

PERUVIAN ECONOMIC ASSOCIATION

A Simulation on the Evolution of Markets: Call Market, Decentralized and Posted Offer

Jean Paul Rabanal

Olga A. Rabanal

Working Paper No. 34, April 2015

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A simulation on the evolution of markets: Call Market, Decentralized and Posted Offer

Jean Paul Rabanal^{*} Olga A. Rabanal[†]

April 3, 2015

Abstract

We apply standard evolutionary dynamics to study of stability of three competing market formats —call market (CM), posted offer (PO) and decentralized market (DM). In our framework, heterogeneous buyers and sellers seek to transact a homogeneous good, which can be done by allocating their time among three different market formats. We study the allocation of time among different formats using simulations of a large (evolutionary) dynamic system. Our results show that (i) the final participation of traders in CM is much higher compared to the two other formats, (ii) the PO can coexist with CM, and (iii) DM vanishes against CM in the long run but can survive against PO, depending on the initial participation conditions.

Keywords: centralized markets, decentralized markets, decentralized bargaining, market design, market formation, evolutionary dynamics.

JEL codes: D40, C78, C73, L10

^{*}Corresponding Author. *Email:* jeanpaulrab@gmail.com. *Address:* Miller College of Business. 2000 W. University Ave. Muncie, IN 47306 *Tel:* 1-765-285-5360.

[†]Department of Economics, Ball State University.

1 Introduction

Online commerce has become an important place for sellers and buyers to rapidly exchange goods and services. Evidence tells us that nowadays most transactions occur in posted offer markets (Surowiecki, 2011 and Einav, Kuchler, Levin, and Sundaresan, 2013). Therefore, we might conclude that posted offer markets are dominant over some market formats. In this paper, we are interested in testing the evolutionary stability of the posted offer format vis-a-vis with other formats that are commonly used for transactions of goods or financial assets.¹

To do so, we apply standard evolutionary ideas to pairwise dynamics of three market formats: posted offer market (PO), decentralized market (DM) and centralized market (CM). Evolutionary principles generally explain the prevalence of certain species through natural selection or survival of the fittest (Maynard Smith and Price, 1979). In economics, “fitness” can be measured as profit and the dynamic process of selecting the best alternative is driven by imitation and/or learning (Friedman, 1991). Therefore, we can assume the evolution of market formats to be driven by profit.

In our framework, heterogeneous buyers and sellers seek to transact a homogeneous good, which can be done by allocating their time among three different market formats. We study the allocation of time among different formats using simulations of a large (evolutionary) dynamic system. We adopt the standard replicator dynamic in continuous time (Taylor and Jonker, 1978; Hofbauer and Sigmund, 1988) to explain trader format participation.

Replicator dynamics are a natural way to model the evolution of format participation. It postulates that format share grows proportionally to its payoff relative to the average payoff. Similar evolutionary principles have been applied to explain the prevalence of some market formats. For example, Lu and McAfee (1996) show that auctions dominate bargaining when the environment is composed of homogeneous buyers and sellers.²

Our simulations allow for strategic behavior of traders by incorporating mark-up rules that govern the formation of bids and asks. The strategy evolves according to simple learning rules. We assume that traders are boundedly rational and that they adjust their mark-up, upwards or downwards, based on the recent trading history. Our mark-up rule is motivated by the early work of Cason and Friedman (1997) and Friedman and Zhan (2007).

We simulate trader behavior over a number of periods and find that (i) trader partici-

¹Gardabe (2012), for example, elaborates on the evolution of posted offer in the primary market of Treasury securities. Notice that there is only one seller (the Treasury) in the market. Our approach assumes a large number of sellers and buyers participating in different market formats.

²Kulti (1997), extending the previous work of Lu and McAfee, compares auction to a directed search PO format and concludes that the two are equivalent.

pation is much higher in the CM compared to the other two formats, (ii) the PO survives, implying that the CM and PO coexist, (iii) the DM disappears or fully unravels against the CM, and lastly (iv) the DM and the PO can coexist when majority of traders initially allocate most of their time to PO.

For the purpose of our analysis, we represent the CM with a call market. In this format, all traders submit sealed bids and asks to the auctioneer, who then unseals the bids (asks) and sorts them from the highest (lowest) to the lowest (highest). The intersection (if there is one) determines the market clearing price. In the case of an interval, we designate the midpoint as the single clearing price. Call market is also defined as a discrete time double auction and is surprisingly efficient at reaching competitive equilibrium (Friedman, 1993), though less so than a continuous double auction (Smith, 1982) according to experimental evidence.

In the PO format, sellers post prices and buyers are randomly allocated a turn to transact in a queue, with each buyer choosing to trade with the seller posting the lowest ask in the market. Experimental literature suggests mixed results for the PO format. According to Ketchman, Smith and Williams (1984), PO converges to the competitive equilibrium prediction, albeit more slowly, and according to Plott (1986) the convergence is less complete.³

In the DM, buyers and sellers are matched using a random protocol and the surplus (if any) is split evenly. Kugler, Neeman and Vulkan (2006) conduct an experiment that put a DM vis-a-vis with a CM. Their findings indicate that the DM unravels as high surplus traders, defined as buyers (sellers) with high (low) values (costs), migrate to the CM.⁴ This is intuitive, as high surplus traders are most vulnerable in the decentralized format, where the possibility of a match with a low surplus trader greatly reduces profitability. In the CM, on the other hand, high surplus traders have a higher expected profit and therefore prefer to trade in the CM format. The role of high surplus traders and the unraveling of DM have also been highlighted by the static theoretical work of Gehrig (1993), Rust and Hall (2003) and Neeman and Vulkan (2010).

