

# PERUVIAN ECONOMIC ASSOCIATION

## Financial Risk Capacity

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# FINANCIAL RISK CAPACITY

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Financial crises seem particularly severe and lengthy when banks fail to recapitalize after large losses. I explain this failure and the consequent depth of financial crises through a model in which banks provide intermediation in markets with informational asymmetries. Large equity losses reduce a bank's capacity to bear further losses. Losing this capacity leads to reductions in intermediation and exacerbates adverse selection. Adverse selection, in turn, lowers profits from intermediation, which explains the failure to attract equity injections or retain earnings quickly. Financial crises are infrequent events characterized by low economic growth that is overcome only as banks slowly recover by retaining earnings. I explore several policy interventions.

KEYWORDS: Financial Crisis, Adverse Selection, Capacity Constraints.

## 1. INTRODUCTION

Financial crises often originate from episodes of extreme bank losses and later develop into prolonged economic declines. Their depth and duration are known to be particularly severe (see [Cerra and Saxena, 2008](#); [Reinhart and Rogoff, 2009](#)). The observation that we can trace financial crises to episodes of bank equity losses suggests that these crises could be mitigated if equity was to be quickly reallocated back to the financial industry following the losses. It is not coincidental that the slow recovery of bank equity has been a major concern for policy makers, academics, and practitioners alike.<sup>1</sup> In fact, during his only television interview at the time, the Chairman of the Federal Reserve, Ben Bernanke, was asked when the crisis would be over, to which he answered, “When banks start raising capital on their own.”<sup>2</sup> Why is it then that the financial sector takes so long to rebuild its equity, thus prolonging financial crises?

This paper offers a new theory, one that speaks to the severity and persistence of financial crises. Under this theory, financial crises occur even though there are no physical changes in the economy and bankers have no impediments to recapitalizing their banks. Instead, crises stem from the in-

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<sup>1</sup>The slow recovery of intermediary capital is the subject of Darrell Duffie's 2010 Presidential Address to the American Finance Association (see [Duffie, 2010](#)) and a centerpiece of the classic treatise, [Bagehot \(1873\)](#).

<sup>2</sup>Quote from “The Chairman,” *60 Minutes*, CBS News, March 15, 2009.

teraction of two mechanisms. One mechanism produces a decline in profitability once banks suffer losses. With the other, bankers fail to coordinate equity injections once profitability is low. Their interaction creates a financial system that struggles to retain profits and to attract fresh equity during crises. This limits the amount of intermediation and hurts growth.

The mechanism that causes a decline in profitability during crises is inherent to financial intermediation. One natural role for banks is to deal with asymmetric information between borrowers and lenders: banks diversify transaction risks caused by asymmetric information.<sup>3</sup> Although financial institutions use their scale to pool assets and diversify those risks, they are not immune to large losses. On the contrary, to perform their job, banks must have the capacity to tolerate financial losses, i.e., the *financial risk capacity*. Under this mechanism, when banks face unexpected equity losses, they lose this capacity and must scale down their operations. In this lies the root of the problem; when banks curtail their activities, borrowers exploit their information advantage, causing intermediation margins to fall. This decline triggers the second mechanism whereby bankers fail to coordinate the recapitalization of financial institutions. Eventually, the financial system recovers. But this only comes through retained earnings, an essentially lethargic process when volumes and margins are low. Economic growth follows this pattern.

Any theory that speaks to the persistence of financial crises must confront why banks take so long to be recapitalized. Without equity entry barriers, the answer must rely on a decline in bank profit margins during a crisis. Otherwise, high margins should attract new equity or at least translate into rapid revenue retention. In either case, high profit margins accelerate recoveries. However, recent macroeconomic theories of financial intermediation cannot explain declines in profitability during crises. This is because those theories place frictions exclusively on the funding side of banks — limits to raise debt and equity.<sup>4</sup> Without frictions in their lending activities, competition arguments suggest that profits from financial intermediation should *rise*, not *fall*, during crises. After all, as with any normal good or service, marginal profits rise when supply is constrained. This paper shows that frictions in the loan market lead to profitability *declines* in a crisis. Furthermore, this decline breaks a natural stabilizing force behind financial markets. Equity injections and retained earnings that are effective stabilizers for moderate financial losses prove to be ineffective when losses are large.

My goal is to provide some first analytic steps that formalize these ideas. I have tried to do so with a minimal set of ingredients. In the model, the reader will find five ingredients: (1) The reallocation of resources across sectors fuels growth. This feature links financial with real activity. (2) Banks face limited liability. This ties financial activity to financial risk capacity. (3) Intermediation is risky. Thus, financial risk capacity evolves stochastically. (4) Financial intermediation is subject to asymmetric information. This friction delivers low profitability when financial risk capacity is low.

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<sup>3</sup>This view is rooted in modern banking theory (see Freixas and Rochet, 2008) and Leland and Pyle (1977), Diamond (1984), or Boyd and Prescott (1986) for formal models.

<sup>4</sup>In most macroeconomic models of intermediation developed after the crisis banks, cannot raise equity because bankers are fully invested specialists who face agency frictions. I review this literature in the next section.

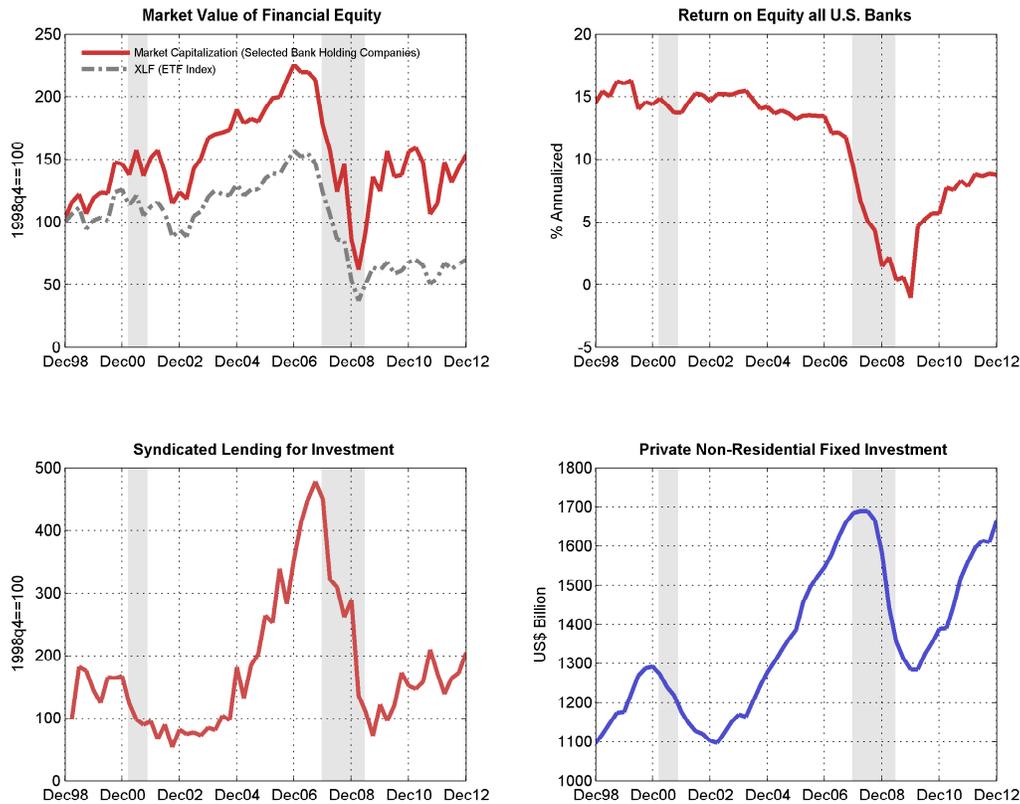


Figure 1: US Banking Indicators and Investment.

(5) Investors inject equity only if they expect to be rewarded by above-average profits.

Together, these ingredients constitute an economy with infrequent, but persistent financial crises. These crises fit the patterns of the 2008-2009 crisis described in Figure 1. The top-left panel shows the evolution of the market capitalization of a group of selected US bank holding companies and the XLF finance stock index. A sharp decline in both variables starts two quarters prior to the Great Recession and continues thereafter. The bottom-left shows a measure of financial intermediation: the flow of syndicated loans with investment purposes. This variable echoes the behavior of bank equity capital. The top-right shows a measure of profitability: the return on bank equity (ROE). This measure shows stable behavior throughout the decade prior, falls dramatically during the recession, and remains low for five years after. The bottom-right shows the decline in non-residential investment, a symptom of the economic deceleration during these years.

The theory attributes the slow recovery of bank equity to the decline in profitability observed after many crises. Of course, by this I do not mean that this is the *only* reason why outside equity does not flow to banks during crises. On the contrary, it is not hard to see how low profit margins would exacerbate other common explanations. For example, reductions in profits can only add stress to agency frictions such as moral hazard (Holmstrom and Tirole, 1997) or limited enforcement (Hart and Moore, 1994). Furthermore, low profits can only worsen the debt-overhang problem introduced by Myers (1977) and recently placed in the context of the crisis by Philippon (2010). Similarly, low profits can only exacerbate asymmetric information between inside and outside investors, a classic

argument in [Myers and Majluf \(1984\)](#) also recently revisited by [Tirole \(2011\)](#) and [Philippon and Skreta \(2011\)](#).

At first, the idea that adverse selection in credit markets worsens during crises may seem to conflict with tighter credit standards or flight-to-quality behavior during downturns (see [Bernanke et al., 1996b](#)). In fact, there is no conflict; there is no reason why adverse selection and flight-to-quality cannot operate in tandem. Adverse selection can worsen within assets of common observed characteristics while banks recompose portfolios between assets of different observed characteristics. What is critical for our discussion is that the profitability decline is not overturned by flight-to-quality. Indeed, static models that couple flight-to-quality with adverse selection show that this is not the case (see [Malherbe, 2013](#)).

The next section contrasts the paper with the literature. Section 3 provides a graphical description of the mechanics of the main model. That model is laid out in Section 4 and characterized in Section 5. Section 6 presents two analytic examples that underscore the role of information asymmetries. Section 7 presents numerical illustrations of the model's dynamics. Section 8 studies two policy experiments. Section 9 concludes. A computational algorithm and proofs are in the Appendix.

## 2. RELATIONSHIP TO THE LITERATURE

The paper is related to two branches of financial macroeconomics. The first branch examines the dynamics of financial intermediation through the evolution of the financial sector's net worth. The second investigates the effects of asymmetric information in financial market intermediation.

Studies in the first branch link the net worth of the financial sector to the amount of financial intermediation through agency frictions. This literature builds on earlier work by [Bernanke and Gertler \(1989\)](#) and [Bernanke et al. \(1996a\)](#) that studies how financial frictions that constrain non-financial firms amplify business cycles. [Holmstrom and Tirole \(1997\)](#) added a second layer. That paper introduced agency frictions to intermediaries that channel resources to non-financial firms. Since the onset of the Great Recession, several papers have incorporated similar intermediaries into state-of-the-art business cycle models. For example, [Gertler and Karadi \(2011\)](#) and [Gertler and Kiyotaki \(2010\)](#) study the business cycle effects of equity losses for intermediaries. This paper is closer to the continuous time models of [He and Krishnamurthy \(2013\)](#) (henceforth [HK](#)), [Brunnermeier and Sannikov \(2014\)](#) (henceforth [BS](#)), and [Adrian and Boyarchenko \(2013\)](#) because those papers also stress the non-linear nature of intermediation dynamics. In [He and Krishnamurthy \(2013\)](#), equity shocks are amplified through a substitution of equity financing toward debt. In [BS](#) and [Adrian and Boyarchenko \(2013\)](#), amplification operates through fire sales.<sup>5</sup> This paper differs from the literature in some important aspects. First, intermediaries do not operate production; they reallocate capital.

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<sup>5</sup>Fire sale phenomena were first described by [Shleifer and Vishny \(1992\)](#). A feedback between losses in intermediary capital and reductions in asset values is also a theme in [Gromb and Vayanos \(2002\)](#) and [Brunnermeier and Pedersen \(2009\)](#). [Maggiore \(2011\)](#) extends this framework to a two-country setup to study current account dynamics. [Diamond and Rajan \(2011\)](#) study strategic behavior by banks to exploit fire sales by their competitors. [Vayanos and Wang \(2011\)](#) introduce asymmetric information into a related setup.

Second, they issue liabilities that become means of payment. Third, frictions they do not limit the ability to raise equity, but do affect their intermediation activities. Thus, non-linear effects result from the interplay among low bank capital, asymmetric information, and low profitability. However, this literature emphasizes the role of pecuniary externalities (see [Lorenzoni, 2008](#); [Bianchi, 2011a](#)) and a similar externality shows up here.

The emphasis on asymmetric information in asset qualities allows me to study the dynamic implications of an idea championed by [Stiglitz and Greenwald \(2003\)](#). That work argues that credit quality deteriorates when banks provide little intermediation, and regards this as being essential to understanding cycles and the behavior of banks in crises. Of course this is not the only paper where credit quality varies over the cycle. [Gennaioli et al. \(2013\)](#) study an environment where intermediaries increase leverage when they can mutually insure against idiosyncratic credit risk. However, their higher leverage increases aggregate credit risk. In [Martinez-Miera and Suarez \(2011\)](#) and [Begenau \(2014\)](#), banks can choose the risk of their assets directly. As an outcome, those models deliver procyclical credit risk, but they cannot explain declines in margins in crises. In this paper, credit risk and returns depend on market conditions — these variables do not have a constant relationship.

Turning to the second branch of the literature, this paper follows work that begins with [Stiglitz and Weiss \(1981\)](#) and [Myers and Majluf \(1984\)](#) that investigates asymmetric information on the side of borrowers. [Carlstrom and Fuerst \(1997\)](#) build on that framework to study credit rationing through the cycle. This paper is closer to [Eisfeldt \(2004\)](#) who studies an asset market with asymmetric information. There, adverse selection adds a cost to insure against investment risks. [Bigio \(2014\)](#) and [Kurlat \(2013\)](#) follow that study with models where assets are also sold under asymmetric information to fund production projects. The novelty here is the interaction between intermediary equity and asymmetric information. This interaction is important because those models lack strong internal propagation: the persistence of adverse selection follows exogenous shocks. Here, low bank equity leads to a persistent aggravation of adverse selection.<sup>6</sup> This connects to the business cycle decompositions in [Christiano et al. \(2012\)](#) and [Ajello \(2012\)](#). Those papers find a prevalence of exogenous shocks that exacerbate asymmetric information. Although those models lack intermediaries, their filtering exercises find that dates associated with stronger adverse selection coincide with dates where financial institutions were in distress.

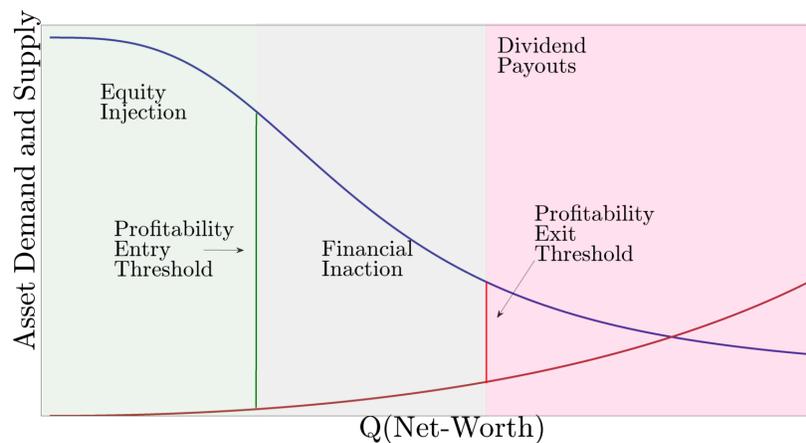
A feedback between net worth and adverse selection is a common theme in the insurance literature. [Winter \(1991\)](#) and [Gron \(1994a,b\)](#) describe insurance market crises where swings in insurance premia and volumes occur after insurers suffer equity losses. The connection with financial markets was emphasized by Darrell Duffie during his Presidential Address to the American Finance Association in 2010 (see [Duffie, 2010](#)).

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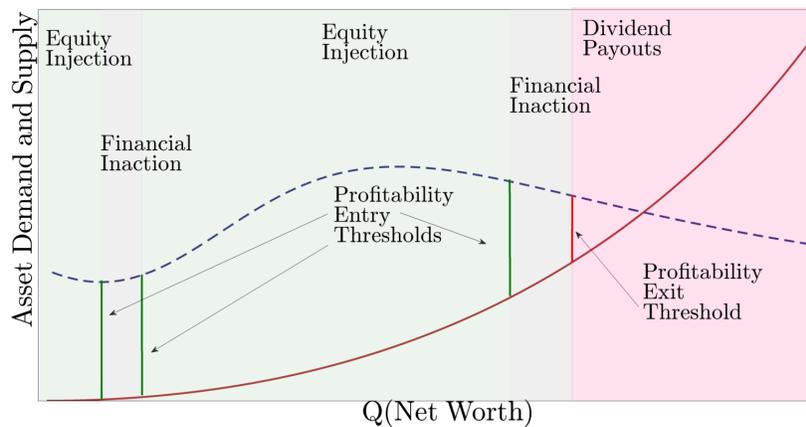
<sup>6</sup>Other models that study lemons markets, such as [Hendel and Lizzeri \(1999\)](#), [Guerrieri and Shimer \(2011a\)](#), [Plantin \(2009\)](#), or [Daley and Green \(2011\)](#), obtain persistence through learning.

## 3. THE MODEL IN A NUTSHELL

In his celebrated Debt Deflation Theory, Irving Fisher compares financial crises to the capsizing of a boat that “under ordinary conditions, is always near a stable equilibrium but which, after being tipped beyond a certain angle, has no longer this tendency to return to equilibrium...”. Eighty years ago, in the aftermath of the Great Depression, Fisher was providing us with a rudimentary description of the non-linear nature of financial crises. The main insight here is that asymmetric information in financial markets can induce these “rocking boat” dynamics. The underlying mechanism can be explained in a nutshell in Figure 2. Panel (2a) illustrates how profitability is a stabilizing force behind financial markets and Panel (2b) how asymmetric information breaks the tendency to return to equilibrium. Figure 2 will be our guide throughout the paper.



(a) Financial Intermediation under Symmetric Information



(b) Financial Intermediation under Asymmetric Information

Figure 2: Financial Intermediation With and Without Asymmetric Information

Let me first discuss Panel (2a). The two curves represent aggregate demand and supply schedules for an asset. In any intermediated market, intermediaries buy assets from suppliers and resell assets to final buyers. For a given aggregate volume of trade  $Q$ , the intermediaries’ marginal profits —

arbitrage — are the distance between the price at the supply schedule to the price at the demand schedule, at that  $Q$ . If some friction imposes a limit on the volume of intermediation, there is a positive arbitrage. In models with financial frictions, the net worth of intermediaries caps  $Q$ . Thus, volumes of intermediation are increasing in the financial sector's net worth, which is why  $Q$  is labelled as a function of net worth in the figure.

The shapes of the demand and supply schedules govern the behavior of marginal profits. In the case of Panel (2a), marginal profits are decreasing in  $Q$  —and thus also in net worth. Conversely, the evolution of net worth is influenced by marginal profits in two ways: directly, by affecting retained earnings, and, indirectly, by attracting outside equity injections or dividends.

To understand this relation, suppose that there is a level of marginal profits below which dividends are paid out. This threshold is the length of the vertical line indicated as the profitability *exit* threshold. Similarly, suppose there is another profitability threshold above which equity injections are attracted. This is the vertical line indicated as the profitability *entry* threshold. Whenever net worth is above the level that induces exit-threshold profits, dividends are paid out. The opposite occurs whenever net worth is below the level that induces entry-threshold profits —injections replenish net worth. Because the entry and exit profit levels are not the same, there is also an intermediate inaction region where intermediaries neither pay dividends nor raise equity. Within that region, equity has a tendency to increase, but only through retained earnings. This simple graph describes an economic force that brings forth financial stability. If anything reduces net worth below (above) the equity entry (exit) point, intermediaries raise (decrease) equity. In that world, intermediation, equity, and profits live in a bounded region.

Asymmetric information alters this stabilization force. This is the situation in Panel (2b). Under asymmetric information, demand may feature a backward-bending portion. The intuition is simple: when intermediaries purchase capital under asymmetric information, both the quantity and the quality of assets increase with the purchase price: the supply schedule is increasing as usual. Now, suppose intermediaries bear all the transaction risks and, thus, receive a price that depends on the quality of the assets they resell. Standard consumer theory dictates that a marginal unit of savings is valued less than the unit before, provided they are assets of the same quality. However, if qualities improve sufficiently fast with quantities, the final price may be increasing in quantities. The result is an effective demand curve that may be back-ward bending. A direct consequence is that marginal profits are also not necessarily decreasing as before. Instead, marginal profits are potentially hump-shaped. In the case of Panel (2b), the hump-shape in marginal profits generates two inaction regions instead of the single region found in Panel (2a).

Let's return to the point of interest: the stability of financial intermediation. Let's assume that net worth is in the inaction region at the right of Panel (2b). In that region, the dynamics of equity and intermediation depend on the size of intermediation losses, as in Fisher's rocking-boat analogy. A shock that produces equity losses, but only sends the economy to the neighboring injection region to the left, will be counterbalanced by quick equity injections. As a result, small shocks are stabilized,

just as in Panel (2a). However, if losses are large enough to send the economy to the inaction region at the left, the economy loses the tendency to return to equilibrium. Because profits are low in that other region, intermediaries lack the individual incentives to inject equity. Unless they coordinate an entry, equity remains low for a while. All in all, large shocks can capsize this economy. Eventually, this economy will recover, but slowly as intermediaries retain earnings.

The next section presents a dynamic environment where similar aggregate demand and supply curves emerge as equilibrium objects. The rest of the paper formalizes the discussion about the implied dynamics and explores policy experiments.

