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## The Political Economy of Growth, Inequality, the Size and Composition of Government Spending

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# The Political Economy of Growth, Inequality, the Size and Composition of Government Spending\*

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## Abstract

This paper develops a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. An analytical solution is derived from majority voting for three government spending categories: public consumption goods and transfers (valued by households), as well as productive government services (complementing private capital in an endogenous-growth technology). Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. In contrast to the previous literature that derives monotonic (typically negative) relations between inequality and growth in one-dimensional voting environments, this paper establishes conditions, in an environment of multi-dimensional voting, under which a non-monotonic, inverted U-shape relation between inequality and growth is obtained. This more general result – that inequality and growth could be negatively or positively related – could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus. The paper also shows that the political-economy equilibrium obtained under multi-dimensional voting for the initial period is time-consistent.

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# 1 Introduction

The relation between inequality and growth has been widely analyzed in the analytical political-economy literature. A typical negative relation between inequality and growth has been established, based on the following mechanism. Higher inequality leads to a larger demand by the median voter for redistributive government spending, which is financed by higher distortionary taxation that reduces (physical or human) capital investment and growth. This negative relationship between inequality and growth is derived in Bertola (1993), Perotti (1993), Alesina and Rodrick (1994), Persson and Tabellini (1994), and Saint-Paul and Verdier (1997).<sup>1</sup> The theoretical framework of the aforementioned papers combines endogenous growth and the endogenous setting of a key policy instrument. However, the empirical literature based on cross-country data provides contradictory or inconclusive results on the sign of the inequality-growth link.<sup>2</sup>

Still within the median-voter framework, Saint-Paul and Verdier (1993) derive a positive relationship between inequality and growth, considering government spending on education as the only public spending category. Education spending has two effects: it is redistributive as the poor benefit from it to a larger extent than the rich and it raises growth because the production function is linear in human capital. This unambiguously positive effect of education on growth is the result of assuming non-distortionary taxation. However, the authors also suggest that it is possible to obtain an inverted U-shaped relationship between growth and inequality when education spending is financed by a sufficiently distortionary tax.

Other studies restrict the relationship between the median and the decisive voter by the inclusion of a wealth bias in the political system. Benabou (1996), Saint-Paul and Verdier (1996), and Lee and Roemer (1998) show that higher inequality may lead to less transfers and therefore higher growth if higher-income agents lobby against redistributive policy or limit access to voting by low-income agents. The latter mechanism can imply a U-shaped relation between inequality and redistribution.

Other studies focus on different mechanisms to majority voting that shape the inequality-growth nexus. Under asset-market imperfections a negative relationship between inequality and growth arises, as derived, for example, in Galor and Zeira (1993) and Benabou (1996). Here individuals endowed with low levels of inherited wealth cannot invest in human capital due to market imperfections. So the initial inequality of wealth continues generation after generation, restricting investment and therefore growth. Gupta (1990), Alesina and Perotti (1995), and Benhabib and Rustichini (1996) focus on the social instability that is

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<sup>1</sup>See Ostry et al. (2014), Aghion et al. (1999), Benabou (1996), and Alesina and Perotti (1994) for a detailed review of the relevant theoretical and empirical literature.

<sup>2</sup>A negative empirical relation is documented in the cross-country studies conducted by Alesina and Rodrik (1994), Persson and Tabellini (1994), Perotti (1994) and Clarke (1992). Barro (2000) establishes a negative relation for low-income countries and a positive relation for high-income countries. A positive relation is reported in panel data studies by Perotti (1996), Benabou (2000), Forbes (2000), Milanovic (2000), Banerjee and Duflo (2003), and Benhabib and Spiegel (1998).

generated by income inequality as the cause of a higher demand for transfers or a lower demand for productive expenditure, which leads to lower growth.

The political-economy literature reviewed so far sets the relevant policy instrument in a one-dimensional policy. However, majority voting in a multi-dimensional setting opens the door to instability of the political equilibrium, due to the lack of a Condorcet winner policy, i.e., there is not a policy that beats any other feasible policy in a pairwise vote. The latter instability can be avoided by restricting the institutional environment for aggregating individual decisions, without restricting individual preferences. Shepsle (1979) and Shepsle and Weingast (1981) follow the latter strategy by developing a structure-induced political equilibrium.<sup>3</sup> In this framework the voting process is based on separate but simultaneous voting by each agent in  $n$  separate committees, one for each of  $n$  voting dimensions. Votes are aggregated for each dimension and the election outcome will be determined by the votes of the corresponding decisive voter. In each dimension, the median voter may vary or not and this depends on the preferences of the agents by government-spending categories. For example, Conde-Ruiz and Galasso (2005) characterize the political equilibrium of multi-dimensional public spending, applying the structure-induced equilibrium. There are two types of public expenditure in their overlapping-generations model: transfer payments from rich to poor and pension payments. In this model, the decisive voter for each expenditure category is different, because pensions are preferred by the older generation and transfers are preferred by the poor (both young and old).

Exogenous growth models do not allow derivation of an analytical solution of the class of dynamic political-economy models considered here because growth is not constant over time and therefore identification and decisions by future median voters are dependent on current policy decisions. Hence dynamic exogenous-growth models, like those by Krusell et al (1997), Krusell and Ríos-Rull (1999), and Azzimonti et al. (2006), are solved numerically. In contrast, the endogenous growth framework allows obtaining analytical solutions of political-economy equilibria. The latter setup allows identification of future median voters because the evolution of state variables preserves the ranking of voters over time.

In this paper we develop a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. Using an endogenous-growth technology, we derive an analytical solution based on majority voting for three government spending categories. We derive conditions under which a non-monotonic, inverted U-shaped relation between inequality and growth is obtained. This result extends the previous analytical literature in a median voter framework that was reviewed above, which derives a monotonic

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<sup>3</sup>An alternative strategy for restricting the institutional environment without restricting individual preferences is adopted in agenda-setter models, where voter groups – the agenda setters – are allowed to submit policy proposals of their interest to general voting. For example, Persson et al. (2000) develop a static model with agenda setters for multi-dimensional voting over three categories of public spending: transfers, public goods, and payments to politicians. Other modeling strategies to avoid political equilibrium instability include probabilistic voting and using intermediate political preferences, as discussed by Persson and Tabellini (2000).

relationship under one-dimensional voting. Our more general result – that inequality and growth could be negatively or positively related – could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus.

Government spending falls on three expenditure categories: public consumption goods, transfers to households, and productive government services, which are financed by distortionary taxes. Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. Household preferences are homogeneous across households, who value positively private and public consumption goods, and benefit from lump-sum transfers from the government. Public consumption enters household utility in per capita terms as a rival and non-excludable consumption good.

Government services raise the productivity of capital and are subject to relative congestion, i.e., they are rival goods.<sup>4</sup> Productive government services are a complement of private capital in an endogenous-growth technology characterized by constant returns.<sup>5</sup>

The composition and aggregate level of spending is chosen endogenously by majority voting in a dynamic context. Here we follow Shepsle (1979) by attaining stability of the political equilibrium as a result of restricting the institutional environment for aggregating individual decisions, without restricting individual preferences. Voting cannot be restricted to aggregate spending or taxation because the median voter expresses a separate political demand for each component of public expenditure. In general equilibrium, the political demand for one government spending category affects the demand for the two other categories. Therefore the political demand for government transfers is lessened by the supply of other government spending categories that are valued by the median voter – a key feature for the main result of our model.

A key feature of this paper is the existence of an endogenous threshold level of inequality at which the median voter chooses a household transfer level of zero. At higher levels of inequality, the relation between inequality and growth is negative, as it is in much of the previous political-economy literature. At levels of inequality below the threshold level, negative transfers (i.e., taxes) would be chosen. This outcome is not sensible, so we restrict transfers to be zero at low levels of inequality. This implies that in the latter case voting is restricted to choosing optimal levels of two remaining spending categories: public consumption goods and productive government services.

This leads to the paper’s main result. In the range where inequality is lower than the aforementioned threshold level and transfers are zero, the relationship between inequality

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<sup>4</sup>Barro and Sala-i-Martin (1992) argue that most public goods are subject to congestion. Therefore pure public goods should be considered as useful but unrealistic benchmarks.

<sup>5</sup>Our model shares with Alesina and Rodrik (1994) and Persson and Tabellini (1994) (as well as other related work on political economy of government size and growth) two features: endogenous growth and public provision of a factor that enhances growth. However, we include two additional categories of government spending, focusing on a multi-dimensional political-economy equilibrium, in contrast to the previous one-dimensional models.

and growth could be positive. While optimal provision of productive services is affected neither by inequality nor by transfers, optimal provision of public consumption goods is affected by inequality. When inequality exceeds the threshold level, lower income concentration leads to a smaller demand for transfers, lower taxation, and higher growth. However, when inequality falls below the threshold level, we obtain an ambiguous relation between income concentration and taxation, which is determined by the degree of substitution between private and public consumption goods in household utility. If private and public consumptions are complements (substitutes), a lower (higher) income concentration leads to a higher (lower) demand for public goods and hence higher (lower) overall taxation, which lowers (raises) growth.

We derive analytical policy functions, which in many cases imply obtaining closed-form solutions for the policy instruments. However, the latter results are obtained by voting at the initial period. Therefore the question is whether our political-economy equilibrium is time consistent. The solution is to implement successive votes; the time consistency is attained when beliefs about future policy rules match the current policy choice. We show that when focusing on Markov-perfect equilibria, the selected policy is stationary, i.e., time is not an argument of the policy function, which is only determined by key state variables. In our model, we derive the Generalized Euler Equation<sup>6</sup> (GEE) for a political equilibrium and conclude that the voting equilibrium attained at time zero satisfies the GEE and therefore is time consistent.

The outline of the paper is as follows. Section 2 describes the model in detail. Section 3 derives an analytical solution for the economic equilibrium, with an exogenous fiscal policy. In section 4 we characterize the full dynamic political-economy equilibrium with analytical solutions for the size and composition of government spending at time zero only, and then prove that this equilibrium is time consistent. Section 5 concludes.

## 2 The Model

The economy is populated by infinitely-lived households which consume a private and a public consumption good. While consumption preferences are homogeneous across households, they are heterogeneous in their initial capital endowment. Production technology reflects the use of capital, supplied by households, and productive services, supplied by the government. The government raises revenue by establishing a constant tax rate on capital and spends on three expenditure categories: public consumption, productive services, and lump-sum payments provided to each household. Next we present the model equations by agents.

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<sup>6</sup>Klein et al (2008) derived a condition called Generalized Euler Equation. This condition captures, in equilibrium, the optimal deviation in the policy variable for the policy maker (in our model, the median voter maximizes utility). The latter deviation should match with the level described by the policy function (this function built on utility-maximizing agents considering the deviation of the policy variable).

## 2.1 Households

Infinitely-lived households differ by initial (physical and human) capital endowment  $k_{i,0}$ . Over time, households accumulate capital by investing. Intertemporal and intratemporal preferences are homogeneous. Intertemporal preferences of representative household  $i$  are reflected by a logarithmic utility function over  $\mathbb{C}_{i,t}$ , a CES aggregate over a private consumption good,  $c_{i,t}$ , and a public consumption good,  $g_{1,t}$ . Household income is the sum of net capital rents (the rental rate of capital,  $r_t$ , times outstanding capital, net of proportional capital income taxes at homogeneous tax rate  $\tau_g$ ) plus government transfer  $g_{3,t}$ .

Representative household  $i$  solves the following intertemporal optimization

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}, g_{1,t}) \quad (1)$$

subject to

$$c_{i,t} + k_{i,t+1} = (1 - \tau_g) r_t k_{i,t} + g_{3,t}, \text{ given } k_{i,0}. \quad (2)$$

Aggregate intratemporal consumption is defined by CES preferences

$$\mathbb{C}_{i,t}(c_{i,t}, g_{1,t}) = \left( a c_{i,t}^\rho + (1 - a) g_{1,t}^\rho \right)^{\frac{1}{\rho}}, \quad (3)$$

where  $\frac{1}{1-\rho}$  is the intratemporal elasticity of substitution between consumption goods ( $1 > \rho > -\infty$ ), and  $a$  is the share parameter ( $0 < a < 1$ ).

