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Abstract

This paper studies an environment in which workers accumulate information about employment contacts made while searching on-the-job. Workers use this search capital to improve wages and insure against job destruction. This behaviour generates voluntary and involuntary job-to-job transitions with both wage hikes and wage cuts. The equilibrium wage distribution becomes less disperse than when workers cannot recall previously met job opportunities. The impact on output depends on depreciation and the extent of on-the-job search, among other factors. If search capital does not depreciate too quickly, the insurance benefits outweigh rent seeking costs and total output is higher with search capital.

Keywords: Search Capital, Turnover, Wage Cuts.
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1 Introduction

A large empirical literature demonstrates that informal employment contacts based on individuals' social or professional networks have a strong influence on their labour market outcomes. Holzer (1988), for example, finds that 66 percent of young workers who accepted a job used informal search channels. Capellari and Tatsiramos (2011) show that informal employment contacts have positive effects on workers' job finding rates, while Brown et al. (2013) show that such contacts lead to better job matches.[1] Theoretical frameworks that followed on from these findings formalise the idea that contacts help alleviate search frictions that arise from imperfect information about the location of jobs and workers and the idea that contacts help mitigate asymmetric information about the quality of applicants in the hiring process (see Topa, 2001, Montgomery 1991 and Galenianos, 2013, among others).

Information flows among the members of a given network lie at the heart of most of these theories. In particular, a prominent assumption made in models that consider employed and unemployed workers' job search is that individuals will always pass along information about job opportunities to their contacts (see Calvó-Armengol, 2004, Calvó-Armengol and Jackson, 2004, Fontaine, 2008, among others) or will pass along such information if the job opportunity is less attractive than the current job (see Mortensen and Vishwanath, 1994, Calvó-Armengol and Jackson, 2007, among others).

Building around these ideas, this paper considers an alternative, complementary environment in which job seekers keep and hold the information they acquire about job opportunities as insurance in the event of a job destruction shock. The mechanism is straightforward. From time to time, workers who engage in on-the-job search encounter firms with job opportunities that are not currently attractive. However, it is in the interest of these workers to establish contacts with such firms. If job matches are subject to destruction shocks, then with positive probability the worker will lose the current job sometime in the future. In the face of such a shock, the worker can approach the accumulated employment contacts (if any) to inquire whether there is still a job available. If so, the worker has the option of taking the job and avoiding unemployment. The crucial insight is that the ability of workers to recall previously met firms allows them to accumulate "search capital", a valuable asset that (partially) insures them against adverse displacement shocks.

The framework is similar to the one proposed by Postel-Vinay and Robin (2002) in which all workers search for jobs and sequential auctions determine wages. The key difference is that we allow workers to keep track of the identity of the firms they encounter during their search process. We also allow for search capital depreciation: there is a positive probability that a worker's employment contact might disappear. In this context, we show that it is in the interest of the firm as well as the worker to establish contacts. With recall, a losing firm in the auction knows it will remain in contact with the worker, at least for some period of time. Because employment for the worker at another firm may end, firms in a multiple bidder auction might at some time in the future become a lone bidder and hire the worker at more favourable,

[1] Topa (2001) and Dustmann et al. (2011), among others, provide further evidence on the importance of search channels. See Ioannides and Loury (2004) and Brown et al. (2013) for a review of the literature.

monopsonistic terms. As a result, with search capital, the option value of losing the worker in a competitive auction is no longer zero (as in Postel-Vinay and Robin, 2002), but positive and depends on the extent of search frictions, job destruction and search capital depreciation.

Search capital generated through on-the-job search has important implications for wages, turnover and welfare. Because workers can immediately recall offers following a displacement, job-to-job transitions occur not only when new work is found but also when the current employer lets the worker go. If on-the-job search generates a contact, the new contact and the current employer bid up the wage. If the new contact wins the auction, voluntary job turnover can occur with a wage jump or a wage cut, depending on the productivity of the poaching firm, among other factors. If a displacement shock subsequently hits, the worker takes employment with the losing firm in the previous auction. As a lone bidder, this firm acts monopolistically and offers a low wage equal to the worker's reservation wage. At this point, the worker experiences an involuntary job-to-job transition with a wage cut accompanied either by a movement between equally productive firms or by a movement to a lower productivity firm. This pattern of behavior provides an explanation for the wide spread observation of voluntary and involuntary job-to-job movements involving pay cuts (see Jolivet et al, 2006, among others).

A further implication of allowing workers to recall contacts accumulated while searching on the job is that the equilibrium wage distribution is less disperse than without recall. The option value of losing a worker in a competitive auction implies firms prefer waiting over bidding up to marginal product; hence, no worker will earn their marginal product in the search capital equilibrium.[2] Furthermore, although firms hire unemployed workers by offering them their reservation wage, the value of the latter is higher in a search capital equilibrium. With search capital, an unemployed worker now has the *option* to continue searching and possibly engaging firms in a competitive auction in the future. A hiring firm must compensate an unemployed worker for this option which increases starting wages relative to an equilibrium without recall. In turn, wage dispersion in a search capital equilibrium is a smooth function of the extent of search frictions, job destruction and search capital depreciation.

Now consider the impact of recall on aggregate equilibrium output. Investment in on-the-job search creates a productive resource for workers. Contacts accumulated through on-the-job search provide back-up employment opportunities that partially insure against costly unemployment from displacement shocks. With the ability to accumulate search capital, on-the-job search is not just rent seeking. Ceteris paribus, output with recall is higher than output with recall. The relevant comparison, however, is not with respect to the recall/no recall distinction but instead with respect to on-the-job search. On-the-job search can be inefficient as employed searchers become rent seekers who not only crowd out the unemployed for jobs but also discourage job creation. We establish that with recall, if search capital from on-the-job search does not depreciate too quickly, the insurance benefits outweigh the costs of rent seeking behaviour and total output is higher with than without search capital.

This paper is closely related to Carrillo-Tudela et al. (2011), which explores the implications of recall by unemployed workers alone. Without on-the-job search, unemployed workers on the

[2] Taylor (1995) finds a similar, related result.

equilibrium path are hired with no contacts. If they subsequently experience a displacement, they become unemployed with no contacts once again. Without on-the-job search, it is the threat of continued search while unemployed which increases the workers' reservation wage, raises the wage offered and hence avoids the Diamond (1971) paradox. In contrast, this paper incorporates on-the-job search as well as firm heterogeneity and optimal firm entry. These extensions create search capital accumulation. With on-the-job search, although unemployed workers again do not hold any contacts in equilibrium, employed workers will hold these contacts, thereby generating the wage dispersion, the rich job-to-job and wage dynamics and the welfare properties described above.[3]

The rest of the paper is as follows. The next section presents the general framework, allowing for firm heterogeneity and recall. In Section 3 we define equilibrium and describe its properties. Section 4 characterises the homogenous firms case to obtain a basic understanding of the model. Section 5 explores the welfare implications of the homogenous case with and without an endogenous number of firms. In Section 6 we analyse wage and job dynamics, wage dispersion and welfare under firm heterogeneity and free entry. Section 7 presents a further discussion and concludes.

2 The Economic Environment

Time is continuous and goes on forever. A unit mass of risk neutral workers and a mass of risk neutral firms with a common discount rate $r > 0$ maximize the expected sum of lifetime consumption and profit, respectively.

Firms operate using a constant return to scale production technology which can accommodate any number of workers. Although many of the basic insights arise with identical workers and firms, the initial specification is more general and allows for differences in productivity per worker across firms. In particular, suppose there are $H \geq 1$ types of firms. Let $\gamma_i > 0$ (where $\sum_{i=1}^{H} \gamma_i = 1$) and $x_i > 0$ (where $x_i > x_j$ for all $i > j$) denote the proportion and productivity respectively of type $i = 1, ..., H$ firms.

Workers are homogeneous and characterized by their employment status and search capital. An unemployed worker obtains and consumes z ($x_1 > z > 0$) units of output. A worker employed in a type i firm at wage w produces x_i and consumes w. The worker's search capital comes from the number, n, and type of each employer contact, excluding the current employer or any firm that the worker might have just met while job hunting. Workers lose their firm contacts at a Poisson rate of $\phi \geq 0$. The latter can be interpreted as the rate at which search capital depreciates.

At rate $\delta \geq 0$, an employed worker is exogenously displaced from the current job. When a displacement occurs, the current employer receives a payoff of zero and the worker can then request new take-it-or-leave-it offers from the firms (if any) that are still in contact with the displaced worker. If the worker accepts a new offer, the worker moves from one employer to the

[3]Kircher (2009) and Wolthoff (2014) explore related environments with multiple bidders.

other without an intervening spell of unemployment. If the worker rejects the offers (or if the worker did not have a contact), the worker becomes unemployed.