## 4. MODEL

### 4.1. *Environment*

Time is discrete and the horizon is infinity. Every period is divided into two stages:  $s \in \{1, 2\}$ . There are two goods: consumption goods (the *numeraire*) and capital goods. There are two aggregate shocks: (1) a total-factor productivity (TFP) shock,  $A_t \in \{A_1, A_2, \dots, A_M\}$ , and (2) a shock,  $\phi_t \in \Phi \equiv \{\phi_1, \phi_2, \dots, \phi_N\}$ , that affects capital depreciation.  $(A_t, \phi_t)$  form a joint Markov process that evolves according to a transition probability  $\chi : (\mathbb{A} \times \Phi) \times (\mathbb{A} \times \Phi) \rightarrow [0, 1]$  with standard assumptions.  $A_t$  is realized during the first stage and  $\phi_t$  during the second stage.  $\phi_t$  is the source of intermediation risk.

**Notation (Sequence Formulation).** If a variable  $y$  changes between stages, I use  $y_{t,s}$  to refer to its value in period  $t$  stage  $s$ . I only use the period subscript if  $y$  does not change values between stages.

**Demography.** There are two populations of agents: producers and bankers. Each population has unit mass.

**Producers.** Producers are identified by some  $z \in [0, 1]$  and carry a capital stock  $k_t(z)$  as an individual state. They have preferences over consumption streams:

$$\mathbb{E} \left[ \sum_{t \geq 0} \beta^t \log(c_t) \right],$$

where  $c_t$  is consumption and  $\beta$  their discount factor.

**Production Activities and Technologies.** At the beginning of the first stage, producers are randomly segmented into two groups: capital-goods producers (k-producers) and consumption-goods producers (c-producers). Producers become k-producers with probability  $\pi$ , independent of time and  $z$ .<sup>7</sup>

C-producers operate a linear technology that produces  $A_t k_t(z)$  units of consumption during the first stage. C-producers lack the possibility of investing directly, but they can buy capital to augment their capital stock. In contrast, k-producers cannot produce consumption goods, but they have access

<sup>7</sup>The real sector is directly borrowed from [Kiyotaki and Moore \(2008\)](#). Random assignments reduce the state-space.

to a linear investment technology that transforms one consumption good into a unit of new capital. The investment technology is operated only during the second stage.

**Fundamental Economic Problem.** The segmentation of production induces the need for trade: On the one hand, k-producers need consumption goods that they don't have to operate their investment technologies. C-producers produce those resources, but they lack access to the investment technology. The fundamental economic problem is that consumption goods must flow from c-producers to k-producers, and capital must flow in the opposite direction. This reallocation fuels growth —this structure is the model's first ingredient.

**Capital.** Capital is homogeneous at the start of a period. However, during the first stage, the capital stock of every producer is divided into a uniform distribution of capital units, each identified by some quality  $\omega \in [0, 1]$ . Capital units can be sold individually.

The quality  $\omega$  and the realization of  $\phi_t$  determine the depreciation rate of each capital unit through the function  $\lambda : [0, 1] \times \Phi \rightarrow \mathbb{R}_+$ . In particular,  $\lambda(\omega, \phi)$  denotes the capital that will remain out of an  $\omega$ -unit given  $\phi$ . Once an  $\omega$ -unit is scaled by  $\lambda(\omega, \phi)$ , it becomes homogeneous capital, which can be merged with other pieces to form a  $t+1$  capital stock. The following period, any capital stock, no matter how it was built, is divided again into different  $\omega$ 's that depreciate depending on the  $t+1$  realization of  $\phi$ -shock. The process is repeated indefinitely.

By the end of the second stage, out of a  $t$ -period stock  $k_t(z)$ , the capital that remains available for  $t+1$  production is  $k_t(z) \int_0^1 \lambda(\omega, \phi_t) d\omega$ . However, this stock will not equal  $z$ 's capital stock at  $t+1$  because  $z$  may sell  $\omega$ -units individually. This decision is summarized by the indicator  $\mathbb{I}(\omega) : [0, 1] \rightarrow \{0, 1\}$  where  $\mathbb{I}(\omega)$  takes a value of 1 if  $\omega$  is sold. Thus, when choosing  $\mathbb{I}(\omega)$ , the producer sells  $k_t(z) \int_0^1 \mathbb{I}(\omega) d\omega$  and keeps  $k_t(z) \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_t) d\omega$ . By assumption, sales take place before the realization of  $\phi_t$ .

If we add the producer's investments and capital purchases,  $z$ 's capital stock evolves according to:

$$(1) \quad k_{t+1}(z) = i + k^b + k_t(z) \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi_t) d\omega.$$

In this expression,  $i$  is  $t+1$  capital created through his investment technology —when investment is an option— and  $k^b$  are his purchases of  $t+1$  capital. Given  $\phi$ , the average quality under a certain quality  $\omega^*$  is

$$\mathbb{E}[\lambda(\omega, \phi) | \omega \leq \omega^*, \phi] \equiv \frac{\int_0^{\omega^*} \lambda(\omega, \phi) d\omega}{\omega^*}.$$

When I do not condition the expectation by  $\phi$ , and use  $\mathbb{E}[\lambda(\omega, \phi) | \omega \leq \omega^*]$  instead, I am referring to the time  $t$  expected value across realizations of  $\phi$ . Finally, I denote the unconditional quality average given  $\phi$  by  $\bar{\lambda}(\phi) \equiv \mathbb{E}[\lambda(\omega, \phi) | \omega \leq 1, \phi]$ .

An assumption provides an interpretation to  $\omega$  and  $\phi$ :

**Assumption 1** (i)  $\lambda(\omega, \phi)$  is increasing in  $\omega$  and (ii)  $\mathbb{E}[\lambda(\omega, \phi) | \omega < \omega^*, \phi]$  is weakly decreasing in  $\phi$ .

The first condition implies that lower  $\omega$ 's are worse because they depreciate faster. The second condition states that the average quality under some cutoff  $\omega^*$  falls with  $\phi$ .

Capital can also be interpreted as efficiency units of production and  $\lambda(\omega, \phi)$  as a permanent productivity shock specific to  $\omega$ . Under this interpretation, the value of a pool of capital with qualities under a cutoff  $\omega^*$  is proportional to  $\mathbb{E}[\lambda(\omega, \phi) | \omega < \omega^*, \phi]$ . As I discuss below, this specification allows for a great degree of flexibility.

**Private Information.** A quality  $\omega$  is known only to its owner. Buyers can only observe the quantity of a pool of sold units,  $k \int_0^1 \mathbb{I}(\omega) d\omega$ , but cannot discern the composition of  $\omega$ 's within that pool. After  $\phi$  is realized, the  $t+1$  capital that remains from that pool is  $k \int_0^1 \mathbb{I}(\omega) \lambda(\omega, \phi_t) d\omega$ . This means that the effective depreciation of that pool is unknown during the first stage, because both the composition of  $\omega$  and the realization of  $\phi$  are unknown. In equilibrium, it will be possible to infer the  $\omega$ 's sold, but it won't be possible to perfectly forecast  $\phi$ . The private information about  $\omega$  and the uncertainty behind  $\phi$  are the model's second and third main ingredients.

**Bankers.** Bankers intermediate the market for capital. Each banker is identified by some  $j \in [0, 1]$ . They have preferences given by:

$$\mathbb{E} \left[ \sum_{t \geq 0} \hat{\beta}^t c_t \right],$$

where  $c_t$  is consumption and  $\hat{\beta}$  a discount factor. Bankers have two sources of wealth. The first is an exogenous endowment of consumption goods  $\bar{e}$  earned every period. The second source are  $n_{t,1}(j)$  consumption goods held in legal institutions called banks. Although  $\bar{e}$  and  $n$  belong to the same commodity space, they have an important legal distinction. The banks' net worth is liable to intermediation losses, but personal endowments are protected by limited liability. Limited liability links the amount of intermediation with a bank's net worth, the model's fourth key ingredient.

If they want, bankers can alter their wealth composition. In particular, a banker can use an amount,  $e_{t,1} \in [0, \bar{e}]$ , as equity injections into his bank. He can also do the opposite. He can transfer  $d_{t,1} \in [0, n]$  as dividends to his personal account. However, when a banker is paid dividends, he must pay a tax  $\tau$ . This tax should be interpreted as an exogenous wedge that emerges from agency frictions or government policies that are not modeled. Thus, after equity injections/dividends, his bank's net worth evolves according to  $n_{t,1} = n_{t-1,2} + e_{t,1} - d_{t,1}$  and his consumption is  $c_{t,1} = (\bar{e} - e_{t,1}) + (1 - \tau) d_{t,1}$ . The role of this tax is to induce a wedge between the marginal cost of equity and the marginal value of dividends. This wedge is essential to obtain inaction regions where bankers neither pay dividends nor inject equity. These inaction regions are the fifth main ingredient of the model.

Finally, bankers face a random exit probability  $\rho$  every period. When I set  $\rho = 1$ , bankers live one

period and those cases allow exact analytic solutions. Most of the analysis is carried out for  $\rho = 0$ , the infinite-horizon case. The reader can ignore this parameter for most of the paper.

**Financial Intermediation.** Banks provide intermediation by buying used capital from k-producers in the first stage and reselling these units in the second stage. Banks fund their paying k-producers with riskless IOUs that they issue. These IOUs are tradeable and entitle the holder to a riskless unit of consumption. These IOUs can be thought of as credit lines or deposits —or inside money, in more general terms. Once k-producers receive IOUs, they immediately buy consumption goods from c-producers because they need those resources to invest. IOUs are redeemed by the end of the period and bear no interest. These flows occur in the first stage and are presented in Panel 3a of Figure 3.

When bankers buy a pool of capital, they cannot distinguish  $\omega$ . Moreover, they will hold on to the pool until  $\phi_t$  is realized and the pool depreciates. After units in the pool depreciate, the pool is resold as homogeneous  $t + 1$  capital. Once they do so, bankers settle all of their IOUs. These flows of funds occur at the second stage and are found in Panel 3b of Figure 3.

A couple of things are worth noticing. The shock  $\phi_t$  is realized between stages so the bankers' assets are risky. Their liabilities aren't. Hence, banks face equity losses if their IOUs exceed the value of their purchased capital pool. If they experience losses, they must draw funds from the bank's net worth to settle debts. In principle, they could be finance losses with their personal endowment, but limited liability protects their personal wealth. Hence, limited-liability constraint (LLC) caps the amount of intermediation a bank can provide: the greater the volume of capital bought, the greater the risk, and the greater need for an equity cushion. Of course, bankers can inject equity to scale up their operations. To do so, they must have incentives.

**Aggregate States.** There are two aggregate quantities of interest, the aggregate capital stock,  $K_t = \int_0^1 k_t(z) dz$ , and  $N_t = \int_0^1 n_t(j) dj$ , the equity of the entire financial system —after injections/dividends. I will show that it is only necessary to keep track of their ratio  $\kappa_{t,s} \equiv N_{t,s}/K_t$ . We will see that  $\kappa_{t,1}$  determines the bankers' capacity to bear financial losses relative to the wealth of producers. Thus, I refer to it as the economy's *financial-risk capacity*. The aggregate state is summarized by  $X_{t,1} = \{A_t, \phi_{t-1}, \kappa_{t,1}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$  and  $X_{t,2} = \{A_t, \phi_t, \kappa_{t,2}\} \in \mathbb{X} \equiv \mathbb{A} \times \Phi \times \mathbb{K}$ .

**Public Information.** At every point,  $X_{t,s}$  and every producer's activity are common knowledge. The latter assumption ensures that, in equilibrium, c-producers are excluded from selling capital. Bankers are informed of their exit at the beginning of the second stage.

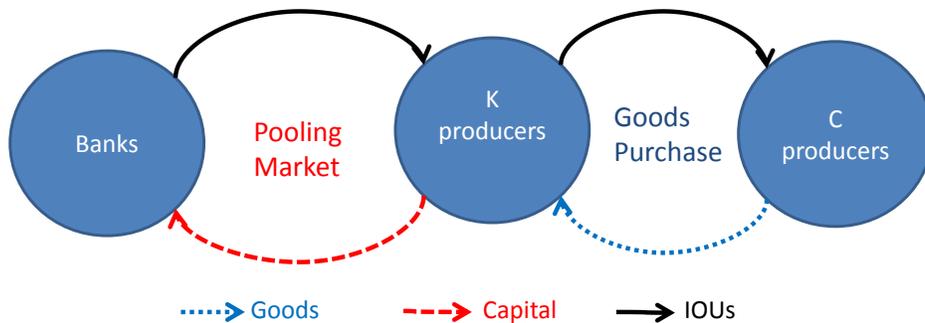
**Markets.** There are two capital markets. The first market is where capital is sold by k-producers under asymmetric information. This market satisfies:

**Assumption 2** *Capital markets are **anonymous** and **non-exclusive**. Banks are **competitive**.*

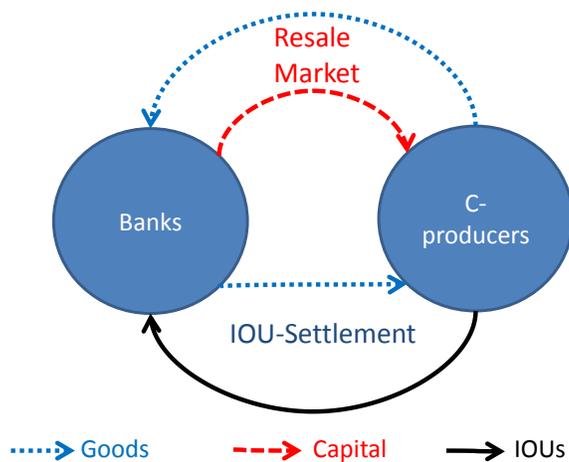
Assumption 2 implies that the first market is a pooling market with a unique pooling price  $p_t$ .<sup>8</sup> The assumption that banks are competitive means that they take prices and qualities as given.<sup>9</sup>

<sup>8</sup>Without anonymity bankers would offer price-quantity menus. With exclusive contracts, bankers would use dynamic incentives to screen.

<sup>9</sup>In principle, intermediaries could use lotteries to ameliorate the lemons problem. This resolution is not possible



(a) Flows of Funds in Stage 1.



(b) Flows of Funds in Stage 2.

Figure 3: Flows of Funds

I refer to this market as the pooling or the first-stage market. The second market, which opens during the second stage, is where bankers sell back all the units purchased during the first stage. This market clears at a price  $q_t$ . I refer to it as the resale market.

**Timing.** At the beginning of the period,  $A_t$  is realized and the production of consumption goods by c-producers takes place. Also, bankers decide on their injections/dividends. Restrained by their

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with transaction costs or if intermediaries lack commitment. [Guerrieri and Shimer \(2011b\)](#) allow separation because capital is sold in different markets that clear at different rates.

adjusted net worth, bankers buy capital under asymmetric information and issue IOUs. In turn, k-producers transfer these funds immediately to c-producers to buy consumption goods.

During the second stage,  $\phi_t$  is realized and capital depreciates accordingly. After capital becomes homogeneous, bankers resell their purchased pools. By the end of the period, bankers settle their IOUs and realize profits/losses. Simultaneously, producers decide on consumption, investment, and second-hand capital purchases.

4.2. Accounting

**Bank Balance Sheets.** During the first stage, bankers hold  $n$  equity in their banks. At the beginning of that stage, their injection/dividend policies alter their banks' balance sheet. Simultaneously, they buy  $pQ$  units of capital and issue  $pQ$  deposits. Through the first stage, their balance sheet evolves as:

<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="width: 50%;">Assets</th> <th style="width: 50%;">Liability</th> </tr> <tr> <td style="text-align: center;"><math>n</math></td> <td style="text-align: center;"><u>Net worth</u></td> </tr> <tr> <td></td> <td style="text-align: center;"><math>n</math></td> </tr> </table>	Assets	Liability	$n$	<u>Net worth</u>		$n$	$\Rightarrow$	<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="width: 50%;">Assets</th> <th style="width: 50%;">Liability</th> </tr> <tr> <td style="text-align: center;"><math>n' = n + e - d</math></td> <td style="text-align: center;"><math>pQ</math></td> </tr> <tr> <td style="text-align: center;"><math>pQ</math></td> <td style="text-align: center;"><u>Net worth</u></td> </tr> <tr> <td></td> <td style="text-align: center;"><math>n' = n + e - d</math></td> </tr> </table>	Assets	Liability	$n' = n + e - d$	$pQ$	$pQ$	<u>Net worth</u>		$n' = n + e - d$
Assets	Liability															
$n$	<u>Net worth</u>															
	$n$															
Assets	Liability															
$n' = n + e - d$	$pQ$															
$pQ$	<u>Net worth</u>															
	$n' = n + e - d$															
Beginning-of-period Balance Sheet		End-of-stage-1 Balance Sheet														

After  $\phi'$  is realized, the value of their purchased capital stock becomes  $q\mathbb{E}[\lambda(\omega, \phi') | \mathbb{I}(\omega) = 1, X'] Q$ , the product of the resale price  $q$  and the fraction of the capital pool that does not depreciate,  $\mathbb{E}[\lambda(\omega, \phi') | \mathbb{I}(\omega) = 1, X'] Q$ . Accounting for these changes in values, the balance sheet evolves to:

<table border="1" style="width: 100%; border-collapse: collapse;"> <tr> <th style="width: 50%;">Assets</th> <th style="width: 50%;">Liability</th> </tr> <tr> <td style="text-align: center;"><math>n'</math></td> <td style="text-align: center;"><math>pQ</math></td> </tr> <tr> <td style="text-align: center;"><math>q\mathbb{E}[\lambda(\omega, \phi')   X] Q</math></td> <td style="text-align: center;"><u>Net worth</u></td> </tr> <tr> <td></td> <td style="text-align: center;"><math>n' + (q\mathbb{E}[\lambda(\omega, \phi')   X] - p) Q</math></td> </tr> </table>	Assets	Liability	$n'$	$pQ$	$q\mathbb{E}[\lambda(\omega, \phi')   X] Q$	<u>Net worth</u>		$n' + (q\mathbb{E}[\lambda(\omega, \phi')   X] - p) Q$	
Assets	Liability								
$n'$	$pQ$								
$q\mathbb{E}[\lambda(\omega, \phi')   X] Q$	<u>Net worth</u>								
	$n' + (q\mathbb{E}[\lambda(\omega, \phi')   X] - p) Q$								
End-of-stage-2 Balance Sheet.									

Note that the gains/losses from financial intermediation are  $(q\mathbb{E}[\lambda(\omega, \phi') | X] - p) Q$ .

4.3. First-Stage Problems

The economy has a recursive representation, so from now on, we drop time subscripts.

**Notation (Recursive Formulation).** I denote by  $n$  the bank's equity at the beginning of the first stage, by  $n'$  after equity injections and dividends alter this variable within a stage, and by  $n''$  the equity after profits/losses. I adopt the same convention for  $\kappa$ . For any other variable  $y$ ,  $y$  denotes its value at the beginning of the stage, and  $y'$  its value in the subsequent stage. To indicate

policy functions, I append terms like  ${}^m(S)$  to a variable chosen by agent  $m$  when his state is  $S$ . I use  $\mathbb{I}(\omega, k, X)$  to refer to a  $k$ -producer's stage-1 decision to sell an  $\omega$ -quality when his state is  $(k, X)$ .

**K-producer's First-Stage Problem.** During the first stage, a  $k$ -producer enters the period with a capital stock  $k$ . At this stage, he decides which qualities to sell:

**Problem 1** (*k-producer's  $s=1$  problem*) *The k-producer's first-stage problem is:*

$$V_1^k(k, X) = \max_{\mathbb{I}(\omega) \in \{0,1\}} \mathbb{E} [V_2^k(k'(\phi'), x, q, X') | X]$$

$$\text{subject to } x = pk \int_0^1 \mathbb{I}(\omega) d\omega \text{ and } k'(\phi') = k \int_0^1 [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi') d\omega, \forall \phi'.$$

The first equation in the constraint set says that the producer holds  $x$  consumption goods obtained selling  $k \int_0^1 \mathbb{I}(\omega) d\omega$  at price  $p$ —and exchanging those IOUs for goods. The second constraint says that capital carried to the next stage, given a  $\phi'$ , is original capital stock minus sales after the depreciations corresponding to that  $\phi'$ .

**C-producer's First-Stage Problem.** The first-stage value function for  $c$ -producers is the same except that they obtain goods by producing them and they don't sell capital.

**Problem 2** (*c-producer's  $s=1$  problem*) *The c-producer's first-stage value function:*

$$V_1^c(k, X) = \mathbb{E} [V_2^c(k'(\phi'), x, q, X') | X]$$

$$\text{where } x = Ak \text{ and } k'(\phi') = k \int_0^1 \lambda(\omega, \phi') d\omega, \forall \phi'.$$

**Banker's First-Stage Problem.** During the first stage, a banker decides whether to inject equity, pay dividends, or leave his bank's equity intact. This decision alters his bank's equity from  $n$  to  $n'$  instantaneously. Once  $n'$  is in place, the banker chooses a volume of capital purchases in the pooling market,  $Q$ . To purchase that amount, he issues  $pQ$ . Under LLC, his bank's liabilities cannot exceed the bank's net worth plus the value of the capital pool bought, for any realization of  $\phi'$ . In other words,

$$pQ \leq q\mathbb{E} [\lambda(\omega, \phi') | \mathbb{I}(\omega) = 1, \phi'] Q + n' \text{ for any } \phi' \in \mathbb{X}.$$

Let  $\Pi(X, X') \equiv q\mathbb{E} [\lambda(\omega, \phi') | \mathbb{I}(\omega) = 1, \phi'] - p$  denote the banker's marginal profit in state  $X'$ . The banker's problem is

**Problem 3** *The banker's first-stage problem is*

$$V_1^f(n, X) = \max_{Q, e \in [0, \bar{e}], d \in [0, n]} c + \mathbb{E} [V_2^f(n' + \Pi(X, X') Q, X') | X]$$

$$\begin{aligned}
(2) \quad & \text{subject to } 0 \leq n' + \Pi(X, X')Q, \quad \forall X' \\
& c = (\bar{e} - e) + (1 - \tau)d \\
& n' = n + e - d.
\end{aligned}$$

The first constraint is the LLC, the second his budget constraints, and the third the definition of  $n'$ .

