varpi(y', z')}{\partial y'} \right] z \, dy'. \]

As \( F(\varpi) = 0 \), (5) implies \( H_\varpi(y', \varpi) = 0 \). (16) then implies \( \frac{\partial H_\varpi(y', \varpi)}{\partial y'} = 0 \) and the previous expression reduces to
\[ \int_{y' = \varepsilon}^{\infty} \int_{z' = \varpi}^{z} \frac{\partial^2 H_\varpi(y', z')}{\partial y' \partial z'} \, dz' \, dy' = \int_{\varepsilon}^{\infty} y \frac{\partial H_\varpi(y', \varpi)}{\partial y'} \, dy'. \]

Now (5) implies
\[ \frac{\partial H_\varpi(\ldots)}{\partial y} = \frac{\phi(\phi + \delta + \lambda_u)F(\varpi)}{y \rho(\lambda_u \delta + \lambda_u(1 - F(\varpi))(\phi + \lambda_u))} \left[ \frac{\lambda_u(1 - F(\varpi))}{\phi(\phi + \delta + \lambda_u)} \right] \frac{\phi(\phi + \delta + \lambda_u)}{\phi(\phi + \delta + \lambda_u) - \rho(\phi + \lambda_u)} \right]. \]

Using this expression and integrating yields
\[ \int_{y' = \varepsilon}^{\infty} \int_{z' = \varpi}^{z} y \frac{\partial^2 H_\varpi(y', z')}{\partial y' \partial z'} \, dz' \, dy' = \left[ \frac{\phi(\phi + \delta + \lambda_u)F(\varpi)}{\lambda_u (1 - F(\varpi))(\phi + \lambda_u)} \right] \times \left[ \frac{\lambda_u(1 - F(\varpi))}{q(\varpi) - \rho} + \frac{\delta \lambda_u}{\phi(\phi + \delta + \lambda_u) - \rho(\phi + \lambda_u)} \right]. \]

Substituting out the expression for the integrals in \( \Omega(z; \varpi) \) and some additional algebra yields (6) in the text.

Proof of Proposition 3:

Consider a firm with productivity \( \varpi \). This firm chooses a \( z \geq z_R \) to maximise
\[ \Omega(z; \varpi) = \tilde{\varepsilon}l(z)(\varpi - z), \tag{17} \]

where \( l(z) \) is given in the text. Let \( z^* = \zeta(\varpi) \) denote the solution to the above maximisation problem (if one exists). Assume the second order condition for a maximum holds. The envelope theorem then implies that \( \Omega'(\zeta(\varpi)) = l'(\zeta(\varpi)) \), which describes a first order differential equation for \( \Omega(\cdot) \) in terms of \( \varpi \) subject to the boundary condition \( \Omega(\zeta(\varpi)) = \tilde{\varepsilon}l(\zeta(\varpi))(\varpi - \tilde{z}) \). Noting that \( F(\zeta(\varpi)) = \Gamma_0(\varpi) \), its solution is given by
\[ \Omega(\zeta(\varpi)) = \tilde{\varepsilon}l(\zeta(\varpi))(\varpi - \zeta(\varpi)) = \tilde{\varepsilon} \int_{x = \tilde{z}}^{\varpi} \frac{\lambda_u(\phi(\phi + \delta + \lambda_u) - \rho(\phi + \lambda_u))}{q(x) - \rho(\phi(\phi + \delta + \lambda_u) - \rho(\phi + \lambda_u))} \, dx, \]

where \( q(x) = \phi + \delta + \lambda_u(1 - \Gamma_0(x)) \). Since \( \zeta(\varpi) = \varpi - \Omega(\zeta(\varpi))/\tilde{\varepsilon}l(\zeta(\varpi)) \), substituting out for \( \Omega(\zeta(\varpi)) \) and \( l(\zeta(\varpi)) \) and some algebra yields (7), the expression for \( \zeta(\varpi) \) in the text.