Unemployed job seekers meet a randomly drawn firm at rate $\lambda \geq 0$. Employed workers meet a randomly drawn firm at rate $s\lambda$, where $s \geq 0$ denotes the worker's exogenous search intensity. To keep the analysis simple, let $n = 0, 1$ and assume that at any point in time, the worker can interact with at most two firms.[4] Upon a meeting, the worker must therefore first decide which two firms to continue a relationship with, that is an employed worker with an existing contact must choose whether or not to incorporate the new contact. To focus on the implications of search capital alone, assume that workers do not pass on information about job opportunities to other workers. Further assume without loss that an unemployed worker or a worker without a contact always proceeds in some way with the new contact.

After choosing how to continue following a contact, the worker then decides whether to hold an auction or simply continue with the current employer, if any, at the current wage. If the worker decides to solicit wage bids, a complete information auction immediately takes place for the worker's services among the firms available. An employed worker adds the firm that loses the auction to the contacts list. If the worker transits from unemployment to employment, however, the newly met firm becomes an employer and does not count as a contact.

2.1 Workers payoffs

Let U_i be the payoff to an unemployed worker who has a contact with a type i firm, $i = 0, 1, ..., H$, where $i = 0$ corresponds to no contact. Let $E_j^i(w)$ denote the payoff to a worker earning wage w in a type $i \geq 1$ firm with contact $j \geq 0$. Let firm auction (pure) strategies or expected bids be given by

$$\Sigma = \{w_j^i\}, i = 1, ..., H, \ j = 0, 1, ...H,$$

where w_j^i is the offer made by a type i firm bidding against a type j firm and again $j = 0$ corresponds to no other contact or bidder. The payoff to unemployment can be written as

$$rU_0 = z + \lambda \left[\sum_{i=1}^{H} \gamma_i \max\{E_0^i(w_0^i), U_i\} - U_0 \right],$$

$$rU_i = z + \lambda \left[\sum_{j=1}^{H} \gamma_j \max\{E_j^i(w_j^i), E_i^j(w_i^j), U_i, U_j\} - U_i \right] + \phi \left[U_0 - U_i \right], \ i = 1, ..., H.$$

and equals the flow payoff z plus the potential expected capital gains of increasing one's search capital from meeting a firm, plus the expected loss from search capital depreciation.

When a worker earning wage w in a type $i \geq 1$ firm with contact $j \geq 1$ meets a type $k \geq 1$ firm, the worker has several choices. Allowing for all combinations, there are six potential bilateral auctions possible among the three firms.[5] In addition, the worker can replace or keep

[4] Carrillo-Tudela et al. (2011) analyze $n > 1$ in this economy without on-the-job search.

[5] To ease notation, ignore the option of opting for a one bidder auction when two are available.

the existing contact j without holding a new auction, thereby still earning wage w at firm i. Allowing the worker to choose unemployment with any of these three potential firms as contacts yields eleven possible outcomes. In this situation, the payoff is given by

$$Q(i,j,k,w)=\max\{E_j^i(w),E_k^i(w),E_j^i(w_j^i),E_k^i(w_k^i),E_i^j(w_i^j),E_k^j(w_k^j),E_i^k(w_i^k),E_j^k(w_j^k),U_i,U_j,U_k\}.$$

Q, coupled with a tie breaking rule, generates a mapping $q:(i,j,k,w)\rightarrow(i',j',D)$ such that (i) $(i',j')\subset(i,j,k)$ (ii) $D\in\{Auction, No\ Auction\}$ and (iii) if $i\notin$, (i',j') then $D=Auction$. The mapping q characterizes an optimal choice and Q its payoff. When a worker characterized by (i,j,w) meets a type k firm, the worker proceeds using the q defined by $Q(i,j,k,w)$.

For $j=0$, there are fewer options but $Q(.)$ is defined in an analogous way. Given this payoff, it follows that

$$\begin{aligned} rE_j^i(w) &= w+s\lambda\left[\sum_{k=1}^{H}\gamma_i Q(i,j,k,w)-E_j^i(w)\right] \\ &\quad +\delta\left[\max\left\{E_0^j(w_0^j),U_j\right\}-E_j^i(w)\right]+\phi\left[\max\left\{E_0^i(w_0^i),E_0^i(w),U_i\right\}-E_j^i(w)\right], \end{aligned}$$

for $i=1,...,H,\ j=1,...,H$. For $j=0$, there is no contact to lose, hence

$$rE_0^i(w)=w+s\lambda\left[\sum_{k=1}^{H}\gamma_i Q(i,j,k,w)-E_0^i(w)\right]+\delta\left[U_0-E_0^i(w)\right].$$

Like the value functions of the unemployed, these equations show that the value of employment equals the flow payoff w plus the expected capital gain from accumulating search capital through on-the-job search, plus the capital losses from displacement as well as from search capital depreciation.

2.2 Type i firm payoffs

Let $J_j^i(w)$ denote the payoff to a type $i=1,...,H$ firm paying a wage w to an employee who currently maintains a contact with a type j firm $j=0,1,...,H$. Likewise let C_j^i denote the value to a type i firm of being the contact for a worker employed at a type j firm. As above $j=0$ corresponds to the situation where the worker does not have a current contact and hence is unemployed.

A type i firm's expected payoff as a lone bidder, that is in an auction with a worker with no other contact, is then given by

$$M_0^i=\max_w\{I[E_0^i(w)\geq U_i](J_0^i(w)-C_0^i)\}+C_0^i,$$

where I is an indicator function that takes the value of one if the firm outbids the worker's value of unemployment, that is the worker's reservation wage defined by $E_0^i(R_i)=U_i$ and zero otherwise.

On the other hand, a type i firm's expected value from engaging in a Bertrand auction against a type j firm bidding w_i^j is

$$M_j^i = \max_w \left\{ \begin{array}{c} I[E_j^i(w) > \max\{E_i^j(w_i^j), U_i, U_j\}](J_j^i(w) - C_j^i) \\ +I[E_j^i(w) = \max\{E_i^j(w_i^j), U_i, U_j\}]\iota_j^i(J_j^i(w) - C_j^i) \\ +C_j^i \end{array} \right\},$$

where ι_j^i denotes the worker's tie breaking rule between equal payoffs in this auction. Depending on productivity differences, some firms (the high productivity ones) may need to outbid the payoff to unemployment rather than the bid of a weak or comparatively very low productivity opponent in order to obtain the worker's services.

The payoff to firm i of winning the auction and employing the worker, $J_j^i(w)$, depends on the subsequent worker's strategy when meeting new firm contacts and then whether to initiate an auction. Suppose a worker currently being paid wage w at firm i with a type j contact meets a type k firm. Firm i continues an attachment of some sort with the worker in four possible ways determined by the mapping q. Let $q_1 = 1$ if $q : (i, j, k, w) \to (i, k, Auction)$, i.e., the worker chooses to hold an auction between the new firm k and firm i; let $q_2 = 1$ if $q : (i, j, k, w) \to (i, k, NoAuction)$, the worker keeps the new firm k as a contact but does not initiate a new auction, thereby staying at firm i at wage w; $q_3 = 1$ if $q : (i, j, k, w) \to (i, j, NoAuction)$, the worker discards the new firm k and does not initiate a new auction and $q_4 = 1$ if $q : (i, j, k, w) \to (i, j, Auction)$, the worker decides for some reason to hold a new i, j auction. Otherwise $q_1 = q_2 = q_3 = q_4 = 0$. It then follows that

$$\begin{aligned} rJ_j^i(w) &= x - w + s\lambda \left[\sum_{k=1}^{H} \gamma_k \left(q_1 M_k^i + q_2 J_k^i(w) + q_3 J_j^i(w) + q_4 M_j^i \right) - J_j^i(w) \right] \\ &\quad + \phi[J_0^i(w) - J_j^i(w)] - \delta J_j^i(w), \end{aligned}$$

where we assume without loss that no auction is called following a ϕ shock in which the worker loses the employer contact and that the workers does not become unemployed with i as the contact. Likewise, the payoff to a type $j = 1, ..., H$ firm of being a contact is given by

$$rC_i^j = s\lambda \left[\sum_{k=1}^{H} \gamma_k \left(q_3 C_j^i + q_4 M_j^i + q_5 M_k^j \right) - C_i^j \right] + \delta(M_0^j - C_i^j) - \phi C_i^j,$$

where $q_5 = 1$ when $q : (i, j, k, w) \to (j, k, Auction)$, the worker chooses an auction between j and k and $q_5 = 0$ otherwise. For contacts who are unemployed, i.e. $i = 0$, assume without loss that the worker accepts the new contact and initiates an auction between firms j and k.

$$rC_0^j = \lambda \left[\sum_{k=1}^{H} \gamma_k M_k^j - C_0^j \right] - \phi C_0^j.$$

3 Equilibrium

Following a worker-firm meeting, the worker first chooses which firms to continue with and whether to open up an auction. If no auction is chosen, activity concludes with the worker employed at the current firm at the existing wage (or unemployed with flow payment z) and the contact, if any, in hand. If an auction is called, the second phase unfolds with firms offering wages to the worker. In the third and last stage of play, the worker chooses an employment or unemployment option. As is standard, the three phased stage game is solved recursively.