#### 4.4. Second-Stage Problems

**K-producer's Second-Stage Problem.** Once k-producers reach the second stage, they hold  $x$  consumption goods and the undepreciated portion of the capital stock they did not sell,  $k$ . They solve:

**Problem 4** (*k-producer's s=2 problem*) *The k-producer's problem in the second stage is:*

$$V_2^k(k, x, q, X) = \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta \mathbb{E} [V_1^j(k', X') | X], \quad j \in \{i, p\}$$

$$\text{subject to } c + i + qk^b = x \text{ and } k' = k^b + i + k.$$

The first constraint is a budget that states that  $x$  can be used to consume  $c$ , invest  $i$ , or purchase capital  $k^b$ . The second is the capital accumulation equation that corresponds to (1). This value function depends on two macroeconomic variables: the current price of capital  $q$  and the aggregate state  $X$ .<sup>10</sup>

**C-producer's Second-Stage Problem.** The problems of the c-producer and the k-producer are the same except that the former is restricted to set  $i \leq 0$  because he lacks investment opportunities.

**Problem 5** (*c-producer's s=2 problem*) *The c-producer's problem at the second stage is:*

$$V_2^c(k, x, q, X) = \max_{c \geq 0, i \leq 0, k^b \geq 0} \log(c) + \beta \mathbb{E} [V_1^j(k', p, X') | X], \quad j \in \{i, p\}$$

$$\text{subject to } c + i + qk^b = x \text{ and } k' = k^b + i + k.$$

**Banker's Second-Stage Problem.** Bankers don't take actions during the second stage, but rather, only realize profits/losses. Thus, their value is  $V_2^f(n'', X) = \hat{\beta} \mathbb{E} [V_1^f(n'', X') | X]$  if they remain in the industry or  $V_2^f(n'') = (1 - \tau) \hat{\beta} n''$  if they exit.

#### 4.5. Market-Clearing Conditions and Equilibrium

**Aggregation.** In every period and stage, there are corresponding measures over capital holdings for k- and c-producers which I denote by  $\Gamma^k$  and  $\Gamma^c$ . By independence, during the first stage, these

<sup>10</sup>The price  $q$  is a function of  $X'$  and its lag because  $Q$  depends on  $X$  and depreciations depend on  $X'$ .

measures satisfy:

$$(3) \quad \int_0^\infty k \Gamma^k (dk) = \pi K \quad \text{and} \quad \int_0^\infty k \Gamma^c (dk) = (1 - \pi) K.$$

These measures evolve according to the producers' decisions and the segmentation of activities. In addition, there is a measure  $\Lambda$  of bank equity. Market clearing at the first stage requires the demand for capital by banks to equal the supply of capital by k-producers:

$$\underbrace{\int_0^\infty Q(n, X) \Lambda(dn)}_{\substack{\text{capital units} \\ \text{purchased by banks}}} = \underbrace{\int_0^\infty k \int_0^1 \mathbb{I}(\omega, k, X) d\omega \Gamma^k (dk)}_{\substack{\text{capital units} \\ \text{sold by k-producers}}}.$$

During the second stage, c- and k-producers' aggregate demand for efficiency units are:

$$D^c(X, X') \equiv \int_0^\infty k^{b,c} (\bar{\lambda}(\phi') k, x^c(k, X), q, X) \Gamma^c (dk)$$

and

$$D^k(X, X') \equiv \int_0^\infty k^{b,k} \left( k \int_0^1 [1 - \mathbb{I}(\omega, k, X)] \lambda(\omega, \phi') d\omega, x^k(k, X), q, X' \right) \Gamma^k (dk).$$

The supply of  $t + 1$  capital during the second stage is the capital bought by banks during the first stage, adjusted for the average efficiency:

$$S(X, X') \equiv \underbrace{\mathbb{E}[\lambda(\omega, \phi') \mid \mathbb{I}(\omega, k, X) = 1, X]}_{\substack{\text{average } \lambda \\ \text{among purchased units}}} \underbrace{\int_0^\infty Q(n, X) \Lambda(dn)}_{\substack{\text{capital units} \\ \text{purchased by banks}}}.$$

The market-clearing condition at the second stage is  $S(X, X') = D^c(X, X') + D^k(X, X')$ . A recursive competitive equilibrium is:

**Definition 1** (Recursive Competitive Equilibrium) *A recursive competitive equilibrium (RCE) is (1), a set of price functions,  $\{q(X, X'), p(X)\}$ ; (2) a set of policy functions for producers  $\{c^j(x, k, q, X), k^{b,j}(x, k, q, X), i^j(x, k, q, X)\}_{j=c,k}$ , a selling decision for k-producers,  $\mathbb{I}^k(\omega, k, X)$ , a set of policy functions for bankers  $Q(n, X), e(n, X), d(n, X)$ ; (3) sets of value functions,  $\{V_1^j(k, X), V_2^j(k, x, q, X)\}_{j=c,k}, \{V_s^f(n, X)\}_{s=1,2}$ ; and (4) a law of motion for the aggregate state  $X$ , such that for any measures  $\Gamma^c, \Gamma^k$ , and  $\Lambda$  satisfying (3), the following hold: (I) The producers' policy functions are solutions to their problems taking  $q(X, X'), p(X)$ , and the law of motion for  $X$  as given. (II)  $Q(n, X), e(n, X), d(n, X)$  are solutions to the banker's problem taking  $q(X, X'), p(X)$ , and the law of motion for  $X$  as given. (III) Capital markets clear during the first and second stages. (IV) The law of motion  $X$  is consistent with policy functions and the transition function  $\chi$ . All expectations are consistent with the law of motion for  $X$  and agent policies.*

This definition does not depend on the distribution of wealth because this economy admits aggregation. This is shown in the characterization of equilibria in the following section. Before proceeding to that characterization, let me discuss the interpretation of this environment.

#### 4.6. Discussion - The Environment

**Interpretation of  $\lambda$ .** In modern economies, firms operate in complex production and financial networks. They produce in multiple interrelated product lines and hold risky claims on others. This amalgamation of physical and financial assets is represented by the collection of  $\omega$  held by producers. The quality  $\omega$  is an ordered index that through  $\lambda$  maps the different attributes of assets into a comparable number, the efficiency units  $\lambda(\omega, \phi)$ . For that reason,  $\lambda(\omega, \phi)$  is also proportional to the intrinsic value of each  $\omega$ . Now, there is no reason to believe that asset values do not change at business cycle frequency —for example, [Bloom \(2009\)](#) shows evidence of increases in return dispersion in recessions. Such distributional changes are represented through the stochastic process of  $\phi_t$ . Moreover, notice that  $\lambda^{-1}(\cdot, \phi)$  is a cumulative distribution function over efficiency units. Hence,  $\phi$  also captures the distributional changes in asset values occurring at business cycle frequencies. Through  $\lambda$ ,  $\omega$ , and  $\phi$ , the model captures these complex forces in a parsimonious way.

Despite its simplicity,  $\lambda$  allows for a great degree of flexibility. Broadly speaking, it enables  $\phi$  to have direct real effects, but also indirect effects through asymmetric information and intermediation risk. It can accommodate several specifications:

**Case 1** *When  $\lambda$  is constant, capital is homogeneous and the economy collapses to an endogenous growth model.*

**Case 2** *When  $\bar{\lambda}(\phi)$  is constant,  $\phi$  does not change the production possibility frontier and  $\phi$  has no real effects.*

**Case 3** *When  $\lambda$  is constant only across  $\omega$ , there is intermediation risk, but no asymmetric information.*

**Case 4** *When  $\lambda$  is constant only across  $\phi$ , there is asymmetric information, but no intermediation risk.*

**Case 5** *When  $\lambda$  varies with both  $\omega$  and  $\phi$ , there is intermediation risk and asymmetric information about risk and returns of  $\omega$ . We have the following subcases:*

(a) *When  $\lambda(\cdot, \phi) - \lambda(\cdot, \tilde{\phi})$  is constant for all  $\{\phi, \tilde{\phi}\}$ , the economy features asymmetric information about the valuations but not about risk. Every  $\omega$  is equally risky, but higher  $\omega$  are associated with higher efficiency units.*

(b) *When  $\mathbb{E}[\lambda(\tilde{\omega}, \tilde{\phi}) | \tilde{\omega} = \omega]$  is constant for all  $\omega$ , there is asymmetric information only about risk. That is, the expected value of efficiency units out of all  $\omega$  is the same, but they vary in risk.*

A final important feature to discuss is that intermediaries cannot observe any attribute of  $\omega$ . This assumption reduces the model's dimensionality at the expense of realism. In actuality, assets have observable and unobservable dimensions of characteristics. With multiple dimensions, prices should be conditional on observables. Furthermore, the portfolio composition of intermediaries across ob-

servables should vary through the cycle. For example, we can speculate that flight-to-quality would occur when financial risk capacity is low. As long as this does not break the non-monotonic behavior of profits, all of the results about the dynamics of  $\kappa$  should go through.

**Role of Banks.** Banks here perform three roles stressed by banking theory (see Freixas and Rochet, 2008, section 1.2). Recall the transactions described in Figure 3. Those flows reflect these roles. First, banks diversify idiosyncratic transaction risks because they pool capital together. Second, they provide liquidity. Third, they provide risk-insurance because they buy capital before the realization of  $\phi$ .

The diversification of idiosyncratic risk follows from the implicit assumption that only banks can buy large pools of capital and dilute idiosyncratic risk. This gives banks an advantage over c-producers, who would otherwise bear the risk of getting a low  $\omega$  when trading with a k-producer directly. This role emerges as an equilibrium outcome in Boyd and Prescott (1986) where banks are coalitions of agents that join together to exploit the Law of Large Numbers.

Banks provide a liquidity service because they create risk-free liabilities. To see this, suppose the contrary: that in exchange for any capital unit, the k-producer receives a claim contingent on the  $\omega$  to be discovered later by the bank. This alternative form of funding creates a liquidity problem. Recall that the k-producer needs to buy consumption goods in the first stage. If he were to use an  $\omega$ -contingent claim as a payment, he would be transferring the  $\omega$ -uncertainty back to the c-producer. This is precisely the problem that banks are there to solve! Instead, because banks offer risk-free IOUs, they mitigate the effects of asymmetric information and make capital more liquid. This financial arrangement is consistent with the view in Gorton (2010) that “the essential function of banking is to create a special kind of debt, debt that is immune to adverse selection.”<sup>11</sup>

A third role for banks is to absorb the risk implied by  $\phi$ . Liquidity provision requires deposits to be independent of  $\omega$ , but it does not require these to be independent of  $\phi$ . This last role must follow from another friction that is not modeled explicitly. For example, this role would emerge if the aggregate state is not contractible.<sup>12</sup> I assume for tractability that the distributions of  $\omega$  and  $\lambda$  are constant across entrepreneurs, but this need not be the case. With a complex cross-section, the aggregate state may not be contractible.

A final observation is that banks do not hold capital throughout periods. The model can be adapted so banks hold capital. However, banks in practice are not in the business of managing businesses.

**Interpretation of Banks.** Let’s turn to the institutional environment that the model is aiming to represent. It should be clear that banks in the model can be interpreted as shadow banks: Their asset side resembles the asset side of security brokers, dealers, or investment banks in practice.

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<sup>11</sup>There are other models where risk free-debt is optimal. In Williamson (1986, 1987), Gorton and Pennacchi (1990) and Diamond and Rajan (2000) deposit funding guarantees monitoring incentives. In DeMarzo and Duffie (1999), issuers precommit to an optimal security design that features a debt-like tranche.

<sup>12</sup>Here, producers are risk-averse and banks are risk neutral. Although the value function of banks is linear, the slope is correlated with  $\phi$ . This is why it is not obvious that intermediaries should bear all aggregate risk.

Their liability side is similar to the funding of money-market funds. I believe the model can also be interpreted as a model of traditional banking. Banks in the model issue deposits, just like banks in practice do. On the asset side, a result in Bigio (2014) shows that sales under asymmetric information are observationally equivalent to collateralized loans, the typical asset in the balance sheet of a traditional bank.

## 5. CHARACTERIZATION

This section shows that the model fits the description of Figure 3. First, we will show that all policy functions are linear in individual states and, as a consequence, that  $\kappa$  is the only endogenous state. We will then obtain effective demand and supply schedules as functions of  $\kappa$  that resemble those in Section 3. We will then be ready to study the rocking-boat dynamics that emerge from this model.

### 5.1. Policy Functions

The demand and supply of capital are obtained by aggregating the individual policy functions of producers so we need to characterize these policies first.

**Producers' Second-Stage Policies.** As a result of homogeneity, the c-producer's policy functions are linear in a state variable interpreted as their wealth. This wealth is the sum of his output and the value of his capital,  $W^c \equiv x + q\bar{\lambda}(\phi')k$ . The solution to his problem is characterized by:

**Proposition 1** *In any RCE, the c-producer's policy functions are  $k^{c'}(k, x, q, X) = \beta \frac{W^c}{q}$  and  $c^c(k, x, q, X) = (1 - \beta)W^c$ . His value function is  $V_2^c(k, x, q, X) = \psi^c(X) + \log(W^c)$ .*

Policy functions for k-producers are also linear in their corresponding wealth. In their case, wealth takes a different form because the investment option changes their valuation of capital. Their wealth is  $W^k \equiv x + q^i \mathbb{E}[\lambda(\omega, \phi') | \omega > \omega^*(X), \phi']k$ , the sum of the funds obtained by selling capital and the replacement cost of the efficiency units they kept. The replacement cost of capital for k-producers is  $q^i = \min\{1, q\}$ , the minimum between the market price of an efficiency unit and the technical rate of transformation. The average efficiency among the units kept is the average depreciation above a cutoff quality  $\omega^*(X)$ . A subsequent result shows that selling decisions are indeed characterized by single  $\omega^*(X)$ , independent of the size of the capital stock. Using the definition of  $W^k$ , the solution to the k-producer's problem is:

**Proposition 2** *In any RCE, the k-producer's policy functions are  $k^{k'}(k, x, q, X) = \beta \frac{W^k}{q^i}$  and  $c^k(k, x, q, X) = (1 - \beta)W^k$ . His value function is  $V_2^k(k, x, q, X) = \psi^k(X) + \log(W^k)$ .*

**Producers' First-Stage Policies.** Replacing the results in Proposition 2 into the k-producer's first-stage value function delivers an analytic expression. Through the definitions of  $x$  and  $\mathbb{E}[\lambda(\omega, \phi') | \omega > \omega^*]$ , we obtain:

$$V_1^k(k, X) = \max_{\mathbb{I}(\omega) \in \{0,1\}} \mathbb{E} \left[ \log \left( p \int_0^1 \mathbb{I}(\omega) d\omega + q^i(X, X') \int_0^1 \lambda(\omega, \phi') [1 - \mathbb{I}(\omega)] d\omega \right) | X \right] + \psi^k(X) + \log(k).$$

There is an important thing to note from  $V_1^k(k, X)$ : the optimal solution to  $\mathbb{I}(\omega)$  is the same across  $k$ -producers, regardless of their wealth. This optimal  $\mathbb{I}(\omega)$  is characterized by:

**Proposition 3** *In any RCE, the  $k$ -producer's first-stage policy function is given by,  $\mathbb{I}^*(\omega, k, X) = 1$  if  $\omega < \omega^*$  and 0 otherwise. The threshold is,*

$$(4) \quad \omega^* = \arg \max_{\tilde{\omega}} \mathbb{E} \left[ \log \left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right) | X \right].$$

Moreover,  $\omega^*$  is unique and increasing in  $p$ .

Proposition 3 confirms that the solution to the  $k$ -producer's sales decision is given by a unique cutoff quality  $\omega^*$  below which all units are sold. This unique cutoff resembles the solution to the lemons problem of [Akerlof \(1970\)](#), with the distinction that there is adverse selection about risky, as opposed to riskless assets.<sup>13</sup> But just as in [Akerlof \(1970\)](#),  $\omega^*$  indicates both the highest quality of capital traded and the volume of intermediation. For that reason, I use threshold quality and volume of intermediation to refer to  $\omega^*$  interchangeably from now on.

The individual supply of capital in the first stage is the inverse of the solution to  $\omega^*$ . We call that supply  $p^s(\omega^*)$  which is increasing in  $\omega^*$  because  $\omega^*$  is increasing in  $p$ . To make further progress, we need to characterize the demand for capital by banks in the first stage. This will give us the equilibrium  $p$  and  $\omega^*$  as functions of  $X$ . For that we need to return to the bankers' problems.

**Bankers' policies.** At the beginning of every period, bankers choose  $e$ ,  $d$ , and  $Q$ . We have:

**Proposition 4** *The banker's value functions are  $V_1^f(n, X) = v_1^f(X)n + \frac{\bar{e}}{1-\beta}$  and  $V_2^f(n, X) = v_2^f(X)n + \beta \frac{\bar{e}}{1-\beta}$ , where  $v_1^f(X)$  and  $v_2^f(X)$  are the marginal value of financial equity in stages 1 and 2, respectively. Furthermore,  $v_1^f(X)$  solves the following Bellman equation:*

$$(5) \quad v_1^f(X) = \max_{Q \geq 0, e \in [0, \bar{e}], d \in [0, 1]} (1 - \tau)d - e + \mathbb{E} \left[ v_2^f(X') \left( \Pi(X, X')Q + n' \right) | X \right]$$

subject to,

$$0 \leq n' + \Pi(X, X')Q, \forall X'$$

$$n' = 1 + e - d,$$

and where  $v_2^f(X) = \hat{\beta}$  if the banker exits, and  $v_2^f(X) = \hat{\beta} \mathbb{E} \left[ v_1^f(X') | X \right]$  otherwise. The value  $V_1^f(n, X)$  is attained by some  $e(X)$ ,  $d(X)$ , and  $Q(X)$  that satisfy:

$$e(n, X) = e^*(X)n, \quad d(n, X) = d^*(X)n, \quad Q(n, X) = Q^*(X)(1 + e^*(X) - d^*(X))n.$$

Proposition 4 is important because it shows that the bankers' policy functions are linear in their individual net worth. This means that all bankers take the same actions that depend on  $X$ . To characterize the coefficients  $\{Q^*(X), e^*(X), d^*(X)\}$ , let's define the first-stage marginal value of

<sup>13</sup>This portfolio problem has an intuitive interpretation: once the  $\omega^*$  units are sold, these become riskless wealth. The remaining fraction remains risky.

inside equity:

$$(6) \quad \tilde{v}(X) \equiv \mathbb{E}[v_2^f(X')] + \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X, X')|X]}{-\min_{\tilde{X}} \Pi(X, \tilde{X})}, 0 \right\}.$$

The formula for  $\tilde{v}$  is intuitive. The first term,  $\mathbb{E}[v_2^f(X')|X]$ , is the marginal value of equity in the next stage. The second term is the shadow value of relaxing the LLC by holding an additional unit of net worth.<sup>14</sup> The solutions to  $\{Q^*(X), e^*(X), d^*(X)\}$  are given by:

**Proposition 5**  $Q^*(X)$  is given by,

$$Q^*(X) = \arg \max_{\tilde{Q}} \mathbb{E} \left[ v_2^f(X') \Pi(X, X') | X \right] \tilde{Q} \text{ subject to } \Pi(X, X') \tilde{Q} \leq 1, \forall X'.$$

In equilibrium,  $\min_{\tilde{X}} \Pi(X, \tilde{X}') < 0$ . The injections/dividend policies,  $\{e^*(X), d^*(X)\}$ , satisfy:  $e = \bar{e}$  if  $\tilde{v}(X) > 1$  and indeterminate at the individual level if  $\tilde{v}(X) = 1$ . In turn,  $d^*(X) = 1$  if  $\tilde{v}(X) < (1 - \tau)$  and indeterminate at the individual level if  $\tilde{v}(X) = (1 - \tau)$ . Otherwise,  $e^*(X) = d^*(X) = 0$  if  $\tilde{v}(X) \in (1 - \tau, 1)$ .

This proposition states that the LLC binds when the value of additional intermediation is positive, when  $\mathbb{E} \left[ v_2^f \Pi | X \right] > 0$ . In turn,  $Q$  is indeterminate when that value is 0, and  $Q = 0$  if that term is negative. The injections/dividend decisions are characterized through binary conditions. When  $\tilde{v} < (1 - \tau)$ , the banker pays dividends because the marginal value of inside equity is below the after-tax benefit of dividends. Conversely, if  $\tilde{v} > 1$ , the banker injects equity because the value of inside equity exceeds the cost in forgone consumption. When  $\tilde{v} \in (1 - \tau, 1)$  there is financial inaction. If  $\tilde{v}$  is either  $(1 - \tau)$  or 1, bankers are indifferent about the scale of dividends and injections, respectively. When the functions  $e^*(X)$  and  $d^*(X)$  are indeterminate at the individual level, I restrict attention to symmetric policies without loss of generality.

## 5.2. Market Prices and Bank Profits

Thus far, we have characterized all of the individual policy rules. We will use these to obtain equilibrium prices. The reader will soon realize how this analytic description fits the graphic description of Section 3.

**Resale Market Price Function.** The linearity of policy rules allows us to express the demands for capital in terms of aggregate quantities without knowledge distributions. The integral across the

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<sup>14</sup>Under LLC, increasing a unit of equity allows the bank to issue up to  $(-\min_{\tilde{X}} \Pi(X, \tilde{X}))^{-1}$  additional IOUs—one can rearrange the LLC and find that the inverse of the worst-case losses are the maximal marginal leverage. In turn, for a given  $X'$ , an additional unit of intermediation adds  $\Pi(X, X')$  to net worth tomorrow, which is valued at  $v_2^f(X')$ . Thus, the value of an additional unit of intermediation is  $\mathbb{E}[v_2^f(X')\Pi(X, X')|X]$ . The shadow value of relaxing the constraint is this amount times the marginal leverage. The *max* operator sets this term to 0 if there is no value from relaxing the constraint.

c-producers' capital stocks yields the aggregate demand for  $t + 1$  capital in stage two:

$$D^c(X, X') = \left[ \frac{\beta (A + q(X, X') \bar{\lambda}(\phi'))}{q(X, X')} - \bar{\lambda}(\phi') \right] (1 - \pi) K.$$

The k-producers' capital demand is obtained similarly. In their case, their individual demands are broken into three tranches that depend on whether  $q$  is above, below, or exactly one. The appendix derives the equilibrium conditions for all three cases. When  $q$  is above one, k-producers do not buy capital because it is cheaper to produce it. To simplify the exposition, I present solutions for  $q(X, X') > 1$ , a condition verified in all of the applications in the paper. Hence, for the rest of the paper,  $D^k(X, X') = 0$ .