Our results, which suggest an eventual absence of trade in the DM (when the competing format is CM), support the work of Neeman and Vulkan (2010) which uses game theoretical framework that relies on assumptions of price distributions. Remarkably, by assuming simple trading rules, we are able to obtain the same results.

The evolution of market formats can also be studied using coordination. Alós-Ferrer,

³Davis and Korenok (2009) discovered that equilibrium predictions can emerge more quickly in a continuous framework. In order to add a continuous framework to the environment, the authors shortened the time allowed for decisions.

⁴Notice that the setup of the DM in Kugler et al. (2006) is meant to increase transaction volume as opposed to surplus, which may have put this format at a disadvantage. Early experiments by Campbell et al. (1991) show that CM and DM can coexist when there is a bid-ask spread in the market.

Kirchsteiger and Walzl (2010) look at selection of market formats as a matter of coordination. This suggests that all market formats, no matter how inefficient, must be part of Nash Equilibria. Along these lines, but incorporating learning dynamics, Alós-Ferrer and Kirchsteiger (2013) theoretically and empirically demonstrate, using laboratory data, that market formats that are non-clearing (like some PO formats) can also be stable. We use alternative dynamics and determine that the PO format can persist for a long time when competing against other formats. Our findings can be viewed as complementary to the work of Alós-Ferrer and Kirchsteiger.⁵

Furthermore, the simulations performed in this paper allow us to study the basin of attraction of each market format. We find that the CM and the PO exhibit the greatest basin of attraction. The intuition behind this result is based on the role of high surplus traders (i.e. sellers with low cost and buyers with high values) and their profitability in these respective formats. DM unravels for every initial condition of participation when the competing format is CM. However, the basin of attraction for DM becomes large when the participation rate in PO is at least 50 percent initially, and the CM format is absent. When analyzing the complete model, where all three formats are present and where traders initially favor the PO, we find that the DM unravels.

The rest of the paper is organized as follows: section 2 describes the general environment of each competing format, section 3 presents the numerical solution and lastly, section 4 follows with a discussion of the results and concludes with a brief summary. The Appendix includes a sketch of the code used to perform our simulations.

2 Environment

The goal of our paper is to evaluate which market format will prevail in the long-run using numerical simulations of an evolutionary model. We build an environment that consists of n buyers, each with some heterogeneous valuation v_i for the homogenous good in the market and n sellers, each with some heterogeneous unit cost c_j of producing this homogenous good. Buyer i 's payoff is $v_i - p$ when he purchases a single indivisible unit of the good at price p and zero otherwise. Seller j 's payoff is $p - c_j$ if she sells a unit at price p and zero otherwise. The values $v_i \in [0, 1]$ and costs $c_j \in [0, 1]$ are privately drawn from a known distribution. In our simulations, we assume a uniform distribution.

The two main endogenous variables in our model are (i) trader bids and asks, and (ii) trader time allocation in each market format. The evolution of these two variables is driven

⁵PO was not observed as an equilibrium market format in Kirchsteiger, Niederle and Potters (2005). They studied the formation of market formats in the laboratory. In their environment, the traders have the option to reveal their information to some traders (from either side of the market). They found that offers are typically directed to all traders of the other side of the market, but to none of the traders of the same side.

by learning and payoff advantages.

We model the trading behavior following the work of Cason and Friedman (1997). They show that in a single call market the BNE bids and asks can be represented as a linear function of the true values.⁶ We adapt the bid and ask functions such that every trader uses a constant mark-up that is related to the true value/cost. Similar mark-up functions were also used in Friedman and Zhan (2007) to compare efficiency in CM. In our environment, traders learn to set the appropriate mark-up using their trading history.

Traders have the option to allocate their time between two different formats. The two available formats in the simulations are chosen from the following possible formats: a centralized market (CM), a posted offer (PO) and a decentralized market (DM). The CM solicits a bid b_i from buyer i and an ask a_j from seller j . The demand revealed in $\{b_i\}$ and the supply revealed in $\{a_j\}$ are then cleared at a uniform equilibrium price p^* . That is, our CM is a call market. Further, with indivisible units, there is often an interval of market clearing prices. When this occurs, we designate the midpoint of the interval as the single clearing price.

The PO format requires that each seller post an ask p_j and buyers are then randomly assigned a place in a queue. When a buyer reaches his turn, he will choose the lowest ask available in the market, and will purchase the good as long as the surplus $v_i - p_j$ from trading is greater than or equal to zero. Once a seller j and a buyer i transact, both leave the market, and the next buyer in the queue takes his turn to see if the next lowest ask can result in a transaction. Our PO format design follows the usual convention. For example, in many online markets the sellers post the terms of trade, and the random buyer protocol is similar to several laboratory environments (Davis and Holt, 1993).⁷

The DM format works via random pair matching in a two-sided market. This approach is meant to give the DM the best possible start when competing against more centralized formats because it allows for the possibility of high surplus trades. Transactions occur in randomly matched pairs when the bid b_i is greater than or equal to the ask a_j . The price (p_{ij}^*) is determined following the work of Kugler et al. (2006) in which the surplus is split evenly between traders. Thus, $p_{ij}^* = (b_i + a_j)/2$.⁸

⁶Cason and Friedman (1997) conducted an experiment using a call market format, allowing players to draw random values and costs each period. Their findings suggest that prices and market efficiency observed is closer to a Bayesian Nash Equilibrium (BNE) than to a CE and that agents do not deviate too much from their actual costs and values. However, it should be noted that as number of players, $n \rightarrow \infty$, the BNE approaches CE.