Next we show that (7) indeed satisfies the first order condition of the firm’s maximisation problem and then that the second order condition for a maximum is indeed met at \( z = z^* \). First
Further, note that differentiation of (7) wrt $p$ implies $\zeta$ satisfies the differential equation

$$\zeta'(p) = \frac{2(p - \zeta(p))\lambda e}{q(p) - \rho},$$

(18)
given the boundary condition $\zeta(p_0) = z_R$. Noting that the first order condition for a maximum implies that for a given $p$

$$l'(z)(p - z) - l(z) = 0$$
at $z^* = z$, using the expression for $l(z)$ and $F'(\zeta(p))\zeta'(p) = \Gamma_0'(p)$, some algebra establishes that (18) is indeed obtained from the above first order condition. Hence the function $\zeta$ implied by (18) satisfies the first order condition for a maximum given the boundary condition $\zeta(p_0) = z_R$. Further, note that $\zeta'(p) > 0$ for all $p \geq p_0$.

Now let $\Omega(\zeta(\tilde{p}); p) = \tilde{\varepsilon}(p - \zeta(\tilde{p}))l(\zeta(\tilde{p}))$ denote the steady state profit of a firm of productivity $p$ by offering a $z = \zeta(\tilde{p})$ and let $\Delta(\tilde{p}) = \tilde{p} - p$. For $\tilde{p} \in (p, \bar{p}]$, the second order condition for a maximum requires that offering such a $z$ should not increase profits or that

$$\left[\frac{d\Omega(\zeta(x); p)}{dx}\right]_{x=\tilde{p}} = \tilde{\varepsilon} \left[\frac{d\zeta(x)}{dz}\right] \left(\zeta'(x)(p - \zeta(x)) - \zeta'(x)l(\zeta(x))\right)_{x=\tilde{p}}$$

$$= \tilde{\varepsilon} \left[\left(x - \zeta(x)\right)\frac{d\zeta(x)}{dz} - l(\zeta(x))\right] \zeta'(x) - \Delta(x) \frac{d\zeta(x)}{dz} \zeta'(x)_{x=\tilde{p}} \leq 0.$$

Since the first order condition implies $(x - \zeta(x))\frac{d\zeta(x)}{dz} - l(\zeta(x)) = 0$ for any $x > p_0$, one obtains that

$$\left[\frac{d\Omega(\zeta(x); p)}{dx}\right]_{x=\tilde{p}} = \varepsilon \Delta(x) \frac{d\zeta(x)}{dz} \zeta'(x)_{x=\tilde{p}} \leq 0$$
is always satisfied. For $\tilde{p} \in [p_0, \bar{p})$ a similar argument shows that

$$\left[\frac{d\Omega(\zeta(x); p)}{dx}\right]_{x=\tilde{p}} = \varepsilon \Delta(x) \frac{d\zeta(x)}{dz} \zeta'(x)_{x=\tilde{p}} \geq 0$$
is always satisfied.

Finally note that a firm with productivity $p = p_0$ will not offer a $z < z_R = \zeta(p_0)$ as doing so will not increase profits. It will strictly decrease profits if $p_0 = \bar{p} > z_R$ and yields the same (zero) profit if $p_0 = z_R$. A firm with $p = \bar{p}$, on the other hand, will not offer a $z > \zeta(\bar{p})$ as doing so does not attract or retain any additional worker, but strictly decreases flow profit $\bar{p} - z$ and hence steady state profits.||

Proof of Theorem 1:

Step 1: The first step to proof existence is to solve for $p_0$. Note that for any $p_0$, $T(z_R; p_0)$ gives the solution to $z_R = z_R(p_0)$ when both (3) and (7) are satisfied. Given $p_0 = \max\{p, z_R(p_0)\}$, we have that $p_0 = \bar{p}$ if and only if $\bar{p} > z_R(p_0)$. Using (8) the latter condition can be expressed as

$$\bar{p} > z_R(p_0) = \frac{b(r + \phi - \rho) + \lambda u(r + \phi - \rho) - (r + \phi)\lambda u}{(r + \phi) + \lambda u(r + \phi - \rho)} \int_{\bar{p}}^p \beta(x)dx$$

$$+ \lambda u(r + \phi - \rho) - (r + \phi)\lambda u \int_{\bar{p}}^p \beta(x)dx.$$
On the other hand, if the above condition does not hold (i.e. $p \leq z_R(p)$), then $p_0 \geq \bar{p}$. In this case (8) implies $p_0$ satisfies $p_0 = \tilde{T}(p_0)$, where

$$\tilde{T}(p_0) = \frac{b(r + \phi - \rho)}{r + \phi} + \frac{\lambda u(r + \phi - \rho) - (r + \phi)\lambda_e}{r + \phi} \int_{z_0}^{p_0} \int_{p_0}^{x} \frac{ds}{(q(s) - \rho)^2} \beta(x)dx.$$ 

Note that $\tilde{T}$ is continuous, bounded and strictly decreasing between $p_0 \in [\bar{p}, \bar{\bar{p}}]$. Since $\tilde{T}(\bar{p}) > \bar{p}$ and $\bar{\bar{p}} \in (b, \infty)$, there exists a unique $p_0^* \in (\bar{p}, \bar{\bar{p}})$ such that $p_0^* = \tilde{T}(p_0^*)$.