Capital supplied by bankers during the second stage,  $S(X, X')$ , is the sum of units sold by k-producers during the first stage scaled by their average quality:

$$S(X, X') = \mathbb{E}[\lambda(\omega, \phi') | \omega < \omega^*(X), \phi'] \omega^*(X) \pi K.$$

The second-stage supply schedule is perfectly inelastic. The quantity depends on the volume bought in the first stage,  $\omega^*(X)$ , but features a random shift because it varies with  $\phi'$ . The price  $q$  is solved out from  $S(X, X') = D^c(X, X')$ :

**Proposition 6** *In equilibrium  $q(X, X')$  is given by,*

$$(7) \quad q(X, X') = \max \left[ \frac{\beta A}{\pi \omega^*(X) \mathbb{E}[\lambda(\omega, \phi') | \omega < \omega^*(X), \phi'] + (1 - \pi)(1 - \beta) \bar{\lambda}(\phi')} \right].$$

There are two things we should notice. First,  $q$  is decreasing in  $\omega^*$  and second, it is increasing in  $\phi'$ . Both outcomes are natural because  $t + 1$  capital is a normal good —we know this from the solution to the producers' problems. This means  $q$  must fall when supply increases, and this is precisely what happens with higher  $\omega^*$  or lower  $\phi'$ . In sum,  $q$  captures a substitution effect.

Now that we have solved for  $q$ , we can reconstruct the price received by banks per unit of intermediation  $Q$ :

$$p^d(X, X') \equiv \underbrace{q(X, X')}_{\text{Substitution Effect}} \underbrace{\mathbb{E}[\lambda(\omega, \phi') | \omega < \omega^*(X), \phi']}_{\text{Composition Effect}}.$$

This price is the product of  $q$ , and the average efficiency below  $\omega^*$ . This average increases with  $\omega^*$ , but declines with  $\phi'$ . This is exactly the opposite direction of the change in  $q$ . Let's analyze the overall effect on  $p^d$ . It is easy to see that a higher  $\phi'$  reduces  $p^d$ . This means that higher  $\phi'$  are associated with lower intermediary profits, the ordering we want. If we inspect what happens as  $\omega^*$  increases, we find that  $p^d$  is potentially non-monotone in  $\omega^*$ . Hence, the functions  $p^d$  and  $p^s$  are the analogues of the demand and supply schedules in Section 3. Just as in that section, the

demand schedule potentially features a backward-bending portion. What we want to do next is to characterize those curves, but now in terms of  $\kappa'$ .

**Pooling Market Price Function.** If we substitute the bankers' and k-producers' first-stage policy rules we obtain the first-stage market clearing :

$$\underbrace{Q^*(X) \int_0^\infty n'(n, X) d\Lambda(n)}_{\text{Demand by Bankers}} = \underbrace{\omega^*(X) \int_0^\infty k\Gamma^k(dk)}_{\text{Supply by k-producers}},$$

so by definition of  $\kappa$ , this condition equals:

$$(8) \quad \omega^*(X) = \frac{\kappa'}{\pi} Q^*(X).$$

We want to obtain a supply schedule as a function of  $\kappa'$ . For this, observe that  $p$  determines  $\omega^*(X)$  and  $Q^*(X)$  through the k-producer and banker problems. Consequently, (8) is an implicit equation in  $p$  whose solutions are indexed by  $\kappa'$ . In general, there may be multiple solutions to that equation. A refinement is needed.

**Pooling Price Multiplicity.** Models that feature asymmetric information often feature multiplicity of equilibrium prices. This one is no exception. Multiplicity arises here because as the pooling price increases,  $\omega^*$ , both the average quality and the quantity of capital traded increase. We already saw that this causes  $p^d$  to be non-monotone, and this implies that worst-case profits can also be non-monotone. Ergo, a single level of financial risk capacity,  $\kappa'$ , can support worst-case losses associated with two different volumes of intermediation  $\omega^*$ .

Although price multiplicity is an interesting phenomenon, it is not my focus. To simplify things, I restrict attention to the highest price-and-volume equilibria. That is, I select the highest  $\{p, \omega^*\}$  pair whose worst-case losses do not exceed that  $\kappa'$  and that deliver a non-negative arbitrage. The volume is the equilibrium  $\omega^*(X)$  and, thus,  $p(X) = p^s(\omega^*(X))$ .

**Profits from Financial Intermediation.** By definition, equilibrium marginal profits are:

$$(9) \quad \Pi(X, X') = p^d(X, X) - p^s(\omega^*(X)).$$

Because  $\omega^*$  is increasing in  $\kappa'$ ,  $p^d$  can be non-monotone in  $\kappa'$  and so can  $\Pi$ . We summarize this observation by:

**Proposition 7** *For sufficiently smooth  $\lambda(\omega, \phi')$ ,  $\Pi(X, X')$  is non-monotone in  $\kappa'$ .*

The behavior of  $\Pi$  is the heart of the model. Without asymmetric information,  $\tilde{v}(X)$  is necessarily decreasing in  $\kappa'$ . Under asymmetric information, a non-monotonicity of  $\Pi$  may be inherited by  $\tilde{v}(X)$ . The shape of  $\tilde{v}(X)$  is critical to generate rocking-boat dynamics.

### 5.3. Evolution of Financial Risk Capacity

**Second-Stage Evolution of Financial Risk Capacity.** Between the first and the second stages,  $\kappa'$  evolves depending on realized profits and the growth rate of the capital stock:

$$\kappa'' = \frac{[1 + \Pi(X, X') Q^*(X)]}{\gamma(X, X')} \kappa'.$$

The numerator is the growth of the net worth of banks. The denominator is the growth rate of the aggregate capital stock:

$$\gamma(X, X') = \pi \underbrace{\beta(p(X) + \mathbb{E}[\lambda(\omega, \phi') | \omega < \omega^*(X), \phi']) \omega^*(X)}_{\text{k-producer capital growth rate}} + (1 - \pi) \underbrace{\beta(A/q(X, X') + \bar{\lambda}(\phi'))}_{\text{c-producer capital growth rate}}.$$

Aggregate investment is the difference between the k-producer's desired t+1 capital holdings, and the amount they did not sell to banks:

$$I(X, X') = \beta p(X) \omega^*(X) \pi K.$$

Recall that the only source of persistent growth is capital accumulation. Since  $I(X, X')$  depends only on  $\kappa'$ , this shows that the financial risk capacity governs the dynamics of investment and growth.

**First-Stage Evolution of Financial Risk Capacity.** At the beginning of the first stage, the economy arrives with some  $\kappa$  inherited from the second stage of the previous period. Depending on the financial policies of banks,  $\kappa$  evolves within period according to:

$$(10) \quad \kappa' = (1 + e^*(X) - d^*(X)) \kappa.$$

Equation (10) links past with current states and determines whether banks will be recapitalized in a given period. A fundamental aspect of this equation is that it is implicit in  $\kappa'$  because the injection\dividend policies of bankers depend on  $\kappa'$  and vice versa. At this point, we know that  $\{e^*(X), d^*(X)\}$  must deliver  $\tilde{v}(X) \in [1 - \tau, 1]$  for any  $X$ . If it were the case that  $\tilde{v}(X) > 1$  for some  $\kappa'$ , bankers would want to inject equity. But this implies that  $\kappa'$  cannot occur in equilibrium. On the contrary, if the economy is at a state where  $\tilde{v}(A \times \phi \times \kappa') > 1$ ,  $\kappa'$  should immediately reflect to some other level  $\tilde{\kappa}' > \kappa'$  with  $\tilde{v}(A \times \phi \times \tilde{\kappa}') = 1$ . The opposite occurs if  $\tilde{v}(X) < (1 - \tau)$ . That state cannot occur in equilibrium. Any such  $\kappa'$  should immediately reflect to some other state  $\tilde{\kappa}' < \kappa'$  where  $\tilde{v}(A \times \phi \times \tilde{\kappa}') = 1 - \tau$ .

We can use equation (10) to obtain the admissible equilibrium values of  $\kappa'$  given some inherited  $\kappa$  from the past. A major message from this paper is that without asymmetric information,  $\tilde{v}(X)$  is necessarily decreasing in  $\kappa'$ . If this is the case, the solution to  $\{e^*(X), d^*(X)\}$  implies that the evolution of financial risk capacity is unique and stable. Instead, in the presence of strong adverse

selection effects,  $\tilde{v}(X)$  is not necessarily monotone and this implies that the economy may be unstable and may feature multiple equilibrium paths for  $\kappa$ .

To understand the source of multiplicity, let's suppose the economy arrives at a certain period with a given  $\kappa$ . By Proposition 5, we know that equity injections (dividends) are indeterminate at the individual level if  $\tilde{v}(X) = 1$  ( $\tilde{v}(X) = 1 - \tau$ ). If  $\tilde{v}(X)$  is non-monotone, it is possible to find two—or more—levels of financial risk capacity,  $\kappa^H$  and  $\kappa^L$ , such that  $\kappa^H > \kappa^L \geq \kappa$  and  $\tilde{v}(A \times \phi \times \kappa^L) = \tilde{v}(A \times \phi \times \kappa^H) = 1$ . Clearly, both  $e^*(A \times \phi \times \kappa^L) = \kappa^L/\kappa - 1$  and  $e^*(A \times \phi \times \kappa^H) = \kappa^H/\kappa - 1$  are equilibrium injection policies. The converse is true about dividends: if two levels of financial risk capacity lower than  $\kappa$  satisfy  $\tilde{v}(X) = 1 - \tau$ ,  $\kappa$  may decrease to either level during the first stage. The only thing determined in equilibrium is that injections should jump to a higher equity and must satisfy  $\tilde{v}(X) = 1$ —and dividends to a lower level and  $\tilde{v}(X) = 1 - \tau$ . Of course, multiplicity can occur only if  $\tilde{v}(X)$  crosses 1 or  $(1 - \tau)$  more than once and for this we need adverse selection effects. If this is the case, multiple paths for  $\kappa$  may be consistent with equilibrium.

**Equilibrium Selection.** One possibility is to introduce a sunspot variable, but I will take a different route. For the rest of the paper, I study a specific equilibrium that I refer to simply as *the equilibrium*. In particular, I focus on the equilibrium that conditions equity injections and dividends on the value of  $\kappa$  inherited from the past.<sup>15</sup> Under this refinement, the solution to the model satisfies:

**Solution 1** *If  $\tilde{v}(A \times \phi \times \kappa) \in (1 - \tau, 1)$ , then  $e^*(X) = d^*(X) = 0$ . If  $\tilde{v}(A \times \phi \times \kappa) > 1$ , then  $e^*(X)$  takes  $\kappa'$  to the closest value greater than  $\kappa$  where  $\tilde{v}(A \times \phi \times \kappa') = 1$ . If  $\tilde{v}(A \times \phi \times \kappa) < (1 - \tau)$ , then  $d^*(X)$  takes  $\kappa'$  to the closest value of  $\kappa'$  lower than  $\kappa$  such that  $\tilde{v}(A \times \phi \times \kappa') = 1 - \tau$ .*

Under this refinement, coordination failures occur in states where bankers fail to inject equity to the highest  $\kappa'$  where  $\tilde{v}(A \times \phi \times \kappa') = 1$  simply because they observe a current low  $\kappa$ . This coordination failure captures a force discussed in the introduction. It is possible that bankers do not inject equity into their banks despite having the resources. I will discuss why this equilibrium is particularly important.

**Equity Value.** Once we know the transition function from  $X$  to  $X''$  within the period, we obtain a recursive expression for  $\tilde{v}(X)$  and  $v_1^f(X)$ :

$$(11) \quad v_1^f(X) = \min \left\{ \max \left\{ \hat{\beta} \mathbb{E} [\tilde{v}(X) | X], (1 - \tau) \right\}, 1 \right\}.$$

The definitions of  $\tilde{v}(X)$  and  $v_2^f(X)$  define a self-map for  $v_1^f(X)$ . This closes the model. We now define a financial crisis.

#### 5.4. Discussion - Equilibrium Selection

Legend has it that during the Panic of 1907, J.P. Morgan locked some of the most prominent Wall Street bankers inside his library in Manhattan (see [Chernow, 2010](#)). He let them go, the

<sup>15</sup>[Szkup \(2014\)](#) incorporates global games into a sovereign debt model and finds equilibria where investors coordinate their actions based on inherited state variables.

story goes, only after they finally agreed on a rescue package for troubled financial institutions.<sup>16</sup> True or not, there is a moral to the story. The selected equilibrium is a representation of what can go wrong if bankers don't coordinate equity injections after they suffer losses: this equilibrium delivers catastrophic and persistent declines in intermediation and economic growth, the hallmarks of a deep financial crisis. From a planner's point of view, this outcome is inefficient and calls for the intervention of a big player like the government or J.P. Morgan. This is why this selection is particularly interesting.<sup>17</sup>

Furthermore, this selection is natural. In this version of the model, good equilibria exist only because I assumed bankers have extremely large endowments. The selected equilibrium would be unique if endowments were finite or if bankers faced convex equity costs. Also, bankers coordinate their policies on an observed variable. This coordination would emerge in equilibrium if bankers faced uncertainty about the endowments of others (e.g., [Szkup, 2014](#)). Overall, the equilibrium selection allows me to obtain rocking-boat dynamics and their policy implications with a minimum set of ingredients.

Coordination failures have a tradition in macroeconomics. Models like those of [Cooper and John \(1988\)](#) and [Kiyotaki \(1988\)](#) feature physical investment complementarities that open the door to similar coordination failures. In models like that of [Cole and Kehoe \(2000\)](#), international investors may fail to coordinate in rolling over the debt of small economies and cause sovereign default crises.

### 5.5. *States of the Financial Industry*

In any RCE, the state space may be divided into four regions. These regions are defined through  $\tilde{v}$ .

**Dividend-Payoff Reflecting Barrier.** When  $\tilde{v}(X) < (1 - \tau)$ , dividend payments instantaneously reduce  $\kappa$  to its closest reflecting barrier, the closest  $\kappa$  that satisfies  $\tilde{v}(X) = (1 - \tau)$ . A dividend-payoff region associated with large  $\kappa$  always exists.<sup>18</sup> However, the following examples show that there may be dividend-payoff regions for intermediate values of  $\kappa$  when  $\Pi$  is non-monotone.

**Equity-Injection Reflecting Barrier.** When  $\tilde{v}(X) > 1$ , equity injections reflect  $\kappa$  toward the closest value such that  $\tilde{v}(X) = 1$ . When  $\Pi$  is non-monotone, there may also be multiple equity-injection barriers.

**Competitive Financial Inaction Region.** There are two other regions characterized by financial policy inaction. The first is a competitive inaction region defined as follows:

**Definition 2** (Competitive Inaction Region) *A state  $X$  is in a competitive inaction region if (a)  $\tilde{v}(X) \in [1 - \tau, 1]$ , and (b)  $\tilde{v}_{\tilde{\kappa}}(X) \leq 0$  for  $\tilde{\kappa} > \kappa$  when  $\tilde{v}_{\kappa}(X)$  is defined.*

Condition (a) says that there is equity/dividend policy inaction in this region. Condition (b)

<sup>16</sup>This passage was written in that library.

<sup>17</sup>Of course, whether a given equilibrium is realistic is an empirical question. Recent work by [Passadore and Xandri \(2014\)](#) describes a procedure to use data to discard equilibria in models that feature multiplicity.

<sup>18</sup>Expected profits must be decreasing for large enough  $\omega^*$ : the substitution effect always dominates the quality effect for  $\omega^*$  approaching 1.

implies that the expected discounted marginal profits are decreasing from that  $\kappa$  and onwards. This means that the incentives to recapitalize banks decrease with  $\kappa$ . This is the only outcome without asymmetric information.<sup>19</sup>

**Financial Crisis Inaction Region.** The complement regions are financial crisis regions. In a financial crisis,  $\kappa$  and  $\Pi$  are low. Low expected profits in these regions discourage equity injections. However, there are  $\kappa$  outside those regions for which  $\tilde{v}$  is higher. This means that within these regions, bankers lack incentives to recapitalize banks individually, but it could be profitable to coordinate to drive  $\kappa$  —and the economy— out of that region.

### 5.6. Solving Equilibria

This section outlines the strategy to compute equilibria. The solution involves two steps. The first is to compute first- and second-stage prices and expected profits for any —possibly off-equilibrium— volume of intermediation given exogenous states  $(A, \phi, \phi')$ . The second step uses these calculations to find *equilibrium* volumes. With this, we can compute  $\tilde{v}(X)$  and the regions of the state space.

**Notation.** Bold letters distinguish equilibrium from off-equilibrium objects. I use  $\mathbf{p}(\omega, \phi)$  to indicate the first-stage supply schedule given  $\phi$  and an off-equilibrium value of  $\omega$ . I use  $\mathbf{q}(\omega, \mathbf{p}, A, \phi')$  to denote a price consistent with second-stage market clearing given  $(\omega, \mathbf{p}, A, \phi')$ . Finally,  $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$  are the corresponding profits given arbitrary prices, volumes, and exogenous states. Thus,  $\mathbf{p}$  and  $\mathbf{q}$  are consistent with the producers' optimal decisions but may violate the bankers' optimality or limited liability.

**Step 1: Off-equilibrium Qualities, Prices, and Profits.** Through Proposition 3, we obtain  $\mathbf{p}(\omega, \phi)$  through the inverse of  $\omega^*$ , the solution to the k-producer's portfolio problem. Through Proposition 6 we obtain  $\mathbf{q}(\omega, \mathbf{p}, A, \phi')$  for any  $(\omega, \mathbf{p}, A, \phi')$ . This is enough to compute  $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$ . This step is carried out once.

**Step 2.1: Equilibrium Volumes, Prices, and Profits.** Guess a function  $\tilde{v}(X')$ . After Step 1, we can find the *equilibrium* volume of intermediation. For each  $X$ , we look for the largest  $\omega$  among the set that yields non-negative expected discounted profits and losses of at most  $\kappa$  :

$$\omega^*(X) = \max \left( \omega : \kappa \leq \min_{\phi} \mathbf{\Pi}(\omega, \mathbf{p}, A, \phi') \omega \text{ and } \mathbb{E}[\tilde{v}(X') \mathbf{\Pi}(\omega, \mathbf{p}, A, \phi') | X] \geq 0 \right).$$

Since  $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi')$  is continuous and  $\omega \in [0, 1]$ , this quantity is well defined.

**Step 2.2: Equilibrium  $\tilde{v}(X)$ .** Given this  $\omega^*(X)$ , we compute

$$\Pi(X, X') = \mathbf{\Pi}(\omega^*(X), \mathbf{p}(\omega^*(X), \phi), A, \phi').$$

We use (11) to update  $\tilde{v}(X)$ . Steps 2.1 and 2.2 are iterated until convergence. Appendix A provides

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<sup>19</sup>In equilibrium  $\tilde{v}(X)$  can potentially feature jumps as a consequence of the highest-price equilibrium refinement. The condition is equivalent to having  $v(X)$  being decreasing above  $\kappa$ , except at the jumps. This condition captures the idea that the quantity effect dominates the quality effect from that level on.

the details.

## 6. ANALYTIC EXAMPLES

This section presents two analytic examples. Their contrast illustrates how equity injections and dividends are stabilizing forces in an economy where intermediation is essential for growth but not under asymmetric information. Only for this section, I assume  $\rho = 1$ .