⁷In the laboratory environments, researchers also analyze the PO format in which the buyers post prices. In this case, the convergence to equilibrium is from below rather than above, as is the case when sellers post prices (David and Holt, 1993).

⁸In order to ensure the best possible format of DM, we test two DM specifications (but omit for brevity) against centralized formats: (i) the random pair matching DM and (ii) the matching market DM as specified by Kugler et al. (2006), where buyers and sellers are sorted, so that the highest cost seller is matched with the highest value buyer, provided that the cost does not exceed the value. The latter approach is meant

For a more intuitive example, consider an environment with 3 buyers and 3 sellers. Suppose that buyers' willingness to pay (WTP) are 10, 8 and 2; and the sellers' cost of producing the good are 1, 2 and 6. The market clearing price in the CM⁹ is any price such that $p \in [2, 6]$. Using our midpoint rule, the actual transaction price is 4. Given this price, two units are exchanged. The buyers with WTP of 10 and 8 receive a surplus equal to 6 and 4, respectively. The sellers with costs 1 and 2 receive a surplus equal to 3 and 2, respectively.

Now assume that two of the lower cost sellers post $p = 4, 5$ in the PO and the seller with cost $c = 6$ posts a price equal to 7. Buyers are selected to queue randomly (with equal probability of 1/3 of being the first in a queue). Suppose that they are ordered as 2, 8 and 10. The first buyer in the queue does not transact since his value is lower than any of the prices posted. The next buyer then transacts at $p = 4$ and the last at $p = 5$. The total profit in PO is identical to the one in CM (and equal to 15), though the division of surplus is not. In CM, buyer surplus is 10 and seller surplus is 5 while in PO buyer surplus is 9 and seller surplus is 6.

The DM is implemented via random matching between buyers and sellers. First, each side of the market is randomly sorted and then matched. For instance, when the traders are truth-telling, the following pairs may be formed: $\{10, 2\}$, $\{8, 6\}$ and $\{2, 1\}$ with prices 6, 7, and 1.5, respectively. The buyer with WTP of 10 receives a surplus equal to 4, the buyer with WTP of 8 receives a surplus equal to 1 and the buyer with WTP of 2 receives a surplus equal to 0.5. The seller with cost of 2 receives a surplus equal to 4, the seller with cost of 6 receives a surplus equal to 1 and the seller with the cost of 1 receives a surplus equal to 0.5.

We allow for trader participation among market formats to evolve. We denote participation in each format as follows: the first market (either CM or PO) as x and the second market as $1 - x$ (either PO or DM). Thus, the vector of participation shares is $S = (x, 1 - x)$. We can interpret participation as the time allocated between markets formats. Alternatively, consider a trader with 100 chances to transact each period and x would then represent the number of times the trader chooses to transact in a particular format. Market participation is subject to evolutionary forces driven by payoff advantage.¹⁰ We use standard continuous time replicator dynamics.

First developed by Taylor and Jonker (1978), replicator dynamics describe biological evolution of phenotypes and were also later shown to describe certain sorts of imitation

to increase the transaction volume in the DM, which it does but at the expense of the trader surplus. Not surprisingly, our results indicate that random pair matching performs better when competing against CM.

⁹Based on previous literature, see Friedman (1993) and Rustichini et al. (1994), we expect that the clearing prices evolve to resemble those in CE, where marginal players reveal their type.

¹⁰Lu and McAfee (1996) also adopt the assumption that payoff advantage is the force that drives the preference of one format over others, when analyzing auctions over bargaining. Here, we specify that the evolutionary dynamics follow replicator dynamics, one of the most studied dynamics in the literature.

processes (Schalg, 1999, and Björnerstedt and Weibull, 1996). Under these dynamics, the growth rate of participation in each format is equal to its payoff relative to the average payoff of the market formats. The average payoff is $\bar{\pi} = x\pi_1 + (1-x)\pi_2$, where π_1 denotes trader profit in the first format and π_2 denotes trader profit in the second format.

When a trader completes a transaction, the profits are calculated using values (costs) and prices observed in the respective market, and when there is no participation or transaction, the profits are zero. We can rewrite the relative payoff for the first format as $\pi_1 - \bar{\pi} = \pi_1 - x\pi_1 - (1-x)\pi_2 = (1-x)(\pi_1 - \pi_2)$. Similar algebra applies to the competing format. Thus, omitting subscripts (i for buyers and j for sellers) our ODE system that governs the evolution of the state variable x can be summarized as

$$\dot{x} = x \cdot (1-x) \cdot (\pi_1 - \pi_2) \quad (1)$$

Notice that replicator dynamics in our environment operates at the individual level. Buyers and sellers are heterogeneous, and each of them can allocate their time to either market.¹¹ Expression (1) tells us that the share in each format changes according to the profit differential between formats. For example, assuming that the first format is CM and the competing format is PO, then the share of time allocated to CM, or x , increases when the payoff differential between the CM and the PO is positive.