The number of households,  $N$ , is exogenous and constant over time. Hence aggregate capital is  $K_t = \sum_{i=1}^N k_{i,t}$ , aggregate consumption is  $C_t = \sum_{i=1}^N c_{i,t}$ , and aggregate public spending is  $G_t = \tau_g Y_t$ .

Therefore the economy's aggregate resource constraint is given by

$$Y_t = C_t + K_{t+1} + G_t, \quad (4)$$

where  $Y$  is aggregate output.

## 2.2 Firms

Technology reflects the use of capital, supplied by households, and productive services, supplied free of charge by the government. Following an AK technology, production is linear in capital. Government productive services are a public good that is rival in the sense that each firm's use of productive services is subject to congestion. Following Turnovsky (1996) the representative firm's per capita output is given by<sup>7</sup>

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<sup>7</sup>Eicher and Turnovsky (2000) present a general specification of production functions with congestion and the conditions that must be fulfilled by these functions to generate endogenous growth.

$$\frac{Y_t}{N} = y_t = A \left[ \left( \frac{G_{2,t}}{k_t} \right)^\sigma \left( \frac{G_{2,t}}{K_t} \right)^{(1-\sigma)} \right]^\gamma k_t, \quad 0 \leq \sigma \leq 1; 0 < \gamma < 1, \quad (5)$$

where  $y_t$  is per capita output,  $G_{2,t}$  is government productive services,  $k_t$  is per capita capital,  $\sigma$  is the congestion parameter associated to aggregate production services,  $\gamma$  is a parameter that reflects the positive but marginally declining productivity of capital with respect to productive services, and  $A$  is the linear capital productivity parameter.

Congestion in the use of government productive services is reflected at both firm level and the economy's aggregate level. If  $0 \leq \sigma \leq 1$ , productive services are rival because their use is associated to  $k$ , the use of capital at firm level, as well as to  $K$ , the economy's aggregate capital. Given  $G_{2,t}$ , a higher stock of firm-level capital and/or a higher stock of aggregate capital reduce the productivity of  $G_2$ .

Let's re-write equation (5) as:

$$y_t = A \left[ h_t \left( \frac{K_t}{k_t} \right)^\sigma \right]^\gamma k_t, \quad (6)$$

where  $h_t \equiv \frac{G_{2,t}}{K_t}$ .

If  $\sigma = 0$ , congestion is proportional and capital productivity declines when  $G_2$  grows at a lower rate than aggregate capital. If  $\sigma = 1$ , there is no congestion, production exhibits constant returns to scale with respect to  $k$  and  $G_2$ , and the latter is non-rival and non-excludible public good.

Firms interact in a perfectly competitive market for capital and take the aggregate capital and productive service as given, where the capital rental rate is determined by the marginal productivity of capital

$$r_t = Ah_t^\gamma - A\gamma h_t^{\gamma-1} h_t \sigma = Ah_t^\gamma (1 - \gamma\sigma). \quad (7)$$

We assume that the governments supply of services,  $G_{2,t}$ , grows proportionally to aggregate capital. Hence the ratio  $h_t$  is time-independent.

We note that the private marginal productivity of capital before taxes is larger than the social marginal productivity of capital. More precisely, a benevolent social planner is aware of the negative congestion externality and therefore chooses a socially optimal level of capital that is consistent with a lower (social) productivity of capital, equal to  $Ah^\gamma(1 - \gamma)$ . If the congestion increases (i.e., when  $\sigma$  declines), the distance of the private marginal productivity with respect to the social marginal productivity is higher due to over-investment of capital.

### 2.3 Government

The government raises tax revenue at zero costs and spends on three expenditure categories. At this stage we assume the tax rate on income,  $\tau_g$ , to be time-invariant.



The government's budget constraint in per capita terms is the following:

$$\tau_g y_t = g_t = \sum_{j=1}^3 g_{j,t}, \quad (8)$$

where  $g_{j,t}$  ( $j = 1, 2, 3$ ) is any of the three expenditure types that were defined above.

In order to simplify derivation of the multi-dimension policy in section 4, we introduce fictitious separate tax rates  $\tau_j$  ( $j = 1, 2, 3$ ) that correspond to each type of government expenditure, satisfying  $g_{j,t} = \tau_j y_t, \forall j$ , and  $\sum_{j=1}^3 \tau_j = \tau_g$ .

Aggregate output of the economy is

$$Y_t = A \left( \frac{G_{2,t}}{K_t} \right)^\gamma K_t = Ah^\gamma K_t. \quad (9)$$

As mentioned above, we assume that the government's expenditure on productive services,  $G_2$ , is proportional to aggregate capital. Considering the latter relation, the budget relation  $G_{2,t} = \tau_2 Y_t$ , and aggregate output equation (9), obtain

$$h \equiv \frac{G_2}{K} = \tau_2 Ah^\gamma. \quad (10)$$

Therefore  $h$  is determined by  $h = \tau_2^{\frac{1}{1-\gamma}} A^{\frac{1}{1-\gamma}}$ .

Considering the government budget constraint, the fictitious tax rates, and the marginal product of capital, per capita output is derived as

$$y_t = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_t \quad (11)$$

Per capita government expenditures on each category are

$$g_{j,t} = \tau_j A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_t, \quad j = 1, 2, 3. \quad (12)$$

Finally, the rental rate of capital is the following

$$r_t = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma). \quad (13)$$

In sum, government spending on productive services raises marginal capital productivity but, as a result of the negative congestion externality due to the use of government services, private marginal capital productivity exceeds social productivity. Finally, a standard result from endogenous-growth theory, our economy's growth rate is time-invariant due to constant returns to capital, the economy's reproducible factor of production.

### 3 Economic Equilibrium

Each household solves her optimization problem, taking as given aggregate government spending, tax rates, and the rental rate of capital. Household  $i$ 's Euler equation determines relative intertemporal private consumption as

$$\frac{c_{i,t+1}}{c_{i,t}} = \beta \left( (1 - \tau_g) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma) \right) \equiv \Theta, \quad (14)$$

where  $\Theta$  is the gross rate of growth of private consumption.

As a result of endogenous constant output growth, private consumption is also a time-invariant function of tax rates and parameters related to technology and preferences. Capital growth both at household  $i$  and the average level is also time invariant, and so is  $\eta_i$ , the ratio of capital held by household  $i$  and the average level of capital

$$\eta_i \equiv \frac{k_{i,t}}{k_t} = \frac{k_i}{k} \quad (15)$$

An important implication of the latter is that the relative ranking of households according to their capital wealth does not change over time and is unaffected by any exogenous or endogenous change in the determinants of capital productivity.<sup>8</sup>

After replacing Euler equation (14), expenditure levels  $g_{j,t}$  as functions of their corresponding fictitious tax rates times output, and the rental rate of capital (13) in the household's budget constraint, we obtain household gross saving as

$$k_{i,t+1} = \Psi_i k_t - c_{i,t}, \quad (16)$$

donde  $\Psi_i \equiv (1 - \tau_g) A^{\frac{1}{\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} \left( (1 - \gamma\sigma) \eta_i + \frac{\tau_3}{1-\tau_g} \right)$ .

Household gross saving in period  $t$ ,  $k_{i,t+1}$ , is a positive function of period- $t$  per capita capital endowment,  $k_t$ , and a negative function of period- $t$  consumption,  $c_{i,t}$ . Hence, as is standard in an endogenous growth model, gross saving and the capital rental rate do not depend on future variables. Output, aggregate (and household) consumption, and aggregate (and household) capital growth at the common and time-invariant rate  $\Theta$ . Hence the relation between consumption and capital is given by

$$c_{i,t} = (\Psi_i - \Theta) k_t \quad (17)$$

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<sup>8</sup>In a neoclassical growth model with exogenous stationary growth but a dynamic time-variant growth path toward the steady-state equilibrium the household ranking by capital endowments is not affected by an exogenous change in tax rates. However, as shown by Krusell and Ríos-Rull (1999), when the tax rate is made endogenous by a political-economy decision in a neoclassical growth model, household rankings are altered in subsequent periods after the endogenous tax change takes place.

**Definition 1.** *Given time-invariant tax rates  $\tau_1, \tau_2, \tau_3$  an initial capital distribution  $k_{i,0}$  for all  $i = 1, \dots, N$ ; then an allocation  $\{c_{i,t}, k_{i,t+1}\}_{t=0}^{\infty}$  represents a competitive equilibrium if and only if there is a price sequence  $\{r_t\}_{t=0}^{\infty}$  such that agents solve (1) subject to budget constraint (2), the capital rental rate is given by (13), and the economy's resource constraint (4) is satisfied.*

The model's competitive economic equilibrium allows to write the agents' utility function in terms of the economy's growth rate  $\Theta$  and initial consumption  $C_{i,0}$ . Hence agent  $i$ 's indirect utility function is given by

$$\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0) \equiv \sum_{t=0}^{\infty} \beta^t \ln(C_{i,0} \Theta^t) = \sum_{t=0}^{\infty} \beta^t \ln C_{i,0} + \sum_{t=0}^{\infty} t \beta^t \ln \Theta, \quad (18)$$

where

$$C_{i,0} = \left( a((\Psi_i - \Theta)k_0)^\rho + (1-a)(\tau_1 A^{\frac{1}{\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0)^\rho \right)^{\frac{1}{\rho}}. \quad (19)$$

## 4 Political-Economy Equilibrium

In this section we derive the political-economy equilibrium, which implies obtaining endogenous tax rates  $\tau_j$  as a result of majority voting. Considering that voter heterogeneity is determined only by differences in initial capital endowments, we could consider applying the version of Median Voter Theorem (MVT) whose political preferences are characterized by the single-crossing property. By the single-crossing property we can sort voters by type and given that policy alternatives can be ordered from smallest to largest, the decisive voter is one whose number of voters who prefer a large alternative is equal to the number of voters who prefer a small alternative, i.e., the decisive voter is that endowed with a level of capital equal to the median value of the households' distribution of capital,  $k_{md}$ , that is  $k_i = k_{md}$ . In this case, there is a Condorcet winner policy that beats any other feasible policy in a pairwise vote and coincides with the preferred policy of the median voter.<sup>9</sup>

However, the standard MVT applies only when the policy variable is one-dimensional. When it is multi-dimensional, as it is our case, majority voting is cyclical, i.e., it is almost always feasible to derive an alternative policy preferred by a majority of voters.<sup>10 11</sup>

To overcome the latter limitation of the standard MVT, Shepsle (1979) and Shepsle and Weingast (1981) propose the Structure-Induced Equilibrium (SIE). This political-economy equilibrium for multi-dimensional decisions results from an issue-by-issue voting game with

<sup>9</sup>See Myerson (1996) for more details of the two versions of MVT.

<sup>10</sup>Shepsle and Weingast (1982) discuss in detail the problem or curse of multi-dimensional majority voting that limits application of the MVT.

<sup>11</sup>Assuming that voters have intermediate preferences the MVT can be applied in multidimensional spaces.

commitment. Agents vote simultaneously but separately for each dimension of the policy under discussion. Votes are then socially aggregated over each issue by the decisive or median vote.

The definition of the SIE for our model is the following.

**Definition 2.** *Structure-Induced Equilibrium.* An Structure-Induced Equilibrium is a policy vector  $\tau^* = (\tau_1^*, \tau_2^*, \tau_3^*)' \in [0, 1]^3$  such that

$$\tau_1^* = \tau_1^{dc}(\tau_2^*, \tau_3^*), \quad \tau_2^* = \tau_2^{dc}(\tau_1^*, \tau_3^*) \quad \text{and} \quad \tau_3^* = \tau_3^{dc}(\tau_1^*, \tau_2^*),$$

where  $\tau_j^{dc} = \arg \max_{\tau_j} \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{dc,0}, k_0)$  for  $j=1, 2, 3$ , and  $k_{dc,0}$  is initial capital of the decisive voter. Considering that the space of alternatives is convex and the  $\tau_j^{dc}$  functions are continuous, then, consistent with the Brouwer fixed-point theorem, an equilibrium is defined for  $\tau_j^*$ , for  $j=1, 2$  and  $3$ .