Since the $E^i_j(w)$ are strictly increasing in w, the worker's best response strategy in an auction has the reservation property for each (i, j) pair. Denote $\Sigma_{-w^i_j}$ as all bids except the particular paired firm $i's$ bid in an auction involving a type j firm. Given $\Sigma_{-w^i_j}$, define R^i_j as the wage that makes the worker indifferent between accepting firm i's offer and the next best opportunity.

$$E^i_j(R^i_j) = \max\{E^j_i(w^j_i), U_i, U_j\},\ i = 1, ...H,\ j = 1, ...H \tag{1}$$

$$E^i_0(R^i_0) = U_i,\ i = 1, ...H.$$

Note that by construction $R^i_j \geq R^i_0$, R^j_0 and that in general reservation wages are not symmetric - R^i_j does not necessarily equal R^j_i.

Given the other firm's bid from $\Sigma_{-w^i_j}$, the worker's best response function to a bid w from firm i is to accept all $w > R^i_j$. For $w^i_j = R^i_j$ assume that the worker's tie breaking rule in an auction is to chose (i) employment over unemployment (ii) the higher productivity firm, that is i if $i > j$ (iii) with equal probability if $i = j$.

The bidding problem, M^i_j, for firm i competing against firm j can be re-written in terms of the reservation wages R as determined by $\Sigma_{-w^j_i}$:

$$M^i_j = \max_w \left\{ I[w > R^i_j](J^i_j(w) - C^i_j) + I[w = R^i_j]\iota^i_j(J^i_j(w) - C^i_j) + C^i_j \right\}. \tag{2}$$

Since $x_i > x_{i'}$ for all $i > i'$, it follows $J^i_j(w) \geq J^{i'}_j(w)$ and $C^i_j \geq C^{i'}_j$. More productive firms can always replicate the behaviour of less productive firms and earn more profit in equilibrium.

Given the tie breaking rule in an i, j auction, any firm $i \leq j$ will bid more than R^i_j, up to the wage that makes i indifferent between hiring and continuing the relationship with the worker as a contact, i.e. up to where the offer w satisfies $J^i_j(w) = C^i_j \leq C^j_i$. On the other hand, if $i > j$, i will win the auction by simply offering R^i_j. The best response strategy for $i > j$ is to offer the reservation bid $w^i_j = R^i_j$ up to where $J^i_j(w^i_j) = C^i_j \geq C^j_i = J^j_i(w^j_i)$.

Suppose $i \leq j$. The Bertrand competition just described implies that in equilibrium firm i increases its bid until $J^i_j(w^i_j) = C^i_j$ whereas firm j bids R^j_i. Since $R^j_i \geq R^j_0$, for all i, it follows that $w^j_i \geq w^j_0 = R^j_0$. The wage offered in a one bidder auction is the reservation wage for continuing search with a contact i, $w^i_0 = R^i_0$, provided $J^i_0(R^i_0) > C^i_0$ implying

$$J^i_0(w^i_0) = J^i_0(R^i_0) = M^i_0.$$

In a two bidder auction, the less productive firm bids up to to the point where it is indifferent between employing and being a contact. The wage offered by firm i in a two bidder auction w^i_j solves

$$J^i_j(w^i_j) = C^i_j = M^i_j.$$

The more productive firm j bids up to this point in the worker's payoff as well and the worker chooses firm j and the offer $w^j_i = R^j_i$. Here

$$J^j_i(w^j_i) = J^j_i(R^j_i) = M^j_i \geq C^j_i$$

with strict inequality for $j > i$. Moreover, the worker's indifference between i and j induced by $j's$ best response implies by construction that $w^i_j = R^i_j$ provided $E^i_j(w^i_j) \geq U_j$.

$$J^i_j(w^i_j) = J(R^i_j) = C^i_j = M^i_j.$$

In this case, $\Sigma = R$. If not, w^i_j solves $J^i_j(w^i_j) = C^i_j$. Using the above arguments we now define an equilibrium.

Definition: An equilibrium is a set of reservation wages and wage offers R, Σ and a mapping q such that

(i) R^i_j satisfied the worker's reservation values (1) in an auction above given offers Σ for $i = 1, ..., H,\ j = 0, 1, ...H$.

(ii) w^i_j solves the firms problem described by M^i_j in (2) given R, $\Sigma_{-w^i_j}$, for $i = 1, ..., H,\ j = 0, 1, ...H$.

(iii) $q : (i, j, k, w) \rightarrow (i', j', D)$ implements $Q(i, j, k, w)$ given Σ, R and the tie breaking rule of choosing strictly higher productivity firms and choosing incumbents among equally productive contacts.

4 Homogeneous Firms

To highlight the main economic mechanisms of the model in a tractable and transparent way, we analyse the case in which all firms are homogeneous ($H = 1$ and $x_1 = x$). In this case it is not necessary to identify firm types. Superscripts can be suppressed and subscripts for contacts are now $\{0, 1\}$.

The expected payoff for a firm in an auction without a competitor is

$$M_0 = \max_w \{I[w \geq R_0](J_0(w) - C_0)\} + C_0.$$

The firm's expected value from an auction with a worker who is unemployed with one contact or is employed with no contacts is

$$M_1 = \max_w \left\{ I[w > R_1](J_1(w) - C_1) + I[w = R_1] \left(\frac{J_1(w) - C_1}{2} \right) \right\} + C_1,$$

where the other bidding firm offers the worker's reservation wage in this auction, $w_{-1} = R_1$, a wage that makes the firm indifferent ($J_1(R_1) = C_1$) about hiring the worker. We conjecture that $R_1 > R_0$. In this situation, the best response strategy (w_0, w_1) is to offer the reservation wages (R_0, R_1). Hence, the firm's strategies (w_0, w_1) imply $M_0 = J_0(w_0)$ and $M_1 = J_1(w_1) = C_1$, where $J_0(w_0)$, $J_1(w_1)$, C_1 and C_0 are given by

$$rJ_0(w_0) = x - w_0 + s\lambda[J_1(w_1) - J(w_0)] - \delta J_0(w_0)$$

$$rJ_1(w_1) = x - w_1 - \delta J_1(w_1)$$

$$rC_1 = \delta(M_0 - C_1) - \phi C_1$$

$$rC_0 = \lambda(M_1 - C_0) - \phi C_0.$$

Given the bidding strategies, the expected value of an unemployed worker with a contact and the expected payoff in a two bidder auction simplifies to

$$rU_0 = z + \lambda \left[E_0(w_0) - U_0\right]$$

$$rU_1 = z + \lambda \left[E_1(w_1) - U_1\right] + \phi \left[U_0 - U_1\right],$$

whereas the expected value of employment at any wage w satisfies

$$rE_0(w) = w + s\lambda \left[\max\{E_1(w), E_1(w_1)\} - E_0(w)\right] + \delta \left[U_0 - E_0(w)\right]$$

$$\begin{aligned} rE_1(w) &= w + s\lambda \left[\max\{E_1(w), E_1(w_1)\} - E_1(w)\right] \\ &\quad + \delta \left[E_0(w_0) - E_1(w)\right] + \phi \left[\max\left\{E_0(w_0), E_0(w)\right\} - E_1(w)\right]. \end{aligned}$$

For $E_1(w_1)$, the worker does not benefit from on-the-job search. Thus when a worker with a job and a contact meets another firm, the worker receives no capital gain and disregards the new contact.

Using the above value functions and firms' indifference condition in an auction with two bidders, we obtain the following result.

Lemma 1: *If all firms have the same productivity, the wage offered in an auction with two bidders is given by*

$$w_1 = \frac{(r+\phi)(r+\delta+s\lambda)}{(r+\phi)(r+\delta+s\lambda)+\delta(r+\delta)}x + \frac{\delta(r+\delta)}{(r+\phi)(r+\delta+s\lambda)+\delta(r+\delta)}w_0.$$

Relative to sequential auction models without recall, the new feature here is that the offered w_1 is strictly below x. With search capital firms have a positive value of holding on to a contact; i.e. $C_1 > 0$. Over time employed workers experience job destruction shocks. Workers will call upon their contact (if they have one) to avoid unemployment. In such a case, the contacted firm is in the desirable situation where it faces no competition from other firms and hence can extract monopsony rents from the worker by paying w_0.[6] This recall implies a positive weight

[6] In the more general case of $n > 1$, the contacted firms will face less competition for the worker than they faced in the last auction and offer the worker a lower wage.

on w_0 in the above equation as the value to the firm of not hiring the worker (and waiting for a job displacement and a subsequent wage of w_0) is decreasing in w_0.

In an auction with just one bidder, the worker (without a contact) gets offered $w_0 = R_0$ making the worker indifferent between accepting the job and searching with a contact, i.e., $E_0(w_1) = U_{1.}$ This condition leads to Lemma 2.