Strategic behavior by traders in order to increase profits involves simple mark-up rules for the bids and asks. In general, the bids for buyer i (b_i) and asks for seller j (a_j) are expressed as

$$b_i = v_i \cdot (1 - \mu_i^M) \quad (2)$$

$$a_j = (1 - \mu_j^M) \cdot c_j + \mu_j^M \cdot c^{max} \quad (3)$$

where c^{max} is the maximum possible cost and μ^M is the mark-up used in format M by sellers and buyers. The bid function is quite standard, while the ask function is written as a convex function to eliminate buyer advantage that occurs when using the standard mark-up $a_j = c_j \cdot (1 + \mu_j^M)$.¹² Furthermore, the standard approach may not be appropriate for ask mark-up in the PO format. A low cost seller does not have an incentive to move to PO if doubling the price, or imposing a hundred percent mark-up, results in an ask that is lower than the prevailing price of the competing format. The convex mark-up allows for greater range in prices.

¹¹We can write the complete model in which three market formats are present, however, the results of the two market formats are sufficient to understand the dynamics between them. Thus, we omit the complete model for sake of brevity.

¹²According to Cervone, Galavotti, and LiCalzi (2009), Friedman and Zhan (2007) who compare efficiency in CM, use a relatively common mark-up, which contains a hidden asymmetry that favors the buyers.

The bids and asks also evolve over time. Specifically, mark-ups are adjusted each period according to the recent trading history. In each format, mark-ups adjust downwards if a trade does not occur in the previous period. When a trade takes place, an upward adjustment occurs only in the PO and the DM. Allowing for such behavior may help the trader obtain greater profit because the good can be traded at a range of prices, unlike in the CM. In the CM, however, there is no upward adjustment because it is unnecessary and could in fact be deleterious. In general, a trader is able to revise the mark-up upward or downward, as necessary, according to the following rule

$$\mu_{t+1}^M = \mu_t^M \pm \Delta_\mu \quad (4)$$

where Δ_μ is constant. We eliminate the subscript i for buyers and j for sellers to express the mark-up in terms of time t . Notice that mark-ups and their adjustment rules are allowed to vary across formats M . In the PO, only sellers post the terms of trade. Therefore, when a transaction occurs in the PO, the transaction price is equal to the ask price and the buyers are truth-telling by default.

Prior to presenting our numerical simulations, it is important to discuss the assumptions and the numerical approach used. Our assumption of bounded rationality in the bids/asks is fairly common (for example, see Gode and Sunder, 1993, for a continuous double auction format).¹³ However, there is not a definitive consensus on the best specification for trader bids/asks. For this reason, we adopt a simple mark-up rule, motivated by Cason and Friedman (1997).

We choose to use numerical simulations to help determine the stationary equilibrium of the participation rates and mark-ups rather than solving for the steady state analytically (a more common approach in literature on competing formats). This is done not only for convenience but also to observe the underlying dynamics without imposing additional constraint. Although our system summarized by equation (1) appears to be quite simple, it involves a large number of heterogeneous buyers and sellers that are boundedly rational and adjust their mark-up myopically. Furthermore, the dynamics apply to each trader, this is not a unique ODE that only govern market participation. Each trader is able to use different mark-ups in each format, and these mark-ups then evolve differently according to the market format (i.e. upward adjustment only in the PO and the DM, and not in the CM). Solving the system analytically requires assuming certain values for mark-ups, values that we would not know without the simulations. Similarly, in order to analytically show

¹³The authors show that even with zero intelligence agents who place random buy and sell offers, a continuous double auction can reach a near 100 percent allocative efficiency. Other approaches include Fano et al. (2014) who use a genetic algorithm, and Alós-Ferrer and Kirchsteiger (2013) who assume learning rules, and that in every period a randomly selected trader is allowed to revise his market choice. Recall that we model the market choice following replicator dynamics.

the basin of attraction we need to make strong assumptions about trader participation and mark-up behavior. This is cumbersome in the PO and the DM, where traders can transact at a range of prices.

In the next section, we begin by analyzing each format separately and where the mark-up is allowed to evolve over time. The first exercise serves to provide insight using simple market dynamics. As we consider more than one format later on, we also allow for the participation shares to adjust over time.

3 Simulation results

In our environment, there are n heterogeneous buyers and n heterogeneous sellers. For our simulations we set $n = 100$. Each buyer valuation v_i and seller cost c_j is independently drawn from a uniform distribution with support $[0, 1]$. Thus, the expected competitive equilibrium (CE) outcomes are $p = 0.5$ and $q = 50$. We allow for mark-up adjustment each trading period and assume that the change in mark-up ($\Delta\mu$) is 0.05. Notice that our system, as specified by equation (1), is in continuous time. We approximate the time derivative using first-order terms and the grid size $dt = 0.1$, which implies that time is (almost) continuous.¹⁴

3.1 Single market outcomes

In order to study a single market environment, we set $\dot{x} = 0$ in the ODE system described by equation (1), and allow mark-ups to adjust as specified by equation (4). We also assume that all traders apply the same initial mark-up ($\mu_0 = 0.3$). All simulation results are based on 100 buyer value (seller cost) draws, where each draw is allowed to run for T trading periods, which vary according to the environment. All tables presented in this paper display results based on the average of these 100 draws.