This equilibrium implies applying a version of the MVT that requires satisfaction of the single-crossing property in each voting dimension. To this end we will define a property of political preferences that will enable us to directly apply the MVT. To this property we called marginal single-crossing utility.

**Definition 3.** *Property of Marginal Single-Crossing Utility.* When the first derivatives of the indirect utility function  $\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0)$  satisfy

$$\frac{\partial^2 \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0)}{\partial k_{i,0} \partial \tau_j} > 0 \text{ or } < 0, \text{ para todo } \tau_j \in [0, 1] \text{ y } k_{i,0} > 0,$$

then this utility function satisfies the property of marginal single-crossing utility.

This monotonicity property implies that marginal utility changes monotonically in each policy dimension. The purpose of this property is to show that the marginal utility of voters shows monotonicity with respect to the variable that identifies the type of agent. Since agents are ordered completely and transitively in space  $\mathbb{R}^+$  (nonnegative real numbers) according to their initial capital endowment, the existence of a Condorcet winner is possible if those agents who prefer a higher value of policy is not located between those who prefer a smaller value of policy. Following Shepsle (1979) and De Donder et al. (2012), we conclude that if indirect utility  $\mathbb{V}$  satisfies the latter property, the decisive voters choice in each policy dimension is the median voters choice, i.e., where each  $\tau_j^{dc} = \tau_j^{md}$ .

Considering that our model is for a dynamic environment, first we assume that voting takes place in the first period under commitment and is therefore maintained over time. Then we will relax this assumption and show that the static policy result is dynamically consistent when assuming a Markov-perfect equilibrium.

#### 4.1 Political-Economic Equilibrium under Voting in Period Zero

Here we assume voting to take place only once, in period  $t = 0$ , when society chooses by majority voting a policy  $\tau^*$  that will be maintained over time. The policy choice results from maximizing indirect utility derived in equation (18).

We show in Appendix A that our indirect utility function satisfies the property of marginal single-crossing utility and we obtain the SIE

$$\tau_1^* = \tau_1^{md}(\tau_2^*, \tau_3^*), \quad \tau_2^* = \tau_2^{md}(\tau_1^*, \tau_3^*) \quad \text{y} \quad \tau_3^* = \tau_3^{md}(\tau_1^*, \tau_2^*).$$

Therefore the social choice by majority voting of the policy variable coincides with the simultaneous choice yet separately (issue by issue), made by the decisive voter, which holds an initial level of capital that corresponds to the median level of the household distribution of capital endowments.

**Lemma 1.** *If the households indirect utility function defined by (18) satisfies the property of single-crossing marginal utility, then a political-economy equilibrium with voting in the initial period is defined by the policy vector  $\tau^* = (\tau_1^*, \tau_2^*, \tau_3^*)' \in [0, 1]^3$  where*

$$\tau_1^* = \frac{(1 - \gamma)(1 - \beta)}{(1 + \phi)},$$

$$\tau_2^* = \gamma,$$

$$\tau_3^* = \frac{(1 - \gamma)(1 - \beta)(\phi - (\phi + \beta)(1 - \gamma\sigma)\eta_{md})}{(1 + \phi)(1 - (1 - \beta)(1 - \gamma\sigma)\eta_{md})},$$

where  $\phi = \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}}$  and  $\eta_{md} = \frac{k_{md}}{k}$ .

*Proof.* See Appendix A. □

Government provision of public consumption goods is financed by tax rate  $\tau_1^*$ , which depends positively on the weight of public consumption goods in household preferences  $(1 - a)$  and the elasticity of substitution between private and public consumption goods  $\frac{1}{1-\rho}$ . When  $a$  is larger (smaller) than 0.5, i.e., when household weight attached to private goods is larger (smaller) than that attached to public goods, then a higher elasticity of substitution between both goods leads to a larger (smaller) demand, and therefore provision of public goods declines (increases). Hence if the degree of substitution between both goods rises, households prefer to reduce demand for the less-valued good. Note that, for now the choice of  $\tau_1^*$  depends only on economic parameters and is therefore independent of the distribution of wealth and voting.

Government provision of productive services is financed by tax rate  $\tau_2^*$ , which is exactly determined by parameter  $\gamma$ , the productivity of capital with respect to government services.

For the social planner, aggregate production is reflected by the function  $Y = AG_2^\gamma K^{(1-\gamma)}$ , where congestion in the use of government services is absent. Hence, from a social welfare perspective, parameter gamma represents the share of government productive services in a Cobb-Douglas production function.<sup>12</sup> It is well-known that in an economy with endogenous growth and where government services are a flow in a Cobb-Douglas production function, the tax rate that maximizes growth is identical to the rate that maximizes social welfare.<sup>13</sup> However, in contrast to the literature that considers productive government flows services in an endogenous-growth model with a Cobb-Douglas technology, in our model  $\tau_2^*$  does not maximize growth because of the presence of other government spending categories.

**Corollary 1.** *In a political-economy model of endogenous growth, Cobb-Douglas technology, CES preferences, and endogenous choice of different government spending categories by majority voting, the tax rate (that finances productive government services) that maximizes welfare of the decisive household is different from the tax rate that maximizes growth.*

*Proof.* This result is different from the result of the previous literature on endogenous growth and Cobb-Douglas technology (quoted in footnote 13), as reflected by the derivative of the growth with respect to  $\tau_2$

$$\frac{\partial \Theta}{\partial \tau_2} = \beta(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}} \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1 - \gamma} - 1 \right). \quad (20)$$

Hence, for an economy with non-zero tax rates  $\tau_1$  and  $\tau_3$ , the  $\tau_2$  tax rate that maximizes growth is given by

$$\tilde{\tau}_2 = (1 - \tau_1 - \tau_3)\gamma$$

It is possible to obtain a value for tax rate  $\tau_3$  equal to zero for a given set of parameter values (in fact, we will consider this case below). However tax rate  $\tau_1$  cannot be zero because  $0 < a, \gamma, \beta < 1$ . Therefore the tax rate that maximizes growth,  $\tilde{\tau}_2$ , is lower than the tax rate  $\tau_2^*$ , selected in the political process.  $\square$

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<sup>12</sup>We have also considered an alternative transfer scheme to our lump-sum transfers. The alternative scheme allows for more distributive transfers, under which households with wealth levels that exceed the average wealth level get zero transfers and those with wealth below the average get transfers that are inversely proportional to their wealth. Hence individual household transfer  $g_3^i$  increases linearly with the reduction of relative wealth  $\eta_i$ . This feature adds an additional distortion to households' saving decisions. When assuming that the ratio  $\eta_i$  is distributed as a Fisk distribution with scale parameter equal to 1 and shape parameter equal to 1, transfers are defined as  $(B(\eta_{md}))^{-1}(1 - \frac{k^i}{k})$ , where  $B(\eta_{md}) \equiv \frac{1}{\eta_{md}} + \ln(\frac{\eta_{md}}{\eta_{1+md}})$ . We show that even though transfers affect the rate of growth, the voted  $\tau_2$  tax rate is equal to  $\gamma$ . For details, see Appendix E.

<sup>13</sup>For example, Barro (1990) and Barro and Sala-i-Martin (1992) derive endogenous-growth models with productive government flow services, for which they show that the level of government services supply that maximizes economic growth also maximizes social welfare. However, Misch et al. (2013) show that for a more general production technology exhibiting a factor substitution elasticity different from one, the latter equivalence is not satisfied anymore.

Therefore Corollary 1 proofs that the indirect utility-maximizing decisive voter, by voting for a  $\tilde{\tau}_2$  tax rate lower than  $\gamma$ , chooses a growth rate lower than the maximum she could attain because she also votes for tax financing of other types of government spending. In terms of her indirect utility, sacrificing growth by voting for a lower  $\tau_2$  is more than offset by higher consumption of private and public goods.

An increase in parameter  $\gamma$  leads to a change in the composition of government spending toward more government productive services, lowering spending on government consumption and transfers (see Lemma 1). This result is due to the fact that the marginal effect of government services on productivity, and hence on growth, rises with  $\gamma$ .

The third government spending category, transfers, with an income share equal to  $\tau_3^*$ , is the only category that depends on  $\eta_{md}$ , the median-average wealth ratio. A reduction of this ratio reflects that the decisive voter is poorer relative to the average household. Let's define the measure of household inequality the term  $1 - \eta_{md}$ . Not surprisingly, a higher level of inequality causes an unambiguous rise in the share of transfers in GDP, defined by  $\tau_3^*$ , although it distorts saving decisions

$$\frac{\partial \tau_3^*}{\partial \eta_{md}} = \frac{-(1-\gamma)(1-\beta)\beta}{(1-(1-\beta)(1-\gamma\sigma)\eta_{md})^2} < 0. \quad (21)$$

This positive (negative) relation between the size of transfers and income concentration ( $\eta_{md}$ ) is well established in the political economy literature. Meltzer and Richard (1981) in a static environment and Krusell and Ríos-Rull in a dynamic environment derive the latter relation. However, we assume that tax rates  $\tau_j$  are restricted to the interval  $[0, 1]$  and all tax rates satisfy this condition other than transfers if and only if the inequality parameter  $\eta_{md}$  falls below a threshold level  $\tilde{\eta}$ .

**Corollary 2.** *For a threshold level  $\tilde{\eta} \equiv \frac{\phi}{(\phi+\beta)(1-\gamma\sigma)}$ , if  $\eta_{md} \in [0, \tilde{\eta}]$  then  $\tau_3^* \in [0, 1]$ .*

*Proof.* Imposing a value of 0 for the numerator of  $\tau_3^*$  (see Lemma 1) determines the threshold level  $\tilde{\eta}$ . Hence if  $\eta_{md}$  tends to zero, then  $\tau_3^*$  tends to  $\frac{(1-\gamma)(\phi+\beta)}{1+\phi} < 1$ . As transfers are monotonic,  $\tau_3^* \in [0, 1]$ .  $\square$

Threshold level  $\tilde{\eta}$  (inequality measure  $1 - \tilde{\eta}$ ) depends negatively (positively) on  $\beta$ . Hence if households get more impatient or  $\beta$  declines, the threshold level of inequality at which transfers are zero rises. More impatience leads to value growth less and therefore transfers attain a value of zero at a lower level of household inequality (Figure 1).

The degree of congestion in the use of public services ( $\sigma$ ) and the share of productive government services in output ( $\gamma$ ) have a positive effect on threshold level  $\tilde{\eta}$ . Higher congestion (or  $\sigma$  decrease) raises the private marginal product of capital (at an intensity determined by the share of government services), hence increasing growth. Therefore the median voter will relinquish obtaining positive transfers at a higher level of inequality because she values growth more (Figure 1).

Finally the coefficient of public consumption in aggregate consumption,  $1 - a$ , has a positive influence on the inequality threshold level. A higher weight of the public consumption good in private utility allows that the median voter demands both a higher transfer and a higher threshold of inequality, i.e., transfers cease to be positive from a higher level of  $\tilde{\eta}$ . In both cases, the goal is to increase the private consumption good.