### 6.1. Example 1 - Risky intermediation without asymmetric information.

The first example is an economy in which financial intermediation is risky, but asymmetric information is not present. Assume that  $\lambda(\omega, \phi) = \lambda^*(\phi)$ , —so all units are of the same quality, but that  $\lambda^*(\phi)$  is decreasing in  $\phi$ . To simplify things, let  $\phi$  take only two values,  $\phi_B > \phi_G$ . All draws are i.i.d. and  $A$  is a constant. Under this example, we have the following:

**Proposition 8** *In any economy without asymmetric information,  $\kappa'$  fluctuates within a unique equilibrium interval  $[\underline{\kappa}, \bar{\kappa}]$ . If  $\kappa \leq \underline{\kappa}$ , then  $e^*(X)$  is such that  $\kappa' = \underline{\kappa}$ . If  $\kappa \geq \bar{\kappa}$ ,  $d^*(X)$  is such that  $\kappa' = \bar{\kappa}$ .  $v(X)$  is decreasing and  $\omega(X)$  is increasing in  $\kappa$ .*

PROOF: From Proposition 6, we know that  $\Pi(\omega, \mathbf{p}, \phi)$  is decreasing in  $\omega$  since quality effects are not present without asymmetric information. Also, as noted earlier,  $\rho = 1$ ,  $v_2^f(X) = (1 - \tau)$ . We can use  $\Pi(\omega, \mathbf{p}, \phi)$  and equation (6) to obtain an expression for the marginal value of equity in terms of any arbitrary  $\omega$ . Call that value  $\tilde{\mathbf{v}}(\omega, \mathbf{p})$ . Without asymmetric information,  $\tilde{\mathbf{v}}(\omega, \mathbf{p}, A, \phi)$  is decreasing in  $\omega$ . Consequently, by Proposition 5, there is a unique interval for  $\omega$  such that  $\tilde{\mathbf{v}}(\omega, \mathbf{p}) \in [(1 - \tau), 1]$ . Correspondingly, since  $\Pi$  is decreasing in  $\omega$ , there is a unique equilibrium interval for  $\kappa$  that determines a unique competitive inaction region. *Q.E.D.*

Figure 4 shows a graphic construction of equilibria. The upper-left panel depicts four curves associated with an arbitrary  $\omega$ . These correspond to the capital-supply schedule,  $\mathbf{p}(\omega, \phi)$ , the marginal value of bank assets in good and bad states,  $\mathbf{q}(\omega, \mathbf{p}, \phi_H) \lambda^*(\phi_H)$  and  $\mathbf{q}(\omega, \mathbf{p}, \phi_L) \lambda^*(\phi_L)$ , and their expected value  $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi) \lambda^*(\phi)]$ . The difference between  $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi) \lambda^*(\phi)]$  and  $\mathbf{p}(\omega)$  is the expected marginal profits of banks. Multiplying this amount by  $\omega$  yields the total expected bank profits  $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi) \lambda^*(\phi) - \mathbf{p}(\omega)]\omega$  normalized by the capital stock. Total expected bank profits are plotted in the bottom-left panel. The bottom-right panel plots the worst-case profits,  $[\mathbf{q}(\omega, \mathbf{p}, \phi_L) \lambda^*(\phi_L)] - \mathbf{p}(\omega)]\omega$ . In equilibrium,  $\kappa$  must be sufficient to sustain the losses induced for the corresponding volumes of intermediation. The top-right panel plots the expected value of bank equity  $\tilde{\mathbf{v}}(\omega, \mathbf{p})$  as a function of  $\omega$ . The horizontal lines in the top-right panel are the marginal costs of injecting equity, 1, and the marginal benefit of dividend pay-offs,  $(1 - \tau)$ . In equilibrium, if a given  $\omega$  is indeed an equilibrium volume of intermediation, bankers must not alter their net worth for those levels of expected profits. Thus, the set of possible equilibrium  $\omega$  is characterized by volumes for which the value of equity falls within the marginal cost of injections and the benefit of dividend payments. The shaded areas of the graphs correspond to this set. Since  $\tilde{\mathbf{v}}(\omega, \mathbf{p})$  is

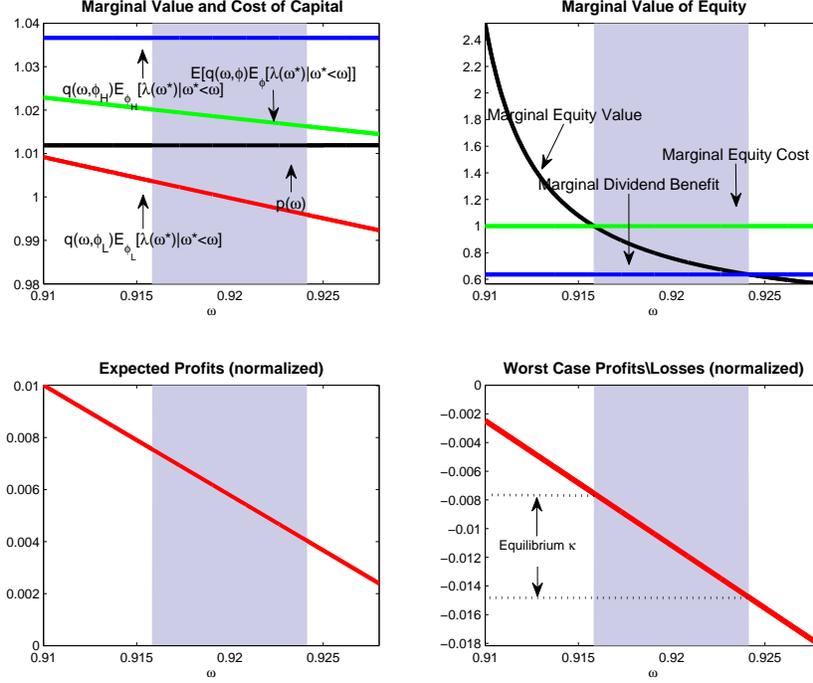


Figure 4: Model without Asymmetric Information  
Equilibrium objects as functions of  $\omega$ .

decreasing in  $\omega$ , the equilibrium set is a unique interval. For each  $\omega$  in that interval, there is an equilibrium  $\kappa$  corresponding to it. We obtain this equilibrium set by computing the maximal losses given each  $\omega$  in the equilibrium set. The bottom-right panel shows this interval for  $\kappa$  is obtained as the image of worst-case losses for the equilibrium  $\omega$ -set.

Figure 5 plots four equilibrium objects. The top-left panel plots  $\omega^*$  as a function of  $\kappa$ —that is,  $\kappa$  before equity injections or dividends. The top-right panel depicts  $\tilde{v}$ . In equilibrium,  $\kappa'$  must be within the inaction region where  $\tilde{v}(\kappa) \in [(1 - \tau), 1]$ . The bottom panel depicts equity injections, dividends, and  $\kappa'$  as functions of  $\kappa$ . In equilibrium,  $e$  and  $d$  adjust to bring  $\kappa'$  to the equilibrium set depicted in Figure 4. The shaded area of the figure is the competitive inaction region. The regions to the right and left of the shaded areas are the dividend payoff and equity injection regions, respectively.

**Dynamics.** Proposition 8 is useful to understand the dynamics of this economy. Recall that, in equilibrium, worst-case losses are always negative. In contrast, expected profits must be non-negative; otherwise, no intermediation would be provided. When  $\phi_B$  is realized, profits are negative and drag down  $\kappa$ . Below  $\underline{\kappa}$ , high expected profits attract equity injections that recapitalize banks and increase the financial risk capacity back to  $\underline{\kappa}$ . Thus, injections stabilize a financial system with low financial risk capacity. When  $\phi_G$  is realized, positive profits increase  $\kappa$ . When  $\kappa$  increases beyond  $\bar{\kappa}$ , dividend payoffs reflect the financial risk capacity downward. Hence, without asymmetric information,  $\kappa$  fluctuates within a unique interval. The next example allows for asymmetric information

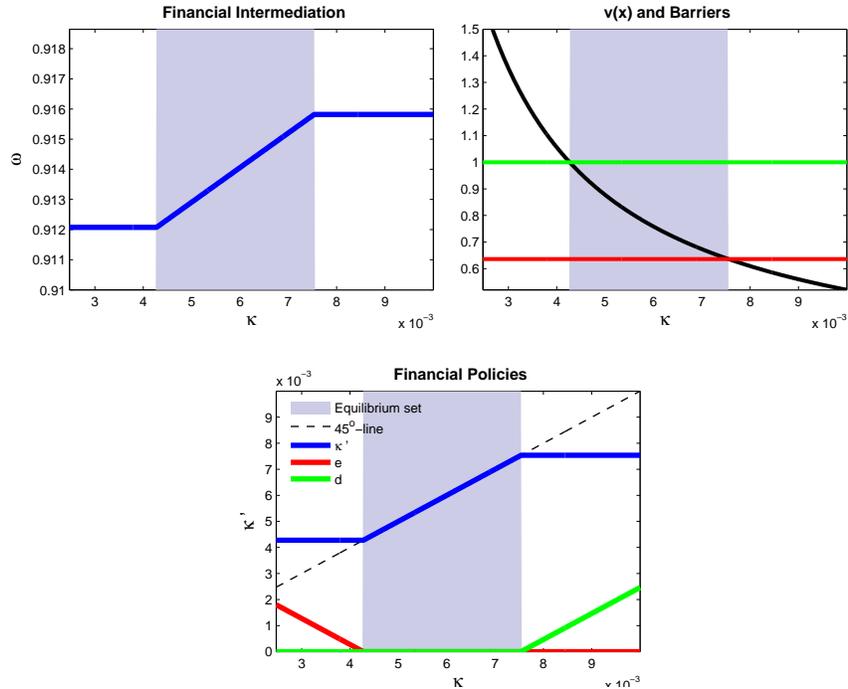


Figure 5: Intermediation without Asymmetric Information  
Equilibrium objects as functions of  $\kappa$ .

and shows how this precludes this stabilizing force.

## 6.2. Example 2 - Risky intermediation with asymmetric information.

In this example, I slightly modify  $\lambda(\omega, \phi)$  to introduce asymmetric information. I fix values for the lower and upper bounds of  $\lambda(\omega, \phi)$ ,  $\lambda_L$  and  $\lambda_H$ . Then, I adopt the following functional form for  $\mathbb{E}[\lambda(\omega, \phi) | \omega < \omega^*] = \lambda_L + (\lambda_H - \lambda_L) F_\phi(\omega^*)$  where  $F_\phi$  is the CDF of a Beta distribution. Thus,  $\phi$  indexes the parameters of that Beta-CDF function such that they satisfy Assumption 1. The rest of the calibration is the same as in the previous example. Because,  $\lambda(\omega, \phi)$  increases with  $\omega$ , this introduces asymmetric information and intermediation risk.

Figure 6 is the asymmetric-information analogue of Figure 4. The upper-left panel shows four curves that correspond to  $\mathbf{p}(\omega, \phi)$ ,  $\mathbf{q}(\omega, \mathbf{p}, \phi_H)$ ,  $\mathbf{q}(\omega, \mathbf{p}, \phi_L) \lambda(\phi_L)$ , and  $\mathbb{E}[\mathbf{q}(\omega, \mathbf{p}, \phi) \lambda(\phi)]$ . Note that  $\mathbf{q}(\omega, \mathbf{p}, \phi_H) \lambda(\phi_H)$  and  $\mathbf{q}(\omega, \mathbf{p}, \phi_L) \lambda(\phi_L)$  are no longer decreasing in  $\omega$ . As volumes increase, the price of capital falls, but the quality improves. The relative strength of either effect governs the shape of the value of the bank's asset position. These forces cause total expected and worst-case profits to be non-monotonic —see the bottom-left and right panels. The same levels of worst-case losses can result in multiple values of  $\omega$ . This implies that a given  $\kappa$  can possibly sustain multiple levels of intermediation. The highest-price refinement implies that, in equilibrium,  $\omega$  is the highest amount of intermediation consistent with bank optimality conditions and the capacity constraint. The top-right panel plots the marginal value of bank equity. There is a difference in the shape of

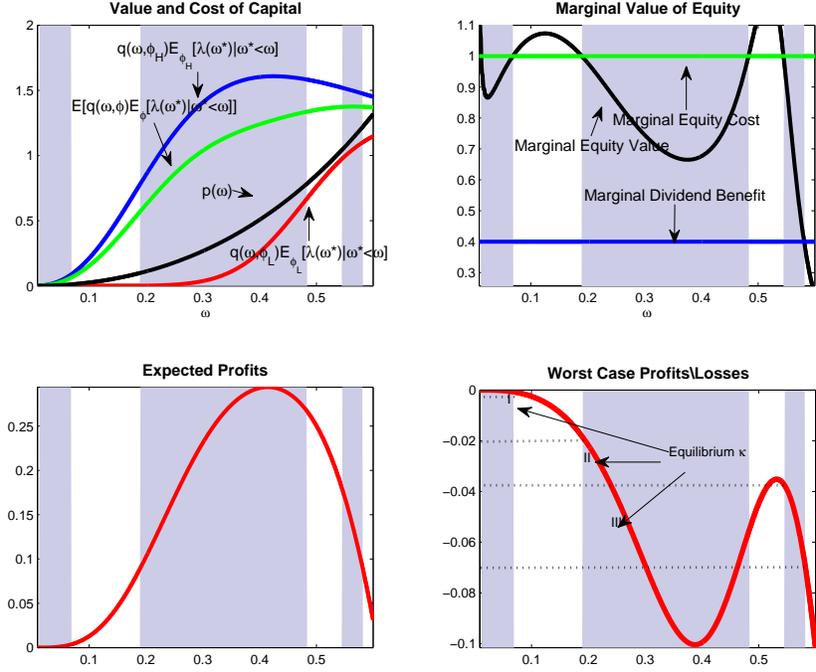


Figure 6: Model with Asymmetric Information  
 Equilibrium objects as functions of  $\omega$

the value of bank equity when asymmetric information is present than without it. With asymmetric information, the marginal value of equity inherits the non-monotonic behavior of profits. Once again, the horizontal lines correspond to the marginal cost of equity and the marginal benefit of dividends. Thus, the non-monotonic profits function leads to multiple inaction regions. In particular, for low levels of intermediation, expected profits are low and equity injections are no longer profitable.

The dynamics in the example are much richer than before. Here, there are three equilibrium inaction intervals identified by the shaded areas. The equilibrium intervals for the financial risk capacity are obtained as the image of the worst-case losses for each equilibrium  $\omega$ -interval. The upper bound of interval II in the figure has a distinctive property: If  $\kappa$  increases slightly at that point, the financial risk capacity can support a much larger volume of intermediation. This happens because there is a sharp increase in intermediation that slightly increases worst-case losses.

Interval III is a financial crisis regime. It is associated with low levels of financial intermediation. This region is an inaction region since bankers do not inject equity here. Note that for larger values of  $\omega$ , equity injections are profitable since  $\tilde{v}$  is above 1. This underscores the nature of the coordination failure faced by banks. Banks choose to maintain their net worth at current levels and engage in less intermediation. Although they are not affected, the economy suffers.

Figure 7 plots the equilibrium objects as functions of  $\kappa$ . The upper-left panel plots the equilibrium financial intermediation. We can observe a discrete jump in the volume of intermediation between the second and the first region. This jump occurs because a slightly larger  $\kappa$  can support a much

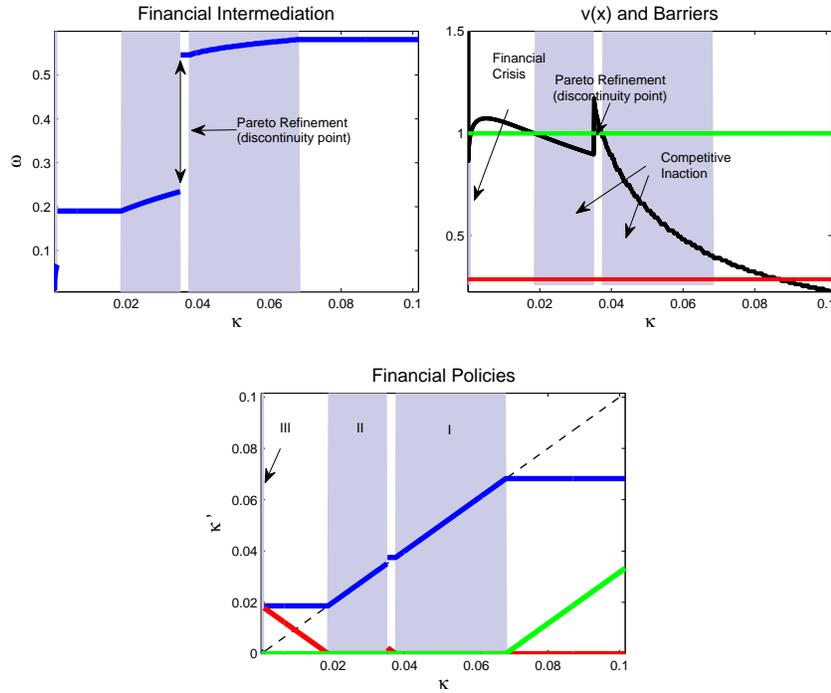


Figure 7: Model with Asymmetric Information  
Equilibrium objects as functions of  $\kappa$ .

larger volume of intermediation and the highest-price refinement selects the equilibrium with the largest volumes. Equity injections between regions I and II are very small: regions I and II are close to each other. Note that to the left of the second region, the marginal value of equity  $\tilde{v}$  is increasing in  $\kappa$ . The financial crisis regime is very small because the volume of intermediation and losses associated with it are also very small. Through this example we have shown:

**Proposition 9** *For sufficiently severe asymmetric information —when  $F_\phi(\cdot)$  varies sufficiently— the return to financial intermediation is non-monotone in  $\kappa$  and there exists a financial crisis regime.*

**Dynamics.** The immediate effects of  $\phi$  on  $\kappa$  are the same as those without asymmetric information. However, the dynamics of the economy are very different. With asymmetric information, a realization of  $\phi = \phi_B$  can drive the financial system to a financial crisis regime. As adverse selection is exacerbated, profitability no longer provides the incentives for capital injections into the banking system. In crisis regimes, the economy may take long to recover so it features rocking-boat dynamics. The economy eventually recovers as banks slowly rebuild their equity through retained earnings. Once  $\kappa$  reaches the equity injection regions between intervals III and II, the banking system attracts new equity injections and jumps immediately to the competitive inaction region II. The following section develops some numerical examples in more depth.

## 7. NUMERICAL EXAMPLES

### 7.1. *Additional Features*

This section presents numerical examples that highlight the model’s non-linear dynamics. I add three additional features. First, banks now pay an operating cost equal to a constant portion  $\psi$  of their equity every period. Also, the technical rate of transformation now differs from 1. Finally, I introduce an equity buffer into the LLC, which now reads:  $0 \leq (1 - \theta) n' + \Pi(X, X') Q$ . I vary  $\theta$  to study capital requirements.

### 7.2. *Parameter Values*

I set the values of  $\beta$  and  $\hat{\beta}$  so that the annualized risk-free rate in the deterministic equilibrium of the model is 3.0%. I set  $\bar{\lambda}(\phi) = \bar{\lambda}$  where  $\bar{\lambda}$  is a constant consistent with an average annual depreciation of the total capital stock of 10%. I set the fraction  $\pi$  to be 0.1 as in [Bigio \(2014\)](#) and the cost of equity,  $\tau$ , to be consistent with the estimates in [Hennessy and Whited \(2005\)](#). I set  $\theta$  to 8% to approximate the capital requirements under Basel-II and  $\psi$  to obtain an average operating cost of 8% of equity. I estimate an  $AR(1)$  process for the  $\log(A_t)$  with mean, lag coefficient, and standard deviation denoted by  $\{\mu_A, \rho_A, \sigma_A\}$ . This process is independent of  $\phi_t$ .

The challenge is to calibrate the process for  $\phi_t$ . I assume  $\phi_t$  follows a Markov chain that takes four values. I need at least four values because I want to analyze responses to big and small negative shocks. Hence, I need at least two shocks above a mean. I adopt the following functional form  $\lambda(\omega, \phi) = \bar{\lambda} F_\phi(\omega^*)$  where  $F_\phi(\omega^*)$  is the CDF of a Beta distribution with parameters  $\{A_\phi, B_\phi\}$ . Thus, for every  $\phi$  there are two values,  $A_\phi$  and  $B_\phi$ . I assume that those values are uniformly spaced between  $[A_L, A_H]$  and  $[B_L, B_H]$ , the only four parameters I have to calibrate  $\{A_L, A_H, B_L, B_H\}$ .

I set the values of  $\{A_L, A_H, B_L, B_H\}$  to obtain the averages of two variables in two regions of the state space —four moments. In particular, I focus on the mean for leverage and the return to bank assets (ROA) and their average values in crisis regions. Leverage and ROA are ideal to calibrate the model because those variables are directly obtained from the values of  $\lambda(\omega, \phi)$ . Recall that ROA equals  $\mathbb{E}[(p^d/p^s - 1) | X]$ . The prices that determine ROA,  $p^d$  and  $p^s$  are only functions of  $\{A_L, A_H, B_L, B_H\}$  and the transition probability of  $\phi$  once we fix the rest of the parameters. Similarly, leverage is the inverse of worst-case losses, which are only a function of  $\{A_L, B_L\}$ . Thus, we can set  $\{A_L, A_H, B_L, B_H\}$  to obtain leverage and ROA for crises and normal times.

The last objects to calibrate are the transition probabilities of  $\phi$ . In equilibrium, the highest two values of  $\phi$  yield equity losses for banks. I set the transition matrix for  $\phi$  to obtain losses once every 20 quarters with an i.i.d probability. Table I is the summary of parameter values.

### 7.3. *Results*

**Invariant Distributions and Moments.** Figure 8 shows four histograms. The two on the left report the invariant distributions of  $\kappa$  and  $\Delta\%K$  in the model. For comparison, the ones on the right

TABLE I  
PARAMETER VALUES

Parameters	Value	Note										
Calibrated												
$\beta$	0.987259	3% annual time discount rate										
$\beta^f$	0.945742	3% annual time discount rate										
$\pi$	0.097342	Cooper et al. (1999)										
$\tau$	0.08	Hennessy and Whited (2005)										
$\psi$	0.08	Bank non-interest expense per net worth										
$\theta$	0.08	Basel-II capital requirements										
Estimated												
$\mu_A$	-0.885	Estimated TFP process										
$\rho_A$	0.993	Estimated TFP process										
$\sigma_A$	0.0083	Estimated TFP process										
Matched												
$A_L$	3.9	To match bank ROA and leverage										
$A_h$	4	To match bank ROA and leverage										
$B_L$	6.2	To match bank ROA and leverage										
$B_H$	5.2	To match bank ROA and leverage										
$\Phi$	<table border="1" style="display: inline-table; vertical-align: middle;"> <tr> <td>0.95</td> <td>0.95</td> <td><math>\otimes</math></td> <td>0.95</td> <td>0.95</td> </tr> <tr> <td>0.05</td> <td>0.05</td> <td></td> <td>0.05</td> <td>0.05</td> </tr> </table>	0.95	0.95	$\otimes$	0.95	0.95	0.05	0.05		0.05	0.05	To match banks' profit transition matrix
0.95	0.95	$\otimes$	0.95	0.95								
0.05	0.05		0.05	0.05								

show the analogues for US data. Bars represent occupation frequencies and the portion painted red in the model's histograms is concentration mass at crisis regions. A salient feature is that occupation times are bimodal. As we read through these histograms, we find that most occupation times occur for high values of  $\kappa$ , but there is a concentration mass for low values of  $\kappa$  coming from crisis regions. The high occupation times from crises arise because exit times are long and arrival rates to that region are high.

Contrasting the model's invariant distribution with those obtained in **HK** and **BS** exposes the different mechanisms at play. In **HK**, the invariant distribution of the analogue of  $\kappa$  is bell shaped. In that model, recoveries out of low  $\kappa$  states are fast because intermediaries become extremely profitable in crises. Banks become profitable because they substitute equity for cheap debt.<sup>20</sup> In **BS**, the corresponding invariant distribution is bimodal, as here. In **BS**, banks cannot substitute debt for equity after banks suffer losses, as in **HK**. This distinction is crucial because bank balance sheets must shrink and intermediaries must fire sell assets after they lose equity. Hence, the bimodal distribution of  $\kappa$  follows from this effect. Nevertheless, intermediation margins are also high during crises, just as in **HK**. If there was free entry into the banking system in **BS**, we would see banks being recapitalized and quick recoveries. This force is different from the one that operates in this

<sup>20</sup>That model distinguishes between inside and outside bank equity. Outside equity is limited by a multiple of inside equity. In addition, there are no restrictions on debt issuances. There, the capital stock is fixed and managed only by bankers. As a result, when intermediaries have low equity, outside equity is substituted by debt. These features imply that intermediaries make higher margins for the same assets under management.