In the CM, the price during the first trading period, $t = 1$, is quite close to p_{CE} and approaches CE nearly instantaneously while quantity traded at $t = 1$ is about half the CE amount but reaches CE by trading period $t = 100$. These results are presented in Table 1, which summarizes prices, quantity and trader surplus for trading periods $t = 1, 100, 500$ and 1,000 after 100 independent draws of v and c . We do not present descriptive characteristics for $t > 1,000$ in Table 1 because the results from higher number of iterations are similar. Note that convergence to CE follows the theoretical predictions (Rustichini et al., 1994), which suggest that marginal traders fully reveal their type.¹⁵

¹⁴We use the term almost continuous because we discretize time in order to numerically solve the ODE. This technique is standard in mathematical packages. For a more technical discussion, see Iserles (2006).

¹⁵This result can be easily derived analytically since the marginal trader does not increase the mark-up in CM. However, this assumption does not hold in other formats. The simulations of the single market also

Table 1: Single market outcomes

Trading period	Price	Quantity	Buyer Surplus	Seller Surplus
<i>Call Market - CM</i>				
1	0.499	28.60	10.18	10.15
100	0.497	50.19	12.51	12.38
500	0.497	50.19	12.51	12.38
1,000	0.497	50.19	12.51	12.38
<i>Posted Offer - PO</i>				
1	0.475	49.96	13.15	11.23
100	0.655	33.85	5.82	10.71
500	0.655	33.92	5.81	10.76
1, 000	0.655	33.91	5.81	10.75
<i>Decentralized - DM</i>				
1	0.501	15.88	4.95	4.93
100	0.495	26.21	5.97	5.84
500	0.495	26.79	6.02	6.02
1,000	0.494	26.60	6.07	5.92
<i>Competitive equilibrium - CE</i>				
	0.497	50.19	12.50	12.40

Note: The results are based on the average of 100 draws.

In the PO, on the other hand, the price never fully approaches p_{CE} . This result is driven by sellers adjusting their mark-up upwards when they are successful. Higher PO price relative to p_{CE} also indicates that sellers receive more surplus than buyers. The quantity traded in the PO is lower than q_{CE} by the terminal trading period $t = 1,000$, which is expected when higher prices prevail. This result is consistent with the experimental work of Plott (1986), who demonstrates that the convergence in PO is less complete. Lastly, in the DM format, we observe that the price is close to p_{CE} , however, the number of transactions never approaches q_{CE} . Since the price is computed as the mid-point of each paired bid and ask, the surplus is similar for both sides of the market.

Next, we examine the evolution of bids and asks. Figure 1 compares the trajectory of bids and asks in CM (left panel) and DM (right panel) formats after 1,000 trading periods. The ask function can be found above the midpoint of the 45 degree line, while the bid function is located right below it. Notice that in the CM, high surplus traders¹⁶ do not continue to change their mark-up because they find success quickly. The intermediate traders move to decrease their mark-up, while the *marginal* traders are forced to reveal their type—their bids/asks appear on the 45 degree line by $t = 1,000$. The extramarginal traders also end up revealing their type because they are not able to exchange any units.

help confirm whether the dynamic system approaches the results observed in a single market, or deviates from it.

¹⁶Recall that high surplus traders are defined as high (low) value (cost) buyers (sellers).

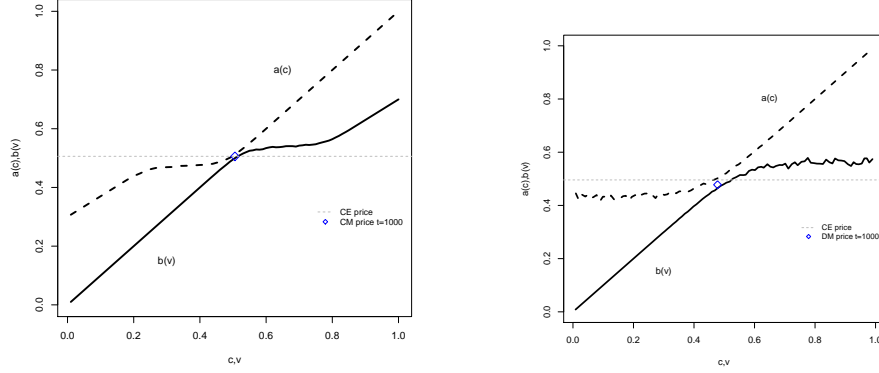


Figure 1: Average bids and asks at $t = 1,000$ in CM (left panel) and DM (right panel)

In the DM, trader behavior is similar to that of the CM. The extramarginals reveal their true value and the intramarginal traders set a price closer to p_{CE} . However, note that the low cost sellers in the DM have a higher mark-up compared to the low cost sellers in the CM. This is because in the CM, it is not profitable to increase the mark-up following a successful trade. In the DM, however, that is not the case, and therefore we observe low cost sellers with higher asks. Likewise, high value buyers decrease their bids significantly in the DM, whereas the same group of buyers in the CM follows a bid function that is closer to their true value.