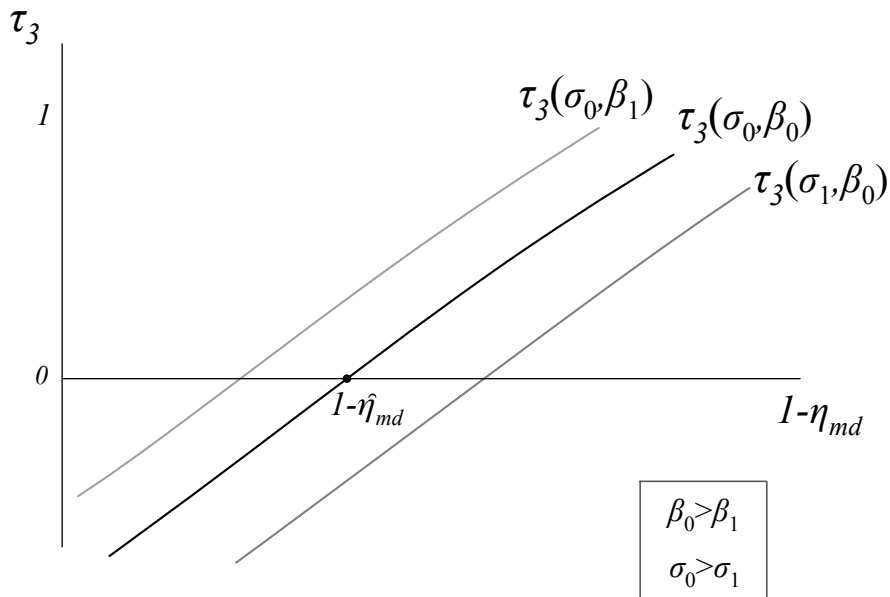


Figure 1: Equilibrium Relations between Inequality and Transfers for different Discount and Congestion Parameters

We show in Appendix A that the marginal utility of transfers is strictly decreasing in household wealth level  $k^i$ . We restrict  $\tau_3$  to be non-negative, because it does not make sense to pay a proportional tax on income to finance negative transfers, i.e., an additional tax. Therefore when income concentration is relatively low, i.e. when  $\eta_{md} > \tilde{\eta}$ , transfers are set to zero and therefore  $\tau_3^* = 0$ .

**Lemma 2.** *Given indirect utility in (18), subject to the condition  $\eta_{md} > \tilde{\eta}$  ( $\tau_3 = 0$ ), then a political economy equilibrium attained under voting in the initial period is defined by vector  $\tau^{**} = (\tau_1^{**}, \tau_2^{**})' \in [0, 1]^2$  where*

$$\frac{a}{1-a} \left( \frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}} (1-\gamma\sigma)(1-\beta)\eta_{md} \right)^\rho = \frac{(1-\beta)(1-\gamma)}{\tau_1^{**}} - 1,$$



$$\tau_2^{**} = \gamma$$

*Proof.* See Appendix B. □

Although transfers are zero, the share of productive services in output is held at  $\gamma$ , i.e., it is determined by technology (5). Also if  $\tau_1^{**} > 0$ , then the voted policy does not maximize growth for the reasons discussed in the case of positive transfers. However, as opposed to the latter case, now it is not possible to derive a closed-form solution for  $\tau_1^{**}$ . The only condition derived from the implicit function for  $\tau_1^{**}$  in Lemma 2 is  $\tau_1^{**} < (1 - \beta)(1 - \gamma)$ .<sup>14</sup> This upper bound on  $\tau_1^{**}$  is a sufficient condition for obtaining a positive level for after-tax household income, hence  $\tau_1^{**} + \tau_2^{**} < 1$ .

A value for  $\eta_{md}$  above the threshold level  $\tilde{\eta}$  implies zero transfers. Therefore we focus on the effect of  $\eta_{md}$  on  $\tau_1^{**}$ . Differentiation of the implicit function for  $\tau_1^{**}$  in Lemma 2 leads to an ambiguous sign of the partial derivative of  $\tau_1^{**}$  with respect to  $\eta_{md}$ . We derive the condition under which the relation between the two latter variables can be signed.

**Corollary 3.** *The effect of income concentration coefficient  $\eta_{md}$  on the level of public consumption goods is determined by the intra-temporal elasticity of substitution between private and public consumption goods in utility.*

- (I) For  $\rho > 0$  (for an elasticity of substitution larger than 1),  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} < 0$ .
- (II) For  $\rho < 0$  (for an elasticity of substitution smaller than 1),  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} > 0$ .

*Proof.* See Appendix C. □

This corollary establishes two cases. We focus on the second case, when a lower concentration coefficient (i.e., a lower level of inequality) leads to a higher tax rate  $\tau_1^{**}$  (to finance a larger level of public consumption goods) and hence to higher overall taxation,  $\tau_g = \tau_1^{**} + \tau_3^{**}$ . This case stands in contrast to the previous literature, which establishes a monotonic relation between income concentration, taxation, and the size of government, a relation that is upheld in case (I) of Corollary 3.

However, in case (II) of Corollary 3, we derive an ambiguous relation between income concentration and taxation. When two conditions are satisfied in our model (income concentration is relatively low, i.e.,  $\eta_{md} > \tilde{\eta}$ , therefore transfers are zero; and private and public consumptions are complements in utility, i.e.,  $\rho < 0$ ), then a reduction in income concentration leads to higher taxation for public goods and hence higher overall taxation.

The reason for our result is the following. In the traditional political-economy literature, less income concentration benefits the median voter who then votes for lower taxation and hence lowers transfers, under governments that spend only on transfers. In contrast, we consider three government-spending categories in our model. When income concentration is

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<sup>14</sup>This inequality condition is derived from the fact that the left-hand side of the implicit equation for  $\tau_1^{**}$  is always positive and therefore the right-hand side is also positive, implying this inequality.

relatively low ( $\eta_{md} > \tilde{\eta}$ ), transfers are set at zero and hence the median household votes only for two government-spending categories. Before she does so, consider first the economic equilibrium, before voting takes place. The median voter benefits from a larger capital endowment and hence higher income, raising her demand for private consumption goods, given the average level of capital. Now consider the political-economy equilibrium, when the median voter casts her vote for tax rates. Voting for tax rate  $\tau_2^{**}$  is unaffected, according to Lemma 2. But voting for tax rate  $\tau_1^{**}$  is determined by the degree of substitution between private and public consumption goods in household utility. When substitution is low ( $\rho < 0$ ), both goods are complementary, and therefore a larger demand for private goods leads also to a larger demand of public consumption goods. Therefore the political-economy equilibrium implies a larger  $\tau_1^{**}$  tax rate.

Now let's analyze the impact of income distribution on growth in our political-economy equilibrium. The endogenous growth rate is determined by

$$\Theta - 1 = \beta(1 - \tau_g)(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}} - 1, \quad (22)$$

where  $(1 - \tau_g) = (1 - \gamma) \left( \beta + \frac{(1-\beta)(\beta+\phi)(1-\gamma\sigma)\eta_{md}}{1-(1-\beta)(1-\gamma\sigma)\eta_{md}} \right)$  if  $\eta_{md} < \tilde{\eta}$  and  $(1 - \tau_g) = (1 - \gamma - \tau_1^{**}(\eta_{md}))$  if  $\eta_{md} \geq \tilde{\eta}$ .

If income concentration is high (i.e., if  $\eta_{md} < \tilde{\eta}$ ), government spending falls on all three spending categories, including transfers. Then a reduction in inequality reduces the distributional conflict and the median voter chooses a lower tax rate to finance transfers.

Consistent with the traditional literature (where government spending falls only on transfers), growth rises when inequality declines

$$\frac{\partial \Theta}{\partial \eta_{md}} = \frac{\beta(1 - \gamma)(1 - \beta)(\beta + \phi)(1 - \gamma\sigma)^2 A^{\frac{1}{1-\gamma}} \gamma^{\frac{\gamma}{1-\gamma}}}{(1 - (1 - \beta)(1 - \gamma\sigma)\eta_{md})^2} > 0 \quad (23)$$

If income concentration is low (i.e., if  $\eta_{md} \geq \tilde{\eta}$ ), government spending falls on two spending categories, excluding transfers. Then a reduction in inequality has an ambiguous effect on the median voters choice of  $\tau_1^{**}$ , and hence on growth, depending on the elasticity of substitution between consumption goods

$$\frac{\partial \Theta}{\partial \eta_{md}} = -(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}}\gamma^{\frac{\gamma}{1-\gamma}} \frac{\partial \tau_1^{**}}{\partial \eta_{md}} \quad (24)$$

where  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} > (<) 0$ , if  $\rho < (>) 0$ .

Figure 2 depicts the three economic and political-economy equilibrium relations between income concentration and growth derived in our model, consistent with equations (23) and (24). When income concentration is relatively high (i.e., if it exceeds threshold level  $1 - \tilde{\eta}_{md}$ ), the median household votes for three tax rates. Then the relation between income concentration and growth is monotonically negative, because higher income concentration leads to higher taxation and hence to lower growth.

However, when income concentration is relatively low (i.e., if it falls below threshold level  $1 - \tilde{\eta}_{md}$ ), the median household votes for two tax rates, excluding transfers. Then the relation between income concentration and growth is ambiguous, depending on consumption good substitution in household utility. If substitution is high ( $\rho > 0$ ), the relation between income concentration and growth is monotonically positive. If substitution is low ( $\rho < 0$ ), the relation is negative.

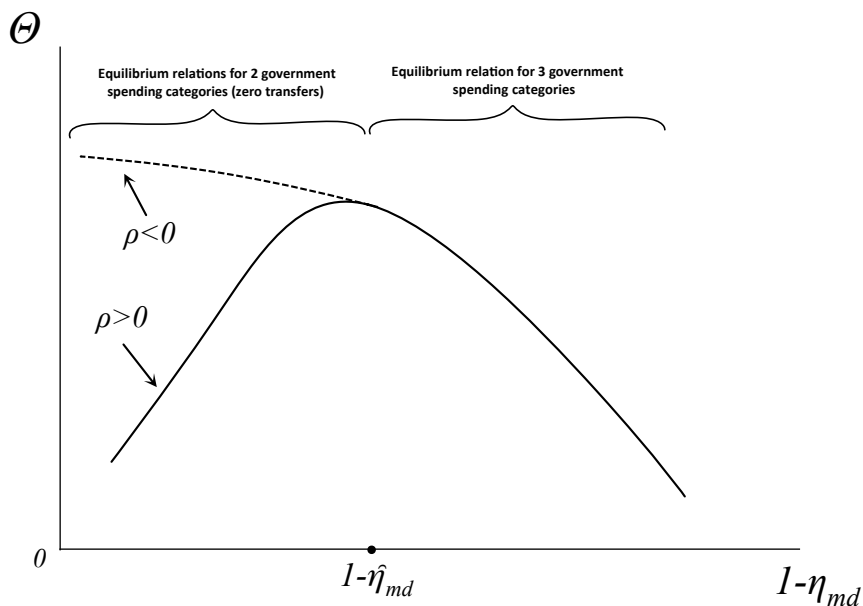


Figure 2: Inequality and Growth in Three Political-Economy Equilibria.

## 4.2 Political-Economic Equilibrium under Sequential Voting

Our previous equilibrium was derived under the assumption that voting takes place once in period zero, and therefore tax rates and government expenditure composition is held constant over time. Now we address the question if the latter policy is time-consistent, i.e., if society would repeat its period-0 choice if allowed to vote again at any period in the future.<sup>15</sup> One way to obtain time-consistent policies is allowing voting to take place in any

<sup>15</sup>Krusell et al. (1997) compare two equilibria, one with voting in period zero and other with sequential voting. A sequential voting can lead that the decisive voter faces more restrictions derived from imposing time consistency, lowering her welfare and hence implying a different vote to voting in period zero.

period  $t$ , under the condition that voting is shaped only by the relevant state variables in period  $t$ , without influence of past variables. This is a Markov-perfect equilibrium.<sup>16</sup>

In order to derive the latter equilibrium we re-write household  $i$ 's indirect utility function in its recursive form. We assume that the policy vector voted in each period follows the rule  $\tau = F(k_{dc}, k)$ , where  $\tau = (\tau_1 \ \tau_2 \ \tau_3)'$ . This policy function is determined by two state variables: capital of the decisive voter  $k_{dc}$ , and society's average capital stock,  $k$ . Households expect that government policies regarding taxation and expenditure follow policy function  $F$ .