Lemma 2: *If all firms have the same productivity, the indifference condition faced by unemployed workers with no contacts yields*

$$w_0 = \varphi w_1 + (1 - \varphi)z,$$

where

$$\varphi = \frac{\lambda[(r + \lambda + \delta) - s(r + \lambda + \phi)]}{(r + \delta + \phi)(r + \lambda + \phi) + \lambda(r + \lambda)} < 1.$$

Note that φ is decreasing with search intensity as there is a "foot-in-the-door" effect at play here. Unemployed workers are prepared to accept a wage below z as an investment for the wage growth that arises from engaging their future employers and poaching firms in Bertrand competition. (See Postel-Vinay and Robin, 2002.) With search capital, however, the possibility of accumulating employment contacts tempers this foot-in-the-door effect thereby increasing w_0. Given that unemployed workers have the option to continue searching and increasing their wage when meeting another contact, a firm must compensate workers for giving up this option. The relative importance of these channels pins down the sign of φ and hence whether w_0 is above z or not.

Combining Lemmas 1 and 2 gives the following.

Proposition 1: *The wages offered in bidding equilibrium are:*

$$w_1 = \alpha x + (1 - \alpha)z \quad \textit{and} \quad w_0 = \beta x + (1 - \beta)z,$$

where

$$\alpha = \frac{(r + \phi)(r + \delta + s\lambda)}{(r + \phi)(r + \delta + s\lambda) + (1 - \varphi)\delta(r + \delta)},$$

$\beta = \varphi\alpha$ *and* $w_1 > w_0$.

The above equations reveal the effects on equilibrium wages of an increase in the rate at which worker meet contacts, s, and the rate at which they loose them, ϕ. For example, differentiation implies that $\partial w_1/\partial\phi > 0$. This result is quite intuitive. As search capital depreciates faster, the firm's value of holding on to contact is lower. There is a higher chance that the worker might not recall the firm by the time a job destruction shock hits the worker. Since this also implies that the firm's value of employing a worker is now higher, firms would be willing to bid more to attract workers.

The sign of $\partial w_0/\partial\phi$ is, however, ambiguous. A higher ϕ increases w_0 through its effect on w_1. On the other hand, a higher ϕ reduces w_0 through its effect on φ. As search capital depreciates faster, the option value to continue searching for another contact becomes less important, thereby decreasing φ. The relative strengths of these forces then pins down the net effect of ϕ on w_0.

Differentiation also establishes that $\lambda(\phi - \delta) + \phi(r + \phi) < 0$ is necessary and sufficient to guarantee $\partial w_1/\partial s < 0$, and is sufficient to guarantee $\partial w_0/\partial s < 0$. When workers are able to hold on to their contacts for a relatively long time (i.e ϕ close to zero), a higher s implies a lower w_0 through the foot-in-the-door effect. This effect in turn increases the firm's value of holding a contact and puts downward pressure on w_1. At higher values of ϕ, these effects are weaker and an increase in s leads to stronger competition between firms in the Bertrand auction, thereby putting upward pressure on w_1 and consequently on w_0.

Derivation of the workers' reservation wages $\{R_0, R_1\}$ completes the characterisation of equilibrium. The above arguments show that $w_0 = R_0$. Solving $E_0(R_0) = U_1$ and $E_1(R_1) = E_1(w_1)$ yields the reservation wages of workers in a two bidder auction. It is straightforward to verify that $w_1 \geq R_1$ is indeed satisfied.[7] Further, $J_0(w_0) > C_0 > 0$ implies firms strictly prefer to hire an unemployed worker at the first meeting rather than keeping the worker as a contact. In a competitive auction, $J_1(w_1) = C_1 > 0$ implies firms are indifferent between hiring the worker and keeping the worker as an employed contact and hence there is no profitable deviation. These arguments establish existence. Showing that there is no equilibrium where firms offer a wage $w_n < R_n$ establishes uniqueness.

Theorem 1: *The reservation strategies* (R_0, R_1) *and the offer strategies* (w_0, w_1) *describe the unique symmetric equilibrium with homogeneous firms. Employed workers without a contact search on-the-job and initiate a two bidder auction when they meet another potential employer. An employed worker with a contact ignores firms met during on-the-job search. This behaviour describes the mapping* q.

5 Implications for Output and Welfare

This section analyses the implications of on-the-job search and search capital on aggregate output. In conventional models with homogeneous agents, employed job seekers are rent seekers who move from job to job for higher pay without an accompanying increase in production. In these environments, on-the-job search can be socially wasteful. A social planner would in general want to avoid this costly reshuffling of workers across employers.[8] Because there are no costs to searching while employed, this inefficiency does not apply here. There are, however, other potential consequences associated with job-to-job turnover. In particular, permitting on-the-job search can reduce output as (a) employed workers compete with (crowd out) unemployed workers for available employment opportunities and (b) firm entry declines due to higher wages, shorter employment spells and less frequent contacts with (more profitable) unemployed job candidates.

In contrast, on-the-job search in the economy presented here takes on a productive aspect - it generates back-up job opportunities that partially insure workers when they become displaced. Establishing contacts with potential employers through job hunting enables workers to avoid

[7] To verify this claim first note that $R_2 = R_2^e(w_1)$. This follows as R_2 solves $E_1(R_2) = U_1$, while $R_2^e(w_1)$ solves $E_1(R_2^e) = E_0(w_1) = U_1$. Next note that the solutions of w_2 and w_1 described in Proposition 1 imply that $E_1(w_2) > E_0(w_1) = U_1 = E_1(R_2)$. Since E_1 is increasing in w, we get that $w_2 > R_2$.

[8] Models with on-the-job search may be more efficient that the competitive outcome without on-the-job search. For example, adding on-the-job search as in Burdett and Mortensen (1998) relieves the outcome of the Diamond paradox (see Diamond, 1971).

costly unemployment spells when they separate from their current employer. The benefits of this insurance depend on the depreciation rate of search capital, so ϕ along with the extent of on-the-job search intensity s become critical parameters for evaluating the impact of on-the-job search in this framework.

To assess these effects we extend the model with homogeneous firms described in the previous sections by first endogenizing the arrival rate through a matching technology and then by allowing firm entry and exit. Let u denote the number of unemployed job seekers, e_0 the number of employed workers without a contact and e_1 the number of employed workers with a contact so that $u + e_0 + e_1 = 1$. In a steady state, flows across these three states balance such that $(\delta + \phi)e_1 = s\lambda e_0$ and $\delta e_0 = \lambda u$ leading to the steady state measures

$$u = \frac{\delta(\delta + \phi)}{(\delta + \phi)(\delta + \lambda) + s\lambda^2} \tag{3}$$

$$e_0 = \frac{\lambda(\delta + \phi)}{(\delta + \phi)(\delta + \lambda) + s\lambda^2} \tag{4}$$

$$e_1 = \frac{s\lambda^2}{(\delta + \phi)(\delta + \lambda) + s\lambda^2}. \tag{5}$$

The $1 - u$ employed workers produce x whereas the unemployed contribute z. Output also includes the costs of firm operations. Let f denote the number of identical firms producing as well as recruiting workers. Firms in this framework are a collection of jobs that can be either vacant or occupied and producing. To be economically active and recruit workers, firms must pay a fixed flow cost k each period. Total steady state output y - the standard measure of welfare in matching models - is the sum of these figures. Hence

$$y - z = (1 - u)(x - z) - kf. \tag{6}$$

5.1 Crowding Out

To focus first on the impact of crowding out in (a) relative to the insurance benefits of creating back-up opportunities, assume that the numbers of firms f is fixed. This specification, which aligns with the conventional notion of the short run, leads to a straight forward determination of λ. Wages and employment follow directly from the equilibrium bids and steady state flows respectively.

Employed workers searching for another job opportunity, the e_0 and e_1 workers, can interfere with the search outcomes of unemployed job seekers, the u jobless workers.[9] To allow for this behaviour, assume search is undirected with random encounters between workers and firms so that unemployed and employed workers are substitutes in the search process.[10] Following

[9] The model specifies that all employed workers search including those with a contact. This specification eases exposition in the general case but does not alter the basic results derived here. Restricting on-the-job search to those without contacts does not materially affect outcomes.

[10] In directed search models with on-the-job search (see, for example, Delacroix and Shi, 2006), employed and unemployed workers search for jobs in different submarkets and any crowing out effect between these workers disappears.

conventional specifications, suppose a Cobb Douglas technology with constant returns to scale governs the way in which job seekers meet potential job opportunities so that the number of work-firm meetings is:

$$m(u+se_0+se_1,f)=m(u+s(1-u),f)=Af^{1/\sigma}(u+s(1-u))^{(\sigma-1)/\sigma},\sigma>1,$$

where A is an efficiency parameter. Employed workers compete with unemployed job seekers in the matching process slowing the escape rate out of unemployment.