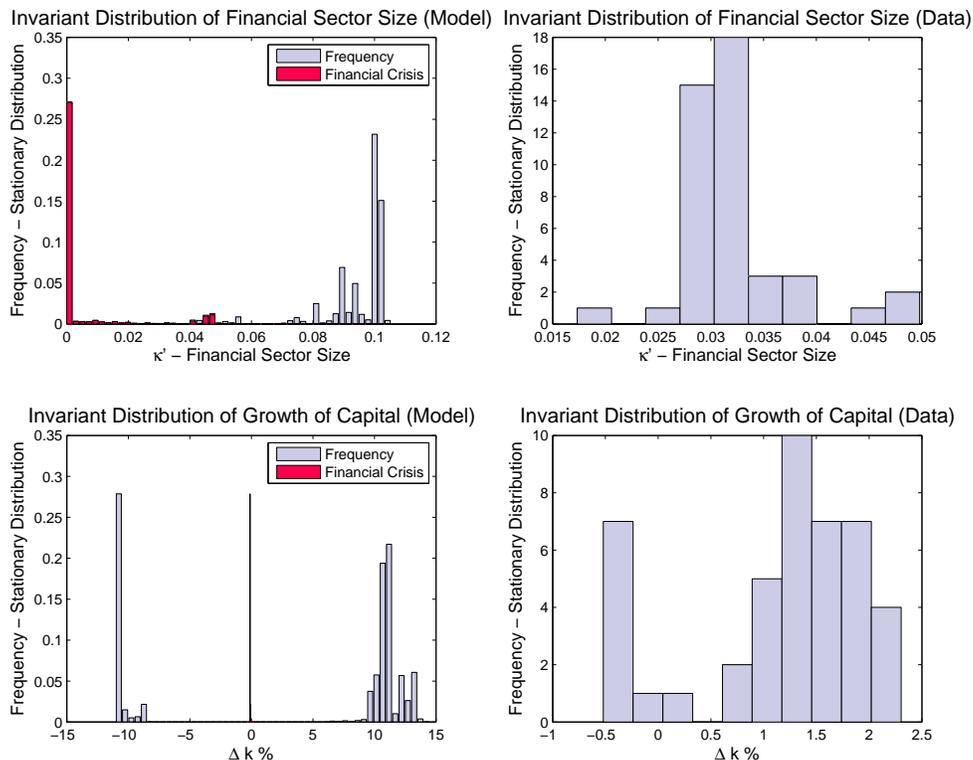


Figure 8: US Banking Indicators and Capital Accumulation.

paper. Here, recoveries take long despite free entry because profitability is low during crises.

Let's now turn to the unconditional and crisis moments reported in Table II. The changes from unconditional moments to crisis moments have the hallmarks of historical financial crises accounts. However, it is clear that the model presents an exaggerated version of reality. A first set of moments reports occupation times: 32.6% of the time the economy is in crisis. [Kindleberger \(1996\)](#) notes that banking crises occurred once every seven years between 1800 and 1990, and more recent work by [Reinhart and Rogoff \(2009\)](#) places that figure at 13% of the years during the national banking era. In the model, exit times from the typical crisis state take 10 quarters. [Reinhart and Rogoff \(2009\)](#) also calculates that countries in its sample took up to a decade to recover from banking crises. The recovery of growth rates during the Great Recession was short in comparison —six quarters— but there there hasn't been a recovery to the pre-crisis trend level.

A second set of moments summarizes economic activity. Average economic growth is close to the historical US growth rate of 2.5% in the model. However, during crises, growth falls to  $-10.0\%$ , a dramatic drop compared to the Great Recession, but not at all in comparison to other international banking crises. For example, [Cerra and Saxena \(2008\)](#) documents that growth fell to  $-8\%$  in a cross-country average of crises. Here, growth follows directly from a decline in investment. Work by [Christiano et al. \(2009\)](#) and [Justiniano et al. \(2010\)](#) finds that investment-specific shocks are important business cycle drivers and interprets these shocks as financial disruptions. In the model,

investment is also tied to capital reallocation. According to [Foster et al. \(2014\)](#), capital reallocation also fell dramatically during the Great Recession.

A third set of moments summarizes the behavior of financial intermediation. In the model, financial risk capacity during crises is about 8% of its unconditional average. We find a similar decline in intermediation measured as the value of capital purchases relative to output (loans and deposits). The historical study in [Jorda et al. \(2010\)](#) documents that this indicator indeed collapsed during many international episodes. The model also predicts spikes in spreads measured during crises. We can measure spreads as  $(q/p^s - 1)$ , the spread between the full information and pooling price of capital. Work by [Ordonez \(2013\)](#) documents long-lasting spikes in spreads in a cross-country study of recessions. We also find that leverage falls during crises, as in [Adrian and Boyarchenko \(2013\)](#). Now, as noted before, the model does not have a robust prediction about leverage.

A last set of moments is the profitability indicators. During normal times banks pay dividends because ROA and ROE are low. During the average crisis, banks no longer pay dividends because the ROA and ROE increases when bank capital is scarce. However, returns are not large enough to provide the incentives to recapitalize banks. We deduce this because the value of financial risk capacity,  $v(X)\kappa$ , falls almost in the same proportion as  $\kappa$ , and this means that  $v(X)$  does not increase enough to induce equity injections. The takeaway from Table II is that the model fits the crisis narratives found in [Mishkin \(1990\)](#) or [Calomiris and Gorton \(1991\)](#), where asymmetric information plays a critical role.

TABLE II  
MODEL MOMENTS

Variable	Unconditional	Crisis
Occupation Times		
Occupation Time	100%	32.6%
Duration (quarters)	-	10.26
Real Activity		
Average Growth Rate	4.3%	-10.3%
Investment/Output	39.9%	-0.935%
Financial Intermediation		
Average $\kappa$	0.0659	0.0048
Loans Output	6.68%	0.0832%
Financial Leverage	6.56	1.99
Financing Premia	39.5%	106%
Profitability Indicators		
Return on Assets	6.94%	16.9%
Return on Equity	31.3%	48.1%
Average Dividend Rate	0.643%	0.0193%
Financial Stocks Index	100%	8.31%

**Rocking-Boat Impulse Responses.** Figure 9 shows the responses to shocks to  $\phi_t$  of different magnitude. To compute responses, I initiate the system from a random state drawn from the

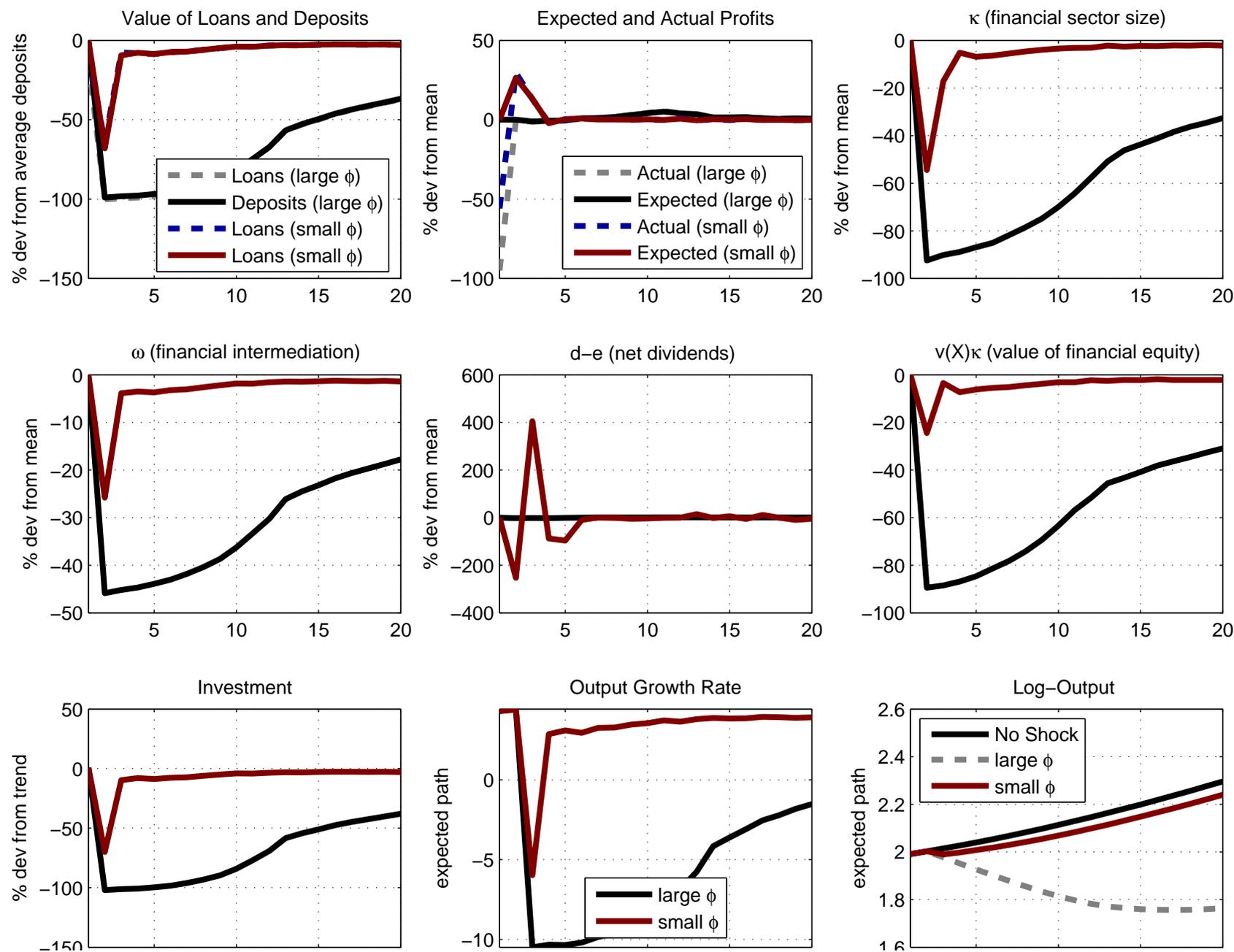
invariant distribution and perform 100,000 simulations. To calculate an impulse response, I simulate the model and compute the differences in the average paths with and without an initial impulse to  $\phi_t$  at time zero. For comparison, each panel shows the responses to the largest and second largest shocks to  $\phi$ .

Let's begin with the top row of Figure 9. The top-left panel shows that bank assets and liabilities (loans and deposits) fall with both shocks the period after the shock is realized. These movements capture the degree of financial disintermediation caused by the equity losses experienced at time zero. Evidently, the impact at  $t + 1$  is greater for the largest shock. What is striking is the difference in persistence: the response to the large shock is extremely persistent, although the shock itself has no memory! In contrast, financial intermediation returns almost to steady state only one period after the shock hits. The variables that we care about, namely, investment and output, follow the same pattern. Why are the dynamics so different after shocks of different size?

Several things occur in tandem, but we have prepared the groundwork to understand these dynamics. The top-middle panel shows realized and expected profits after the shocks. On impact, both shocks generate unexpected losses. However, note how expected profits rebound for the small shock the period after the shock hits. For the large shock, things are rather different: expected profits return to zero, but never undo the negative losses of the previous period. The behavior of profits translate to  $\kappa$ . In the top right, we observe that  $\kappa$  remains "locked" after the large shock whereas it rebounds after the small shock.

This striking difference follows from the interaction between the substitution and quality effects described earlier. Given that financial risk capacity falls with either shock, equilibrium volumes  $\omega^*$  must also fall for both shocks. This causes a simultaneous decline in volumes and qualities of intermediated assets. Now, for the large shock, the decline in  $\omega^*$  is so dramatic that it overcomes the substitution effect. This doesn't happen for the small losses and this is the root of the problem. Because the quality effect dominates with the large shock, marginal profits decline dramatically if equity injections do not compensate the equity losses —recall Figure (2b). This explains why expected profits rebound after the small shock, but they never do for the large shock.

Now let's observe the middle panel. We see very large equity injections after the small shock, but we don't see them for the large shock. The reason for this difference is the second mechanism; the low profits after the large shock lead to the failure to coordinate equity injections and vice versa. Hence, the drop in  $\kappa$  is contained by equity injections when the shock is small, but this stabilization mechanism breaks down for large shocks. The decline in the value of bank capital,  $v(X)\kappa$ , plotted in the middle right represents a crash in the market capitalization of banks, as in 1. Beyond this, we find severe effects on real economic activity shown at the bottom. The collapse in financial intermediation translates into an investment decline. Since growth is fueled only by investment, growth follows the same pattern. The long-lasting impact on growth after the large shock creates a large shift in the trend of output, a concurrent theme during the last crisis.

Figure 9: Impulse Response to Large and Small  $\phi$ .

## 8. FINANCIAL STABILITY POLICIES

**Two Externalities.** This section discusses the effects of capital requirements. Two externalities merit their use. First, when bankers purchase capital, they consider risks and rewards, but act as price takers. They fail to internalize that, on aggregate, they affect  $\kappa$ . Although this is also true in frictionless models, here  $\kappa$  affects prices. In turn, prices affect the LLC so there is a pecuniary externality. Hence, a planner that controls  $Q$  directly, but is subject to the same constraints and equity/dividend policies, would consider the law of motion of  $\kappa$  in his decision. In particular, the planner may want to limit intermediation to avoid large losses and low prices.

The second externality is produced by the coordination failures. Coordination failures occur only in crises. Although a planner may not control equity injections directly, he may want to limit intermediation to make sure the economy falls to equity injection regions instead of crisis regions.

**Effects of Capital Requirements.** The impact of capital requirements can be studied through  $\theta$ . Capital requirements have two effects. The first effect is the direct decrease in intermediation for a given  $\kappa$  by tightening of the LLC. The second is the effect on the dynamics of  $\kappa$ . Both effects show in the marginal value of bank equity, which now is:

$$\underbrace{\tilde{v}(X)}_{\text{Dynamic Effect}} = \hat{\beta} \left[ \mathbb{E}[v_2^f(X')] + \underbrace{(1-\theta)}_{\text{Limit on Leverage}} \max \left\{ \frac{\mathbb{E}[v_2^f(X')\Pi(X, X')]}{-\min_{\tilde{X}} \Pi(X, \tilde{X})}, 0 \right\} \right].$$

In summary, a social planner will trade off the cost of a lower growth rate against the reduction in the probability of a crisis. Furthermore, a planner will also consider the change in the dynamics of  $\kappa$ .

The following two numerical exercises illustrate how these effects balance out and are motivated by ongoing policy debates on the optimal regulation of financial institutions (see [Admati et al., 2011](#)).

**Invariant Distribution after Tightening of Capital Requirements.** This section describes the effects of an increase in  $\theta$  from 0.08 (Basel-II scenario) to 0.18 (Basel-III scenario).<sup>21</sup> Figure 10 presents the invariant distribution of  $\kappa$  for both scenarios. Table III shows the corresponding occupation and exit times obtained from these distributions. The Basel-III distribution has the bimodal shape of Basel-II, although there are some key differences. The distribution under Basel-III has a lower mass concentrated at the bottom because exit times are faster and the likelihood of entering a crisis state is lower. Second, the distribution under Basel-III is wider. The higher average levels of  $\kappa$  under Basel-III result from higher intermediation margins. In competitive inaction regions, a constraint in the quantity generates higher profits. Thus, although leverage is lower, the value of equity can increase due to higher margins and this generates higher average equity values. Through these measures, the economy seems more resilient under Basel-III.

<sup>21</sup>Similar exercises are performed in [Begenau \(2014\)](#) and [Bianchi \(2011b\)](#) in the context of international capital flows.

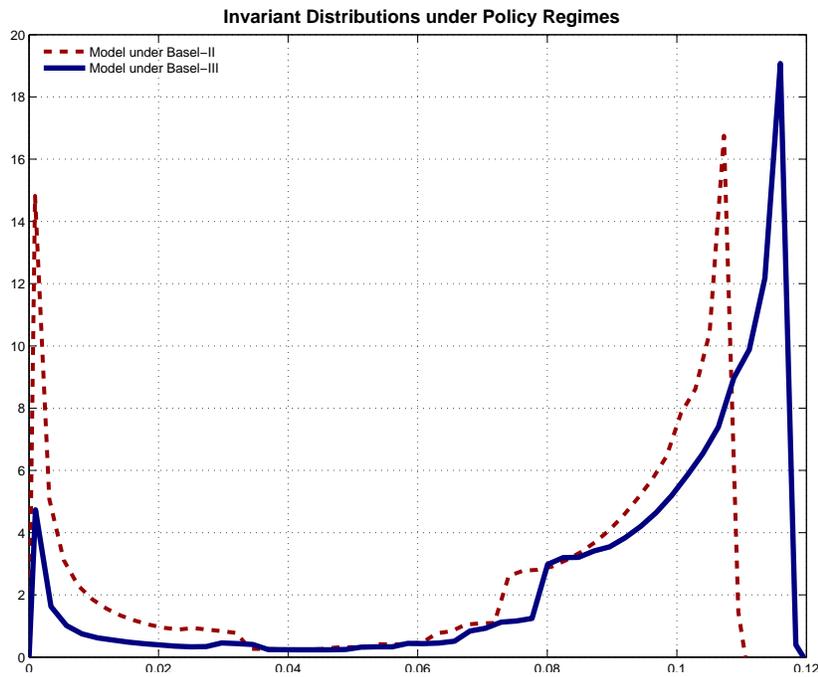


Figure 10: Invariant Distributions under Basel-II and III.

TABLE III

COMPARISON OF MOMENTS UNDER BASEL-II AND III IN THE MODEL.

THE TABLE PLOTS SOME MOMENTS CORRESPONDING TO VALUES OF  $\theta$  EQUAL TO 0.08 (BASEL-II) AND 0.18 (BASEL-III).

Variable	Basel-II		Basel-III	
	Unconditional	Crisis	Unconditional	Crisis
Occupation Time	100%	32.6%	100%	19.77%
Duration (quarters)	-	10.26	-	3.89
Average Growth Rate	4.3%	-10.3%	9.3%	-9.61%
Average $\kappa$	0.0659	0.0048	0.100	0.01

**Timing of Capital Requirements.** There is another important lesson: the timing of capital requirements matters. I illustrate this through an unexpected increase to Basel-III once the economy is in a typical Basel-II crisis. To show this, in Figure 11 I compare the average growth rate that results from the policy change with the growth rate if the economy remains under Basel-II.

There is a surprising result. The recovery under Basel-II is faster even though exit times are lower under Basel-III. Why? Exit times from the typical Basel-III crisis are shorter because the average Basel-III crisis is less severe. However, if the economy is already in a Basel-II crisis, the policy change only prolongs the decline because capital requirements depress intermediation, exacerbate adverse selection, and hurt growth. The model warns policy makers not to do in bad times what they should have done in good times. In turn, this result suggests the use of pro-cyclical requirements, also proposed by [Kashyap and Stein \(2004\)](#).

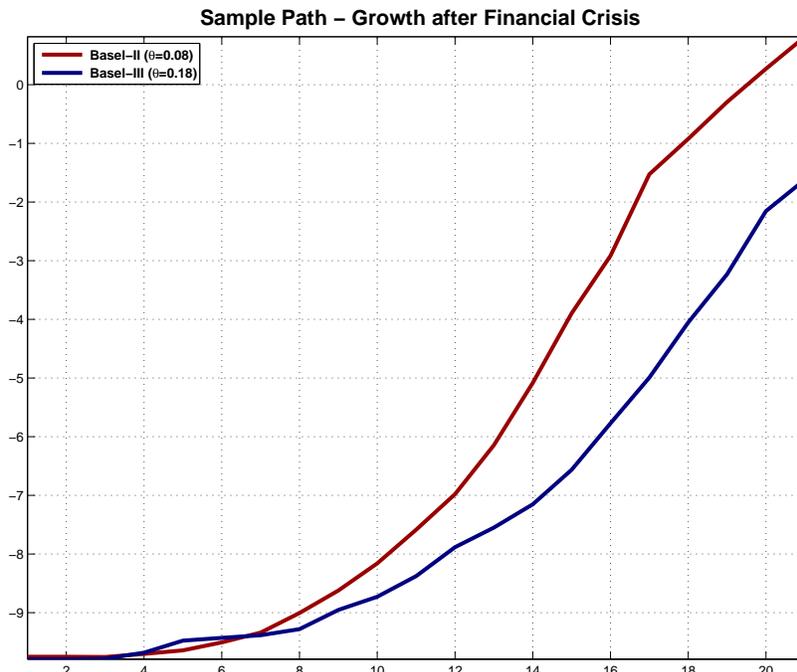


Figure 11: Expected Growth Recovery Path Under Basel-II and -III.

## 9. CONCLUSIONS

This paper provides a theory about risky financial intermediation under asymmetric information. The central message is that financial markets where asymmetric information is a first-order friction are likely to be more unstable than otherwise. The source of instability is low profitability generated when intermediation is low. This force can also lead to coordination failures to recapitalize banks after large losses, even when resources are available. The financial crises that emerge are deep and long-lasting.

The nature of asymmetric information and financial contracting is deliberately stark here. However, I hope this can be a first step to illustrate forces that may be introduced in models with more complex financial markets. I tried to make the case that declines in profit margins during crises are fundamental to understand the nature of financial crises.