Considering the latter, households (including the decisive household) solve the following problem

$$\max_{\{k_{i,t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t), g_1(\tau_t, k_t)) \quad (25)$$

where  $\mathbb{C}_i$  is defined by (3),  $c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t) = \psi_0(\tau)k + \psi_1(\tau)k_i - k_{i,t+1}$ , and  $g_{1,t}(\tau_t, k_t) = \psi_2(\tau)k_t$ .<sup>17</sup>

Define function  $\mathcal{H}^i(\tau, k_i, k)$  as the optimal saving response of household  $i$ . This function describes the response of households to a "one-shot deviation" on the part of the decisive voter in the following sense. If the current capital stock is  $k$ , current policy is  $\tau$ , and the household  $i$  expects that future policy will be determined by the policy function  $F(k_{dc}, k)$ , then savings will be given by  $\mathcal{H}^i(\tau, k_i, k)$ .

The latter optimal response satisfies household  $i$ 's first-order condition

$$\left( \frac{\mathbb{C}_{i,t+1}(\tau_{t+1}, k_{i,t+1}, k_{i,t+2}, k_{t+1})}{\mathbb{C}_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t)} \right)^\rho \left( \frac{c_{i,t+1}(\tau_{t+1}, k_{i,t+1}, k_{i,t+2}, k_{t+1})}{c_{i,t}(\tau_t, k_{i,t}, k_{i,t+1}, k_t)} \right)^{1-\rho} = \beta \psi_1(\tau_{t+1}) \quad (26)$$

where  $k_{i,t+1} = \mathcal{H}^i(\tau_t, k_{i,t}, k_t)$ ,  $\tau_{t+1} = F(k_{i,t+1}, k_{t+1})$  and  $k_{i,t+2} = \mathcal{H}^i(\tau_{t+1}, k_{i,t+1}, k_{t+1})$ .

It is not possible to simplify (26) further because consumption growth rate is not necessarily constant over time because it depends on current and future tax rates. In addition, it is not yet possible to assure that aggregate capital growth is equal to consumption growth, which also implies that aggregate consumption growth  $\mathbb{C}$  is equal to the growth rate of private consumption  $c$ . Hence it is not possible to simplify the latter first-order condition as we did in deriving equation (14).<sup>18</sup> As future tax rates determine current consumption,

<sup>16</sup>Time inconsistency arises in the absence of a mechanism or technology that allows commitment regarding the public policy choice. We do not consider sustainable equilibrium or reputation mechanisms in this model because we assume that governments implement the decisive voter's tax choice in any period and at zero cost. We also assume that current governments are not linked in any way to future governments, i.e., we do not consider government reelection.

<sup>17</sup> $\psi_0(\tau) \equiv A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}} \tau_{3,t}$ ;  $\psi_1(\tau) \equiv (1 - \tau_{g,t})(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}}$  y  $\psi_2(\tau) \equiv \tau_{1,t}A^{\frac{1}{1-\gamma}} \tau_{2,t}^{\frac{\gamma}{1-\gamma}}$ .

<sup>18</sup>For the special case of logarithmic aggregation function, the ratio of marginal utilities of private consumption  $c_{t+1}/c_t$  would not depend on public consumption good and the first-order condition would be simplified.

this opens the door to potential time inconsistency: the policy maker could change her tax policy after households have taken their current-period decisions. In our model, the policy maker is the median voter, who could face an incentive to revise his future tax choice in order to maximize his utility.

Considering optimal saving response  $\mathcal{H}^i(\tau, k_i, k)$ , the median voter chooses tax rates  $\tau_j$  by solving the following problem expressed in its recursive form

$$v(k_{md}, k) = \max_{\tau_j} \ln \mathbb{C}_{md}(c_{md}, g_1) + \beta v(\mathcal{H}^{md}(k_{md}, \tau), \mathcal{H}(k, \tau)) \quad (27)$$

where  $\mathcal{H}^{md}(k_{md}, \tau)$  and  $\mathcal{H}(k, \tau)$  are the optimal saving responses of the median voter household and the average household, respectively.

We derive the first-order condition on for the political-economy problem of the median household in casting her decisive vote for tax rates, known as the Generalized Euler Equation (GEE), as defined in Klein et al. (2008).<sup>19</sup> The GEE reflects agreement between the median household's decision in period  $t$  with his own decision in period  $t + 1$  in setting tax rates  $\tau_{j,t+1}$ . Note that for this result to hold we assume (with Klein et al. 2008) that the choice of taxes in period  $t$  takes place before the economic choice of all households in period  $t$ . In other word, it is required to assume a weak sense of lack of commitment or that a intra-period commitment is needed because the government (i.e., the median voter) has to commit to her political choice after households take their saving decisions.

**Definition 4.** *Markov-perfect Political-Economy Equilibrium.*

Let  $\tau = [\tau_1, \tau_2, \tau_3]' \in [0, 1]^3$  the policy vector implemented in any period and  $k_{md,0}$  as the median value of the initial distribution of capital. A Markov-perfect equilibrium is defined as a policy function  $F : \mathbb{R}_+^2 \rightarrow [0, 1]^3$  such that the following equations hold: (i)  $F(k_{md}, k) = \arg \max_{\tau} v(k_{md}, k)$  where  $\tau = F(k_{md}, k)$  and  $k' = \mathcal{H}^i(\tau, k_i, k)$ ; and (ii)  $\mathcal{H}^i(\tau, k_i, k)$  satisfies the condition (26).

Generally the literature has solved Markov-perfect equilibrium with numerical techniques, the absence of an analytical solution is due to the higher-order derivatives of decision rules. Both the policy functions and their derivatives have to satisfy the first-order conditions for both the current and future periods.<sup>20</sup>

In our case we conjecture the following solution for the policy function  $F(k_{md}, k) = \tau^*$ , that is, the policy of tax rates and expenditure composition that is constant over time, verifying if this policy satisfies the Markov-Perfect equilibrium definition. Our conjecture is based on analyzing first-order condition (26) for household  $i$ . If tax rates are constant over time, then individual consumption and gross saving, and the corresponding average levels, grow at a common rate determined by equation (14). Hence the latter variables do not depend on future capital, so that current saving is determined only by current, not by

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<sup>19</sup>See Appendix D.

<sup>20</sup>Klein et al. (2008) propose a method of local approximations and compare its result with an alternative method based on Chebyshev polynomials to evaluate first-order conditions for the steady-state equilibrium.

future variables. Note that this result also depends on having endogenous growth. If we had a technology of declining returns to capital, the dependence of current variables on future capital could not be avoided, and therefore the policy maker's (the median voter's) temptation to deviate from previous policy decisions would still be present.<sup>21</sup>

**Proposition 1.** *In an economy characterized by logarithmic inter-temporal preferences, CES intra-temporal preferences over private and public consumption goods, and constant marginal productivity of capital, the political-economy equilibrium with voting over the size and composition of government spending in period zero coincides with a Markov-perfect equilibrium of sequential voting.*

*Proof.* Full proof is provided in Appendix D. □

The idea of the proof is simple. Define  $\tau^*$  as the time-invariant policy to be chosen. In a model of endogenous growth, consumption, investment and public spending growth at a common constant rate  $\Theta(\tau^*) - 1$  (see equation 14). Therefore optimal gross household saving is  $k_{i,t+1} = \mathcal{H}^i(\tau, k_i) = \Theta(\tau^*)k_{i,t}$ .

As function  $\mathcal{H}^i(\tau^*, k_i)$  is twice differentiable, replace  $\mathcal{H}'^i$  and  $\mathcal{H}''^i$  in the GEE derived in Appendix D obtaining

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_j} \Theta_{\tau_j^*} k - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*'}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k \right\} = 0 \quad (28)$$

We derive the corresponding derivatives and obtain

$$(1 - \beta) \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \frac{\Theta_{\tau_j^*}}{\Theta(\tau_j^*)} = 0 \quad (29)$$

Equation (29) coincides with the first-order condition of the political-economy equilibrium with voting in period zero. Considering that optimal government spending and its composition (i.e., tax rates) are independent of  $k_i$  and  $k$ , our conjecture about  $F$  is valid and, therefore, the equilibrium with voting in the initial period is time-consistent.

## 5 Conclusions

In this paper we have developed a dynamic general-equilibrium political-economy model for the optimal size and composition of public spending. An analytical solution has been derived from majority voting for the optimal level of three government-spending categories: public consumption goods and transfers (valued by households), as well as government productive services (complementing private capital in an endogenous-growth technology).

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<sup>21</sup>Using numerical simulations, Krusell et al. (1997) illustrate how the choice of a Markov-Perfect equilibrium is constant in a model of endogenous growth.

Inequality is reflected by a discrete distribution of infinitely-lived agents that differ by their initial capital holdings. In contrast to the previous literature that derives monotonic (typically negative) relations between inequality and growth in one-dimensional voting environments, we have established conditions, in an environment of multi-dimensional voting, under which a non-monotonic, inverted U-shape relation between inequality and growth is obtained.

A key feature of this paper is the existence of an endogenous threshold level of inequality at which the median voter chooses a household transfer level of zero. At higher levels of inequality, the relation between inequality and growth is negative, as it is in much of the previous political-economy literature. At levels of inequality below the threshold level, negative transfers (i.e., taxes) would be chosen. This outcome is not sensible, so we restrict transfers to be zero at low levels of inequality. This implies that in the latter case voting is restricted to choosing optimal levels of two remaining spending categories: public consumption goods and productive government services.

This leads to the paper's main result. In the range where inequality is lower than the aforementioned threshold level and transfers are zero, the relationship between inequality and growth could be positive. While optimal provision of productive services is affected neither by inequality nor by transfers, optimal provision of public consumption goods is affected by inequality. When inequality exceeds the threshold level, lower income concentration leads to a smaller demand for transfers, lower taxation, and higher growth. However, when inequality falls below the threshold level, we obtain an ambiguous relation between income concentration and taxation, which is determined by the degree of substitution between private and public consumption goods in household utility. If private and public consumptions are complements (substitutes), a lower (higher) income concentration leads to a higher (lower) demand for public goods and hence higher (lower) overall taxation, which lowers (raises) growth.

The latter main result extends the previous analytical literature that derives a monotonic relationship under one-dimensional voting. Our more general result that inequality and growth could be negatively or positively related could be consistent with the ambiguous or inconclusive results documented in the empirical literature on the inequality-growth nexus. We have derived analytical policy functions that characterize the political-economy equilibrium under multi-dimensional voting, which in many cases implied obtaining closed-form solutions for the policy instruments. The latter results were obtained by voting at the initial period. Then we showed that when focusing on Markov-perfect equilibria, the policy voted at time zero is stationary, i.e., it is time consistent.