Although the number of agents in the economy is fixed, the number of meetings occurring between firms and job seekers of both types depends on the extent of on-the-job search captured by s. Fixing f is not equivalent to fixing the number of vacancies as firms can maintain contacts as they hire other workers. If a firm meets a worker with a job and becomes the contact of the worker, this relationship does not impede the firm's capacity to hire other workers at any point. The value functions are consistent with this set up as is the exogenous specification for the decay of human capital ϕ which is independent of firm matching and hiring rates.[11]

The corresponding arrival rate is

$$\lambda=m(u+s(1-u),f)/(u+s(1-u))=A[f/(u+s(1-u))]^{1/\sigma}. \tag{7}$$

Implicit differentiation and manipulation gives

$$\frac{\partial\lambda}{\partial s}=\frac{-\lambda^2[\delta+\phi+s\lambda-(1-s)\lambda u]}{\sigma[(\delta+\phi)(\delta+s\lambda)+s^2\lambda^2]-(1-s)(\delta+\phi+2s\lambda)\lambda u},$$

which is negative as both the denominator and the numerator term in brackets are positive. Taken together, total differentiation of unemployment above gives

$$\begin{aligned}\frac{\partial u}{\partial s} &= \frac{-u}{(\delta+\phi)(\delta+\lambda)+s\lambda^2}\left[\lambda^2+(\delta+\phi+2s\lambda)\frac{\partial\lambda}{\partial s}\right]\\ &= \frac{-\lambda^2 u}{(\delta+\phi)(\delta+\lambda)+s\lambda^2}\\ &\quad\times\frac{(\sigma-1)[(\delta+\phi)(\delta+s\lambda)+s\lambda^2]-(\delta+\phi)(\phi+2s\lambda)-s^2\lambda^2}{\sigma[(\delta+\phi)(\delta+s\lambda)+s^2\lambda^2]-(1-s)(\delta+\phi+2s\lambda)\lambda u}.\end{aligned}$$

As u governs output for a fixed f in (6), the term

$$\sigma-2+\frac{(\delta+\phi)(\delta-\phi-s\lambda)}{(\delta+\phi)(\delta+s\lambda)+s\lambda^2} \tag{8}$$

determines the impact of on-the-job search on unemployment, output and welfare. For sufficiently small ϕ and small s, allowing workers to increase the extent of on-the-job search with recall via s lowers unemployment. Since the number of firms is fixed, output in (6) consequently rises with s. On the other hand, if employed workers do not hold onto their contacts sufficiently well (for ϕ large), crowding out outweighs backstopping from recall. As $\phi\to\infty$ recall disap-

[11] Although specifying a fixed number of jobs is conceptually awkward given recall, naive specifications with $u=v$ yield similar results to those found here.

pears, the model converges to a standard model in which on-the-job search reduces output and welfare. It is worth noting that under (i) symmetric matching ($\sigma = 2$) coupled with (ii) equal destruction of jobs and contacts ($\delta = \phi$) the insurance component of on-the-job search increases output for all $s > 0$. At $s = 0$, there is no effect.

A similar but simpler exercise establishes that $\partial y/\partial\phi < 0$. This result is quite natural. A decline the insurance component from and increased ϕ lowers output.

5.2 Job Creation

With a fixed number of agents, workers and firms have limited opportunities to adjust behavior in response to wage changes as s or ϕ vary. Wages do not allocate resources along familiar margins such as participation but instead solely determine the share of the match rents between the worker and the firm. Suppose there is a change in the effectiveness of on-the-job search, that is, in s. Wages characterized in Proposition 1 change accordingly. Job tenure and the matching rate λ, as just demonstrated, change as well inducing further wage adjustments. Without a participation decision, workers and firms have no choice but to accept the wage outcome.

Allowing a participation response through entry and exit enables firms to react to wages (and matching) thereby further altering employment and output. This response, which aligns with conventional notions of the long run, causes the interactions among the endogenous λ, u, e_0, e_1, w and now f to become more complex.[12]

Wages determine profits. Firms respond to profits over time through their decision to be economically active, that is the entry and exit margin. With contacts and hired workers present, the zero profit condition in this model differs from the familiar expression relating to the value of a vacancy. See Pissarides (2001). Recall that firms use a constant returns to scale production technology that enables them to acquire and maintain contacts independently of the number of workers currently working with the firm. As such, entry and exit become tied to the payoffs of employing a given worker and of holding contacts. In a steady state of this economy, there are $e_0 + e_1$ workers at any point in time evenly allocated across f firms each paying operating costs k. Steady state profit flow at each firm is thus given by

$$\pi = (x - w_0)e_0/f + (x - w_1)e_1/f - k.$$

We assume that participation through entry or exit drives flow profits to zero - $\pi = 0$ and determines the amount of job creation in the economy.[13] Substitution and manipulation then gives

$$\frac{kf}{x-z} = \frac{\lambda(1-\varphi\alpha)(\delta+\phi) + s\lambda^2(1-\alpha)}{(\delta+\lambda)(\delta+\phi) + s\lambda^2}. \tag{9}$$

A free entry equilibrium further includes a (λ, f) pair solving equations (7) and (9). Wages follow from Proposition 1 whereas employment and unemployment levels follow from equations

[12] It is possible but conceptually less transparent and meaningful to fix λ and let f be endogenous. Doing so restricts matching to be independent of unemployment, employment and the number of firms. Moreover, as wages are now allocating resources, the impact of parameter changes works through wages as well as employment.

[13] We assume free-entry requires flow profits to be zero purely to keep the analysis tractable. Such a condition can be derived when interpreting r as a "death" shock for firms instead of an interest rate.

(3)-(5). Equation (6) yields output.

Analytic outcomes and hence comparative static results are elusive for any free entry equilibrium solution. We therefore numerically solve the model. Table 1 describes the parameter values adopted here which roughly correspond to similar parameter values found in the literature.

Table 1: Parameters for Homogeneous Firms

Parameter	x	z	r	δ	k	σ	A
Value	10	5	0.01	0.1	3	2	0.5

Using Table 1 parameters, the model is simulated for varying on-the-job search activity, captured by $0 \leq s \leq 1$, and for varying ability to recall, $0 \leq \phi \leq 0.8$. Given the degree of abstraction, these simulations should not be considered a precise quantitative exercise. On the other hand, the chosen parameters generate results in line with observed outcomes. In particular, for $0.1 \leq s \leq 0.5$ and $0.05 \leq \phi \leq 0.4$, unemployment averages approximately 8.5%, the exit rate from employment into unemployment is 0.063 and the employment to employment turnover rate is 0.095 with just under 40% of all these job-to-job transitions involving a wage cut.[14]

Table 2 presents several equilibrium outcomes for varying values of s and for $\phi = 0, \delta/4, \delta, 4\delta$. From Table 2, the $s = \phi = 0$ parametrization yields the fewest firms f and the highest wages, in this case w_0. Despite high unemployment - due in part to the absence of the insurance aspect on-the-job search - employed workers enjoy their highest mean wage in this scenario. For $s = 0$, unemployed search is the only potential way in which workers can generate search capital. Search terminates when a worker accepts a job so workers become reluctant to give up continued unemployed search with a contact. They require substantial compensation to accept a job and forgo the prospect of Bertrand wages. On the other hand, firms would receive little from being held as the contact of an unemployed worker. Unlike an employed worker as a contact, an unemployed worker as a contact would only re-visit the firm after meeting another potential employer and deciding to initiate competitive not monopolistic bidding. An employed worker as contact - and there are no such workers when there is no on-the-job search - would return to the firm following a displacement shock and without another bidder in hand.

When workers with perfect recall begin to search on the job, they become more willing to accept a lower wage. The effect in this parameterization is striking as w_0 falls rapidly and firms start to beneficially hold employed workers as contacts. Entry follows suit. Unemployment falls drastically - with the insurance component now occurring in this case - and total production which is proportional to u rises. Total net output, however, falls due to the added costs of firm operations outweighing the increased production.

When $\phi > 0$, contacts are lost from time to time which lowers the potential insurance contribution of recall. The economy, however, responds less vigorously when moving away from $s = 0$. At $s = 0$ and $\phi > 0$, the diminished ability to recall (relative to $\phi = 0$) leads to a reduction in the wage offered to those workers with no contacts, w_0. Profits rise, entry increases and unemployment is lower than at $\phi = 0$. Moreover, when on-the-job search increases from

[14]Based on the interest rate r, outcomes correspond to quarterly observations. As some values for ϕ and s might yield high elasticities or extreme values with respect to the endogenous variables, the comparison of model outcomes with observations is reported for s and ϕ in this range.