## REFERENCES

- Admati, Anat R., Peter M. DeMarzo, Martin F. Hellwig, and Paul Pfleiderer**, “Fallacies, Irrelevant Facts, and Myths in the Discussion of Capital Regulation: Why Bank Equity is Not Expensive,” March 2011. Unpublished. 39
- Adrian, Tobias and Nina Boyarchenko**, “Intermediary Leverage Cycles and Financial Stability,” 2013. New York Federal Reserve, Staff Report No. 567. 4, 36
- Ajello, Andrea**, “Financial Intermediation, Investment Dynamics and Business Cycle Fluctuations,” 2012. 5
- Akerlof, George A.**, “The Market for ‘Lemons’: Quality Uncertainty and the Market Mechanism,” *Quarterly Journal of Economics*, 1970, 84 (3), 488–500. 20
- Alvarez, Fernando and Nancy L. Stokey**, “Dynamic Programming with Homogeneous Functions,” *Journal of Economic Theory*, 1998, 82 (1), 167 – 189. 47
- Bagshot, Walter**, *Lombard Street: A description of the money market*, Wiley, 1873. 1
- Begenau, Juliane**, “Capital Requirements, Risk Choice, and Liquidity Provision in a Business Cycle Model,” 2014. Unpublished Manuscript. 5, 39
- Bernanke, Ben and Mark Gertler**, “Agency Costs, Net Worth and Business Fluctuations,” *The American Economic Review*, March 1989, 79 (1), 14–31. 4
- , ———, and **Simon Gilchrist**, “Agency Costs, Net Worth and Business Fluctuations,” *The Review of Economics and Statistics*, February 1996, 78 (1), 1–15. 4
- , ———, and **Simon Gilchrist**, “The Financial Accelerator and the Flight to Quality,” *The Review of Economics and Statistics*, 1996, 78 (1), pp. 1–15. 4
- Bianchi, Javier**, “Overborrowing and Systematic Externalities in the Business Cycle,” 2011. *American Economic Review*, forthcoming. 5
- , “Overborrowing and Systemic Externalities in the Business Cycle,” *The American Economic Review*, 2011, 101 (7), 3400–3426. 39
- Bigio, Saki**, “Endogenous Liquidity and the Business Cycle,” 2014. forthcoming *American Economic Review*. 5, 19, 33
- Bloom, Nicholas**, “The Impact of Uncertainty Shocks,” *Econometrica*, 2009, 77 (3), 623–685. 17
- Boyd, John H and Edward C Prescott**, “Financial intermediary-coalitions,” *Journal of Economic Theory*, 1986, 38 (2), 211 – 232. 2, 18
- Brunnermeier, Markus K. and Lasse Heje Pedersen**, “Market Liquidity and Funding Liquidity,” *The Review of Financial Studies*, 2009, 22 (6), pp. 2201–2238. 4
- and **Yuliy Sannikov**, “A Macroeconomic Model with a Financial Sector,” *American Economic Review*, 2014, 104 (2), 379–421. 4, 34
- Calomiris, Charles W. and Gary Gorton**, “The Origins of Banking Panics: Models, Facts, and Bank Regulation,” in R. Glenn Hubbard, ed., *Financial Markets and Financial Crises*, University of Chicago Press, 1991. 36
- Carlstrom, Charles T. and Timothy S. Fuerst**, “Agency Costs, Net Worth, and Business Fluctuations: A Computable General Equilibrium Analysis,” *The American Economic Review*, 1997, 87 (5), pp. 893–910. 5
- Cerra, Valerie and Chaman Saxena**, “Growth Dynamics: The Myth of Economic Recovery,” *American Economic Review*, 2008, 98 (1), 439–57. 1, 35
- Chernow, Ron**, *The House of Morgan: An American Banking Dynasty and the Rise of Modern Finance*, New York: Grove Press, 2010. 25
- Christiano, Lawrence, Roberto Motto, and Massimo Rostagno**, “Financial Factors in Business Cycles,” 2009. Unpublished Manuscript. 35
- , ———, and ———, “Risks,” 2012. Unpublished. 5
- Cole, Harold L. and Timothy J. Kehoe**, “Self-Fulfilling Debt Crises,” *The Review of Economic Studies*, 2000,

67 (1), pp. 91–116. 26

**Cooper, Russell and Andrew John**, “Coordinating Coordination Failures in Keynesian Models,” *The Quarterly Journal of Economics*, 1988, 103 (3), pp. 441–463. 26

———, **John Haltiwanger, and Laura Power**, “Machine Replacement and the Business Cycle: Lumps and Bumps,” *The American Economic Review*, 1999, 89 (4), 921–946. 34

**Daley, Brendan and Brett Green**, “Waiting for News in the Market for Lemons,” 2011. forthcoming in *Econometrica*. 5

**DeMarzo, Peter and Darrell Duffie**, “A Liquidity-Based Model of Security Design,” *Econometrica*, 1999, 67, 65–100. 18

**Diamond, Douglas and Raghuram Rajan**, “Fear of Fire Sales, Illiquidity Seeking, and the Credit Freeze,” *Quarterly Journal of Economics*, May 2011, 126 (2), 557–591. 4

**Diamond, Douglas W.**, “Financial Intermediation and Delegated Monitoring,” *The Review of Economic Studies*, 1984, 51 (3), pp. 393–414. 2

——— and **Raghuram G. Rajan**, “A Theory of Bank Capital,” *The Journal of Finance*, 2000, 55 (6), pp. 2431–2465. 18

**Duffie, Darrell**, “Presidential Address: Asset Price Dynamics with Slow-Moving Capital,” *The Journal of Finance*, 2010, LXV (4). 1, 5

**Eisfeldt, Andrea L.**, “Endogenous Liquidity in Asset Markets,” *The Journal of Finance*, 2004, 59 (1), 1–30. 5

**Foster, Lucia, Cheryl Grim, and John Haltiwanger**, “Reallocation in the Great Recession: Cleansing or Not?,” Working Paper 20427, National Bureau of Economic Research August 2014. 36

**Freixas, Xavier and Jean-Charles Rochet**, *Microeconomics of Banking*, second ed., Cambridge, MA: MIT Press, 2008. 2, 18

**Gennaioli, Nicola, Andrei Shleifer, and Robert W. Vishny**, “A Model of Shadow Banking,” *The Journal of Finance*, 2013, 68 (4), pp. 1331–1363. 5

**Gertler, Mark and Nobuhiro Kiyotaki**, “Chapter 11 - Financial Intermediation and Credit Policy in Business Cycle Analysis,” in Benjamin M. Friedman and Michael Woodford, eds., *Handbook of Monetary Economics*, Vol. 3 of *Handbook of Monetary Economics*, Elsevier, 2010, pp. 547 – 599. 4

——— and **Peter Karadi**, “A model of unconventional monetary policy,” *Journal of Monetary Economics*, 2011, 58 (1), 17 – 34. Carnegie-Rochester Conference Series on Public Policy: The Future of Central Banking April 16-17, 2010. 4

**Gorton, Gary**, *Slapped by the invisible hand: the panic of 2007*, New York, New York: Oxford University Press, 2010. 18

——— and **George Pennacchi**, “Financial Intermediaries and Liquidity Creation,” *The Journal of Finance*, 1990, 45 (1), pp. 49–71. 18

**Gromb, Denis and Dimitri Vayanos**, “Equilibrium and welfare in markets with financially constrained arbitrageurs,” *Journal of Financial Economics*, 2002, 66 (2-3), 361 – 407. 4

**Gron, Anne**, “Capacity Constraints and Cycles in Property-Casualty Insurance Markets,” *The RAND Journal of Economics*, 1994, 25 (1), 110–127. 5

———, “Evidence of Capacity Constraints in Insurance Markets,” *Journal of Law and Economics*, 1994, 37 (2), 349–377. 5

**Guerrieri, Veronica and Robert Shimer**, “Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality,” 2011. Unpublished Manuscript. 5

——— and ———, “Dynamic Adverse Selection: A Theory of Illiquidity, Fire Sales, and Flight to Quality,” 2011. Unpublished Manuscript. 12

**Hart, Oliver and John Moore**, “A Theory of Debt Based on the Inalienability of Human Capital,” *The Quarterly Journal of Economics*, 1994, 109 (4), pp. 841–879. 3

- He, Zhiguo and Arvind Krishnamurthy**, “Intermediary Asset Pricing,” *The American Economic Review*, 2013, 103 (2), pp. 732–770. 4, 34
- Hendel, Igal and Alessandro Lizzeri**, “Adverse Selection in Durable Goods Markets,” *The American Economic Review*, 1999, 89 (5), pp. 1097–1115. 5
- Hennessy, Christopher A. and Toni M. Whited**, “Debt Dynamics,” *The Journal of Finance*, 2005, 60 (3), 1129–1165. 33, 34
- Holmstrom, Bengt and Jean Tirole**, “Financial Intermediation, Loanable Funds, and the Real Sector,” *The Quarterly Journal of Economics*, August 1997, 112 (3), 663–691. 3, 4
- Jorda, Oscar, Moritz Schularick, and Alan M. Taylor**, “Financial Crises, Credit Booms, and External Imbalances: 140 Years of Lessons,” Working Paper 16567, National Bureau of Economic Research December 2010. 36
- Justiniano, Alejandro, Giorgio E. Primiceri, and Andrea Tambalotti**, “Investment shocks and business cycles,” *Journal of Monetary Economics*, 2010, 57 (2), 132 – 145. 35
- Kashyap, Anil K. and Jeremy C. Stein**, “Cyclical Implications of the Basel-II Capital Standard,” *Federal Reserve Bank of Chicago Economic Perspectives*, 2004, pp. 18–31. 40
- Kindleberger, Charles P.**, *Manias, Panics and Crashes: A History of Financial Crises*, Mcmillan, 1996. 35
- Kiyotaki, Nobuhiro**, “Multiple Expectational Equilibria under Monopolistic Competition,” *The Quarterly Journal of Economics*, 1988, 103 (4), pp. 695–713. 26
- and **John Moore**, “Liquidity, Business Cycles, and Monetary Policy,” 2008. Unpublished Manuscript. 8
- Kurlat, Pablo**, “Lemons Markets and the Transmission of Aggregate Shocks,” *The American Economic Review*, 2013, 103 (4), pp. 1463–1489. 5
- Leland, Hayne E. and David H. Pyle**, “Informational Asymmetries, Financial Structure, and Financial Intermediation,” *The Journal of Finance*, 1977, 32 (2), pp. 371–387. 2
- Lorenzoni, Guido**, “Inefficient Credit Booms,” *Review of Economics Studies*, July 2008, 75 (2), 809–833. 5
- Maggiore, Matteo**, “Financial Intermediation International Risk Sharing, and Reserve Currencies,” November 2011. Job Market Paper. 4
- Malherbe, Frederic**, “Self-Fulfilling Liquidity Dry-Ups,” *The Journal of Finance*, 2013. 4
- Martinez-Miera, David and Javier Suarez**, “A macroeconomic model of endogenous systemic risk taking,” 2011. Unpublished Manuscript. 5
- Mishkin, Frederic S.**, “Asymmetric Information and Financial Crises: A Historical Perspective,” Working Paper 3400, National Bureau of Economic Research July 1990. 36
- Myers, Stewart C.**, “Determinants of corporate borrowing,” *Journal of Financial Economics*, 1977, 5 (2), 147 – 175. 3
- and **Nicholas S. Majluf**, “Corporate financing and investment decisions when firms have information that investors do not have,” *Journal of Financial Economics*, 1984, 13 (2), 187 – 221. 4, 5
- Ordonez, Guillermo**, “The Asymmetric Effects of Financial Frictions,” *Journal of Political Economy*, 2013, 121 (5), pp. 844–895. 36
- Passadore, Juan and Juan Xandri**, “Robust (conditional) Predictions in a Model of Sovereign Debt,” 2014. Unpublished. 26
- Philippon, Thomas**, “Debt Overhang and Recapitalization in Closed and Open Economies, 2010,” 2010. forthcoming in the inaugural issue of the IMF Economic Review. 3
- and **Vasiliki Skreta**, “Optimal Interventions in Markets with Adverse Selection,” 2011. American Economic Review, forthcoming. 4
- Plantin, Guillaume**, “Learning by Holding and Liquidity,” *Review of Economic Studies*, 2009, 76, 395–412. 5
- Reinhart, Carmen and Kenneth Rogoff**, *This time is different*, Princeton, NJ: Princeton University Press, 2009. 1, 35

- Shleifer, Andrei and Robert W. Vishny**, “Liquidation Values and Debt Capacity: A Market Equilibrium Approach,” *The Journal of Finance*, 1992, 47 (4), pp. 1343–1366. 4
- Stiglitz, Joseph and Bruce Greenwald**, *Towards a New Paradigm in Monetary Economics (Raffaele Mattioli Lectures)*, Cambridge University Press, 2003. 5
- Stiglitz, Joseph E. and Andrew Weiss**, “Credit Rationing in Markets with Imperfect Information,” *American Economic Review*, 1981, 71 (3), 393–410. 5
- Stokey, Nancy L., Robert E. Lucas Jr., and Edward C. Prescott**, *Recursive Methods in Economic Dynamics*, Massachusetts: Harvard University Press, 1989. 51
- Szkup, Michal**, “Preventing Self-Fulfilling Debt Crisis,” 2014. Unpublished Manuscript. 25, 26
- Tirole, Jean**, “Overcoming Adverse Selection: How Public Intervention Can Restore Market Functioning,” January 2011. *American Economic Review*, forthcoming. 4
- Vayanos, Dimitri and Jiang Wang**, “Liquidity and Asset Prices under Asymmetric Information and Imperfect Competition,” 2011. Unpublished Manuscript. 4
- Williamson, Stephen D.**, “Costly monitoring, financial intermediation, and equilibrium credit rationing,” *Journal of Monetary Economics*, 1986, 18 (2), 159 – 179. 18
- , “Financial Intermediation, Business Failures, and Real Business Cycles,” *Journal of Political Economy*, 1987, 95 (6), pp. 1196–1216. 18
- Winter, Ralph A.**, “The Liability Insurance Market,” *The Journal of Economic Perspectives*, 1991, 5 (3), 115–136.

# Appendix

## *Not for Publication*

### APPENDIX A: ALGORITHM

The following algorithm computes equilibria.

1. Build a grid for the state space values of  $A \times \phi$  and the transition function  $\chi$ .
2. Build a grid on the unit interval for  $\omega$ .
3. For all possible values of  $A \times \phi$  and  $\omega$  on the grid, solve for  $\{\mathbf{p}(\omega, A, \phi), \mathbf{q}(\omega, \mathbf{p}, A, \phi), \mathbf{\Pi}(\omega, A, \phi)\}$ . This step is performed once.
  - Discretize a set of points between  $p_L = \min_{\phi} \lambda(0, \phi)$  and  $p_H = \max_{\phi} \lambda(1, \phi)$ . Solve the optimal portfolio problem assuming  $\mathbf{q}^c(\omega, \mathbf{A}, \phi) = 1$ . To do this, find a value for  $\omega^*$  in (4) for each possible value of  $A \times \phi$  and  $\mathbf{p}$  on the grid. Interpolate over the grid for  $\omega$  to obtain  $\mathbf{p}(\omega, A, \phi)$ .
  - Using this  $(\mathbf{p}(\omega, A, \phi), \omega)$ , find  $\mathbf{q}(\omega, \mathbf{p}, A, \phi)$ ,  $\mathbf{\Pi}(\omega, \mathbf{p}, A, \phi)$  using the expressions for these variables, equations (7) and (9). Then, check if  $\mathbf{q}(\omega, \mathbf{p}, A, \phi) \geq 1$ .
  - For values where the condition fails, solve  $\mathbf{p}(\omega, A, \phi)$ ,  $\mathbf{q}(\omega, A, \phi)$  jointly using (4) and (7). Finally, find  $\mathbf{\Pi}(\omega, A, \phi)$  using (9).
4. Guess a candidate function for  $\tilde{v}$ .
5. Compute the set  $\omega^o$  using the candidate function  $\tilde{v}$ .
  - Compute, for each  $\omega$  on the grid, the value of  $\mathbb{E}[\tilde{v}(X') \mathbf{\Pi}(\omega, \mathbf{p}(\omega, A, \phi), \phi') | X]$ , where  $\mathbf{p}(\omega, A, \phi)$  is found in step 3.  $\omega^o$  is the set of values of  $\omega$  that yield a zero for  $\mathbb{E}[\tilde{v}(X') \mathbf{\Pi}(\omega, \mathbf{p}(\omega, A, \phi), \phi') | X]$ .
6. Compute the set  $\omega^{\kappa}$ .
  - For each  $\omega$  in the grid, compute  $\kappa = \min_{\phi} \mathbf{\Pi}(\omega, \mathbf{p}(\omega, A, \phi), \phi) \omega$ .
7. Compute  $\omega^*(X)$ .
8. Define  $p(X) = \mathbf{p}(\omega^*(X), \mathbf{A}, \phi)$ ,  $\Pi(X) = \mathbf{\Pi}(\omega^*(X), \mathbf{A}, \phi)$  and  $q(X) = \mathbf{q}(\omega^*(X), \mathbf{p}(\omega^*(X), \mathbf{A}, \phi), \mathbf{A}, \phi)$ .
9. Compute the transition function for  $X$ .
10. Update the  $\tilde{v}(X)$  iterating the Bellman equation for  $\tilde{v}(X)$  until convergence.
11. Iterate steps 4-10 until convergence.
12. Compute  $v(X), d(X), e(X)$ .

## APPENDIX B: PROOFS

## B.1. Proof of Propositions 1, 2, and 3

PROOF: The proof of propositions 1, 2, and 3 is presented jointly. The idea of the proof is to transform the producer's problem into a consumption-savings problem with log-preferences and linear constraints. For this, one has to solve the quality sales decisions under asymmetric information first. This is done showing the problem is homogeneous. Once this is done, one can use the dynamic programming arguments for homogenous objectives in [Alvarez and Stokey \(1998\)](#) to argue that all Bellman equations have unique solutions. I proceed by guess and verify.

Define  $W^p \equiv w^p k \equiv (A + q\bar{\lambda}(\phi')) k$  and  $W^i \equiv w^i k \equiv (p\omega^* + q^c \mathbb{E}[\lambda(\omega, \phi') | \omega < \omega^*]) k$  as in the main text. The guess for the c-producer's policy function is  $k^{p'} = \beta \frac{W^p}{q}$  and  $c^p = (1 - \beta) W^p$  and that his value function is of the form  $V_2^p = \psi^p(X) + \frac{1}{(1-\beta)} \log W^p$  where  $\psi^p(X)$  is a function of the aggregate state. For k-producers the guess is that  $k^{i'} = \beta \frac{W^i}{q^c}$  and  $c^i = (1 - \beta) W^i$  and that their value function is of the form  $V_2^i = \psi^i(X) + \frac{1}{(1-\beta)} \log W^i$  where  $\psi^i(X)$  is, again, a function of the aggregate state.

Consider the k-producer's problem during the first stage. Substituting the guess for  $V_2^i$  and his constraints yields:

$$V_1^i = \max_{\mathbb{I}(\omega) \in \{0,1\}} \mathbb{E} \left[ \psi^i(X) + \log \left( \left( p \int_0^1 \mathbb{I}(\omega) d\omega + q^c \mathbb{E}[\lambda(\omega, \phi') | \mathbb{I}(\omega) = 1] \right) k \right) | X \right].$$

From this expression, we show that choosing  $\mathbb{I}(\omega)$  is identical to choosing a cutoff  $\omega^*$  under which all units of quality lower than this cutoff are sold. We show this by arguing that an optimal  $\mathbb{I}^*(\omega)$  must be monotone decreasing. Suppose not and assume the optimal plan is given by some  $\mathbb{I}'(\omega)$  whose value cannot be attained by any monotone decreasing policy. It is enough to show that the producer can find another candidate  $\mathbb{I}(\omega)$  that integrates to the same number, that is monotone decreasing and that makes his value weakly greater. Thus, let's compare  $\mathbb{I}'(\omega)$  with some  $\mathbb{I}(\omega)$  that integrates to the same number in  $[0, 1]$ .

Since  $\mathbb{I}'(\omega)$  and  $\mathbb{I}(\omega)$  integrate to the same number, the amount of IOUs obtained by the k-producer during the first stage is the same:  $p \int_0^1 \mathbb{I}(\omega) d\omega = p \int_0^1 \mathbb{I}'(\omega) d\omega$ . Now, since  $\mathbb{I}(\omega)$  is monotone decreasing and  $\lambda(\omega, \phi)$  is monotone increasing in  $\omega$ ,

$$\int [1 - \mathbb{I}'(\omega)] \lambda(\omega, \phi') d\omega \leq \int [1 - \mathbb{I}(\omega)] \lambda(\omega, \phi') d\omega$$

implying that any optimal can be attained by some  $\mathbb{I}^*(\omega)$  monotone decreasing. This shows that  $V_1^i$  is attained by some monotone decreasing function.

Since  $\mathbb{I}^*(\omega)$  is monotone decreasing, it is also equivalent to choosing a threshold  $\omega^*$ . Substituting this threshold into the objective yields an expression for the optimal cutoff rule:

$$(12) \quad \omega^*(X) = \arg \max_{\tilde{\omega}} \mathbb{E} \left[ \log \left[ p\tilde{\omega} + q^c(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right] | X \right].$$

This proves Proposition 3 provided that  $V_1^i$  takes the shape of our guess. I now return to the second stage problems. Taking the solution to (12) as given, we know that the optimal plan for a k-producer sets  $x = p(X) \omega^*(X) k$ . Using the optimal policy for  $\omega^*$  and these definitions, one can write the second stage Bellman equation without reference to the first stage Bellman equation. To do this, one can substitute for  $x$  and  $k$  in the second stage Bellman equation to rewrite the k-producer's problem as

$$\begin{aligned} \max_{c \geq 0, i, k^b \geq 0} \log(c) + \beta \mathbb{E} [\pi V_1^i + (1 - \pi) V_1^p | X] \\ c + i + qk^b = p\omega^*(X) k \text{ and } k' = k^b + i + k \int_{\omega^*(X)}^1 \lambda(\omega, \phi') d\omega. \end{aligned}$$

Now, since  $(V_1^i, V_1^p)$  are increasing in  $k'$ , and  $k^b$  and  $i$  are perfect substitutes, an optimal solution will set  $i > 0$  only if  $q \geq 1$  and  $k^b > 0$  only if  $q \leq 1$ . This implies that substituting the k-producer's capital accumulation equation into his budget constraint simplifies his problem to:

$$\max_{c \geq 0, k'} \log(c) + \beta \mathbb{E} [\pi V_1^i(k', X') + (1 - \pi) V_1^p(k', X') | X] \quad \text{s.t. } c + q^c k' = w^i k.$$

The same steps allow one to write the c-producer's problem as,

$$\max_{c \geq 0, k'} \log(c) + \beta \mathbb{E} [\pi V_1^i + (1 - \pi) V_1^p | X] \quad \text{s.t. } c + qk' = w^p k.$$

Replacing the definitions of  $V_1^i$  and  $V_1^p$  into the objective above, and substituting our guess yields  $V_2^i$  and  $V_2^p$ , we obtain:

$$\max_{c \geq 0, k'} \log c + \frac{\beta}{(1 - \beta)} \log k' + \tilde{\psi}^i(X) \quad \text{s.t. } c + q^c k' = w^i k$$

and

$$\max_{c \geq 0, k'} \log c + \frac{\beta}{(1 - \beta)} \log k' + \tilde{\psi}^p(X) \quad \text{s.t. } c + qk' = w^p k.$$

respectively. In this expressions  $\tilde{\psi}^i(X)$  and  $\tilde{\psi}^p(X)$  are functions of  $X$  which don't depend on the policy decisions. Taking first-order conditions for  $(k', c)$  in both problems leads to:

$$\begin{aligned} c^i &= (1 - \beta) w^i(\omega^*, X) k \quad \text{and} \quad k^{i'} = \frac{\beta}{q^c} w^i(\omega^*, X) k \\ c^p &= (1 - \beta) w^p(X) k \quad \text{and} \quad k^{p'} = \frac{\beta}{q} w^p(X) k. \end{aligned}$$

These solutions are consistent with the statement of Propositions 1 and 2. To verify that the guess for our value functions is the correct one, we substitute the optimal policies:

$$\begin{aligned} & \log(1 - \beta) w^i(\omega^*, X) k + \frac{\beta}{(1 - \beta)} \log \frac{\beta}{q^c} w^i(\omega^*, X) k + \tilde{\psi}^i(X) \\ &= \frac{\log w^i(\omega^*, X) k}{(1 - \beta)} + \psi^i(X) = \frac{\log W^i(k, \omega^*, X)}{(1 - \beta)} + \psi^i(X) \end{aligned}$$

for some function  $\psi^i(X)$ . The same steps lead to a similar expression for c-producers. This verifies the initial guess.