**Step 2:** Given a $p_0 \leq \bar{\bar{p}}$ always exists, $z_R$ is described by the unique solution to (8). $\zeta(.)$ is then characterised by (7) in Proposition 2 and $F(\zeta(p)) = \Gamma_0(p)$ for all $p \in [p_0, \bar{\bar{p}}]$, where $\Gamma_0$ is given by (4). Furthermore, since Proposition 2 implies no firm with productivity $p \in [p_0, \bar{\bar{p}}]$ will offer a different $z$, as doing so yields lower steady state profits, this establishes existence of a unique Market Equilibrium.]

**Proof of Lemma 2:**

Since frictional wage dispersion concerns wage dispersion that is not driven by difference in abilities, without loss of generality consider the case in which all workers enter with initial productivity $\varepsilon = 1$. Next note that $H(\infty, z)$ describes the distribution of $z$ across employed workers given the offer distribution $F$. Using integration by parts and $z = z_R$, it can be easily shown that the average $z$ earned by employed workers, $z^M$, is given by

$$z^M = z_R + \int_{z_R}^{\infty} [1 - H(\infty, z)]dz. \quad (19)$$

Putting $y = \infty$ in (5) implies

$$H(\infty, z) = \frac{(\phi + \delta)F(z)}{q(z)}.$$ 

Since $r$ and $\rho$ are typically of the same order of magnitude (see section 5), we follow Hornstein, et al. (2007) and approximate $H(\infty, z)$ by

$$H(\infty, z) \simeq \frac{(r + \phi - \rho + \delta)F(z)}{q(z) + r - \rho}.$$ 

Solving for $1 - F(z)$ and using (3) yields

$$z_R \simeq \frac{b(r + \phi - \rho)}{r + \phi} + \frac{\lambda u(r + \phi - \rho) - (r + \phi)\lambda_e}{(r + \phi)(r + \phi + \delta + \lambda_e - \rho)} \int_{z_R}^{\infty} [1 - H(\infty, z)]dz$$

$$\simeq \frac{b(r + \phi - \rho)}{r + \phi} + \frac{\lambda u(r + \phi - \rho) - (r + \phi)\lambda_e}{(r + \phi)(r + \phi + \delta + \lambda_e - \rho)} (z^M - z_R).$$

Dividing both sides by $z^M$ we have the expression in the text.]

**B Simulation Procedure**

To simulate the model we compute the employment histories of 10,000 workers for each iteration of the minimisation process done in the simulation minimum distance procedure. In simulating
the employment histories we assume that all workers start unemployed and experience different
types of shocks during their lifetime depending on the worker’s employment status.

When unemployed, workers face a retirement and a job offer shock, where both process are
Poisson with rates $\phi$ and $\lambda_u$, and the shocks are mutually exclusive. What is important here is
to capture the time spent in unemployment for each individual. To obtain the unemployment
duration, we draw two random numbers, $r1 \in [0, 1]$ and $r2 \in [0, 1]$, using a uniform distribution
and then exploit the fact that the inter arrival time between events in a Poisson process follows
an exponential distribution with parameter equal to the rate of the process. That is, the duration
until the worker receives a job offer is determined by $tu = -\log(1 - r1)/\lambda_u$ and the duration
until the worker experience a retirement shock is $td = -\log(1 - r2)/\phi$. Since this a competing
risk model the unemployment duration of an individual is given $\min\{td, tu\}$. If the individual
retires, $td < tu$, then he leaves the sample. If the individual becomes employed, $td \geq tu$, then a
firm productivity is sampled from $\Gamma$ by, once again, choosing a random number between 0 and
1 and using the inverse of $\Gamma$ to recover the corresponding productivity $p$. The latter then allows
us to compute the corresponding $z$ and $\theta$.