## 6 References

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## Appendix A: Politico-Economic Equilibrium with Voting at Time Zero

We define the utility function of the decisive voter

$$\mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0) = \ln(\mathbb{C}_{i,0}) \sum_{t=0}^{\infty} \beta^t + \ln(\Theta) \sum_{t=0}^{\infty} t\beta^t = \mathbb{M}_0 \ln(\mathbb{C}_{i,0}) + \mathbb{M}_1 \ln(\Theta)$$

The first-order conditions are

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_1} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_1} = 0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_2} = 0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_3} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_3} = 0$$

Given  $\mathbb{C}_{i,0} = \left( ac_t^\rho + (1-a)g_{1,t}^\rho \right)^\frac{1}{\rho}$ , (10) and (11), we get

$$\frac{\partial c_{i,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0$$

$$\frac{\partial c_{i,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1 - \gamma} - 1 \right) \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\gamma}{1 - \gamma} \frac{\tau_3}{\tau_2} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ \frac{\gamma}{1 - \gamma} \frac{\tau_1}{\tau_2} \right]$$

$$\frac{\partial c_{i,0}}{\partial \tau_3} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ 1 - (1 - \beta) \frac{k_{i,0}}{k_0} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_3} = 0$$

$$\frac{\partial \ln \mathbb{C}_0}{\partial \tau_j} = \frac{1}{\mathbb{C}_0} \left( ac_{i,0}^{\rho-1} \frac{\partial c_{i,0}}{\partial \tau_j} + (1-a)g_{1,0}^{\rho-1} \frac{\partial g_{1,0}}{\partial \tau_j} \right), \text{ for } j = 1, 2 \text{ y } 3.$$

Taking into account (10) we obtain

$$\frac{\partial \ln \Theta}{\partial \tau_2} = \frac{(1 - \gamma\sigma) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1 - \gamma} - 1 \right)}{(1 - \tau_1 - \tau_2 - \tau_3)(1 - \gamma\sigma) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_j} = \frac{-(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}}{(1 - \tau_1 - \tau_2 - \tau_3)(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}}, \text{ para } j = 1 \text{ y } 3.$$

Replacing  $\frac{\partial \ln \mathbb{C}_0}{\partial \tau_j}$  and  $\frac{\partial \ln \Theta}{\partial \tau_j}$  in the first-order conditions

$$ac_{i,0}^{\rho-1} \left( (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + (1 - a)g_0^{\rho-1} - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^\rho \frac{1}{\Psi k_0} = 0$$

$$ac_{i,0}^{\rho-1} \left[ \mu \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\tau_3}{\tau_2} \frac{\gamma}{1 - \gamma} \right] + (1 - a)g_0^{\rho-1} \frac{\tau_1}{\tau_2} \frac{\gamma}{1 - \gamma} + \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^\rho \frac{\mu}{\Psi k_0} = 0$$

$$ac_{i,0}^{\rho-1} \left( 1 - (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^\rho \frac{1}{\Psi k_0} = 0$$

where  $\Psi = (1 - \tau_1 - \tau_2 - \tau_3)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}}$  and  $\mu = \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1 - \gamma} - 1 \right)$ .

*Marginal Single-Crossing Condition.*

When multiplying both sides of later equations by  $k_0$ , it is straightforward to show that  $\frac{\partial^2 \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}) k_0}{\partial k_{i,0} \partial \tau_j} > 0$  for  $j = 1, 2$  and  $\frac{\partial^2 \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}) k_0}{\partial k_{i,0} \partial \tau_3} < 0$ .

From the first-order conditions, the following results are obtained

$$\tau_1^* = \frac{(1 - \gamma)}{(1 + \phi) \frac{\mathbb{M}_0}{\mathbb{M}_1} (1 + \frac{\mathbb{M}_1}{\mathbb{M}_0})}$$

$$\tau_2^* = \gamma$$

$$\tau_3^* = \frac{(1 - \gamma) \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \left( 1 - (1 + \phi) \frac{\mathbb{M}_0}{\mathbb{M}_1} (1 + \frac{\mathbb{M}_1}{\mathbb{M}_0}) \right) + \phi \right)}{(1 + \phi) \frac{\mathbb{M}_0}{\mathbb{M}_1} (1 + \frac{\mathbb{M}_1}{\mathbb{M}_0}) \left( 1 - (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right)}$$

where  $\phi = \left( \frac{a}{1-a} \right)^{\frac{1}{1-\rho}}$ .

*Convergence of ratio  $\mathbb{M}_0 / \mathbb{M}_1$ .*

Given that  $0 < \beta < 1$  we know that  $\mathbb{M}_0 \equiv \sum_{t=0}^{\infty} \beta^t = \frac{1}{1-\beta}$  and  $\mathbb{M}_1 \equiv \sum_{t=0}^{\infty} t\beta^t = \frac{\beta}{(1-\beta)^2}$ .

$$\begin{aligned}
\mathbb{M}_1 &= 0 + \beta + 2\beta^2 + 3\beta^3 + \dots \\
&= \beta(1 + 2\beta + 3\beta^2 + 4\beta^3 + \dots) \\
&= \beta(1 + \beta + \beta^2 + \beta^3 + \dots \\
&\quad \beta + \beta^2 + \beta^3 + \dots \\
&\quad + \beta^2 + \beta^3 + \dots )
\end{aligned}$$

Let  $S = (1 + \beta + \beta^2 + \beta^3 + \dots)$  then

$$\begin{aligned}
\mathbb{M}_1 &= \beta(1 + \beta + \beta^2 + \beta^3 + \dots)S \\
&= \beta \frac{1}{1-\beta} \frac{1}{1-\beta}.
\end{aligned}$$

Therefore, if we replace the ratio  $\frac{\mathbb{M}_1}{\mathbb{M}_0} = \frac{\beta}{1-\beta}$  in the political equilibrium, we obtain

$$\tau_1^* = \frac{(1-\gamma)(1-\beta)}{(1+\phi)}$$

$$\tau_2^* = \gamma$$

$$\tau_3^* = \frac{(1-\gamma)(1-\beta)(\phi - (\phi + \beta)(1-\gamma\sigma)\eta_{md})}{(1+\phi)(1-(1-\beta)(1-\gamma\sigma)\eta_{md})}$$

## Appendix B: Politico-Economic Equilibrium with Voting at Time Zero and with $\tau_3 = 0$

We define the new utility function of the decisive voter

$$\mathbb{V}(\tau_1, \tau_2; k_{i,0}, k_0) \equiv \sum_{t=0}^{\infty} \beta^t \ln(\mathbb{C}_{i,0} \Theta^t) = \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t \beta^t \ln \Theta$$

where  $\mathbb{C}_{i,0} = \left( a \tilde{c}_{i,0}^\rho + (1-a)(\tau_1 A^{\frac{1}{\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0)^\rho \right)^{\frac{1}{\rho}}$ ;  $\tilde{c}_{i,0} = (1 - \tau_1 - \tau_2) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma) k_{i,0} - \tilde{\Theta} k_{i,0}$ ; and  $\tilde{\Theta} = \beta(1 - \tau_1 \tau_2)(1 - \gamma\sigma) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}$ .

The first-order conditions are

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_1} + \mathbb{M}_1 \frac{\partial \ln \tilde{\Theta}}{\partial \tau_1} = 0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1 \frac{\partial \ln \tilde{\Theta}}{\partial \tau_2} = 0$$

After the corresponding substitutions, we get

$$ac_{i,0}^{\rho-1} \left( (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + (1 - a)g_0^{\rho-1} - \frac{\beta}{1 - \beta} \mathbb{C}_{i,0}^\rho \frac{1}{\tilde{\Psi} k_0} = 0$$

$$ac_{i,0}^{\rho-1} \left[ \tilde{\mu} \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\tau_3}{\tau_2} \frac{\gamma}{1 - \gamma} \right] + (1 - a)g_0^{\rho-1} \frac{\tau_1}{\tau_2} \frac{\gamma}{1 - \gamma} + \frac{\beta}{1 - \beta} \mathbb{C}_{i,0}^\rho \frac{\tilde{\mu}}{\tilde{\Psi} k_0} = 0$$

where  $\tilde{\Psi} = (1 - \tau_1 - \tau_2)A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}$  y  $\tilde{\mu} = \left( \frac{(1-\tau_1-\tau_2)}{\tau_2} \frac{\gamma}{1-\gamma} - 1 \right)$ .

With a little algebra, we get

$$\frac{a}{1 - a} \left( \frac{(1 - \gamma - \tau_1^{**})}{\tau_1^{**}} (1 - \gamma\sigma)(1 - \beta)\eta_{md} \right)^\rho = \frac{(1 - \beta)(1 - \gamma)}{\tau_1^{**}} - 1$$

$$\tau_2^{**} = \gamma$$

### Appendix C: Differentiation of the equation that characterizes the relationship between $\tau_1^{**}$ and $\eta_{md}$

Given equation  $\frac{a}{1-a} \left( \frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}} (1 - \gamma\sigma)(1 - \beta)\eta_{md} \right)^\rho = \frac{(1-\beta)(1-\gamma)}{\tau_1^{**}} - 1$ , the left-hand side is always positive because parameters  $a, \beta, \gamma, \sigma$  y  $\tau_1$  belong to  $[0, 1]$ . Given the latter, the value of  $\tau_1^{**}$  must satisfy the following condition

$$\tau_1^{**} < (1 - \beta)(1 - \gamma).$$

Now we differentiate the equation that characterizes  $\tau_1^{**}$  and obtain

$$\frac{\partial \tau_1^{**}}{\partial \eta_{md}} = \frac{\frac{a}{1-a} \rho \left( \frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}} \right)^\rho ((1 - \gamma\sigma)(1 - \beta))^\rho (\eta_{md})^{\rho-1}}{-\frac{(1-\beta)(1-\gamma)}{(\tau_1^{**})^2} + \frac{a}{1-a} \frac{\rho(1-\gamma)}{(\tau_2^{**})^2} \left( \frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}} \right)^{\rho-1} ((1 - \gamma\sigma)(1 - \beta))^\rho (\eta_{md})^\rho}$$

The denominator of  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}}$  can be re-written taking the equation that characterizes  $\tau_1^{**}$  and we get

$$\frac{\partial \tau_1^{**}}{\partial \eta_{md}} = -\frac{\frac{a}{1-a} \rho \left( \frac{(1-\gamma-\tau_1^{**})}{\tau_1^{**}} \right)^\rho ((1 - \gamma\sigma)(1 - \beta))^\rho (\eta_{md})^{\rho-1} (\tau_1^{**})^2 (1 - \tau_1^{**} - \gamma)}{(1 - \gamma) [(1 - \beta)(1 - \gamma)(1 - \rho) + (\rho - (1 - \beta))\tau_1^{**}]}$$

The sign of the numerator depends on the value of  $\rho$ . Now let's analyze the sign of the denominator

$$(1 - \beta)(1 - \gamma)(1 - \rho) + (\rho - (1 - \beta))\tau_1^{**}$$

Case I:  $(1 - \beta) < \rho < 1$ , the first term of the denominator is positive and the term  $\rho - (1 - \beta)$  is also positive. Therefore the denominator is positive and  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} < 0$ .

Case II:  $0 < \rho < (1 - \beta)$ , the first term of the denominator is positive and the term  $\rho - (1 - \beta)$  is negative. By contradiction, if  $\tau_1^{**} > \frac{(\rho-1)}{(\rho-(1-\beta))}(1 - \beta)(1 - \gamma)$  then the denominator is negative. However, given that  $\frac{(\rho-1)}{(\rho-(1-\beta))} > 1$ , this contradicts the restriction  $\tau_1^{**} < (1 - \beta)(1 - \gamma)$ . Therefore the denominator can not be negative and  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} < 0$ .

Case III:  $-\infty < \rho < 0$ , the first term of the denominator is positive and the term  $\rho - (1 - \beta)$  is negative. If  $\tau_1^{**} < \frac{(\rho-1)}{(\rho-(1-\beta))}(1 - \beta)(1 - \gamma)$ , the denominator and numerator are negative and therefore the  $\frac{\partial \tau_1^{**}}{\partial \eta_{md}} > 0$ .

#### Appendix D: Politico-Economic Equilibrium with Sequential Voting.

The median voter faces the following problem in recursive version

$$v(k_{md}, k) = \max_{\tau} \ln \mathbb{C}_{md}(c_{md}, g_1) + \beta v(\mathcal{H}^{md}(k_{md}, \tau), \mathcal{H}(k, \tau))$$

where  $\mathcal{H}^{med}(k_{md}, \tau)$  and  $\mathcal{H}(k, \tau)$  are optimal decisions of gross saving of the average and median agent, respectively;  $c_{md}$  and  $g_1$  are the consumption function of the median agent and the supply function of the public good, respectively. The latter functions depend on the political variable and the state variables

$$c_{md} = \psi_0(\tau)k + \psi_1(\tau)k_{md} - \mathcal{H}^{med}(k_{md}, \tau)$$

and

$$g_1 = \psi_2(\tau)k,$$

where  $\psi_0(\tau) \equiv (1 - \tau)A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}(1 - \alpha + \frac{\tau_3}{1-\tau})$ ,  $\psi_1(\tau) \equiv (1 - \tau)\alpha A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}$ , and  $\psi_2(\tau) \equiv \tau_1 A^{\frac{1}{\alpha}}\tau_2^{\frac{1-\alpha}{\alpha}}$ .