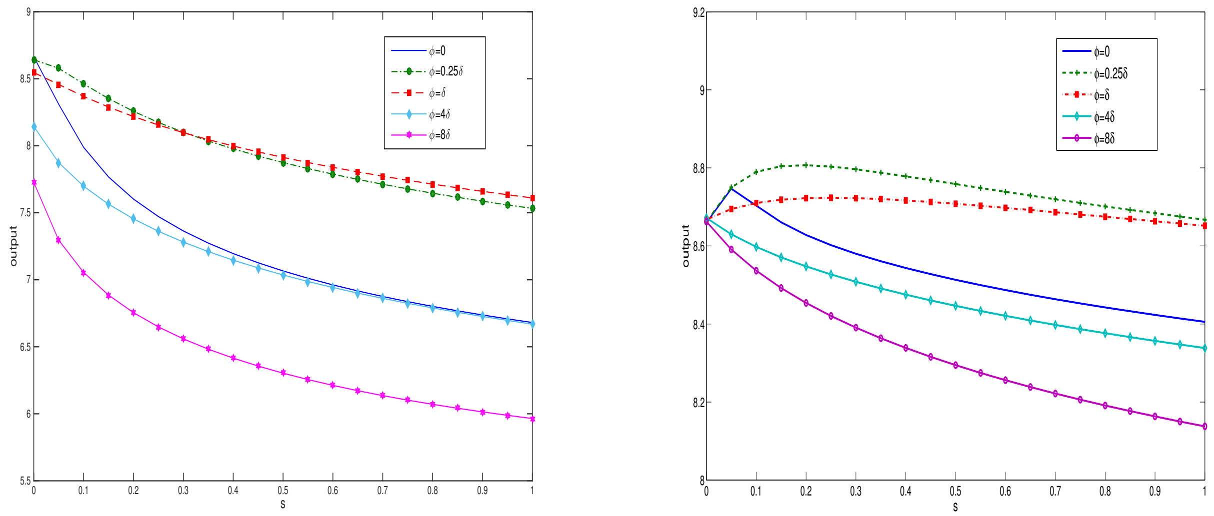
Table 2: Numerical Simulations by Search Intensity and Recall: Homogeneous Firms

s	**0.00**	**0.05**	**0.15**	**0.25**	**0.50**	**0.75**	**0.85**	**1.00**
				$\phi = 0$				
λ	0.624	1.047	0.955	0.838	0.668	0.575	0.548	0.514
f	0.215	0.465	0.675	0.781	0.922	1.001	1.025	1.055
w_0	9.251	8.448	7.624	7.130	6.324	5.763	5.574	5.316
w_1	9.319	8.648	8.069	7.776	7.381	7.160	7.094	7.011
Mean wage	9.251	8.517	7.886	7.567	7.137	6.897	6.826	6.735
u	0.138	0.059	0.041	0.037	0.033	0.032	0.031	0.031
				$\phi = 0.25\delta$				
λ	0.695	0.774	0.721	0.652	0.540	0.475	0.455	0.430
f	0.243	0.324	0.434	0.505	0.616	0.687	0.710	0.740
w_0	9.166	8.843	8.274	7.809	6.873	6.103	5.823	5.425
w_1	9.382	9.215	8.981	8.827	8.585	8.430	8.381	8.317
Mean wage	9.166	8.931	8.602	8.385	8.043	7.825	7.756	7.665
u	0.126	0.090	0.069	0.062	0.055	0.052	0.051	0.050
				$\phi = \delta$				
λ	0.873	0.799	0.688	0.616	0.511	0.451	0.433	0.410
f	0.313	0.357	0.424	0.475	0.564	0.625	0.646	0.673
w_0	8.952	8.687	8.184	7.727	6.718	5.823	5.487	5.000
w_1	9.501	9.475	9.420	9.376	9.295	9.238	9.219	9.194
Mean wage	8.952	8.818	8.604	8.444	8.164	7.969	7.905	7.819
u	0.103	0.095	0.088	0.084	0.079	0.076	0.075	0.074
				$\phi = 4\delta$				
λ	1.352	1.088	0.873	0.764	0.619	0.541	0.517	0.488
f	0.504	0.581	0.673	0.735	0.837	0.904	0.925	0.954
w_0	8.377	7.939	7.290	6.765	5.689	4.796	4.471	4.010
w_1	9.682	9.711	9.729	9.735	9.741	9.743	9.743	9.743
Mean wage	8.377	8.113	7.797	7.586	7.239	7.011	6.938	6.842
u	0.069	0.077	0.083	0.087	0.091	0.093	0.093	0.094

this point, production rises as the insurance component kicks in. The increase in the number of firms is less pronounced so that total output and welfare rise. As ϕ rises still further, the pattern continues with output falling with s.

If firms are homogeneous, total net output is maximised when workers do not search on the job and there is perfect recall, even though the number of firms is the lowest and unemployment is the highest relative to parameterisations where $s > 0$. See the left panel in Figure 1. As mentioned, the main reason for this result is that as firms enter, the costs of firm operations outweigh the increased production. This feature, however, does not occur when firms differ in their productivities. As discussed in the next section, with firm heterogeneity and free-entry, total output is maximised at positive values of s and ϕ. Note as well that with homogeneous firms, there is not a consistent ordering in the relationship of output with respect to recall as s varies. The lines denoting output for different ϕ intersect in several places with output associated with better recall falling faster as s increases. In contrast, given a fixed number of firms, output was found to always be higher with lower levels of ϕ.

Figure 1: Output as a function of s and ϕ



(a) Homogeneous Firms

(b) Heterogeneous Firms

6 Wages, Turnover and Output Under Firm Heterogeneity

To explore the model with firm heterogeneity, suppose $H = 2$. In this setting, workers who meet a potential employer continue with the contact if and only if the new option strictly improves payoffs. They do not replace existing contacts with similar ones. Workers with a type 1 employer and a type 1 contact do not change arrangements when they meet another type 1 firm. This assumption does not affect the individual worker's payoff, but it does alter the firm's return to job creation as this behaviour by the worker alters the likelihood of the firm separating from an existing contact or worker as well as the likelihood of being asked to bid after a meeting takes place.[15] Under firm heterogeneity workers can improve the "quality" of their search capital by moving from type 1 firms to type 2 firms.

With two types of firms there are four different bids in competitive two firm auctions. Firms offer reservation wages R_j^i, $i = 1, 2,\ j = 1, 2$. There are also two wages offered in a monopolistic auction, R_0^1 and R_0^2. Because the worker's contacts can come and go without a wage change from an auction, some of these R_j^i wages will appear in non (i, j) employment states. For example, a worker earning R_2^2 might lose the current contact and then find a type 1 firm contact so that the payoffs to $E_0^2(R_2^2)$ and $E_1^2(R_2^2)$ need to be accounted for. Likewise a worker earning R_1^1 who loses contact will continue with this same wage. Similar outcomes apply for workers earning R_1^2 and R_2^2. Accounting for all the possible observed contact changes without auctions as well as the off the equilibrium path payoffs to $E_2^1(R_2^1), U_1, U_2$ yields fifteen Bellman equations. For firms, the

[15]Workers might want to collectively commit to swapping contacts around and thereby lowering the firm's payoff to holding a contact. It is uncertain if subsequent firm exit offsets these gains. In any case, an ϵ small switching costs rule out this behavior.

same accounting exercise yields a corresponding twelve $J^i_j(w)$ equations - three of which are off the equilibrium path, along with C^1_1, C^1_2 and C^2_2.

These thirty Bellman equations for the thirty unknowns E^i_j, J^i_j, C^i_j are linear in the six unknown R^i_j. Fortunately, the six equilibrium restrictions

$$U^i_0 = E^i_0(R^i_0), i = 1, 2$$

$$J^i_i(R^i_i) = C^i_i, i = 1, 2 \quad and \quad J^1_2(R^1_2) = C^1_2$$

and

$$E^1_2(R^1_2) = E^2_1(R^2_1)$$

are likewise linear in the R^i_j. An equilibrium corresponds to the solution of these 36 linear equations in 36 unknowns. Although analytically cumbersome, it is straightforward to numerically compute the solution to the system for given parameter values.

Table 3: Parameters for Heterogeneous Firms

Parameter	x_2	x_1	γ_1	z	r	δ	k	σ	A
Value	10.5	9.5	0.5	5	0.01	0.1	3	2	0.5

To ease comparability with the homogeneous firm case, we present the results for the case with free-entry, suitably adapting (7) and (9) to endogenise λ and f.[16] Using the parameters in Table 3, the model is simulated for varying on-the-job search activity, captured by $0 \leq s \leq 1$, and for varying ability to recall, $\phi = [0, 0.25\delta, \delta, 4\delta]$. Again, given the degree of abstraction, these simulations should not be considered a precise quantitative exercise but viewed as illustrative examples.

This extension has significant implications for output. As shown in the right panel of Figure 1, with firm heterogeneity and free-entry total output is maximised at positive values for s and for ϕ. For the range of parameter values considered, output is maximised around $\phi = 0.25$ and $s = 0.2$. As mentioned above, this result contrasts with the homogeneous firms case in which $\phi = s = 0$ maximise output for similar parameter values. When ϕ is sufficiently small, total output exhibits a non-monotonic relationship with s and is maximised when s is around 0.2. A similar result is obtained when $\phi = 0$, although in this case output is maximised when s is around 0.05. Note as well that except for the $\phi = 0$ case, the ordering of output in this figure with respect to recall does not change with s. For $\phi > 0$ in the figure, the lines do not intersect. For the positive values of ϕ, output is higher with better recall, the same relationship for homogeneous firms without free entry but unlike the homogeneous free entry case in the left panel. In addition, note that output is neither maximized or minimized when $\phi = 0$ at any s.