Given the individual becomes employed in a firm with productivity $p$, the value of $z(p)$ is
computed using (7) and the value of $\theta = z(p)/p$. This individual now faces three shocks: a
retirement shock, a job offer shock and a displacement shock. All these shocks follow Poisson
process with rates, $\phi, \lambda_e$ and $\delta$, respectively. As in the case of unemployed workers, what is
important is the duration of the job and the employment spells, where the latter is defined as the
sum of job spells that start with the worker transiting from unemployment to employment
and end with the worker becoming unemployed or leaving the labour market. We use the same
procedure as before to obtain the durations until the worker receives a job offer $tj$, receives a
displacement shock, $tu$, and receives a retirement shock, $td$. The job duration until the worker
experiences one of these three events in then $\min\{tj, tu, td\}$. If the worker becomes unemployed,
$tu = \min\{tj, tu, td\}$, then the procedure described above for unemployed workers is repeated.
If the worker leaves the labour market, $td = \min\{tj, tu, td\}$, then he drops from the sample.
If the worker receives an outside offer, $tj = \min\{tj, tu, td\}$, a new $p'$ is drawn using the same
procedure described above and the values of $z(p)$ and $z(p')$ are compared. If $z(p) \geq z(p')$ the
worker stays employed in his current job, while if $z(p) < z(p')$ the worker moves to the new
firm and we repeat the process given a the new firm productivity $p'$. During this procedure
we calculate the labour market experience of workers as the sum of employment spells. This
information can then be used to compute workers’ wages at each point in which an event has
occurred taking into account that workers accumulate human capital at rate $\rho$.

The above procedure generates the full labour market histories of workers for an average life
of $1/\phi$ months. However, the BHPS sample is restricted to workers that in 1991 were between 16
and 30 years of age and by 2004 were between 30 and 44 years of age. Hence one needs to create
a sample of the simulated data that resembles that of the BHPS in terms of the age structure
and has the same variance of actual experience (this is crucial for the variance decomposition
exercise). It is only after creating such a sample that we compute the average wage-experience
profiles by using an OLS regression on log wages on a constant a quadratic on experience and
tenure. Using this sample we also compute all the other moments targeted in the calibration as described in Table 1. In particular, using this sample we compute the $Mm$ ratio in the simulation such that it is consistent with the way we compute the $Mm$ ratio in the data. We now detail such a procedure.

**Estimating of the Mean-min ratio:** Following Hornstein, et al. (2007) we first estimate the wage equation

$$\log w_{ijt} = \beta X_{it} + \eta_{ijt}, \quad (20)$$

for each year of the sample period and skill group using OLS, where $X$ is a vector of covariates consisting of a quadratic in actual experience, quadratic on tenure, a dummy for marital status, 8 regional dummies, 8 (one-digit) occupational dummies, 8 (one-digit) industry dummies and cohort dummies, where $\eta$ denotes white noise and is assumed to be normally distributed. The second step is to eliminate unobserved worker heterogeneity from wages by using the individual residuals $\hat{\eta}_{it}$ and their individual specific mean $\eta_i = \sum_{t=1}^{N_i} \hat{\eta}_{it} / N_i$. The vector $\{\eta_i\}_{i=1}^{N}$ then captures the wage variation due to fixed unobserved individual factors. Finally, we use the estimated distribution of transformed wages, $\tilde{w}_{it} = \exp(\hat{\eta}_{it} - \eta_i)$, across individuals and time to calculate the $Mm$ ratio for each skill group.

For each skill group, we estimate a set of three $Mm$ ratios using the minimum observed wage, the wage at the first percentile, and fifth percentile. Given that the wage data has already been trimmed by 5 percent on each side when performing the OLS regressions and that the minimum observed wage is still very noisy for the medium and high skilled categories, we use as a target the $Mm$ ratio obtained from averaging the ones obtained for the first and fifth percentile.

As pointed out by Hornstein, et al. (2007) the danger with their approach is that one may underestimate the amount of frictional wage dispersion when controlling for those worker characteristics that also provide information on generate wage dispersion due to productivity differentials among workers. Further, by introducing a polynomial on experience and tenure in (20) one is reducing the effects of on-the-job search and human capital accumulation on wage dispersion. However, in the data this reduction is not very strong and hence the downward bias does not have a mayor impact on the estimated parameters. Indeed, when estimating (20) without controlling for experience or tenure effects, the resulting average $Mm$ ratios are 1.57, 1.48 and 1.54, for low, medium and high skilled workers, respectively. These $Mm$ ratios are only slightly different than the ones used as targets in the simulations, reported in Table 1.