To obtain the generalized Euler equation (GEE) of the median voter, we derive the first-order conditions

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta v'_k \mathcal{H}_{\tau_j} + \beta v'_{k_{md}} \mathcal{H}_{\tau_j}^{md} = 0 \quad j = 1, 2, 3.$$

Now we derive the Bellman equation with respect to  $k$  and  $k_{md}$

$$v_k = \frac{\partial \ln \mathbb{C}_{md}}{\partial k} + \beta v'_k \mathcal{H}_k \quad j = 1, 2, 3.$$

$$v_{k_{md}} = \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} + \beta v'_{k_{md}} \mathcal{H}_{k_{md}}^{md} \quad j = 1, 2, 3.$$

From the first-order conditions we obtain

$$\beta v'_k = \left( -\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} - \beta v'_{k_{md}} \mathcal{H}_{\tau_j}^{md} \right) \frac{1}{\mathcal{H}_{\tau_j}} \quad j = 1, 2, 3.$$

Then  $v_k$  is

$$v_k = \frac{\partial \ln \mathbb{C}_{md}}{\partial k} - \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} - \beta v'_{k_{md}} \mathcal{H}_{\tau_j}^{md} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} \quad j = 1, 2, 3.$$

From  $v_k$  and  $v_{k_{md}}$  we get

$$\beta v'_{k_{md}} = \left( v_{k_{md}} - \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} \right) \frac{1}{\mathcal{H}_{k_{md}}^{md}} \quad j = 1, 2, 3.$$

Now  $v_k$  can be expressed as

$$v_k = \frac{\partial \ln \mathbb{C}_{md}}{\partial k} - \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} - \left( v_{k_{md}} - \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} \right) \frac{\mathcal{H}_{\tau_j}^{md}}{\mathcal{H}_{k_{md}}^{md}} \frac{\mathcal{H}_k}{\mathcal{H}_{\tau_j}} \quad j = 1, 2, 3.$$

Shifting the latter equation one period forward

$$v'_k = \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} - \left( v'_{k_{md}} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \right) \frac{\mathcal{H}_{\tau'_j}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \quad j = 1, 2, 3.$$

Replacing the latter equation in the first-order conditions, we obtain

$$\begin{aligned} \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \frac{\mathcal{H}_{\tau'_j}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \right\} \mathcal{H}_{\tau_j} + \dots \\ \beta v'_{k_{md}} \left( \mathcal{H}_{\tau_j}^{md} - \frac{\mathcal{H}_{\tau'_j}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \mathcal{H}_{\tau_j} \right) = 0 \quad j = 1, 2, 3. \end{aligned}$$

From the latter equation we obtain an expression for  $\beta v'_{k_{md}}$ , and we replace in  $v_{k_{md}}$  as follows

$$v_{k_{md}} = \frac{\partial \ln \mathbb{C}_{md}}{\partial k_{md}} - \left[ \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau'_j} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \frac{\mathcal{H}_{\tau'_j}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau'_j}} \right\} \mathcal{H}_{\tau_j} \right] \times$$



$$\left\{ \mathcal{H}_{\tau_j}^{md} - \frac{\mathcal{H}_{\tau_j'}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}^{md}}{\mathcal{H}_{\tau_j}^{md}} \mathcal{H}_{\tau_j} \right\}^{-1} \mathcal{H}_{k'_{md}}^{md}. \quad j = 1, 2, 3.$$

Shifting the latter equation one period forward and replacing in the first-order conditions we get the GEE of the median voter

$$\Omega_j + \beta \left[ \frac{\partial \ln \mathcal{C}'_{md}}{\partial k'_{md}} - \Omega'_j \times \left\{ \mathcal{H}_{\tau_j}^{md} - \frac{\mathcal{H}_{\tau_j'}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}^{md}}{\mathcal{H}_{\tau_j}^{md}} \mathcal{H}_{\tau_j} \right\}^{-1} \mathcal{H}_{k'_{md}}^{md} \right] \left( \mathcal{H}_{\tau_j}^{md} - \frac{\mathcal{H}_{\tau_j'}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}^{md}}{\mathcal{H}_{\tau_j}^{md}} \mathcal{H}_{\tau_j} \right) = 0 \quad j = 1, 2, 3.$$

where

$$\Omega_j \equiv \frac{\partial \ln \mathcal{C}_{md}}{\partial \tau_j} + \beta \left\{ \frac{\partial \ln \mathcal{C}'_{md}}{\partial k'} - \frac{\partial \ln \mathcal{C}'_{md}}{\partial \tau_j'} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau_j'}} + \frac{\partial \ln \mathcal{C}'_{md}}{\partial k'_{md}} \frac{\mathcal{H}_{\tau_j'}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}}{\mathcal{H}_{\tau_j'}} \right\} \mathcal{H}_{\tau_j}$$

*Case: Generalized Euler Equation with  $\mathbf{F}(\mathbf{k}_i, \mathbf{k}) = \tau^*$*

Now we derive a particular outcome of the GEE. Given that tax rate  $\tau^*$  is constant, all variables grow at the same rate and we obtain

$$\begin{aligned} k' &= \Theta(\tau^*)k, \\ c' &= \psi_0(\tau^*)k + \psi_1(\tau^*)k_i - \Theta(\tau^*)k, \text{ and } y \\ \mathcal{C}'_i/\mathcal{C}_i &= c'_i/c_i = \Theta(\tau^*). \end{aligned}$$

The derivatives of functions  $\mathcal{H}^i$  and  $\mathcal{H}$  are

$$\begin{aligned} \mathcal{H}_{\tau_j^*} &= \Theta_{\tau_j^*} k \\ \mathcal{H}_{\tau_j^*}^i &= \Theta_{\tau_j^*} k_i \\ \mathcal{H}_{\tau_j^{*'}} &= \Theta_{\tau_j^{*'}} k' \\ \mathcal{H}_{\tau_j^{*'}}^i &= \Theta_{\tau_j^{*'}} k'_i \\ \mathcal{H}_{k'} &= \mathcal{H}_{k'_i}^i = \Theta(\tau^{*'}) \end{aligned}$$

If we substitute the latter derivatives into the GEE, the GEE is reduced to the expression  $\Omega_j$  since  $\mathcal{H}_{\tau_j}^{md} - \frac{\mathcal{H}_{\tau_j'}^{md}}{\mathcal{H}_{k'_{md}}^{md}} \frac{\mathcal{H}_{k'}^{md}}{\mathcal{H}_{\tau_j}^{md}} \mathcal{H}_{\tau_j} = 0$ . This simplification is possible because the conjecture that the tax rate is constant allows us to deduce that individual and aggregate saving grow at the same rate and, therefore, it is redundant to consider  $k_i$  and  $k$  in the value function. The ratio  $k_i/k$  is constant for all  $t$  and, therefore,  $k$  is a linear transformation of  $k_i$ .

The GEE is

$$\frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \left\{ \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k - \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*'}} + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k \right\} = 0$$

We derive the corresponding derivatives and obtain

$$\begin{aligned} \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} k' (\mathbb{C}'_{md})^{-\rho} \{ a(c'_{md})^{\rho-1} \psi_0(\tau') + (1-a)(g'_1)^{\rho-1} \sigma(\tau') \} \\ &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} \{ a(c'_{md})^{\rho-1} \psi_0(\tau') k' + (1-a)(g'_1)^{\rho-1} \sigma(\tau') k' \} \end{aligned}$$

$$\begin{aligned} \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} k'_{md} (\mathbb{C}'_{md})^{-\rho} \{ a(c'_{md})^{\rho-1} \psi_1(\tau') \} \\ &= \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} \{ a(c'_{md})^{\rho-1} \psi_1(\tau') k'_{md} \} \end{aligned}$$

Therefore

$$\frac{\partial \ln \mathbb{C}'_{md}}{\partial k'} \Theta_{\tau_j^*} k + \frac{\partial \ln \mathbb{C}'_{md}}{\partial k'_{md}} \eta_{md} \Theta_{\tau_j^*} k = \frac{\Theta_{\tau_j^*}}{\Theta(\tau)} (\mathbb{C}'_{md})^{-\rho} (\mathbb{C}'_{md})^\rho = \frac{\Theta_{\tau_j^*}}{\Theta(\tau)}$$

Moreover

$$\begin{aligned} \frac{\partial \ln \mathbb{C}'_{md}}{\partial \tau_j^{*'}} &= (\mathbb{C}'_{md})^{-\rho} \left\{ a(c'_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau'} k' + (1-a)(g'_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau'} k' \right\} \\ &= (\mathbb{C}'_{md})^{-\rho} (\Theta(\tau))^\rho \Theta(\tau) (\Theta(\tau))^{\rho-1} \left\{ a(c_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau'} k + (1-a)(g_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau'} k \right\} \\ &= (\mathbb{C}'_{md})^{-\rho} \left\{ a(c_{md})^{\rho-1} \frac{\partial \psi_0}{\partial \tau} k + (1-a)(g_1)^{\rho-1} \frac{\partial \sigma}{\partial \tau} k \right\} = \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} \end{aligned}$$

Finally, the GEE of the median voter is

$$(1-\beta) \frac{\partial \ln \mathbb{C}_{md}}{\partial \tau_j^*} + \beta \frac{\Theta_{\tau_j^*}}{\Theta(\tau_j^*)} = 0.$$

## Appendix E: Politico-Economic Equilibrium with Voting at Time Zero and Conditional Transfers

Agent  $i$  solves the following intertemporal problem

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,t}(c_{i,t}, g_{1,t})$$

subject to

$$c_{i,t} + k_{i,t+1} = (1 - \tau_g) r_t k_{i,t} + g_{3,t}^i, \text{ dado } k_{i,0},$$

Now transfers  $g_{3,t}^i$  are financed by a linear tax rate  $\tau_3$  (all agents pay without exception) and those resources are distributed only to the poor. The poor are those endowed with a capital stock below mean,  $k_i \leq k$ . Furthermore, the distribution of transfers to the poor will be inversely proportional to  $k_i/k$ . Transfers must satisfy the following condition

$$g_{3,t}^i = g \left( 1 - \frac{k_{i,t}}{k_t} \right), \text{ where } g > 0$$

Tax revenue should be equal to transfers delivered, i.e.,  $\tau_3 y_t = \int_0^k g(1-s)f(s)ds$ . Assuming that ratio  $\eta = \frac{k_i}{k}$  follows a Fisk or log-logistic distribution with scale parameter  $\alpha = \eta_{md}$  and shape parameter  $\beta = 1$ . In this case, the probability distribution and cumulative distribution functions are  $f(s) = \frac{1}{(\eta_{md}+s)^2}$  and  $F(s) = \frac{s}{\eta_{md}+s}$ , respectively. Transfers are equal to

$$\begin{aligned} \int_0^k g(1-s) \frac{1}{(\eta_{md}+s)^2} ds &= g \left\{ \int_0^k \frac{1}{(\eta_{md}+s)^2} - \int_0^k \frac{s}{(\eta_{md}+s)^2} \right\} ds \\ &= g \left\{ \frac{1}{\eta_{md}} + \log \left( \frac{\eta_{md}}{1+\eta_{md}} \right) \right\} = gB(\eta_{md}) \end{aligned}$$

Given that transfers must be equal to tax revenue then  $g = \tau_3 y_t (B(\eta_{md}))^{-1}$ . Finally, transfers are determined by

$$g_{3,t}^i = \tau_3 y_t (B(\eta_{md}))^{-1} \left( 1 - \frac{k_{i,t}}{k_t} \right)$$

As shown in subsection 2.2, marginal productivity of capital is  $A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma)$  and production per capita is  $y_t = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k$ .