Heterogeneity also reveals a rich picture of wage and job dynamics, more so than in the homogeneous case. Table 4 shows statistics related to job mobility and wage dispersion, where "Prop. vol. J2J wr", "Prop. vol. J2J wc" and "Prop. inv. J2J wc" refer to the proportions

[16] To solve for the free-entry equilibrium we follow an iterative procedure to find equilibrium outcomes. For any (s, ϕ) pair, solve the 36 linear equations for an arbitrary matching rate λ. The corresponding steady state employment levels imply a new arrival rate. Iterating until a fixed point emerges finds the equilibrium.

of voluntary or involuntary job-to-job transitions with wage cuts or wage rises. Figure 2 shows the resulting equilibrium wages for positive values of ϕ. With two productivity types, all four potential job-to-job transitions ($1 \to 1, 1 \to 2, 2 \to 2, 2 \to 1$) occur in equilibrium. Both the $1 \to 1$ and the $2 \to 2$ transitions involve wage cuts (following displacement shocks) or wage hikes (following the arrival of a new bidder who wins the bidding contest) as occurred with homogeneous firms. Transitions from high to low productivity firms, the $2 \to 1$ job movements, generate wage cuts.[17] These transitions follow exclusively from a displacement shock at a relatively high wage. Workers who move from $1 \to 2$ may leave behind a low (monopolistic) or high (competitive) wage at the low productivity firm. Note, however, that the wage cuts from a $1 \to 2$ transition have a steeper expected wage profile than the expected wage growth following a wage cut from $2 \to 1$ transitions.[18]

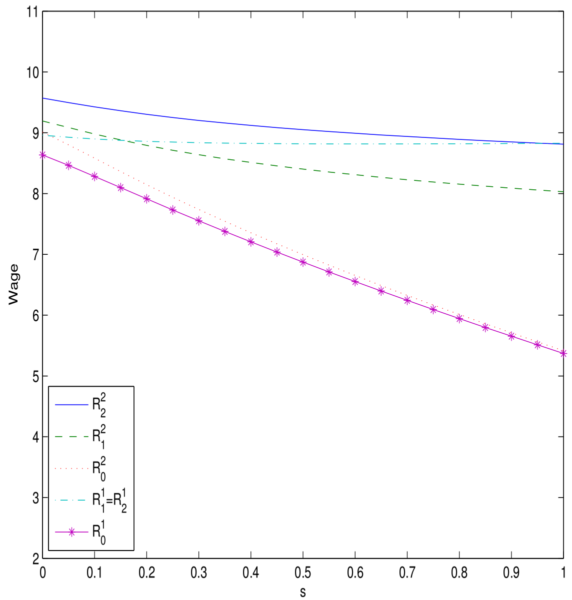
Table 4 shows that as s increases, the proportion of job-to-job transitions that involve a wage cut increases whereas the proportion of job-to-job transitions that involve a wage rise decreases. To understand these results, first note that since the number of employed workers holding a contact increases with s, a higher proportion of those workers who get hit by a job destruction shock will call upon their employment contact to avoid unemployment and earn R_0^1 or R_0^2. A positive relationship arises between s and the number of involuntary job-to-job transitions with wage cuts. Furthermore, as employed workers meet contacts more often there will be more workers earning R_1^1 meeting type 2 firms. A higher s implies these workers can engage the newly met type 2 employer with another type 2 firm in an auction at a faster rate. The difference between R_1^2 and R_1^1 decreases with s (see Figure 2), leading to more voluntary job-to-job transitions with wage cuts. At the same time, however, s decreases the number of workers employed in a type 1 or type 2 firm with no contact, thus slowing down the increase in the number of job-to-job transitions with wage rises relative to the increase in the number of job-to-job transitions with wage cuts. These differentials lead to a faster increase in the number of job-to-job transitions with wage cuts than with wage rises.

In contrast, as ϕ increases, the proportion of involuntary job-to-job transitions with wage cuts decreases, whereas the proportion of voluntary job-to-job transitions with wage cuts or wage rises increases. A higher rate of search capital depreciation implies that those workers hit by the job destruction shock are now more likely to become unemployed as opposed to experiencing involuntary job-to-job transitions with wage cuts. Although a higher ϕ also leads to a lower number of employed workers experiencing voluntary job-to-job transitions (for a given s) through its effects on λ, the decrease in the number of involuntary job-to-job transitions is much larger.

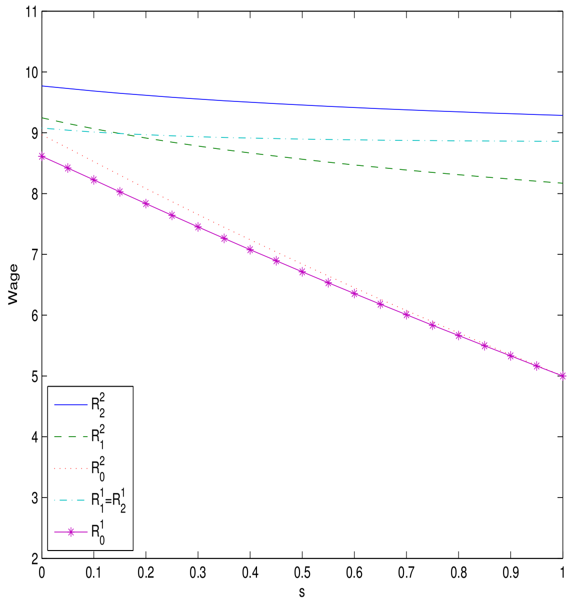
A decrease in wage compression and an increase in wage inequality accompany these employment dynamics. Figure 2 shows that as s increases, both R_0^1 and R_0^2 decrease sharply relative to R_2^2 leading to an increase in the range of wages paid. The decrease in R_0^1 and R_0^2 implies

[17]Under some parameterisations, involuntary transitions from high to low productivity firms could involve a wage hike. Consider a worker who experiences wages $w_0^1 \to w_2^1 \to w_1^0$ where $w_0^1 > w_2^1$. This event does not occur in the specifications examined here.

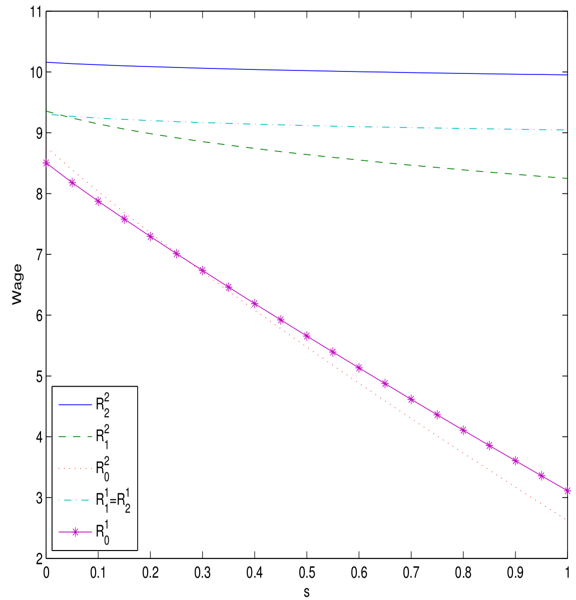
[18]The $1 \to 2$ job-to-job movements with wage jumps accompany a jump in productivity so that wage gains do not necessarily depend on transitions between equally productive firms as above.



(c) $\phi = 0.25\delta$



(d) $\phi = \delta$



(e) $\phi = 4\delta$

Figure 2: Wages earned in equilibrium

Table 4: Numerical Simulations by Search Intensity and Recall: Heterogenous Firms