The last claim in Proposition 3 is shown using the implicit function theorem. Define,

$$\Omega(\tilde{\omega}, p; X) \equiv \mathbb{E} \left[ \log \left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right) | X \right].$$

The optimal cutoff is a maximum of  $\Omega(\tilde{\omega}, p; X)$ . The Theorem of the Maximum asserts that  $\tilde{\omega}$  is continuous in  $p$  since  $\Omega(\tilde{\omega}, p; X)$  is a continuous function. I use the Implicit Function Theorem to show that  $\omega^*(p)$  is increasing in  $p$ . Since the objective is continuous and differentiable, the first order condition is necessary for an interior solution:

$$\mathbb{E} \left[ (p - q^i(X, X') \lambda(\tilde{\omega}, \phi')) \left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)^{-1} | X \right] = 0.$$

The second derivative is,

$$\begin{aligned} \frac{\partial^2 \Omega(\tilde{\omega}, p; X)}{\partial \tilde{\omega}^2} &= \mathbb{E} \left[ -q^i(X, X') \lambda_{\tilde{\omega}}(\tilde{\omega}, \phi') \left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)^{-1} \middle| X \right] - \\ &\quad \mathbb{E} \left[ \left( p - q^i(X, X') \lambda(\tilde{\omega}, \phi') \right)^2 \left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)^{-2} \middle| X \right] \end{aligned}$$

because  $\lambda_{\tilde{\omega}}(\tilde{\omega}, \phi')$  is positive for any  $\phi'$ . This shows the objective is concave and therefore, there is a unique maximum. Let's assume the maximum is interior for some  $p$ . Then, it solves  $\partial \Omega(\tilde{\omega}, p; X) / \partial \tilde{\omega} = 0$ . By the implicit function theorem, we have that

$$\frac{\partial \tilde{\omega}}{\partial p} = - \frac{\partial \Omega(\tilde{\omega}, p; X) / \partial p}{\partial^2 \Omega(\tilde{\omega}, p; X) / \partial \tilde{\omega}^2}.$$

It suffices to show  $\partial \Omega(\tilde{\omega}, p; X) / \partial p > 0$ . This expression is:

$$\mathbb{E} \left[ \frac{1}{\left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)} - \frac{p(\tilde{\omega} - q^i(X, X') \lambda_{\tilde{\omega}}(\tilde{\omega}, \phi'))}{\left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)^2} \middle| X \right]$$

and arranging terms yields:

$$\mathbb{E} \left[ \frac{\left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right) - p(\tilde{\omega} - q^i(X, X') \lambda_{\tilde{\omega}}(\tilde{\omega}, \phi'))}{\left( p\tilde{\omega} + q^i(X, X') \int_{\tilde{\omega}}^1 \lambda(\omega, \phi') d\omega \right)} \middle| X \right] > 0$$

which is enough to guarantee a positive  $\frac{\partial \tilde{\omega}}{\partial p}$ . If there is any  $p$  for which  $\omega^* = 1$ , then by continuity of  $\omega^*$ ,  $\omega^*$  must also be increasing at that point also. This proves Proposition 3.

*Q.E.D.*

## B.2. Proof of Lemma 4 and Proposition 5

PROOF: Lemma 4 and Proposition 5 are proven jointly here. We begin by guessing that  $V_1^f(n, X) = v_1^f(X)n$ , and  $V_2^f(n, X) = v_2^f(X)n$ , where  $v_2^f(X) = \beta^F \mathbb{E} \left[ v_1^f(X) R^b \right]$  if the banker remains alive, and  $v_2^f(X) = \beta^F R^b n$  if he dies.

Plugging this guess into the bankers problem yields:

$$\max_{Q \geq 0, e \in [0, \bar{e}], d \in [0, 1]} (1 - \tau) d - e + \beta^F R^b \mathbb{E} \left[ v_1^f(X) \left( \Pi(X, X') Q + n' \right) \middle| X \right]$$

subject to

$$\begin{aligned} - \min_{X'} \Pi(X, X') Q &\leq n' \\ n' &= n + e - d. \end{aligned}$$

Assume that the optimal solution to this problem is characterized by some  $e^*(n, X)$  and  $d^*(n, X)$  yet to be determined. In equilibrium,  $\Pi(X, X')$  is finite. Hence,  $\mathbb{E} \left[ v_2^f(X') \Pi(X, X') \right]$  is also finite, provided that the problem has a solution. If  $\mathbb{E} \left[ v_2^f(X') \Pi(X, X') \right] > 0$  and  $-\min_{X'} \Pi(X, X') \leq 0$ , the banker would set  $Q^* = \infty$ . But this would imply that in equilibrium  $\Pi(X, X') \leq 0$  for any  $X'$  because there cannot be a future state where firms provide infinite intermediation and there are positive profits. Hence, it is the case that if  $\mathbb{E} \left[ v_2^f(X') \Pi(X, X') \right] >$

$0 \rightarrow -\min_{X'} \Pi(X, X') > 0$ . Now if this is the case,

$$(13) \quad Q^* = \frac{n'}{-\min_{X'} \Pi(X, X')} > 0.$$

If  $\mathbb{E}[v_2^f(X') \Pi(X, X')] < 0$ , the producer optimally sets  $Q^*$ . If  $\mathbb{E}[v_2^f(X') \Pi(X, X')] = 0$ ,  $Q^*$  is indeterminate (but finite). Thus, in either case,  $\mathbb{E}[v_2^f(X') \Pi(X, X') Q^*] = 0$ .

This implies that for any optimal policy,

$$\mathbb{E}[v_2^f(X') \Pi(X, X') Q^*] = \max \left\{ \frac{v_2^f(X') \Pi(X, X') n'}{-\min_{X'} \Pi(X, X')}, 0 \right\}.$$

Thus, one can substitute this expression into the objective of the firm and express it without reference to  $Q$  :

$$\max_{e \in [0, \bar{e}], d \in [0, n]} (1 - \tau) d - e + (n + e - d) \beta^F R^b \mathbb{E} \left[ v_2^f(X') + \max \left\{ \frac{v_2^f(X') \Pi(X, X')}{-\min_{X'} \Pi(X, X')}, 0 \right\} \middle| X \right]$$

where I also used the definition of  $n'$ . Now, it is clear from this expression that  $(e, d)$  are the choices of a linear program. Consequently, any optimal financial policy satisfies

$$\begin{aligned} e &> 0 \text{ only if } \beta^F \left[ \mathbb{E}[v_2^f(X')] + \max \left\{ \frac{\mathbb{E}[v_2^f(X') \Pi(X, X')]}{-\min_{\tilde{X}} \Pi(X, \tilde{X})}, 0 \right\} \right] \geq 1 \text{ and} \\ d &> 0 \text{ only if } \beta^F \left[ \mathbb{E}[v_2^f(X')] + \max \left\{ \frac{\mathbb{E}[v_2^f(X') \Pi(X, X')]}{-\min_{\tilde{X}} \Pi(X, \tilde{X})}, 0 \right\} \right] \leq (1 - \tau). \end{aligned}$$

If the inequalities are strict, it is clear that  $e = \bar{e}$  and  $d = n$ . By linearity,  $e$  and  $d$  are indeterminate when the relations hold with equality, and equal 0 if these are not satisfied. This implies that  $e(n, X) = e^*(X)n$ ,  $d(n, X) = d^*(X)n$  are solutions to the banker's problem without loss in generality.

We now use these results to show that the value function is linear in  $n$ . Substituting the optimal policies into the objective we obtain:

$$\left[ (1 - \tau) d^*(X) - e^*(X) + (1 + e^*(X) - d^*(X)) \beta^F R^b \mathbb{E} \left[ v_2^f(X') + \max \left\{ \frac{v_2^f(X') \Pi(X, X')}{-\min_{X'} \Pi(X, X')}, 0 \right\} \middle| X \right] \right] n,$$

which is a linear function of  $n$ .

Returning to the optimal quantity decision, it is clear that  $Q$  can be written as,

$$Q^*(X) n = \left\{ \frac{1 + e^*(X) - d^*(X)}{-\min_{X'} \Pi(X, X')} \right\} n.$$

and clearly  $Q^*(X) = \arg \max_{\tilde{Q}} \mathbb{E}[v_2^f(X') \Pi(X, X') | X] \tilde{Q}$  subject to  $\Pi(X, X') \tilde{Q} \leq n'$ . This proves Proposition 6.

We are ready to show that  $v_1^f(X)$  solves the functional equation described in the body of the text. Define

$$\tilde{v}(X) = \beta^F R^b \mathbb{E} \left[ v_2^f(X') + v_2^f(X') \max \left\{ \frac{\Pi(X, X')}{-\min_{X'} \Pi(X, X')}, 0 \right\} \middle| X \right]$$

as the marginal value of equity in the bank, and note that

$$v_1^f(X) = \max_{d^*(X) \in [0,1], e \geq 0} (1 - \tau) d^*(X) - e^*(X) + (1 + e^*(X) - d^*(X)) \tilde{v}(X).$$

If  $\tilde{v}(X) \in ((1 - \tau), 1)$ , then  $v_1^f(X) = \tilde{v}(X)$  because  $(d^*(X), e^*(X)) = 0$ . If  $\tilde{v}(X) \leq (1 - \tau)$ , then  $e^*(X) = 0$  and we have that

$$(1 - \tau) d^*(X) + (1 - d^*(X)) \tilde{v}(X) = (1 - \tau).$$

Finally, if  $\tilde{v}(X) = 1$ , then,  $v_1^f(X) = 1$ . This information is summarized in the following functional equation for  $v_1^f(X)$ :

$$v_1^f(X) = \min \left\{ \max \left\{ \beta^F R^b \mathbb{E} \left[ v_2^f(X') \left\{ 1 + \max \left\{ \frac{\Pi(X, X')}{-\min_{X'} \Pi(X, X')}, 0 \right\} \right\} | X \right], (1 - \tau) \right\}, 1 \right\}$$

which equals

$$\min \left\{ \max \left\{ \mathbb{E} \left[ \left( \rho + (1 - \rho) v_1^f(X'') \right) \beta^F R^b \left\{ 1 + \max \left\{ \frac{\Pi(X, X')}{-\min_{X'} \Pi(X, X')}, 0 \right\} \right\} | X \right], (1 - \tau) \right\}, 1 \right\}.$$

This functional equation determines the slope of the banker's value function,  $v_1^f(X)$ . It can be shown that the solution to this functional equation is unique. Assumptions 9.18-9.20 of [Stokey et al. \(1989\)](#) are satisfied by this problem. It remains to be shown that Assumption 9.5 (part a) is also satisfied. By assumption,  $X$  is compact, so the only piece left is that  $X$  is countable. Because the transition function for the state is an endogenous object, it depends on an aggregate state,  $\kappa$ . It will be shown that although  $(d, e)$  are not uniquely defined, there is unique mapping from  $\phi$  to  $\kappa'$ . By exercise 9.10 in [Stokey et al. \(1989\)](#), together these assumptions ensure that there is a unique solution to the this functional equation. *Q.E.D.*

### B.3. Proof of Proposition 6

PROOF: To obtain an expression for  $q$ , fix any sequence of states  $(X, X')$  and call  $\bar{\omega} \equiv \omega(X)$ . Assume  $q > 1$  so that  $D^i = 0$ . Market clearing in stage 2 requires  $D^p(X, X') = S(X) = \mathbb{E}[\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega} \pi K$ . By Proposition 1, we can integrate across the c-producer's policy functions to obtain an expression for  $D^p(X, X')$  as a function of  $q$ :

$$\beta \int \left[ \frac{W^p(k, x, X)}{q} - \bar{\lambda}(\phi') k \right] \Gamma^c(dk) = \beta \frac{A + q \bar{\lambda}(\phi')}{q} (1 - \pi) K.$$

By market clearing,  $q$  is such that:

$$\left[ \beta \frac{A + q \bar{\lambda}(\phi')}{q} - \bar{\lambda}(\phi') \right] (1 - \pi) K = \mathbb{E}[\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega} \pi K.$$

Manipulating this expression leads to the value of  $q$  that satisfies market clearing:

$$q = \frac{\beta A (1 - \pi)}{\mathbb{E}[\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega} \pi + (1 - \pi) (1 - \beta) \bar{\lambda}(\phi')}.$$

Recall now that this expression is valid only when  $q > 1$ , because capital good producers are not participating in the market. Thus, the expression is only true for values of

$$(14) \quad \beta A \left[ \frac{\pi}{(1-\pi)} \mathbb{E} [\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega} + (1-\beta) \lambda(\phi') \right]^{-1} > 1.$$

If  $q = 1$ , then it must be the case that the total demand for capital must be larger than the supply provided by financial firms. In this case,  $D^i(X, X')$  is obtained again by integrating across the demand for k-producers' capital given in Proposition 1. Thus, for a stage one price  $p$ , this demand is given by

$$D^i + I = \beta p \bar{\omega} \pi K - (1-\beta) \mathbb{E} [\lambda(\omega, \phi') | \omega \leq \bar{\omega}] (1-\bar{\omega}) \pi K \text{ for } q = 1.$$

The corresponding condition is

$$(15) \quad \beta p \bar{\omega} - (1-\beta) \mathbb{E} [\lambda(\omega, \phi') | \omega \leq \bar{\omega}] (1-\bar{\omega}) \pi + [\beta A - (1-\beta) \bar{\lambda}(\phi')] (1-\pi) \geq \pi \mathbb{E} [\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega}$$

where the aggregate capital stock has been canceled from both sides. If the condition is satisfied, then  $q = 1$ , and

$$D^i(q, p) = \pi \mathbb{E} [\lambda(\omega, \phi') | \omega \leq \bar{\omega}] \bar{\omega} - [\beta R - (1-\beta) \bar{\lambda}(\phi')] (1-\pi)$$

and

$$I = [\beta p \bar{\omega} - (1-\beta) \mathbb{E} [\lambda(\omega, \phi') | \omega > \bar{\omega}] (1-\bar{\omega})] \pi - D^i(q, p).$$

If (14) and (15) are violated, this implies  $q < 1$  and  $I = 0$ . The corresponding market clearing-condition is obtained by solving  $q$  from

$$\begin{aligned} & \left[ \frac{\beta p \bar{\omega}}{q} - (1-\beta) \mathbb{E} [\lambda(\omega, \phi') | \omega > \bar{\omega}] (1-\bar{\omega}) \right] \pi + \left[ \frac{\beta A}{q} - (1-\beta) \bar{\lambda}(\phi') \right] (1-\pi) \\ & \geq \pi \mathbb{E} [\lambda(\omega, \phi') | \omega < \bar{\omega}] \bar{\omega}. \end{aligned}$$

We can collect the terms where  $q$  shows in the denominator to obtain,

$$\frac{\beta(p\bar{\omega}\pi + A)}{q} = \pi \mathbb{E} [\lambda(\omega, \phi') | \omega < \bar{\omega}] \bar{\omega} + (1-\beta) [-\mathbb{E} [\lambda(\omega, \phi') | \omega < \bar{\omega}] \bar{\omega} \pi + \bar{\lambda}(\phi')].$$

The solution is given by

$$q = \frac{\beta(\pi p \bar{\omega} + A)}{(\beta \mathbb{E} [\lambda(\omega, \phi') | \bar{\omega} < \bar{\omega}] \bar{\omega} \pi + (1-\beta) \bar{\lambda}(\phi'))}$$

The formula in Proposition 6 corresponds to this expression. Moreover, the demand function is weakly decreasing so for each  $p, X$  there will be a unique  $q$  satisfying the market clearing condition. We can express the profit function in the following way:

$$\Pi(X, X') = \max \left\{ (1-\pi) \beta A \frac{\pi \mathbb{E}_{\phi'} [\lambda(\omega) | \bar{\omega} < \bar{\omega}] \bar{\omega}}{\pi \mathbb{E} [\lambda(\omega, \phi') | \bar{\omega} < \bar{\omega}] \bar{\omega} + (1-\beta) \bar{\lambda}(\phi')}, \tilde{\pi}(X, X') \right\}$$

where

$$\tilde{\Pi}(X, X') = \min \left\{ 1, (\pi p^i + A(1 - \pi)) \frac{\pi \mathbb{E} [\lambda(\omega, \phi') | \bar{\omega} < \bar{\omega}] \bar{\omega}}{\left( \frac{(1-\beta)\lambda}{\beta} + \pi \bar{\omega} \mathbb{E} [\lambda(\omega, \phi') | \bar{\omega} < \bar{\omega}] \right)} \right\}.$$

Since both functions are increasing in  $\mathbb{E} [\lambda(\omega, \phi') | \omega < \bar{\omega}]$ , the conditional expectation, we know by Assumption **A1**, that these functions are decreasing in the shock  $\phi'$ . Thus,  $\Pi(X, X')$  is decreasing in  $\phi'$ . *Q.E.D.*

#### B.4. Proof of Proposition 7

PROOF: Take two values of financial risk capacity,  $\kappa^L < \kappa^H$ . Fix any exogenous state  $A \times \phi$  and denote by  $X^L$  and  $X^H$  the corresponding aggregate states for these two levels of financial risk capacity. By the last claim in Proposition 3, it is enough to show that  $Q^*(X^L) \leq Q^*(X^H)$  in any RCE to argue that  $\omega^*$  and  $p$  are increasing in  $\kappa$ . This follows from the market-clearing condition in the first stage,  $Q^* \kappa = \omega^*$ . We show by contradiction that  $Q^*(X^H) > Q^*(X^L)$  cannot be part of a RCE equilibrium as  $\rho \rightarrow 1$ . Optimality of the banker's problem requires to solve:

$$Q^*(X) = \arg \max_{\tilde{Q}} \mathbb{E} \left[ v_2^f(X') \Pi(X, X') | X \right] \tilde{Q} \text{ subject to } \Pi_{\min \phi'}(X, X') \tilde{Q} \leq 1, \forall X'$$

so either the constraint binds or  $\mathbb{E} \left[ v_2^f(X') \Pi(X, X') | X \right] = 0$ . Assume the constraint binds when the financial risk capacity is low,  $\kappa^L$ . This implies that:

$$\begin{aligned} \mathbb{E} \left[ v_2^f(X^{L'}) \Pi(X^L, X^{L'}) | X^L \right] &> 0 \text{ and} \\ \Pi_{\min \phi'}(X^L, X^{L'}) \tilde{Q}(X^L) &= 1. \end{aligned}$$

In turn, market clearing implies that  $\min_{\phi'} \Pi(X^L, X^{L'}) \tilde{Q}(X^L) N = \min_{\phi'} \Pi(X^L, X^{L'}) \omega^*(X^L) K$ , which in turn implies that  $\min_{\phi'} \Pi(X^L, X^{L'}) \omega^*(X^L) = \kappa$ . Thus,  $\omega^*(X^L)$  is feasible for  $\kappa^H$  since  $\min_{\phi'} \Pi(\omega^*(X^L), A, \phi) \omega^*(X^L) < \kappa^o$  implies  $\Pi(\omega^*(X^L), A, \phi) \tilde{Q}(X^L) < 1$ . Therefore, if a RCE features  $Q^*(X^L) > Q^*(X^H)$ , it must be that  $\mathbb{E} \left[ v_2^f(A, \phi, \kappa^H + \Pi(\omega^*(X^L), A, \phi)) \Pi(\omega^*(X^L), A, \phi) | X^L \right] < 0$ . However, for  $\rho \rightarrow 1$ ,  $v_2^f \rightarrow \beta^F R^b$  pointwise, there always exists some  $\rho$  sufficiently close to 1, such that  $\mathbb{E} \left[ v_2^f(X^{L'}) \Pi(X^L, X^{L'}) | X^L \right] > 0$  also implies:

$$(16) \quad \mathbb{E} \left[ v_2^f(A, \phi, \kappa^H + \Pi(\omega^*(X^L), A, \phi)) \Pi(\omega^*(X^L), A, \phi) | X^L \right] \geq 0.$$

In this case,  $Q^*(X^H) < Q^*(X^L)$  cannot hold in a highest price equilibrium.

Assume that given  $X^L$ ,  $\mathbb{E} \left[ v_2^f(X^{L'}) \Pi(X^L, X^{L'}) | X^L \right] = 0$  so bankers are not constrained. Hence,  $Q^*(X^L)$  is not binding for financial risk capacity  $\kappa^L$ . Then, if a RCE features  $Q^*(X^H) < Q^*(X^L)$ , it must be that  $\mathbb{E} \left[ v_2^f(A, \phi, \kappa^H + \Pi(\omega^*(X^L), A, \phi)) \Pi(\omega^*(X^L), A, \phi) | X \right] < 0$ . However for  $\rho = 1$ ,  $v_2^f \rightarrow \beta^F R^b$ , the two conditions cannot hold at the same time.

*Q.E.D.*