In the PO, the dynamics are slightly different. The starting price is lower than p_{CE} because low cost sellers —given the initial mark-up $\mu_0 = 0.3$ — transact at lower prices than in the CM. Initial low prices encourage more trading in the PO. However, prices increase over time as mark-up adjust upward following each successful trade (as opposed to the CM, where a successful trade results in same price next period) and the prices eventually surpass p_{CE} . By the terminal period, sellers receive higher surplus relative to buyers and the number of units exchanged is lower compared to q_{CE} .

Prior to analyzing the (evolutionary) stability of competing market formats, notice that trader surplus can provide a good hint as to which market format will survive in the long-run. Trader surplus is much higher in the CM compared to the DM (see market outcomes in Table 1). Therefore, we expect that the traders will eventually migrate from DM to CM regardless of the initial condition. Also, the PO exhibits a higher surplus relative to the DM, but lower than that of the CM. Thus, the basin of attraction for the CM should be bigger compared to the PO. In the next section, we present the numerical solution for competing market formats.

3.2 Two formats

In the baseline scenario, we assume that all players set the initial mark-up equal to 0.3. The mark-up is allowed to adjust over time following equation (4). Recall that mark-up adjustment is five percentage points, $\Delta\mu = 0.05$. The number of trading periods, T , is allowed to vary between 10,000 and 100,000, depending on the format. We allow for this variations because in some cases it takes a significant number of iterations to observe complete unraveling.

We begin by analyzing the CM (first market, whose share is represented by x in equation 1) and the DM ($1 - x$) formats. The simulation results are shown in Table 2. Notice that the transaction volume in CM is quite invariant while in the DM, the transaction volume decreases to zero by $t = 5,000$. Furthermore, the basin of attraction that explains the dominance of CM over DM includes a wide range of initial conditions. We give DM the best possible chance of survival by assuming that traders allocate most of their time to DM. We observe that even with the initial allocation where $1 - x = 0.99$, traders move from DM to CM, so that at $t = 50,000$, the quantity transacted in DM is equal to zero.

Table 2: Centralized (CM) vs. Decentralized (DM)

Trading period	Price CM	Quantity CM	Price DM	Quantity DM
<i>Initial share in CM $x_0 = 0.5$</i>				
1	0.498	29.20	0.50	17.48
100	0.500	50.22	0.50	26.00
1,000	0.500	50.22	0.49	5.50
5,000	0.500	50.22	—	0.00
10,000	0.500	50.22	—	0.00
CE	0.499	50.22	0.50	50.22
<i>Initial share in CM $x_0 = 0.01$</i>				
1	0.495	28.13	0.50	16.02
100	0.495	49.52	0.50	26.37
1,000	0.496	49.52	0.50	11.76
5,000	0.496	49.52	0.51	0.47
10,000	0.496	49.52	0.20	0.05
50,000	0.496	49.52	—	0.00
CE	0.496	49.52	0.50	49.52

Note: The results are based on the average of 100 draws.

Simulations results from Table 2 confirm earlier theoretical work (Rust and Hall, 2003, and Kugler et al., 2010) and experimental evidence (Kugler et al., 2006). As the literature has emphasized (and we corroborate using a dynamic model), high surplus traders strongly prefer the CM format, which ultimately leads to the collapse of the DM. This is clearly depicted in Figure 2, which shows two random samples of trader participation in CM at

$t = 100$ and $5,000$ for sellers (top panel) and buyers (bottom panel). The x-axis presents each trader, sorted according to surplus. High surplus traders (highest value/lowest cost) are on left while the low surplus traders (lowest value/highest cost) are on the right. The y-axis shows the participation rate in the CM, with the initial participation rate of one percent in the CM (or 99 percent in the DM).

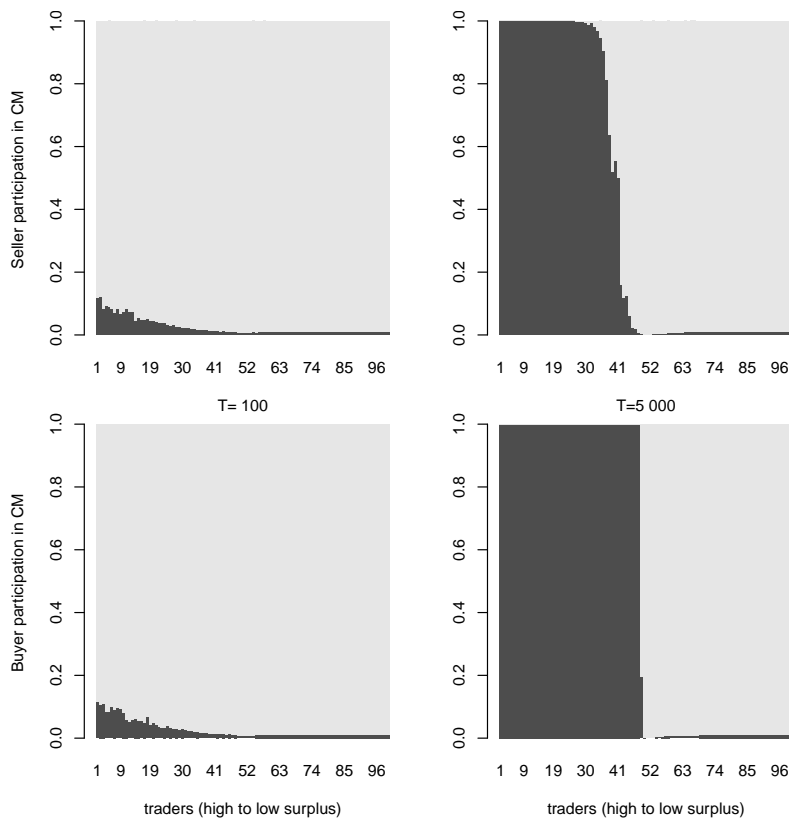


Figure 2: Evolution of seller (top) and buyer (bottom) participation rates at trading periods $t = 100$ and $5,000$. Traders are sorted from high surplus (high value/low cost) to low surplus (low value/high cost)