s	**0.00**	**0.05**	**0.15**	**0.25**	**0.50**	**0.75**	**0.85**	**1.00**
				$\phi = 0$				
λ	0.476	0.657	0.671	0.603	0.487	0.421	0.401	0.377
f	0.157	0.255	0.376	0.431	0.501	0.540	0.552	0.567
u	0.174	0.103	0.069	0.062	0.056	0.054	0.054	0.053
Prop. vol. J2J wr	0.000	0.351	0.361	0.361	0.358	0.356	0.355	0.354
Prop. vol. J2J wc	0.000	0.024	0.053	0.061	0.064	0.063	0.062	0.060
Prop. inv. J2J wc	0.000	0.624	0.586	0.578	0.577	0.582	0.584	0.586
Mean wage	9.909	8.896	8.093	7.791	7.428	7.223	7.160	7.078
Gini	0.011	0.006	0.020	0.028	0.043	0.053	0.056	0.060
				$\phi = 0.25\delta$				
λ	0.483	0.509	0.500	0.469	0.403	0.358	0.344	0.326
f	0.160	0.190	0.244	0.286	0.354	0.396	0.409	0.426
u	0.171	0.140	0.111	0.099	0.087	0.081	0.080	0.078
Prop. vol. J2J wr	0.000	0.409	0.386	0.384	0.378	0.373	0.372	0.370
Prop. vol. J2J wc	0.000	0.000	0.048	0.061	0.069	0.069	0.068	0.067
Prop. inv. J2J wc	0.000	0.591	0.565	0.556	0.553	0.558	0.560	0.563
Mean wage	9.874	9.515	9.082	8.819	8.428	8.189	8.114	8.016
Gini	0.011	0.014	0.023	0.032	0.050	0.063	0.068	0.073
				$\phi = \delta$				
λ	0.504	0.484	0.445	0.413	0.357	0.321	0.310	0.295
f	0.168	0.185	0.216	0.241	0.288	0.322	0.333	0.348
u	0.165	0.156	0.144	0.138	0.129	0.124	0.122	0.121
Prop. vol. J2J wr	0.000	0.504	0.455	0.442	0.425	0.416	0.413	0.410
Prop. vol. J2J wc	0.000	0.000	0.054	0.068	0.080	0.083	0.082	0.082
Prop. inv. J2J wc	0.000	0.496	0.491	0.490	0.494	0.501	0.504	0.508
Mean wage	9.780	9.638	9.432	9.279	9.004	8.806	8.740	8.651
Gini	0.010	0.020	0.036	0.050	0.076	0.095	0.101	0.110
				$\phi = 4\delta$				
λ	0.569	0.520	0.459	0.419	0.359	0.322	0.311	0.296
f	0.194	0.213	0.241	0.263	0.301	0.329	0.338	0.351
u	0.149	0.155	0.161	0.165	0.170	0.173	0.174	0.175
Prob. vol. J2J wr	0.000	0.661	0.605	0.575	0.537	0.517	0.512	0.505
Prob. vol. J2J wc	0.000	0.038	0.075	0.093	0.112	0.117	0.118	0.118
Prob. inv. J2J wc	0.000	0.301	0.320	0.332	0.351	0.366	0.371	0.377
Mean wage	9.513	9.451	9.349	9.263	9.089	8.949	8.900	8.831
Gini	0.008	0.029	0.060	0.083	0.128	0.161	0.172	0.187

the foot-in-the-door effect becomes stronger as workers are able to meet firms and call auctions more frequently. Note, however, that at low values of ϕ or s, these two wages are still above z, which implies that unemployed workers' willingness to accept a low wage to start employment is still relatively weak compared to the compensation firms have to pay these workers for the value of continued unemployed search.

In Postel-Vinay and Robin (2002), the foot-in-the-door effect creates a negative relationship between entry wages and the productivity of the firm. Here, Figure 2 shows that search capital can generate situations in which there is a *positive* relationship between entry wages and the productivity of the firm; i.e. $R_0^2 > R_0^1$. In this situation, type 2 firms must now compensate workers more than type 1 firms for the value to continued unemployed search. As in the case of homogeneous firms, Figure 2 also shows that an increase in ϕ decreases wage compression for given values of s. In this case, a higher ϕ implies that the reduction in R_0^1 and R_0^2 is larger than the reduction in R_2^2.

The extent of wage inequality in our model follows from job-to-job transitions as well as from hiring workers out of unemployment. Low productivity firms pay lower average wages but the range of wages at each type overlap. High wages in low productivity firms are above the low wage in high productivity firms. Depending on parameters, low wage workers in low productivity firms earn less than low wage workers in high productivity firms. To measure this wage inequality, we compute the Gini coefficient implied by our comparative statics. Table 4 shows that this measure of wage inequality increases with s and ϕ, reflecting the reduction in wage compression.

7 Discussion

A prominent feature of labor markets in many OECD economies is that wage cuts accompany a large proportion of job-to-job transitions. See Jolivet, Postel-Vinay and Robin (2006). For example, between 1993-2013, 35 percent of all job-to-job transitions observed the UK Labour Force Survey (LFS) came with a wage cut. Models like Postel-Vinay and Robin (2002) and Burdett and Coles (2010) rationalise these wage cuts as the outcome of optimal decisions. In these models, workers who move voluntarily to a new job accept a lower starting wage as an investment for a higher future wage growth within the firm. Although appealing, Postel-Vinay and Robin (2002) and more recently Bagger, Fontaine, Postel-Vinay and Robin (2014), show that this mechanism by itself falls short in explaining the extent of job-to-job transitions with wage cuts. Connolly and Gottshalk (2008) present further evidence showing that an important proportion of job transitions that involve a wage cut do not lead to faster wage growth in subsequent employment.

This evidence suggests that a substantial proportion of the wage cuts observed in the data are associated with involuntary job-to-job transitions. Indeed, again using the UK LFS, we find that among all job-to-job transitions with wage cuts, 25 percent are involuntary transitions, 45 percent are voluntary transitions and the remainder are transitions due to other reasons. To reconcile the extent of involuntary mobility, several studies introduce an exogenous and time-invariant reallocation shock (Jolivet, Postel-Vinay and Robin, 2006, among others). After a shock, the worker has no alternative but to move to a randomly drawn job and face a potential wage cut.[19]

The framework developed in this paper puts forward the concept of search capital which

[19] All the statistics reported using the UK LFS are based on a sample of male workers between 16 and 65 years old and female workers between 16 and 60 years old. To compute the probabilities of wage cuts and rises with job-to-job transitions we pooled the 5-quarter LFS for the 1993-2013 period. We use the term job-to-job transitions to refer to employer-to-employer transitions, as in the data we consider the latter. Carrillo-Tudela, Hobijn, She and Visschers (2014) provide further details of the LFS and definitions of mobility due to voluntary, involuntary and other reasons.

provides an endogenous explanation of involuntary job mobility with wage cuts. Workers who experience a job destruction shock can call upon their existing employment contacts and avoid unemployment. Since the new auction has a lower number of bidders than the auction that gave these workers their last wage, these workers start their new job at a lower wage. Hence, involuntary job-to-job transitions with wage cuts are a natural outcome of search capital. Since voluntary job mobility also occurs, job-to-job transitions with wage rises co-exist with voluntary and involuntary transitions with wage cuts. Further, given that the amount of search capital is correlated with time spent in employment, a more general version of our model allowing $n > 1$ also implies that young workers would have (i) a higher probability of experiencing unemployment and (ii) a lower probability of experiencing an involuntary employer to employer transition with a wage cut than more senior workers.

These two features are consistent with empirical evidence. It is well established that young employed workers face higher unemployment risk than more senior workers. For example, the quarterly employment to unemployment transition rate of young workers in the UK LFS is more than twice as high as the transitions rates of prime-age and old workers. A second feature found in the LFS data is less well known. Conditional on experiencing an employer separation that leads to either another job or to unemployment, the probability of an involuntary employer to employer transition with a wage cut is 3 percent for young workers, while this probability is 6.2 percent and 8.2 percent for prime-age and old workers.[20]

The framework also has implications for the distribution of wages. In the homogeneous firm case, two mass points describe the wage distribution. Lemmas 1 and 2 imply that, relative to Postel-Vinay and Robin (2002), wages become more compressed with search capital. In particular, Lemma 1 shows that the wage offered in an auction with two bidders is strictly below a worker's marginal productivity as the firms' outside options when engaging in Bertrand competition are no longer zero. With search capital, a firm that was unable to poach the worker in the Bertrand competition has the option of hiring this worker in the future following a job destruction shock and paying the monopsony wage. This prospect reduces the highest wage paid by firms. Lemma 2 further shows that firms must compensate unemployed workers for forgoing the option of continued search with a contact rather than forgoing search without a contact, which implies that the lowest wage paid by firms increases. The extent of wage compression, however, depends on the parameters of the model, particularly on s and ϕ. For example, Table 1 shows that for the homogeneous firm case with free entry an increase in s or ϕ decreases w_1 and w_0, but increases the difference between w_1 and w_0. For a given (finite) ϕ, an increase in the rate at which employed workers accumulate contacts yields a very strong foot-in-the-door effect for unemployed workers, one that outweighs the increase in the firm's value of holding a contact and therefore generates less compressed wages. However, as discussed above, slower search capital depreciation (lower ϕ) leads to more compressed wages. In the heterogeneous firm case, these considerations manifest themselves in richer wage dispersion and wage dynamics.

The impact of search capital on output and welfare depends on the the extent of on-the-job

[20] Young workers are those between 16-30 years old, prime-age workers are those between 31-50 years and old workers are those between 51-65 years for men and between 51-60 years for women. For the employment to unemployment transition rates we use the 2 -quarter LFS.

search, the ability of workers to retain contacts over time, and the firm's ability to enter and exit the market. It also depends on firm heterogeneity. Although capital generated through on-the-job search insures the worker against productivity shocks, the counterproductive consequences of this rent seeking of this search yields ambiguous effects under free entry of equally productive firms. Increasing the levels of on-the-job search tends to lower output whereas the impact of reducing the depreciation rate of search capital yields not clear pattern. With two types of firms and free entry, a more discernable pattern emerges. For low and moderate depreciation rates, total output in this case is maximised at positive and substantial values of s. Total output for the most part also increases as depreciation rates of search captial fall.

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