Each agent solves his optimization problem taking as given equilibrium levels of public spending, tax rates, and factor prices. The Euler equation for each agent  $i$  is

$$\Theta \equiv \frac{c_{i,t+1}}{c_{i,t}} = \beta \left( (1 - \tau_g) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (1 - \gamma\sigma) - \tau_3 A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} (B(\eta_{md}))^{-1} \right).$$

Since we are dealing with a model of endogenous growth,  $\Theta$  reflects that growth will be affected by transfers unlike the model where transfers are *lump sum* (See Appendix A).

Replacing the Euler equation, government expenditure and marginal productivity of capital in the agent's budget constraint, we obtain

$$c_{i,t} = (\Psi - \Theta)k_t$$

$$\text{where } \Psi \equiv (1 - \tau_g)A^{\frac{1}{1-\gamma}}\tau_2^{\frac{\gamma}{1-\gamma}} \left( (1 - \gamma\sigma)\eta_i + \frac{\tau_3}{(1-\tau_g)}(B(\eta_{md}))^{-1}(1 - \eta_i) \right).$$

The competitive economic equilibrium allows us to write the utility function of the agent depending on  $\Theta$  and  $\mathbb{C}_0$ . Therefore, the problem faced by the decisive agent is the following

$$\max_{\tau_1, \tau_2, \tau_3} \mathbb{V}(\tau_1, \tau_2, \tau_3; k_{i,0}, k_0) \equiv \sum_{t=0}^{\infty} \beta^t \ln(\mathbb{C}_{i,0}\Theta^t) = \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t\beta^t \ln \Theta$$

$$\text{where } \mathbb{C}_{i,0} = \left( a((\Psi - \Theta)k_0)^\rho + (1 - a)(\tau_1 A^{\frac{1}{\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0)^\rho \right)^{\frac{1}{\rho}}.$$

Let  $\mathbb{M}_0 = \sum_{t=0}^{\infty} \beta^t$  and  $\mathbb{M}_1 = \sum_{t=0}^{\infty} t\beta^t$  then the first-order conditions are

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_1} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_1} = 0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_2} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_2} = 0$$

$$\mathbb{M}_0 \frac{\partial \ln \mathbb{C}_0}{\partial \tau_3} + \mathbb{M}_1 \frac{\partial \ln \Theta}{\partial \tau_3} = 0$$

Given  $\mathbb{C}_{i,0}$ ,  $\Theta$  and  $\Psi$ , we obtain

$$\frac{\partial c_{i,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_1} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0$$

$$\frac{\partial c_{i,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1 - \gamma} - 1 \right) \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\gamma}{1 - \gamma} \frac{\tau_3}{\tau_2} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_2} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ \frac{\gamma}{1 - \gamma} \frac{\tau_1}{\tau_2} \right]$$

$$\frac{\partial c_{i,0}}{\partial \tau_3} = A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0 \left[ (B(\eta_{md}))^{-1} (1 - (1 - \beta)) \frac{k_{i,0}}{k_0} + (\beta - 1)(1 - \gamma\sigma) \frac{k_i}{k} \right]$$

$$\frac{\partial g_{1,0}}{\partial \tau_3} = 0$$

$$\frac{\ln \mathbb{C}_0}{\partial \tau_j} = \frac{1}{\mathbb{C}_0} \left( ac_{i,0}^{\rho-1} \frac{\partial c_{i,0}}{\partial \tau_j} + (1 - a)g_0^{\rho-1} \frac{\partial g_0}{\partial \tau_j} \right), \text{ para } j = 1, 2 \text{ y } 3.$$

$$\frac{\partial \ln \Theta}{\partial \tau_1} = \frac{-(1 - \gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}{((1 - \tau_1 - \tau_2 - \tau_3)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_2} = \frac{\left[ \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1-\gamma} - 1 \right) (1 - \gamma\sigma) - \frac{\gamma}{1-\gamma} \frac{\tau_3}{\tau_2} (B(\eta_{md}))^{-1} \right] A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}{((1 - \tau_1 - \tau_2 - \tau_3)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

$$\frac{\partial \ln \Theta}{\partial \tau_3} = \frac{- \left\{ (1 - \gamma\sigma)A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} + (B(\eta_{md}))^{-1} \right\}}{((1 - \tau_1 - \tau_2 - \tau_3)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}}}$$

Substituting the latter equations in the first-order conditions, we obtain

$$ac_{i,0}^{\rho-1} \left( (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + (1 - a)g_0^{\rho-1} - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1 - \gamma\sigma)}{((1 - \tau_g)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

$$ac_{i,0}^{\rho-1} \left[ \mu \left( (1 - \beta)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) + \frac{\tau_3}{\tau_2} \frac{\gamma}{1-\gamma} (B(\eta_{md}))^{-1} \left( 1 - (1 - \beta) \frac{k_i}{k} \right) \right] +$$

$$(1 - a)g_0^{\rho-1} \frac{\tau_1}{\tau_2} \frac{\gamma}{1-\gamma} + \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1 - \gamma\sigma)\mu - \frac{\tau_3}{\tau_2} (B(\eta_{md}))^{-1}}{((1 - \tau_g)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

$$ac_{i,0}^{\rho-1} \left( (B(\eta_{md}))^{-1} (1 - (1 - \beta)) \frac{k_{i,0}}{k_0} + (\beta - 1)(1 - \gamma\sigma) \frac{k_{i,0}}{k_0} \right) -$$

$$\frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^{\rho} \frac{(1 - \gamma\sigma) + (B(\eta_{md}))^{-1}}{((1 - \tau_g)(1 - \gamma\sigma) - \tau_3(B(\eta_{md}))^{-1}) A^{\frac{1}{1-\gamma}} \tau_2^{\frac{\gamma}{1-\gamma}} k_0} = 0$$

where  $\mu = \left( \frac{(1 - \tau_1 - \tau_2 - \tau_3)}{\tau_2} \frac{\gamma}{1-\gamma} - 1 \right)$ .

After some algebra and considering that  $\frac{M_1}{M_0} = \frac{\beta}{1+\beta}$  (see appendix A), we obtain the following

$$\begin{aligned}\tau_1^* &= \frac{(1-\gamma)(1-\beta)}{\beta(1+\phi)} \left( 1 + (1+\beta) \frac{\zeta + \phi \nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma) + (B(\eta_{md}))^{-1})\beta(1+\phi)} \right)^{-1} \\ \tau_2^* &= \gamma \\ \tau_3^* &= \frac{(1-\gamma)\phi(1+\beta)\nu^{\frac{1}{1-\rho}} \left( 1 + (1+\beta) \frac{\zeta + \phi \nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma) + (B(\eta_{md}))^{-1})\beta(1+\phi)} \right)^{-1}}{\beta(1+\phi)} + \\ & (1-\gamma)(1-\beta)(1-\gamma\sigma) \frac{k_i}{k} \left( \frac{\left( 1 + (1+\beta) \frac{\zeta + \phi \nu^{\frac{\rho}{1-\rho}}}{((1-\gamma\sigma) + (B(\eta_{md}))^{-1})\beta(1+\phi)} \right)^{-1}}{\beta(1+\phi)} - 1 \right)\end{aligned}$$

where  $\phi \equiv \left(\frac{a}{1-a}\right)^{\frac{1}{1-\rho}}$ ,  $\chi \equiv (B(\eta_{md}))^{-1}(1 - (1-\beta)\frac{k_i}{k}) - (1-\beta)(1-\gamma\sigma)\frac{k_i}{k}$ ,  
 $\nu \equiv (1-\beta)(1-\gamma\sigma)\frac{k_i}{k} + \chi \frac{(1-\gamma\sigma)}{(1-\gamma\sigma) + (B(\eta_{md}))^{-1}}$ ,  $\zeta \equiv \frac{(1-\beta)(1-\gamma\sigma)\frac{k_i}{k} + \chi(1-\gamma\sigma)}{(1-\beta)(1-\gamma\sigma)\frac{k_i}{k} + \chi \frac{(1-\gamma\sigma)}{(1-\gamma\sigma) + (B(\eta_{md}))^{-1}}}$ .

For every unit of tax revenue, the amount allocated to productive services is equal to  $\gamma$ .

### Appendix F: Politico-Economic Equilibrium with Voting at Time Zero under Constant Returns of Capital

In this context, we face an endogenous growth model a la Romer (1986). The production function for a representative firm is

$$y_t = Ak^\alpha \bar{k}^{1-\alpha},$$

where  $\bar{k}$  is the aggregate stock of capital and Romer assumes that it is proportional to aggregate stock of knowledge. There are only two types of public expenditures:  $g_1 = \tau_1 y_t$  is public good consumption and  $g_3 = \tau_3 y_t$  are lump-sum transfers distributed to all individuals.

Agent  $i$  solves the following intertemporal problem

$$\max_{\{c_{i,t}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t \ln C_{i,t}(c_{i,t}, g_{1,t})$$

subject to

$$c_{i,t} + k_{i,t+1} = (1 - \tau_g)(w_t + r_t k_{i,t}) + g_{3,t}, \text{ given } k_{i,0},$$

where the agent's income is taxed at a linear tax  $t_g = \tau_1 + \tau_3$ .

Since firms operate in competitive factor market, factor prices are determined by

$$\begin{aligned} w_t &= (1 - \alpha)A\bar{k} \\ r_t &= \alpha A \end{aligned}$$

The Euler equation for each agent is

$$\Theta = \beta(1 - \tau_1 - \tau_3)\alpha A$$

Replacing the Euler equation in the budget constraint, we get

$$c_{i,t} = (1 - \tau_1 - \tau_3)A\bar{k} \left( (1 - \alpha) + \alpha \frac{k_i}{\bar{k}} + \frac{\tau_3}{1 - \tau_1 - \tau_3} \right) - \Theta k_i$$

The utility function of the decisive voter is

$$\mathbb{V}(\tau_1, \tau_3; k_{i,0}, k_0) \equiv \sum_{t=0}^{\infty} \beta^t \ln(\mathbb{C}_{i,0} \Theta^t) = \sum_{t=0}^{\infty} \beta^t \ln \mathbb{C}_{i,0} + \sum_{t=0}^{\infty} t \beta^t \ln \Theta$$

where  $\mathbb{C}_{i,0} = \left( ac_{i,0}^\rho + (1 - a)(\tau_1 A \bar{k}_0)^\rho \right)^{\frac{1}{\rho}}$ ;  $c_{i,0} = (1 - \tau_1 - \tau_3)A((1 - \alpha)\bar{k}_0 + \alpha k_{i,0} + \frac{\tau_3}{1 - \tau_1 - \tau_3} \bar{k}_0) - \Theta k_{i,0}$ ; and  $\Theta = \beta(1 - \tau_1 - \tau_3)\alpha A$ .

The first-order conditions are

$$ac_{i,0}^{\rho-1} \left( -(1 - \alpha) - \alpha(1 - \beta) \frac{k_i}{\bar{k}} \right) + (1 - a)g_{1,0}^{\rho-1} - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^\rho \frac{\alpha}{((1 - \tau_1 - \tau_3)\alpha A + 1)\bar{k}_0} = 0$$

$$ac_{i,0}^{\rho-1} \alpha \left( 1 - (1 - \beta) \frac{k_i}{\bar{k}} \right) - \frac{\mathbb{M}_1}{\mathbb{M}_0} \mathbb{C}_{i,0}^\rho \frac{\alpha}{((1 - \tau_1 - \tau_3)\alpha A + 1)\bar{k}_0} = 0$$

Given that  $\frac{\mathbb{M}_1}{\mathbb{M}_0} = \frac{\beta}{1 + \beta}$  (see appendix A) and after the corresponding substitutions, we obtain

$$\tau_1^* = \frac{1 - \beta}{1 + \phi},$$

$$\tau_3^* = 1 - \frac{\beta}{\alpha \epsilon_i} - \tau_1^*,$$

where  $\epsilon_i \equiv 1 - (1 - \beta) \frac{k_{i,0}}{\bar{k}_0}$ .