At $t = 100$, high surplus traders are more likely to move to the CM compared to other traders. The marginal traders (localized around the trader numbered 50) move progressively to the CM. This is illustrated with the dark grey region declining in area as we move toward low surplus traders. When $t = 5,000$ most high surplus traders move to allocate all or nearly all their time to CM.

Next, we compare PO (first market, whose share is represented as x) and DM formats (see Table 3) using a range of initial conditions. We find that PO dominates DM for nearly all initial conditions. However, note that when traders initially favor PO, DM is able to survive in the long run (i.e. when the initial share in PO is 50 percent or higher). Table 3 shows results for $t = 30,000$ and $x_0 = \{0.01, 0.10, 0.40, 0.50, 0.99\}$. We present the results in terms of CE. For example, $q_{CE} = 50$ and therefore 66 percent when $x_0 = 0.50$

(see column 3) indicates that the traded quantity in PO is approximately $0.66 \times 50 = 33$ units. In particular, we find that when $x_0 \geq 0.50$, more transactions occur in the DM in the long run. The prices in PO (about $1.29 \times 50 \approx 64$) are higher compared to p_{CE} (approx. 50). Recall that in PO sellers post prices and that mark-up adjust upward following each successful trade.

Table 3: Posted Offer (PO) vs. Decentralized (DM)

Initial share in PO (%)	Price PO (% CE)	Quantity PO (% CE)	Price DM (% CE)	Quantity DM (% CE)
1	129.6	66.1	—	0
10	128.6	65.9	—	0
40	128.5	66.2	—	0
50	128.8	66.7	128.8	0.2
99	129.5	58.7	127.2	2.7

Note: The results are based on the average of 100 draws, at $t = 30,000$. CE =competitive equilibrium.

Our results are related to the work of Alós-Ferrer and Kirchsteiger (2013) who find that some non-clearing markets can disappear against other non-clearing markets. Similarly, some of these non-clearing markets can also coexist. We complement their results by showing that the basin of attraction for PO is quite large and that when traders favor PO too much, DM can survive in the long run.

One might wonder what happens when the CM format is also available to traders. Does the DM survive when all three formats interact? We can answer this question by referring to the analysis of the CM and DM environment. Recall that CM dominates DM, so traders migrate quite fast to CM (in particular the high surplus traders), negatively affecting the chances of successful transactions in the DM. We confirm this result by analyzing a system that incorporates an additional market format. In this model, the traders can decide how much time to allocate to CM, DM or PO.¹⁷ We assume that traders initially allocate most of their time to PO (95 percent), followed by DM (four percent) and CM (one percent). Our results indicate that DM unravels and that the evolution of CM and PO formats follows qualitatively the results of the pairwise comparison between CM and PO formats, which we analyze in greater below.

For our last analysis, we compare the CM (whose participation rate is x) and the PO formats. The results are summarized in Table 4. We find that both formats survive in

¹⁷We can write the ODE system for three competing markets that governs the evolution of the state variables x (participation at CM) and y (participation at PO) as

$$\begin{aligned}\dot{x} &= x \cdot y \cdot (\pi_{CM} - \pi_{PO}) + x \cdot (1 - x - y) \cdot (\pi_{CM} - \pi_{DM}) \\ \dot{y} &= y \cdot x \cdot (\pi_{PO} - \pi_{CM}) + y \cdot (1 - x - y) \cdot (\pi_{PO} - \pi_{DM})\end{aligned}$$

where $1 - x - y$ is the participation in DM.

the long run. The marginal traders participate in both formats, while high surplus traders almost immediately move to the CM. Thus, the quantity exchanged in CM is significantly higher than in PO.

Table 4 shows results for two initial conditions, when traders favor CM ($x_0 = 0.99$) and when they favor PO ($x_0 = 0.01$). To establish convergence, we significantly increase the number of trading periods. The results in the long run ($t = 100,000$) show that prices and quantities converge to the CE quite fast in the CM, and that few marginal traders remain in PO. We present the results for two extreme initial participation allocations because we are interested in analyzing the basin of attraction of each format. The simulations indicate that the basin of attraction is quite large in each format. When traders favor a particular format initially, the less favorable format is still able to survive in the long run.

Table 4: Centralized (CM) vs. Posted Offer (PO)

Trading period	Price CM	Quantity CM	Price PO	Quantity PO
<i>Initial share in CM $x_0 = 0.99$</i>				
1	0.50	28.70	0.44	41.49
1,000	0.50	50.58	0.52	8.95
10,000	0.50	50.58	0.50	1.07
40,000	0.50	50.58	0.51	0.24
80,000	0.50	50.58	0.52	0.16
90,000	0.50	50.58	0.52	0.13
100,000	0.50	50.58	0.52	0.13
CE	0.50	50.58	0.50	50.58
<i>Initial share in CM $x_0 = 0.01$</i>				
1	0.50	28.34	0.45	41.56
1,000	0.54	46.20	0.60	25.56
10,000	0.55	45.95	0.54	4.29
40,000	0.52	45.92	0.52	3.36
80,000	0.52	45.92	0.51	3.07
90,000	0.51	45.92	0.51	3.03
100,000	0.51	45.92	0.51	3.02
CE	0.50	50.01	0.50	50.01

Note: The results are based on the average of 100 draws.

Our results of the pairwise comparison between formats can be summarized as follows.

Result 1. *The decentralized format fully unravels when the competing format is a call market.*

Result 2. *The decentralized format can coexist with a posted offer format when the initial participation in the posted offer is at least 50 percent.*

Result 3. *The two more centralized formats of the three, a call market and posted offer, coexist.*

Result 4. *The basin of attraction of a call market and posted offer is large.*

4 Conclusion

In this paper, we study the evolutionary dynamics of three different market formats, allowing for a range of initial conditions. Given our results, we can conclude that: (i) the centralized market (CM) is clearly preferred to the two other market formats, (ii) the posted offer (PO) never fully unravels, thus suggesting that the CM and the PO can coexist as part of a stationary equilibrium, (iii) the decentralized market (DM) completely unravels against the CM and (iv) the DM can survive when competing against the PO, depending on the initial conditions.

We show how market formats evolve toward more centralized formats and provide further support for the theoretical work of Rust and Hall (2003) and Neeman and Vulkan (2010), and the experimental evidence of Kugler et al. (2006), who find that traders (especially high surplus) prefer centralized formats to decentralized formats. Remarkably, we are able to obtain results consistent with the previous literature using simple learning rules for trader bid and ask strategies.

Our model also includes a posted offer market, which theoretically is deemed similar to other centralized formats, though, empirically (especially according to laboratory evidence) shows slower convergence rate and sometimes less efficient outcomes relative to other centralized formats such as continuous double auction or call markets. We study the basin of attraction of each format and find that the CM has a wide basin of attraction. This result relies on the high surplus obtained by some traders in CM. Interestingly, when comparing the PO and CM formats, we find that the basin of attraction for PO is quite large for marginal traders. Also, when we analyze the PO and DM formats, we find that when traders initially allocate at least 50 percent of their time to PO, DM survives in the long-run. The intuition of this result relies on idea that non-clearing markets are easier to invade. In this case, the alternative format can provide more favorable terms of trade.

Although our results show strong support for a particular format (CM), we believe that we can improve our conclusion in a number of ways. First, the unit good in our model is homogeneous. Therefore, it would be interesting to see if these findings hold in an environment with heterogeneous goods. In particular, when applying our model to financial markets, it is important to distinguish the type of service or good exchanged. Perhaps, this is one of the main reasons why over-the-counter (OTC) markets, considered to be decentralized in nature, are preferred for some financial transactions. Second, an experiment would be an ideal way to test the direct interaction of CM, PO and DM formats. An interaction amongst all three could surely bring fresh insight into the dynamics of

trading formats.

Appendix

The simulation of the two-market format is performed in R. The code is available upon request. Here, we include a description of the algorithm.

- From a uniform distribution $[0,1]$, we draw values for buyers and costs for sellers.
- The bid/ask functions depend on the values/costs and the mark-up.
- In the CM, there is a unique price that clears the market. If there is an interval of prices, then we select the mid-point.
- We compute the profits for each buyer/seller in the CM.
- In the PO, we compute the ask function for sellers since they are posting prices. The buyers transact based on their true values.
- We then order the sellers and randomly select a buyer to transact with the first seller (the lowest cost). You can consider this random selection of buyers as a queue.
- If the buyer's value is greater than cost, then a trade occurs. The process is then repeated for the second seller, and the next randomly selected buyer.
- If a trade does not occur, then a new buyer is randomly selected from the pool for a chance to transact with the first seller. This continues until a trade occurs, or there are no buyers left in the queue.
- We compute the profits for each buyer/seller in the PO.
- In the DM, we use the bids/ask functions and then randomly pair buyers and sellers.
- The transaction price in each pair is the mid-point between the bid and ask, provided that the difference is positive. If the ask exceeds the bid, then trade does not occur.
- We then compute the profits for each buyer/seller in the DM.
- In the next round, we adjust the mark-ups as follows: $\mu_{t+1} = \mu_t \pm \Delta\mu$.
 - In the CM, a successful trade implies no change in mark-up the following period while an absence of trade indicates a decrease (–) in mark-up the following period for sellers, and an increase (+) for buyers.
 - In the PO, only sellers adjust mark-ups. However, following a successful trade a seller will increase (+) his mark-up, and decrease (–) if unsuccessful.

- In the DM, behavior is similar to that in the PO, except that the buyers also adjust mark-up. Thus a successful trade will force a seller to increase (+) his mark-up and a buyer to decrease (–) his bid. The opposite is true for an unsuccessful trade.
- We adjust the market participation as indicated by equation (1).
- Traders have a chance to transact only when participation shares $(x, 1 - x)$ are greater than zero. Otherwise, they leave each applicable market until the next round.
- We move to the next round until the end of period is achieved.

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